Throughput Scaling Laws for Vehicular Ad Hoc Networks

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***Abstract*—This paper investigates throughput scaling laws for Vehicular Ad Hoc Networks (VANETs). We show that the road geometry greatly affects the throughput of a VANET. To this end we introduce the notion of sparseness to capture the geometrical properties of roads. We then start by addressing scaling laws for single roads. We shall see that even a single road can have very different scaling behaviors based on its path trajectory. Scaling laws for more complex systems such as downtown grids and general road systems are studied next. Here, the concept of road-connectivity plays a major role in determining the scaling behavior. In our analysis we account for a spectrum of node distributions that represent different vehicular traffic conditions. We also introduce the distance-limited throughput, a notion of throughput especially introduced for VANET-specific applications, and see how it scales in a single road system and in the presence of infrastructure. Our results are obtained by combining geometrical analysis, network flow arguments, and the probabilistic study of VANETs.**

***Index Terms*—Road geometry, scaling laws, vehicular ad hoc networks (VANETs).**

I. INTRODUCTION

TODAY, the impresive perspectives promised by Vehicular Ad hoc Networks (VANETs) have made it a worldwide focal area of research. Ubiquitous connectivity on roads, improved safety of driving, and reduced traffic congestion along with many enterprize applications are just a few to name when it comes to what VANETs have to offer. Based on the target region, VANET applications can be classified into one-to-one (unicast), one-to-a-zone (geocast), or one-to-many (broadcast) [1]. Unicast communication is mainly used for enterprize and convenience applications, whereas geocast and broadcast cover safety applications. Many studies address the routing challenges that arise in vehicular environments for each of the above modes of communications [2], [3]. Considerable efforts have also been dedicated to the study of other layers of the VANET protocol stack. Physical layer considerations for vehicle-to-vehicle communications are studied in [4] in which a MIMO model is used to characterize the physical channel between vehicles. A multi-channel media access protocol for general and safety applications has been proposed in [5]. As for application layer security, [6] studies a privacy-preserving secure vehicular communications scheme.

Despite the increasing amount of research on protocol design and development, to the best of our knowledge there is currently a void of a rigorous mathematical framework to study the throughput scaling laws of VANETs. There exists a considerable amount of literature on the scaling laws of unicast [7], [8], [9] and broadcast [10] in wireless networks and some papers further address the effect of mobility [11]. However, our results show that VANETs feature some unique characteristics that makes their scaling laws different from that of other wireless networks.

We show that the road geometry plays a significant role in determining the fundamental scaling laws of VANETs. As it will be seen, even a single isolated road (e.g. rural road) can have very different throughput scalings just based on its path trajectory. Such a phenomenon was not observed in the ordinary analysis of wireless networks, where it is believed that as long as the deployment region has smooth boundaries the scaling laws do not change. Thus, there is a need to categorize roads based on their geometric properties. To this end we define road *sparseness* in section II-A. As we shall see, sparsity is a measure of how dense a road system is on the plane. In section III We shall study throughput scaling laws for sparse single roads, grids and general road systems. For general road systems we shall see that *road-connectivity* is an important element in characterizing the scaling behavior. We shall also study the throughput scaling of non-sparse general road systems in the special case where obstacles such as buildings fill the empty spaces confined by roads.

Another issue is the unique mobility paradigm that exists in VANETs. Numerous numerical and simulation-based studies study the effect of vehicle mobility on the performance of VANETs [12], [13]. In [13] the authors use MITSIMLab [14] to generate vehicle trajectories which are then fed into an Integer Programming formulation to derive the maximum number of concurrent transmissions in a VANET. As for analysis, the effect of mobility on the capacity of conventional Mobile Ad Hoc Networks (MANETs) has been studied in [11], [15]. A common assumption in such studies that report an increase in throughput due to node mobility is that at each slot a node is equally likely to be at any location within its legitimate domain . Hence the network topology changes over time-scale of packet delivery time. This assumption does not hold in VANETs where the topology change speed of the network is restricted by the physically bounded speed of individual vehicles and also the interdependency of vehicle movements. In fact we shall argue that vehicle mobility cannot guarantee improving the throughput scaling of a VANET.

Finally, note that in the study of throughput it is usually assumed that each source node has a random destination chosen uniformly from the available nodes in the network. Yet in VANETs, applications such as safety are more interested to communicate with vehicles that are in their vicinities.

Manuscript received September 9, 2011; revised February 17 and May 10, 2012; accepted May 17, 2012. The associate editor coordinating the review of this paper and approving it for publication was L. Libman.

This work was supported by the NSF under CCF 0728970 and CCF 0844725.

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Digital Object Identifier 10.1109/TWC.2012.062012.111681

TABLE I
SUMMARY OF NOTATIONS AND PARAMETERS

$\mathcal{R}$	Road system
L	Length (Hausdorff one-dimensional measure) of $\mathcal{R}$
n	Expected number of vehicles in $\mathcal{R}$ (scaling parameter)
$n(t)$	Number of vehicles in $\mathcal{R}$ at time t
κ	Vehicle density
$u(n)$	Number of roads in $\mathcal{R}$
$m(n)$	Road-connectivity of $\mathcal{R}$ (number of parallel roads in $Grid(m)$)
$h(n)$	Length of each road in $Grid(m)$
$\phi(n)$	Distance between adjacent roads in $Grid(m)$
P	Per-node transmission power
β	Physical model SINR threshold
α	Path-loss exponent
N	Noise power
W	Physical model transmission rate
$\lambda(n)$	Per-node throughput
$\lambda_D^T(n)$	Per-node distance-limited throughput with $T(n)$ RSUs

To address this issue we introduce the notion of *distance-limited* throughput and see how it scales with the support of infrastructure in a sparse single road. In [16], we studied the distance-limited throughput of single roads and downtown grids for a set of Poisson distributed nodes and using the Protocol model for successful transmission. We further studied the effect of deploying roadside units on the throughput scaling of single roads in [17]. In [18] we took an initial effort to address the throughput scaling of straight line single roads under the Physical model.

The current paper integrates and builds upon our prior work to study the throughput scaling of a variety of road systems from single roads to road systems with general geometry under the more realistic Physical model. We also show that our results hold true for a wide range of node distributions corresponding to different traffic conditions. Furthermore, we study the effect of obstacles (such as buildings) on the throughput scaling of such networks.

The rest of the paper is organized as follows; Section II embodies the model description and preliminaries. Section III is dedicated to the study of throughput scaling laws for VANETs with different road geometries. Later in this section we study VANETs with infrastructure support. The paper is finally concluded in section IV.

II. Model and Preliminaries

In this section we introduce the preliminaries of our study regarding the road geometry, node distribution, and communications model specifications and also discuss the effect of mobility on our results. Table I summarizes the notations used in the paper.

A. Road geometry and node distribution specifications

Here we elaborate on the specifications of the deployment region. In our model nodes[1] are placed on roads. Here we deal with the *extended* model [19] for analyzing scaling laws as opposed to the *dense* model. In the latter model, the density of nodes in a bounded region goes to infinity whereas in the former, network size goes to infinity while the density is fixed. We use the extended model because at any part of the road, the density of vehicles is assumed to be a bounded positive number as in reality the density is limited by the physical size of the vehicles.

Let n denote the scaling parameter, being the expected number of nodes in the network. We are interested in the fundamental limits of the network as n grows large. We define a *road system*, $\mathcal{R}$, as a collection of possibly intersecting single roads, i.e. $\mathcal{R} = \{\mathcal{R}^1, \mathcal{R}^2, \cdots, \mathcal{R}^{u(n)}\}$. In general the number of roads, $u(n)$, and the length of each road, $H(\mathcal{R}^i)$, are both functions of n. Each road in the set is characterized by a parameterized smooth continuous curve on the plane denoted by $\mathcal{R}^i(s) = (x_n^i(s), y_n^i(s))$ where $s \in [0,1]$ is the parameter of the curve. The subscript n denotes dependence on the scaling parameter n. The curve represents the trajectory of the road. The length of each segment of the curve is obtained using the Hausdorff one-dimensional measure $H(.)$ [20]. For the length of each road we have:

$$H(\mathcal{R}^i) = \int_{s=0}^{s=1} \sqrt{\left(\frac{d}{ds}x_n^i(s)\right)^2 + \left(\frac{d}{ds}y_n^i(s)\right)^2}\, ds \qquad (1)$$

We consider nodes to initially be placed on the roads according to a homogeneous Poisson Point Process (P.P.P) of density κ. Hence the vehicle spacings between adjacent nodes is exponentially distributed. Each node then independent from other nodes and from the initial process, chooses its target speed from a common distribution function $f_V(v)$. Empirical measurements show that $f_V(v)$ follows a truncated Normal distribution [21]. We assume that vehicles can overtake one another to reach their targeted speed. Hence, based on [22, Theorem 9.14], at anytime t the location of the nodes still form a P.P.P. of the same density.

Available literature in traffic flow theory asserts that the Poisson node distribution is a justified assumption for low to moderate traffic conditions [23], [24]. Empirical measurements conducted on roads also confirm this assertion [21], [25]. Though during heavy flow periods, vehicles behave in a car-following manner in which at any instant of time the spacing between them is better represented by a Positive Normal distribution [26, Ch. 2]. This distribution is used since in this regime drivers tend to maintain a constant spacing with their leader, yet driver error would cause some variation about this constant spacing. Also, for the intermediate state where some vehicles engage in car-following while others do not interact, a composite distribution consisting of an exponential and a shifted exponential distribution for vehicle spacings has been proposed [27], [28]. We shall show that our results in this paper hold true for any distribution of vehicle spacings which does not have a heavier tail than the exponential distribution i.e. the family of distributions for which $\mathbb{P}(\mathfrak{X} \geq \mathfrak{x}) = O(e^{-\kappa \mathfrak{x}})$ where $\mathfrak{X}$ denotes the spacing between two adjacent vehicles. As the Positive Normal distribution belongs to the above family of distributions[2], we conclude that our results hold true under different traffic conditions.

[1]Due to the context of our study, the terms "node" and "vehicle" are used interchangeably hereafter.

[2]This is shown using the approximation $\int_x^\infty e^{-\frac{y^2}{2}}dy \approx \frac{1}{x}e^{-\frac{x^2}{2}}$ as $x \to \infty$.

Although our primary assumption in this paper is that all vehicles are equipped with communication devices, our results can easily be generalized to when market penetration is in its early stages. To this end consider a light to moderate traffic scenario where the process of vehicles is best represented by a P.P.P. Here if we assume that each vehicle is equipped with probability p independent from others, then the process of the equipped vehicles is a thinning of the original process and hence is also a P.P.P. with parameter κp [29]. Furthermore, it is shown in [25] through empirical measurements, that even during heavy traffic the P.P.P. assumption for the distribution of equipped vehicles is still valid for low market penetration rates.

For the total length of the road system we have $L = H(\mathcal{R}) = \frac{n}{\kappa} = \Theta(n)$. At any time t, the number $n(t)$ of vehicles that reside on the road system is $n(1+o(1))$. That is, using the Chernoff bound [30] we can show that for any $\epsilon > 0$, there exists a fixed number ξ such that:

$$\mathbb{P}\{|n(t) - n| > \epsilon n\} < e^{-\xi n} \tag{2}$$

To measure how dense the road system is, we introduce the *sparseness* condition in Definition 1. In the literature, sparsity mainly denotes scarceness in terms of node density. However we use the term to denote that the roads are not highly dense on the plane. We need to introduce some terminology before stating the definition. For any point Y on the roads, assume $B(Y,r)$ and $C(Y,r)$ to be the closed ball and circle with radius r centered at Y, respectively. Let $l(Y,r)$ denote the Hausdorff one-dimensional measure (combined length) of the segments of the roads inside $B(Y,r)$. Further, let $z(Y)$ be the number of times that $C(Y,r)$ intersects with the road curves, and A_r be the segments of the roads consisting of points with $z(Y) > 2$.

Definition 1. *Let $r = \Theta(\ln n)$. A road system is sparse if for all points Y on the roads:*

1) *$z(Y)$ is bounded.*
2) *For any constant $\sigma > 0$, there exists a constant $c_\sigma^Y > 0$ such that $l(Y, \sigma r) < c_\sigma^Y r$.*
3) *The combined length of A_r is $o(L)$.*

Moreover, if $z(Y) \leq 2$ for all Y, then the system is said to be *highly sparse*. See Figure 1 for examples of highly sparse, sparse, and non-sparse roads. As we shall later see, condition 2 is needed in order to have a bounded interference term from all concurrently transmitting nodes, which shall help us in deriving achievable lower bounds for throughput. Further, condition 3 implies that most transmissions occur *along* the road trajectory. To make the definitions more clear, here we elaborate on two special classes of roads systems, namely single roads and downtown grids.

a) Single roads: The simplest system consists of a single road such as an inter-state (see Figure 1). Here we have $\mathcal{R} = \{\mathcal{R}^1\}$. The trajectory of the single road is parameterized as $\mathcal{R}^1(s) = (x_n^1(s), y_n^1(s)) = (anx^1(s), any^1(s)), s \in [0,1]$ where a is a constant. Here $(x^1(s), y^1(s)), s \in [0,1]$ is a bounded-length parameterized curve which is then scaled with n to transform into the extended road $\mathcal{R}^1(s), s \in [0,1]$. Replacing this in (1) we have, $H(\mathcal{R}^1) = an \int_{s=0}^{s=1} \sqrt{(\frac{d}{ds}x^1(s))^2 + (\frac{d}{ds}y^1(s))^2} ds = abn$, where b is a constant denoting the value of the integral. On the other hand we saw earlier that $H(\mathcal{R}^1) = \frac{n}{\kappa}$. Hence $a = \frac{1}{\kappa b}$.

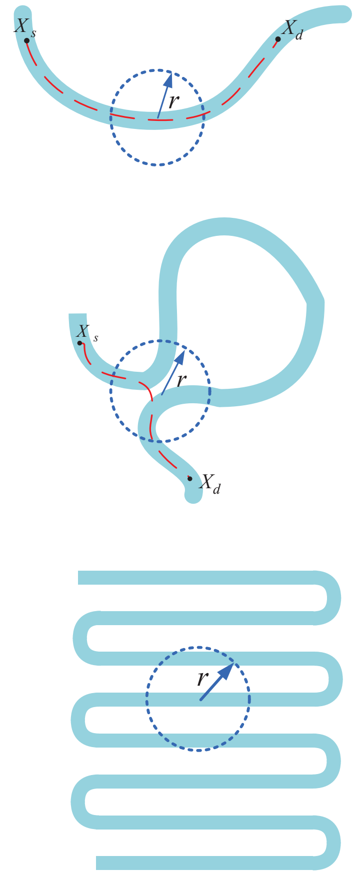

Fig. 1. (top) A single highly sparse road with throughput $\Theta(\frac{1}{n})$. (middle) A single sparse road with throughput $\Omega(\frac{1}{n})$. (bottom) A single non-sparse road with throughput as high as $\Theta(\frac{1}{\sqrt{n \ln n}})$. Here X_s is the source node and X_d the destination node.

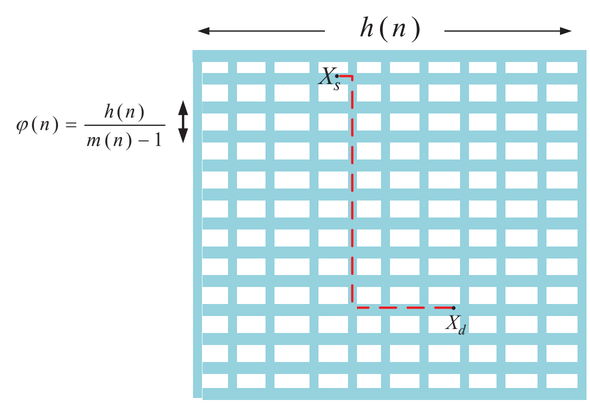

Fig. 2. $Grid(m)$.

b) Downtown grids: We now consider a case in which the road system consists of several roads. In particular, we consider a grid geometry that can appear in downtowns of cities such as Manhattan. We define $Grid(m)$ as a group of $m(n)$ parallel roads each of length $h(n)$ and distance between adjacent roads $\phi(n)$, intersected with another group of $m(n)$ parallel roads with the same length per road and distance between adjacent roads. We assume that the two groups of roads are orthogonal to each other (see Figure 2). Moreover $Grid(m)$ can be represented as a set $\mathcal{R} = \{\mathcal{R}^1, \cdots, \mathcal{R}^{m(n)}, \mathcal{R}^{1_\perp}, \cdots, \mathcal{R}^{m_\perp(n)}\}$. If the leftmost vertical road is placed on the y axis, the i^{th} vertical road can be parameterized as $\mathcal{R}^i(s) = (x_n^i(s), y_n^i(s)) = ((i-1)\phi(n), sh(n)), s \in [0,1]$. For the special case of sparse grids, we shall later see that $h(n) = \omega(\sqrt{n \ln n})$ and $\phi(n) = \omega(\ln n)$. Note that all other roads in the grid can be parameterized in a similar manner[3].

B. Communications model specifications

Within the road system, each source vehicle chooses a destination vehicle which is the node closest to a uniformly chosen random location on $\mathcal{R}$. If the destination vehicle moves out of the road but the first vehicle is still on the road, it will choose another target vehicle at random. A throughput of $\lambda(n)$ denotes the number of bits per second that every source node can send to its destination with high probability (w.h.p.)[4]. We are interested in the scaling of $\lambda(n)$ as n grows arbitrarily large. Throughout the paper when addressing random values, we adopt the following probabilistic variation of order notation. That is $f(n) = O(g(n))$ if there exists a positive constant μ such that:

$$\lim_{n\to\infty} \mathbb{P}(f(n) \leq \mu g(n)) = 1$$

Further, $f(n) = \Omega(g(n))$ if $g(n) = O(f(n))$. Finally $f(n) = \Theta(g(n))$ if $f(n) = O(g(n))$ and $f(n) = \Omega(g(n))$.

In our analysis we consider the Physical model [7] for successful transmissions. This model assumes that all nodes employ a common transmission power P for all their transmissions. Node X_i can successfully transmit to node X_j at time t, if the signal-to-interference-plus-noise ratio (SINR) at the receiver is no less than a threshold β, i.e. assuming $\tau(t)$ is the set of simultaneously transmitting nodes at time t, we must have:

$$\frac{P|X_i - X_j|^{-\alpha}}{N + \sum\limits_{k\in\tau(t), k\neq i} P|X_k - X_j|^{-\alpha}} \geq \beta \tag{3}$$

Where N is the noise power and α the path-loss exponent. If the above condition is satisfied, every successful transmitter-receiver pair can communicate at a rate of W bits/second.

C. Effect of mobility

Here we elaborate on the effect of mobility on our results. As discussed before, vehicles move with an arbitrary speed that is a bounded real number. In [11] it is shown that for a scaling network a constant per-node throughput is attainable if as well as having large buffer sizes and allowing for unbounded delays, the process of each nodes' location over time is stationary and ergodic with stationary distribution uniform on the plane. This means that at each slot, each node is equally likely to be at any location in the network. The idea there is to exploit the multiuser diversity. There, the packets of a source node are distributed among all other nodes of the network acting as relays. Since at each instant at least one node is close enough to the destination, a constant per-node throughput is achievable.

However, the location of a vehicle at time t does not follow a stationary uniform distribution on the road. To clarify this consider a road system which consists of only a single road. Two arbitrary vehicles have a distance of $\Theta(n)$ at a specific time. Further, the bounded speed assumption ensures that this distance is preserved over time in the scaling sense. Hence, such vehicles shall never get close enough which hinders utilizing multiuser diversity in this network. For general road systems, the same argument holds for vehicles on the same road. Thus in our model, mobility cannot guarantee increase in throughput.

Moreover due to the bounded speed of vehicles, the topology of the scaling network changes much more slowly than the packet delivery time. This way and by allowing for dynamic routing protocols in which the routes need to be slowly adjusted as the vehicles move, we are still able to use the same methodology introduced in [7] for static networks. This strategy of analyzing the equivalent static topology in networks where nodes do not move significant distances during packet transmit times has been used in many prior studies such as in [31].

III. THROUGHPUT SCALING OF ROAD SYSTEMS

In this section, we present our main results regarding throughput scaling laws of single roads, grids, and general road systems. We shall derive tight $\Theta(.)$ scalings for sparse single roads and grids by proposing both an achievable scheme and an upper bound. For general sparse road systems we shall prove an achievable lower bound. We then study the effect of roadside units on the distance-limited throughput of sparse single roads. The proofs are brought in the Appendix at the end of the paper. Generally, our results are obtained by combining network flow arguments, geometrical tools that capture the road structures, and also the tools developed in the analysis of scaling laws of wireless networks such as in [32].

Before stating the results we need a lemma that is used throughout our paper in proving the achievable throughput scalings. The lemma addresses spatial reuse in the network. It basically states that in a sparse road system, per-hop transfer of packets can be scheduled at a rate which does not go to zero as $n \to \infty$.

Lemma 1. *Consider a sparse road system $\mathcal{R} = \{\mathcal{R}^1, \cdots, \mathcal{R}^u\}$ where $H(\mathcal{R}^i) = \ell_i$, $i \in \{1, 2, ..., u\}$. Suppose that roads are divided into segments of lengths $\eta \ln n$, where η is a positive constant. Then there exists a positive constant integer M such that one node from each segment can transmit to a node in an adjacent segment at a rate of $\frac{W}{M}$.*[5]

A. Throughput scaling of single roads

Here we initially address the throughput scaling of the single road system $\mathcal{R} = \{\mathcal{R}^1\}$. The throughput of the highly

[3]We shall hereafter drop the dependency of $u(n)$, $m(n)$, $h(n)$, and $\phi(n)$ on n for ease of representation.

[4]Note that $\lambda(n)$ is a random value since the node locations and the choice of destinations are random.

[5]Note that M depends on the Physical model parameters β and α, and also the road geometry but not on the scaling parameter n.

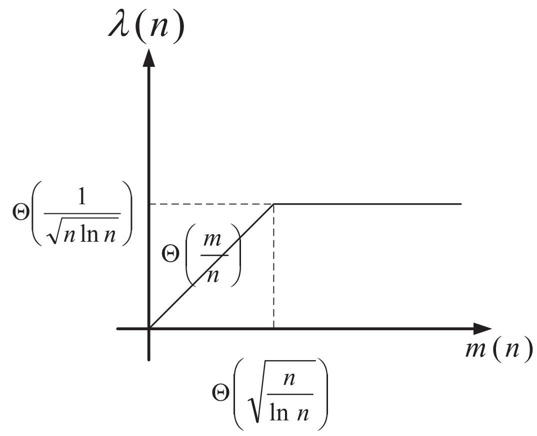

Fig. 3. Throughput of $Grid(m)$. The throughput increases linearly with m until $m = \Theta(\sqrt{\frac{n}{\ln n}})$. After that the throughput remains constant with respect to m.

sparse and sparse single road VANET (Figure 1 (top) , (middle)) is characterized by the following theorem.

Theorem 1. *Consider the road system $\mathcal{R} = \{\mathcal{R}^1\}$. If the road is highly sparse then $\lambda(n) = \Theta(\frac{1}{n})$. If the road is sparse, then $\lambda(n) = \Omega(\frac{1}{n})$.*

This result may seem somewhat trivial, as it brings to mind the throughput of a line network. Nevertheless, it is important to note that the road trajectory can significantly affect the scaling law. Indeed, if the sparseness condition does not hold, the throughput of a single road can be as large as $\Theta(\frac{1}{\sqrt{n\ln n}})$. Figure 1 (bottom) shows an example of such roads.

Note that in highly sparse single roads as a packet is always routed *along* the road trajectory from the source to its destination, and since the source-destination distance is $\Theta(L)$, the same scaling result proposed by Theorem 1 for a randomly chosen destination, also holds true for broadcast communications.

B. Throughput scaling of grid road systems

The following theorem addresses the throughput scaling of $Grid(m)$ defined earlier as the road system $\mathcal{R} = \{\mathcal{R}^1, \cdots, \mathcal{R}^m, \mathcal{R}^{1_\perp}, \cdots, \mathcal{R}^{m_\perp}\}$.

Theorem 2. *Consider $Grid(m)$:*

- *If it is sparse, then $m = O(\sqrt{\frac{n}{\ln n}})$ and $\lambda(n) = \Theta(\frac{m}{n})$.*
- *If $m = \Omega(\sqrt{\frac{n}{\ln n}})$, then $\lambda(n) = \Theta(\frac{1}{\sqrt{n\ln n}})$.*

Theorem 2 states that the throughput increases linearly with m until $m = \Theta(\sqrt{\frac{n}{\ln n}})$. After this point the throughput scaling is equivalent to that of random ad hoc networks. Figure 3 shows the throughput of a grid road system as a function of m.

C. Throughput scaling of general road systems

So far we discussed single roads and downtown grids. We now discuss a general scenario that includes downtown grids as a special case. To state the result, we need to develop a new concept. In particular, we define the concept of *road-connectivity*. This is somewhat similar to edge-connectivity in graph theory. Note that edge-connectivity shows the size of the minimum cut in a graph. It also relates to the number of disjoint paths that exist between two nodes.

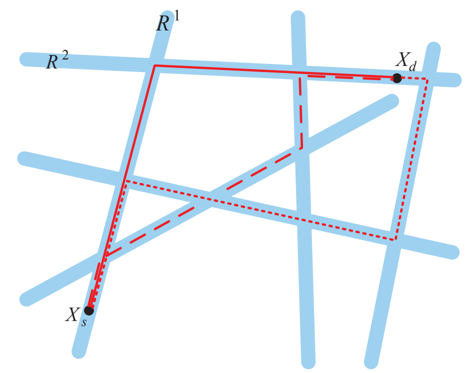

Fig. 4. A general road system with 6 roads. There are 3 road-disjoint paths between X_s and X_d.

Consider two vehicles X_s and X_d located on roads R^1 and R^2 respectively (see Figure 4). We call two different paths from X_s to X_d *road-disjoint* if the only common roads between them are R^1 and R^2. We say that a road system has road-connectivity m if there are at least m road-disjoint paths between any two points in the network. In what follows, we initially show via Lemmas 2 and 3 that when the road system is sparse, road-connectivity cannot grow faster than $\Theta(\sqrt{\frac{n}{\ln n}})$. Theorem 3 then presents the main result on the throughput scaling of sparse general road systems.

Lemma 2. *Let $\mathcal{L}$ be the subset of $\mathbb{R}^2$ containing the road curves and $H(.)$ be the Hausdorff one-dimensional measure. Let $A_1, A_2, ..., A_i \subseteq \mathcal{L}$ be sections of the roads satisfying the following property: For all $I \subseteq \{1, 2, ..., i\}$ with $|I| > c$, we have $\bigcap_{j\in I} A_j = \emptyset$. Then*

$$\sum_{j=1}^{i} H(A_j) \leq cH(\bigcup_{j=1}^{i} A_j).$$

Lemma 3. *Consider a sparse road system $\mathcal{R} = \{\mathcal{R}^1, \cdots, \mathcal{R}^u\}$ with road-connectivity m. Let $H(\mathcal{R}^i) = \ell_i$, $i \in \{1, 2, ..., u\}$ where $\delta < \frac{\ell_i}{\ell_j} < \gamma$ for some fixed constant numbers δ and γ and for any $i, j \in \{1, 2, ..., u\}$. Then:*

$$m = o(\sqrt{\frac{n}{\ln n}})$$

Theorem 3. *Consider a sparse road system $\mathcal{R} = \{\mathcal{R}^1, \cdots, \mathcal{R}^u\}$ with road-connectivity m. Let $H(\mathcal{R}^i) = \ell_i$, $i \in \{1, 2, ..., u\}$ where $\delta < \frac{\ell_i}{\ell_j} < \gamma$ for some fixed constant numbers δ and γ and for any $i, j \in \{1, 2, ..., u\}$. Then:*

$$\lambda(n) = \Omega(\frac{m}{n})$$

As we shall see in the proof, the constructive lower bound is achieved by deploying a probabilistic routing strategy. The above theorem simply states that the throughput is determined by the road-connectivity of the system. Thus, Figure 3 can be used for general systems, where m shows the road-connectivity. Note that when the road system is sparse, it is shown that the road-connectivity cannot grow faster than $\Theta(\sqrt{\frac{n}{\ln n}})$. Thus, the throughput does not have to grow faster than $\lambda(n) = \Theta(\frac{1}{\sqrt{n \ln n}})$.

We now consider the case where the *faces* of the road system, i.e the empty spaces confined between roads, are filled with obstacles such as buildings. Note that the bandwidth allocated by the FCC for Dedicated Short Range Communications (DSRC) in VANETs has a center frequency of 5.9 GHz and a bandwidth of 75 MHz. Waves in this range typically suffer high penetration loss when passing through obstacles [33]. Hence we can argue that when obstacles fill the empty spaces between roads, transmissions occur *along* the roads no matter how dense the road system is on the plane. This way the sparseness condition does not need to hold and we can prove the following theorem:

Theorem 4. *Consider an obstacle-filled road system $\mathcal{R} = \{\mathcal{R}^1, \cdots, \mathcal{R}^u\}$ with road-connectivity m. Let $H(\mathcal{R}^i) = \ell_i$, $i \in \{1, 2, ..., u\}$ where $\delta < \frac{\ell_i}{\ell_j} < \gamma$ for some fixed constant numbers δ and γ and for any $i, j \in \{1, 2, ..., u\}$. Then:*

$$\lambda(n) = \Omega(\frac{m}{n})$$

When

$$m = o(\frac{n}{\ln n})$$

As we shall see in the proof, here the restriction on m is not a result of applying the sparseness condition but from the need to bound the number of information paths that pass through an arbitrary segment of the road in the probabilistic routing strategy.

D. Effect of RoadSide Units (RSUs) on distance-limited throughput

At this point we allude to an extension of our analysis which specially gains importance in a practical VANET setting. We first introduce the notion of distance-limited throughput and then study how the deployment of RoadSide Units (RSUs), affects this notion of throughput in a sparse single road system.

Up until now we have considered the case in which it is assumed that every source node communicates with a randomly chosen destination which, in the case of single roads, is a distance $\Theta(n)$ away. In many applications of VANETs, however, nodes usually need to communicate with other nodes that are within a certain distance $D(n)$ from them. For example, in accident warning systems, a vehicle would need to exchange messages with vehicles that are in its vicinity. To this end we define the *distance-limited* throughput as the per-node feasible throughput when vehicles need to communicate with others that are within distance $D(n)$ from them. Moreover, let $\lambda_D^T(n)$ be the distance-limited throughput of a system with $T(n)$ RSUs. Some studies in the literature discuss the throughput of wireless networks with infrastructure [34]. Theorem 5 addresses $\lambda_D^T(n)$ for a sparse single road system. The corresponding analysis for a general road system is analytically involved and left for future work. Here we assume that the RSUs do not generate new information and only serve to help the communication between vehicles. It is assumed that the nodes can communicate with each other using a channel with a bounded bandwidth $W_1 < \infty$, and they can communicate with the RSUs using a channel with a bounded bandwidth $W_2 < \infty$ that does not interfere with W_1.

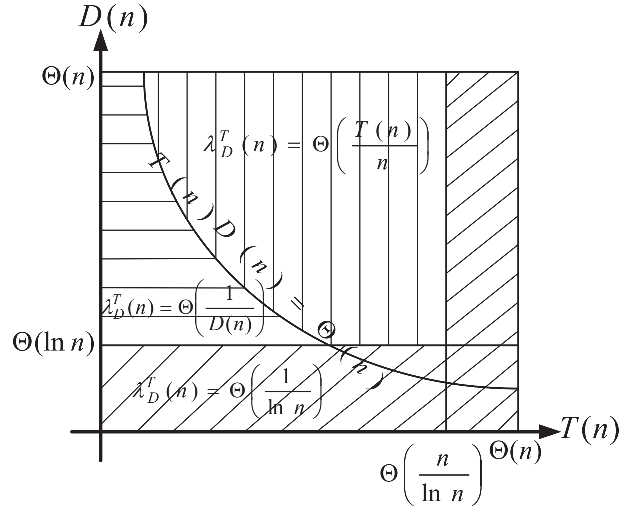

Fig. 5. Throughput regions of a single road system with $T(n)$ RSUs.

Theorem 5. *Consider a sparse single road system $\mathcal{R} = \{\mathcal{R}^1\}$ with $T(n)$ RSUs. Assume $r = \Theta(\ln n)$. Then,*

- *If $D(n) = \Omega(r)$ and $T(n)D(n) = O(n)$, then $\lambda_D^T(n) = \Theta(\frac{1}{D(n)})$.*
- *If $D(n) = O(r)$ or $T(n) = \Omega(\frac{n}{\ln n})$, then $\lambda_D^T(n) = \Theta(\frac{1}{\ln n})$.*
- *If $D(n) = \Omega(r)$ and $T(n)D(n) = \Omega(n)$ and $T(n) = O(\frac{n}{\ln n})$, then $\lambda_D^T(n) = \Theta(\frac{T(n)}{n})$.*

In Figure 5 we have illustrated the throughput regions for a single road VANET with $T(n)$ RSUs. Based on this result, having more than $\Theta(\frac{n}{\ln n})$ RSUs cannot increase the capacity beyond $\Theta(\frac{1}{\ln n})$. Along the same lines as this theoretic result, a recent trace-driven simulation study of taxis in urban Shanghai [35] reveals that the delivery ratio increases with the number of RSU up until a threshold value beyond which the addition of extra RSUs has little effect on performance.

IV. Conclusion

In this paper we studied throughput scaling laws for unicast communications in VANETs. We showed that the VANET throughput scaling differs significantly from the known results obtained for MANETs. In particular, it was observed that the road geometry plays an important role in determining the throughput of VANETs. We studied scaling laws for various road geometries such as single roads, grids and general road systems. We further defined the distance-limited throughput and studied how it scales in the presence of RoadSide Units (RSUs) in a single road system. The effect of RSUs on the throughput scaling of general road systems is an interesting open problem as is the throughput scaling of geocast and broadcast communications for such systems. We believe that although the present work is a theoretical study of VANETs' scaling behavior, it nonetheless provides useful insights on practical issues such as the design of efficient routing algorithms, optimal deployment of RSUs, and the effect of traffic flow conditions on the achievable throughput of such wireless networks.

APPENDIX

Proof of Lemma 1

Proof: Construct the interference graph G in the following way. Let the vertex set χ be the set of midpoints of segments of length $s_n = \eta \ln n$ on $\mathcal{R}$. Clearly, as we are using the extended model, the cardinality of χ grows unbounded with the size of the network. Now centered at the vertices, draw circles with radius σs_n where σ is a positive constant (see Figure 6 (top)). Connect vertices in χ that are in the same circle. Due to condition 2 of sparseness, for every vertex $Y_i \in \chi$, $l(Y_i, \sigma s_n) < c_\sigma^{Y_i} \ln n$ where $c_\sigma^{Y_i}$ is a positive constant. Hence the degree of vertex Y_i can be at most $\Delta(Y_i) = \lceil \frac{c_\sigma^{Y_i} \ln n}{\eta \ln n} \rceil = b_i$. Now let $M = \sup_{Y_i \in \chi} \Delta(Y_i) + 1 = \sup_{Y_i \in \chi} b_i + 1$. M is a bounded number as b_i's are all bounded. It is shown in [36] that every graph can be colored with one more than the maximum vertex degree. Hence the graph G is M-colorable. What is left to show is that for large enough M, concurrent transmissions from nodes within segments of the same color group are successfully received at the receiver in the neighboring segment (when only one node per segment transmits). Then, as there are M color groups, one node per segment can transmit at a rate of $\frac{W}{M}$ using a TDMA strategy, which marks the end of the proof.

To this end we find the maximum interference and show that, for a sparse road system, it can be kept bounded even when the number of interferers goes to infinity. Figure 6 (bottom) shows a worst case configuration of concurrently transmitting nodes from an arbitrary color group that leads to maximum interference (and hence minimum SIR[6]) at the receiver node X_j corresponding to the transmitter node X_i in the center. The minimum SIR as given by (3), is:

$$\frac{P(2s_n)^{-\alpha}}{6P(\sigma s_n - s_n)^{-\alpha} + \sum_{k=1}^{\infty} 12(2k+1)P(\sqrt{3}k\sigma s_n - s_n)^{-\alpha}}$$
$$= \frac{1}{6(\frac{2}{\sigma-1})^\alpha + 12(\frac{2}{\sqrt{3}\sigma})^\alpha \sum_{k=1}^{\infty} \frac{2k+1}{(k-\frac{1}{\sqrt{3}\sigma})^\alpha}}$$

Where the first term in the denominator denotes the contribution of the first tier of interferers and the summation represents the contribution from the rest of the tiers. Now if the summation in the denominator converges, then by choosing σ sufficiently large (and hence M sufficiently large), the SIR can be made larger than the threshold β and hence successful reception is guaranteed due to the Physical model assumption. We can show that the summation converges for $\alpha > 2$. Note that:

$$\sum_{k=1}^{\infty} \frac{2k+1}{(k-\frac{1}{\sqrt{3}\sigma})^\alpha} = \sum_{k=1}^{\infty} \frac{1}{(k-\frac{1}{\sqrt{3}\sigma})^\alpha} + 2\left(\sum_{k=1}^{\infty} \frac{1}{(k-\frac{1}{\sqrt{3}\sigma})^{\alpha-1}} + \frac{1}{\sqrt{3}\sigma}\sum_{k=1}^{\infty} \frac{1}{(k-\frac{1}{\sqrt{3}\sigma})^\alpha}\right)$$

In which each of the summations individually converge. For example:

$$\sum_{k=1}^{\infty} \frac{1}{(k-\frac{1}{\sqrt{3}\sigma})^\alpha} \le \frac{1}{(1-\frac{1}{\sqrt{3}\sigma})^\alpha} + \int_{1-\frac{1}{\sqrt{3}\sigma}}^{\infty} \frac{dx}{x^\alpha}$$
$$= \frac{1}{(1-\frac{1}{\sqrt{3}\sigma})^\alpha} + \frac{(1-\frac{1}{\sqrt{3}\sigma})^{-\alpha-1}}{\alpha-1}$$

■

[6]Here we consider an interference-limited network where the power of noise can be neglected compared to the power of interference. This is a justified assumption for vehicular networks that are not faced with power limitations. Hence the Signal to Interference Ratio (SIR) is addressed instead of SINR.

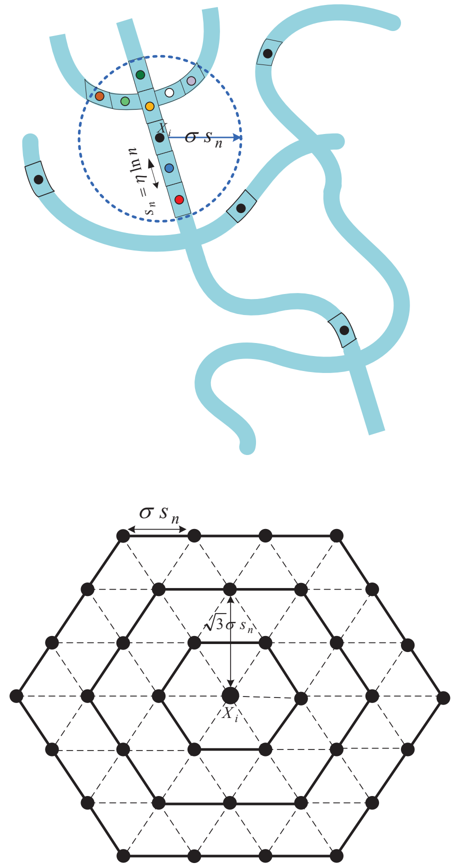

Fig. 6. (top) Sketch of the proof of Lemma 1. Single transmitters within segments of the same color group can successfully transmit at the same time. (bottom) A worst case configuration of all concurrently transmitting nodes of the same color group (here black) used in deriving the least SIR for the center node X_i's transmission.

Proof of Theorem 1

Proof: Let us first consider highly sparse single roads. Here we consider a general case where the vehicle spacings has a non-heavy tail distribution i.e. is asymptotically bounded above by the exponential distribution with density κ. To prove a $\Theta(.)$ scaling for highly sparse roads we need to prove an achievability and an upper bound. To show the achievability we provide a routing strategy. Divide the road into segments of length $\frac{\ln n}{\kappa}$. With this selection, each segment contains at least one node with high probability. To show this let Q_i denote the event that any specific segment is empty. Then we have for Q, the event that there exists at least one empty segment:

$$\mathbb{P}(Q) = \mathbb{P}\left(\bigcup_{i=1}^{\frac{n}{\ln n}} Q_i\right) \le \sum_{i=1}^{\frac{n}{\ln n}} \mathbb{P}(Q_i) \le \frac{n}{\ln n} e^{-\ln n} = \frac{1}{\ln n} = o(1)$$

Where the first inequality follows from the union bound and the second inequality is due to the non-heavy-tail distribution of inter-vehicle spacings and the void probability of Poisson processes [29]. Now, divide the segments into M non-interfering groups. As the road geometry is sparse, based on Lemma 1 one node in each segment gets to transmit at a rate of $\frac{W}{M}$. For each source-destination pair, route the messages along the road through adjacent segments[7]. As a standard method of analysis (which we shall also use for grids and general road systems), we find the number of routes that pass through each segment. For a sparse single road, each segment has to support at most $\Theta(n)$ routes. Hence each segment can handle a rate less than $\Theta(n\lambda(n))$. This can be accommodated by all segments if $\Theta(n\lambda(n)) \leq \frac{W}{M}$ and hence the claimed scaling is achievable.

The upper bound results from the fact that transmissions consume length along the roads. Suppose X_i is transmitting to X_j, while X_k is transmitting to X_l at the same time. For X_i's transmission to be successful we must have:

$$|X_j - X_l| \geq |X_k - X_j| - |X_k - X_l| \tag{4}$$

$$\geq \beta^{\frac{1}{\alpha}}|X_i - X_j| - |X_k - X_l| \tag{5}$$

Where the first inequality is due to the triangle inequality and the second due to the Physical model assumption. Similarly for X_k's transmission to be successful we must have:

$$|X_j - X_l| \geq \beta^{\frac{1}{\alpha}}|X_k - X_l| - |X_i - X_j| \tag{6}$$

Adding the (5) and (6) we obtain:

$$|X_j - X_l| \geq \frac{\beta^{\frac{1}{\alpha}} - 1}{2}(|X_i - X_j| + |X_k - X_l|)$$

Which means that segments of $\mathcal{R}$ within circles of radius $\frac{\beta^{\frac{1}{\alpha}}-1}{2}$ times the transmitter-receiver distances and centered at the receivers are disjoint[8]. As the road system is highly sparse, transmissions occur along the roads. This way and using the notation $d_{ij} = |X_i - X_j|$, we have:

$$\sum_{(i,j)\in\tau(t)} 2(\frac{\beta^{\frac{1}{\alpha}} - 1}{2})d_{ij} \leq L$$

Here $\tau(t)$ represents the set of active transmitter-receiver pairs at time t. Hence the total bit meters per second transported in the network satisfies:

$$W \sum_{(i,j)\in\tau(t)} d_{ij} \leq \frac{WL}{\beta^{\frac{1}{\alpha}} - 1}$$

As there are n nodes in the network and the average distance between each source-destination pair is $L - o(1)$, the result follows:

$$\lambda(n) \leq \frac{W}{(\beta^{\frac{1}{\alpha}} - 1)n}$$

As we shall later see, deriving the throughput upper bound for grids follows from the same argument except that the distance between each source-destination pair is different there.

[7]Traffic passing through each segment can be handled by any node in that segment.

[8]We consider a straight road in our derivation. Since we are finding an upper bound, the result holds for other trajectories of highly sparse roads as well.

For sparse single roads the above achievable bound still holds. However, the upper bound is not necessarily true as information paths can take other routes rather than just along the road trajectory like the path shown in Figure 1 (middle). ∎

Proof of Theorem 2

Proof: Note that the total length of the roads is $L = 2mh$ (see Figure 2). If $m = O(1)$, then the result is trivial and can be shown similar to the proof of Theorem 1. Thus we assume $m = \omega(1)$. Note that the sparseness condition implies that $\phi = \frac{h}{m-1} = \omega(\ln n)$. This is because, if $\phi = \Theta(\ln n)$, then the combined length of the segments of road with $z(Y) > 2$ would be $O(L)$ which contradicts the sparseness condition 3. Also note that due to the geometry of the grid we have $\frac{n}{\kappa} = 2mh$. Hence we conclude:

$$m = o(\sqrt{\frac{n}{\ln n}}).$$

We now derive the achievable rate. Each road is divided into segments of length $\frac{\ln n}{\kappa}$. In the intersections, the segments consist of four parts of length $\frac{\ln n}{4\kappa}$. Again, it can be shown that there is at least one node in each segment[9]. The routing strategy is deterministic and works as follows. Assume a source vehicle located at point X_s chooses another vehicle located at X_d as its destination. The information is transferred through the closest vertical road to X_s, and the closest horizontal road to X_d (see dashed route in Figure 2). The packets are transferred from each segment to the neighboring segments until they reach the destination. To find the achievable throughput, we need to obtain the number of paths that pass through an arbitrary segment w.h.p. Based on the routing strategy, the number of information paths traveling through the segment is at most $\Theta(\frac{n}{m})$. Also as the grid is sparse, due to Lemma 1 we can divide the segments into M non-interfering groups such that one node in each segment can transmit at a rate of $\frac{W}{M}$. This way, through similar reasoning as in the achievability proof of Theorem 1, we have $\lambda(n) = \frac{W}{M}\Omega(\frac{m}{n})$.

For the upper bound, note that two randomly chosen points have a distance of $\Theta(h) = \Theta(\frac{L}{2m})$. Hence through similar steps as in Theorem 1, we have that $\lambda(n) = O(\frac{m}{n})$. In sum from the achievability and the upper bound we conclude that when $m = o(\sqrt{\frac{n}{\ln n}})$, $\lambda(n) = \Theta(\frac{m}{n})$. This corresponds to the first part of the throughput curve given in Figure 3. For the case where $m = \Omega(\sqrt{\frac{n}{\ln n}})$, divide the plane to the cells as in [32]. It follows from similar arguments as in [32] that the throughput is $\Theta(\frac{1}{\sqrt{n \ln n}})$. ∎

Proof of Lemma 2

Proof: Consider a square S_0 with side b that embodies the set $\mathcal{L}$. Define $sh_b(.) : \mathbb{R}^2 \mapsto \mathbb{R}^2$, as $sh_b(x, y) = (x+b, y)$. For any measurable set $\Xi \in \mathbb{R}^2$, we have $H(sh_b(\Xi)) = H(\Xi)$. Define

$$\mathcal{L}^1 = \textstyle\bigcup_{j=1}^{i} A_j, \qquad \mathcal{L}^2 = sh_b(\mathcal{L}^1),$$
$$\mathcal{L}^3 = sh_b(\mathcal{L}^2) = sh_b^2(\mathcal{L}^1), \qquad \cdots, \qquad \mathcal{L}^c = sh_b^{c-1}(\mathcal{L}^1)$$

[9]Note that it can be shown that starting from a non-heavy-tail distribution of density κ, node deployment shall preserve its stochastic properties over time based on grid mobility models such as the Manhattan mobility model [37].

Now define functions $f_j : \mathbb{R}^2 \mapsto \mathbb{R}^2, j \in \{1, 2, ..., i\}$ in the following way. $f_j(x, y) = (x + tb, y)$, where x belongs to t of the sets $A_1, A_2, ..., A_{j-1}$. We conclude from the assumptions of the lemma that

$$\bigcup_{j=1}^{i} f_j(A_j) \subseteq \bigcup_{k=1}^{c} \mathcal{L}^k. \tag{7}$$

For all $j \neq k$, we have $f_j(A_j) \cap f_k(A_k) = \emptyset$. Moreover, since i is a countable number, we have $H(f_j(A_j)) = H(A_j)$. We conclude

$$H(\bigcup_{j=1}^{i} f_j(A_j)) = \sum_{j=1}^{i} H(f_j(A_j)) = \sum_{j=1}^{i} H(A_j). \tag{8}$$

Combining Equations (7) and (8), we obtain

$$\sum_{j=1}^{i} H(A_j) \leq H(\bigcup_{k=1}^{c} \mathcal{L}^k) = cH(\mathcal{L}^1) = cH(\bigcup_{j=1}^{i} A_j).$$

■

Proof of Lemma 3

Proof: Since the system has road-connectivity m, we conclude that each road intersects with at least m other roads and also $u \geq m$. Thus the number of intersections in the system is $i(n) \geq m^2$. Number the intersections in the system from 1 to $i(n)$. Remember that for any point Y on a road, we say that Y is contradicting point if $z(Y) > 2$. Note that any intersection will create a segment of length at least $\Theta(\ln n)$ consisting of contradicting points. For the jth intersection let A_j be the segments of the road consisting of the contradicting points due to the intersection. Then $A_j = \Omega(\ln n)$. since we assume that $z(Y)$ is bounded at any point in the network, there exists a constant $c > 0$ such that $z(Y) < c$ for all Y. That means that for all $I \subseteq \{1, 2, ..., i(n)\}$ with $|I| > c$, we have $\bigcap_{j \in I} A_j = \emptyset$. Now using Lemma 2 we conclude

$$\sum_{j=1}^{i(n)} H(A_j) \leq cH(\bigcup_{j=1}^{i(n)} A_j). \tag{9}$$

However, from the sparseness condition we conclude that $H(\bigcup_{j=1}^{i(n)} A_j) = o(n)$. Thus, we have

$$\sum_{j=1}^{i(n)} H(A_j) = o(n). \tag{10}$$

On the other hand, we have $\sum_{j=1}^{i(n)} H(A_j) \geq i(n)\Theta(\ln n)$. Thus we have:

$$\sum_{j=1}^{i(n)} H(A_j) = \Omega(m^2 \ln n). \tag{11}$$

combining Equations (10) and (11), we conclude $m = o(\sqrt{\frac{n}{\ln n}})$. ■

Proof of Theorem 3

Proof: We first study the achievability. First note that by the definitions of road-connectivity we have $u \geq m$. The algorithm works in the following way. First, as usual, divide the road into segments of length $\Theta(\frac{\ln n}{\kappa})$. For any source-destination pair consider a set of m road-disjoint paths. Choose one of the paths at random and use multi-hop communications between the segments to send the messages from the source to the destination. Following our general method of analysis, we address the number of paths that pass through each segment w.h.p. To this end, define Boolean random variables ψ_{ij} in the following way. $\psi_{ij} = 1$ if and only if the routing path starting at node j uses at least one segment of road i in the network. We claim that $\mathbb{P}\{\psi_{ij} = 1\} = O(\frac{1}{m})$. Indeed, if node j or its destination are on road i, then $\psi_{ij} = 1$ with probability one. This event occurs with probability $\Theta(\frac{1}{u})$. On the other hand, if node j and its destination are not on the road i, then $\mathbb{P}\{\psi_{ij} = 1\} \leq \frac{1}{m}$, because road i can be in at most one of the m road-disjoint paths from node j to its destination. Thus,

$$\begin{aligned}\mathbb{P}\{\psi_{ij} = 1\} &\leq \Theta(\frac{1}{u}) + (1 - \Theta(\frac{1}{u}))\frac{1}{m} \\ &= O(\frac{1}{m})\end{aligned}$$

where we used $u \geq m$. Thus we conclude that $E\psi_{ij} = \rho(1 + o(1))\frac{1}{m}$, where ρ is a positive constant number. Define the random variables ψ_i as

$$\psi_i = \sum_{j=1}^{n} \psi_{ij}.$$

We have $E\psi_i = \rho(1 + o(1))\frac{n}{m}$. This is the expected number of information paths which pass through any segment of road i. As the routing strategy is probabilistic we need to show that the actual number of paths does not asymptotically diverge from the expected value. To do this define the event $E_i = \{\psi_i \leq (1 + \frac{1}{\rho})E\psi_i\}$. Hence we need $\bigcap_{i=1}^{u} E_i$ to occur with high probability. Define $F_i = E_i^c$. Using a form of the Chernoff-Hoeffding bound [38], we have:

$$\begin{aligned}\mathbb{P}\{F_i\} = \mathbb{P}\{\psi_i > (1 + \frac{1}{\rho})E\psi_i\} &< \exp\left(-\frac{E\psi_i}{3\rho^2}\right) \\ &= \exp\left(-\frac{n(1 + o(1))}{3\rho m}\right) \\ &\leq \exp\left(-\frac{n}{6\rho m}\right).\end{aligned}$$

Using the union bound:

$$\mathbb{P}\{\bigcup_{i=1}^{u} F_i\} \leq n \exp\left(-\frac{n}{6\rho m}\right)$$

By Lemma 3, we can write $m = \sqrt{\frac{n}{\ln n}}\frac{1}{w(n)}$, where $w(n) \to \infty$ as n goes to infinity. Thus we conclude

$$\begin{aligned}\mathbb{P}\{\bigcup_{i=1}^{u} F_i\} &\leq n \exp\left(-\frac{n}{6\rho m}\right) \\ &= n \exp\left(-\frac{\sqrt{n \ln n}w(n)}{6\rho}\right) \\ &\leq n \exp\left(-\frac{\ln n w(n)}{6\rho}\right) \\ &= nn^{-\frac{w(n)}{6\rho}} = o(1)\end{aligned} \tag{12}$$

Since we have $\mathbb{P}\{\bigcap_{i=1}^{u} E_i\} = 1 - \text{Prob}\{\bigcup_{i=1}^{u} F_i\}$, we conclude $\bigcap_{i=1}^{u} E_i$ occurs with high probability. This shows that each segment of the roads has to support at most $\Theta(\frac{n}{m})$ paths. Using the coloring Lemma 1, we can divide the segments into M non-interfering groups such that one node from each segment gets to transmit at a rate of $\frac{W}{M}$. Thus we conclude $\lambda(n) = \frac{W}{M}\Omega(\frac{m}{n})$. ∎

Proof of Theorem 4

Proof: The proof goes along the same lines as the proof of Theorem 3. Though this time the road-connectivity m does not need to satisfy the sparseness condition but rather should be such that $\mathbb{P}\{\bigcup_{i=1}^{u} F_i\} = o(1)$ which according to (12) happens when:

$$m = o(\frac{n}{\ln n})$$

∎

Proof of Theorem 5

Proof: There are several factors limiting the throughput and in each region the dominant factors determine the achievable throughput.

Consider the first region. Note that each source is a distance $\Theta(D(n))$ from its randomly chosen destination. However since $T(n)D(n) = O(n)$, we do not necessarily have an RSU in all sections of length $\Theta(D(n))$ of the road. So in this case what limits the distance-limited throughput is the ad hoc throughput of the network, which can be shown to be $\Theta(\frac{1}{D(n)})$ similar to the proof of Theorem 1. The second region represents the case where the distance of each source node from its randomly chosen destination or the nearest RSU is $O(r)$. Note that here, a throughput higher than $\Theta(\frac{1}{\ln n})$ is not achievable since there is always a segment of length $\Theta(r)$ that has $\Theta(\ln n)$ receiver nodes. In the third region, each source has a distance of $\Theta(D(n))$ from its randomly chosen destination. However since $T(n)D(n) = \Omega(n)$, each receiver can always reach an RSU distance $\Theta(\frac{n}{T(n)})$ away. Hence through similar reasoning as in Theorem 1, the upper bound of $\lambda_D^T(n) = O(\frac{T(n)}{n})$ follows. Further, note that each segment of length $\Theta(\ln n)$ has to support at most $\Theta(\frac{n}{T(n)})$ routes. Since the road is sparse, Lemma 1 asserts that $\lambda_D^T(n) = \frac{W_1}{M}\Omega(\frac{T(n)}{n})$ is achievable. Thus in sum $\lambda_D^T(n) = \Theta(\frac{T(n)}{n})$. ∎

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