On the Impact of Dynamic Jamming on End-to-End Delay in Linear Wireless Networks

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Abstract—The effectiveness and straightforward implementation of jammers make them an essential security threat for wireless networks. In particular, in order to increase the battery lifetime of the jammers and add spatial and temporal randomness to the jamming signal, it is advantageous for the adversaries to use duty cycling strategies. In this case, the state of each wireless link is governed by the dynamics of multiple stochastic jammers affecting each link. Since it is possible that each jammer can interrupt communication over multiple links, the up-down dynamics of different links can be spatially and/or temporally correlated. Dynamic networks have been considered widely in the literature; however, most analyses have ignored these challenging dependencies. In this paper, we consider communication over a linear network in the presence of duty cycling jammers. We model the process associated to each link in both slotted time and continuous-time regimes, and evaluate the exact end-to-end latency of the network in each case.

I. INTRODUCTION

Wireless networks, due to their broadcast nature, are susceptible to many security attacks. Among them, passive eavesdropping attacks have attracted a lot of attention in the literature (e.g. see [1] and references therein). Nevertheless, active jamming attacks are a key component of any security scheme, as jamming can severely disrupt the network performance, and thus are of interest in this work. In particular, jamming the physical layer is one of the simplest and most effective attacks, as any cheap radio device can broadcast electromagnetic radiation to block the communication channel [2]. They may not only alter the interference level of the channel as commonly assumed in modeling of the malicious jammers, but also can greatly impact receiver operation by compressing the dynamic range of the receiver's front-end [3].

Now consider a multi-hop wireless network such as a mesh network. In order to maximize the area under the jamming attack, instead of using a few jammers, an adversary can spread many jammers at arbitrary and random locations in the network. In this case, the jammers rely on battery power, which is limited, and hence it is essential for the jammers to seek methods to reduce their energy consumption and increase their jamming lifetime. A simple way to increase the battery lifetime of the jammers, while keeping their design simple, is applying random duty cycling [4]. Duty cycling strategies, where each node of the network switches frequently between an active
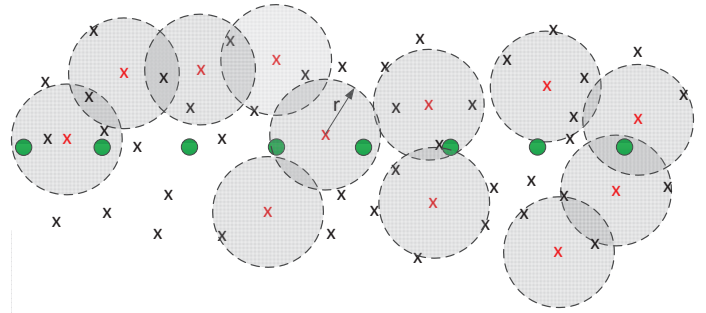

Fig. 1. The linear network in the presence of on-off jammers. The network nodes are shown by green circles, the silent jammers by black crosses, and the active jammers by red crosses. The jammers switch between active and silent modes frequently. Each active jammer can preclude reception in an area of radius r around it.

mode and a sleep mode, have been extensively used as a means of energy conservation in wireless networks [5]. An important performance metric in networks with duty cycling nodes is the end-to-end latency for transmitting messages. Because of duty cycling, some nodes along the transmission path of a message may be in sleep mode and hence the message has to wait for those nodes to become active. Network latency with duty cycling nodes, in both exact and asymptotic regimes has been studied in the literature [6]; however, wireless communication in the presence of duty cycling jammers and the impact of them on the network latency is not investigated.

This paper considers multi-hop wireless communication in a linear network in the presence of such random duty cycling jammers (Figure 1). Using duty cycling jammers in a network would enable the adversary to interfere with the communication of many nodes over a large area with low power and complexity. We assume that each jammer, when it is active, affects an area with a fixed radius around it such that the receivers located in that area cannot receive any message. Because of the stochastic nature of the jammers, and the fact that each jammer can possibly affect multiple nodes, the processes that describe the up and down links across the network are potentially temporally and/or spatially correlated. As a result, the network behaves like a dynamic graph with stochastic links. While dynamic networks, with a variety of applications in different fields, have widely been studied in the literature [7], [8], only a few studies have considered such

dependencies [9].

In this work, the exact end-to-end latency of the transmission of a message for the cases of synchronous and asynchronous jammers is characterized. In the first part of the paper, we assume that the time is slotted, and in every time slot, each jammer, which can affect multiple nodes and thus cause correlation between the dynamics of different links, is active with probability p and silent with probability $1 - p$. In the second part of the paper, we assume that each jammer changes its state asynchronously and at random times, such that the duration of an active period and a silent period are independent exponential random variables with rates λ and μ, respectively. Therefore, unlike the first part, in the second part we model the dynamics of each link by a continuous time renewal process and find the latency of the network. The theoretical framework developed in this work can be applied to other systems such as channels with interference and duty cycling networks.

The rest of the paper is organized as follows. Section II describes the system model. The latency under synchronous and asynchronous on-off jamming is studied and characterized in Sections III and IV, respectively. In Section V, conclusions and ideas for future work are discussed.

II. SYSTEM MODEL

Consider a system consisting of $n + 1$ nodes $V = \{v_0, v_1, v_2, ..., v_n\}$ forming a linear network. Without loss of generality we assume that consecutive nodes are located at unit distance from each other. The node v_0 transmits a message to the node v_n hop by hop via the intermediate nodes. A number of jammers are present in the two-dimensional area that contains the linear network. The jammers are distributed in the area according to a homogeneous Poisson point process with density σ. Each jammer decides independently of other jammers to send a jamming signal or be silent. Suppose when a jammer is active, it can preclude message reception in a region of radius r around itself. Hence, a link is "open" when there is no active jammer in the region of radius r around the receiving node of the link, and it is "closed" otherwise. We assume that when the message arrives at a node and finds the next link open, the waiting time at this node is zero. We assume that the propagation delay and the queuing delay are zero. Hence, in the absence of jammers the end-to-end latency of the network, i.e. the time that a message can travel from the source to the destination, is zero. Our goal is to find the latency of communication between v_0 and v_n in the presence of the duty cycling jammers.

Cases of both synchronous and asynchronous jammers are considered. In the first part, we assume that the time is divided into time slots. In every time slot, each jammer is in ON state (sends the jamming signal) with probability p and is in OFF state (silent) with probability $1 - p$. At the beginning of each time slot each jammer chooses its state independently of its previous state and of the state of the other jammers. In the second part of the paper, we assume that each jammer changes its state asynchronously and at random times. The duration of time that each jammer is in ON state is an exponential random variable with rate μ and the duration of time that each jammer is in an OFF state is an independent exponential random variable with rate λ. The end-to-end latency of the hop-by-hop transmission of a message from the source (v_0) to the destination (v_n) for each case is investigated in the following sections.

III. SYNCHRONOUS ON-OFF JAMMERS

A. Synchronous On-Off Jammers with Small Jamming Range

In this section we assume that the jammers change their state simultaneously and independently at the beginning of each time slot such that each jammer is on with probability p and is off with probability $1 - p$. In order to gain insight into the problem, we first assume that the jamming range of each jammer is small such that $r < 1/2$. In this case, the set of jammers affecting different nodes have no jammers in common. This corresponds to a scenario where jammers operate in a low power regime or in an area with high signal attenuation. The average latency of sending a message from v_0 to v_n is characterized in the following lemma.

Lemma 1. The latency of a linear n-hop network in the presence of on-off jammers distributed according to a Poisson point process with density σ and with jamming radius $r < 1/2$ is,

$$E[W] = n(e^{\frac{p}{1-p}\sigma\pi r^2} - 1).$$

Proof. The proof is presented in APPENDIX A. $\square$

B. Synchronous On-Off Jammers with Arbitrary Jamming Range

Similar to the previous section, here we assume that the jammers are synchronous. However, the transmission range of each jammer can be large and thus each jammer can affect multiple nodes. Suppose that the process describing the state of the jth link is denoted by $l_j = \{l_j(k)\}_{k=-\infty}^{k=\infty}$, where k is the discrete time. Let $l_j(k) = 1$ when at time slot k the link between v_{j-1} and v_j is open (all the jammers in the region of radius r around v_j are off) and $l_j(k) = 0$ otherwise. Since the jamming range can be large ($r > 1/2$), some jammers can affect multiple nodes and thus some dependency between the processes of different links exists. The following lemma shows that the assumption that the network is linear plays a critical role in the analysis of the network in the presence of jammers with arbitrary jamming range.

Lemma 2. The event that a link is open given that the last visited link is open is independent of the state of all other previous links.

Proof. We need to show that

$$p(l_j(k) = 1|l_{j-1}(k) = 1, l_{j-2}(k), \ldots, l_1(k))$$
$$= p(l_j(k) = 1|l_{j-1}(k) = 1),$$

for $j = 2, \ldots, n - 1$. Because of the linearity of the network, all the jammers common between jamming region of the jth
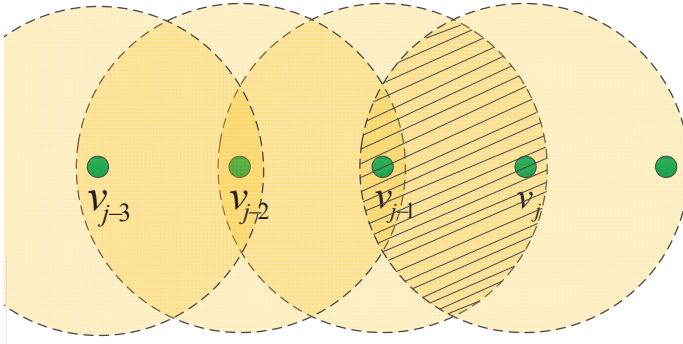
Fig. 2. Because of the linearity of the network, all the jammers common between jamming region of the jth node and any of its previous system nodes are a subset of the jammers common between the jth node and the $(j-1)^{th}$ node (jammers in the shaded area).

node and any of its previous system nodes are a subset of the jammers common between the jth node and the $(j-1)^{th}$ node (jammers in the shaded area in Figure 2). Hence, the probability of a link being open given that its previous link is open is independent of the state of all other previously visited links. □

In this case, the following theorem shows the end-to-end latency of the network.

Theorem 1. Latency of a linear n-hop network in the presence of synchronous independent on-off jammers with density σ is:

$$E[W] = e^{\frac{p}{1-p}\sigma\pi r^2} - 1 + (n-1)(e^{\frac{p}{1-p}\sigma\pi r^2} - e^{\frac{p}{1-p}\sigma A_c})$$

where p is the probability of a jammer being on, r is the jamming radius of each jammer, and, A_c is the area of the common jamming region between nodes v_j and v_{j-1}, which is given by,

$$A_c = 2r^2 \cos^{-1}(\frac{1}{\sqrt{2}r}) - \sqrt{2r^2 - 1}$$

Proof. The proof is presented in APPENDIX B. □

IV. ASYNCHRONOUS ON-OFF JAMMERS

In this section, unlike the previous section that all the jammers were changing their states simultaneously, each jammer switches between ON state (when it transmits the jamming signal) and OFF state (when it is silent) independently from the other nodes. Also, the durations of ON states and OFF states for each jammer are no longer fixed, and are two independent exponential random variables with rates μ and λ, respectively. This model, in particular, describes the simple jammers that are spread over an area without any connection to each other and without any synchronization between them, and switch between ON and OFF modes to save energy and increase their jamming lifetime.

A. Asynchronous On-Off Jammers with Small Jamming Range

We show the random process describing dynamics of the jth link (the link between v_{j-1} and v_j) by $l_j = \{l_j(t)\}_{t=-\infty}^{t=\infty}$, where t is the continuous time. Let $l_j(t) = 1$ when the link l_j is open (all the jammers in the region of radius r around v_j are off) at time t, and $l_j(t) = 0$ when l_j is closed or down (at least one jammer in the region of radius r around v_j is on) at time t.

Consider the link l_j. Let us denote the number of jammers affecting v_j by random variable M. Given that $M = m$, the state of link l_j is governed by m independent random duty-cycling jammers.

Suppose the process describing l_j given that m jammers are affecting v_j is denoted by E_j such that E_j has m states $\{0,\ldots,m\}$, and is in state i when i jammers are on and $m - i$ jammers are off. Since the durations of ON states and OFF states of each jammer are exponentially distributed, E_j is a continuous time Markov chain with state space $\mathcal{S} = \{0, 1, \ldots, m\}$. When E_j is in state i, each of i active jammers become silent at rate μ, and each of $m - i$ silent jammers become active at rate λ (Figure 3). Hence, the birth and death rates are:

$$\lambda_i = (m-i)\lambda$$

$$\mu_i = i\mu$$

This birth-and-death process is a classical stochastic process, and is called Ehrenfest process[1]. The transition probabilities of E_j are,

$$\begin{cases} p_{i,i+1} = \frac{(m-i)\lambda}{(m-i)\lambda+i\mu}, & 0 \leq i < m \\ p_{i,i-1} = \frac{i\mu}{(m-i)\lambda+i\mu}, & 0 < i \leq m \\ p_{i,j} = 0, & |j-i| > 1 \end{cases} \quad (1)$$

The following theorem characterizes the average end-to-end latency of a linear network in the presence of asynchronous on-off jammers. To gain some intuition, we first consider the case that the jammers affecting each node are independent from the jammers affecting the other nodes, i.e. $r < 1/2$.

Theorem 2. The average end-to-end latency of a linear network in the presence of asynchronous on-off jammers is,

$$E[W] = \frac{n}{\mu} \sum_{m=0}^{\infty} \sum_{j=1}^{m} \sum_{i=j}^{m} \sum_{k=j}^{m} \frac{\binom{m}{i}\binom{m}{k}}{j\binom{m}{j}} \frac{\rho^{i+k-j}}{(1+\rho)^m} \frac{e^{-\sigma\pi r^2}(\sigma\pi r^2)^m}{m!},$$

where $\rho = \frac{\lambda}{\mu}$.

Proof. Proof is presented in APPENDIX C. □

B. Asynchronous On-Off Jammers with Arbitrary Jamming Range

Now consider the case that the transmission range of the jammers is large and thus each jammer can interfere with the reception of message at multiple nodes. Hence, some

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[1]This process is the continuous version of the Ehrenfest urn model, which was introduced by Paul and Tatyana Ehrenfest to describe the heat exchange between two bodies in the kinetic theory of gases [10].

dependency between the processes of different links exists and thus the result of the previous section is not valid. The following theorem characterizes the end-to-end latency of the network in this scenario.

Theorem 3. The expected end-to-end latency of a linear n-hop network in the presence of Poisson distributed jammers with arbitrary transmission range r is given by

$$E[W] = \frac{1}{\mu} \sum_{m=0}^{\infty} \sum_{j=1}^{m} \sum_{i=j}^{m} \sum_{k=j}^{m} \frac{\binom{m}{i}\binom{m}{k}}{j\binom{m}{j}} \frac{\rho^{i+k-j}}{(1+\rho)^m} \frac{e^{-\sigma\pi r^2}(\sigma\pi r^2)^m}{m!}$$

$$+ \frac{n-1}{\mu} \sum_{m_c=0}^{\infty} \sum_{m=0}^{\infty} \sum_{j=1}^{m} \sum_{i=j}^{m} \sum_{k=j}^{m+m_c} \frac{P_i P_k}{j P_j} \frac{e^{-\sigma\pi r^2}(\sigma A)^m (\sigma A_c)^{m_c}}{m! m_c!}$$

where,

$$P_i = \frac{\binom{m}{i}\rho^i}{(1+\rho)^m},$$

$$P_s = \frac{\binom{m+m_c}{s}\rho^s}{(1+\rho)^{m+m_c}}, \ s \in \{j,k\},$$

$$A_c = 2r^2 \cos^{-1}(\frac{1}{\sqrt{2}r}) - \sqrt{2r^2-1},$$

and $A = \pi r^2 - A_c$.

Proof. Proof is presented in APPENDIX D. □

V. CONCLUSION AND FUTURE WORK

In this paper, multi-hop communication over a linear wireless network in the presence of synchronous and asynchronous duty cycling jammers is considered. In the case of synchronous jammers, the dynamics of each link are modeled as a discrete time process, and the exact expected end-to-end message delay is evaluated. In the case of asynchronous jammers, the dynamics of each link are modeled by a continuous time birth-and-death process, and the exact expected latency of sending a message is characterized. Modeling the link dynamics of a 2D grid network in the presence of stochastic jammers and computing the end-to-end latency are currently under investigation.

APPENDIX A

Suppose that the area of the jamming region around v_1 is A. Let us denote the number of jammers in this region by M. Since the jammers are distributed according to a Poisson point process,

$$P(M = m) = \frac{e^{-\sigma A}(\sigma A)^m}{m!}$$

The proof of Lemma 1 follows.

Proof. Since $r < \frac{1}{2}$, the jamming regions around different nodes do not overlap and thus the jammers affecting each node are independent from the jammers affecting the other nodes. Hence, the end-to-end latency is,

$$E[W] = \sum_{i=0}^{n-1} E[W_i].$$

where $E[W_i]$ is the mean waiting time at v_i, i.e. it is the expected number of time slots that takes until v_i can transmit the message to v_{i+1}. Using the uniformity of the network,

$$E[W_i] = E[W_0], \text{ for } i = 1, \ldots, n-1.$$

Thus, it suffices to find the expected waiting time at v_0.

The waiting time at v_0 given that $M = m$ jammers exist in the region around v_1 follows a geometric distribution,

$$p(W_0 = k | M = m) = (1 - (1-p)^m)^k (1-p)^m.$$

Thus, using the law of total probability,

$$p(W_0 = k) = \sum_{m=1}^{\infty} p(W_0 = k | M = m) P(M = m)$$

$$= \sum_{m=1}^{\infty} (1 - (1-p)^m)^k (1-p)^m \frac{e^{-\sigma A}(\sigma A)^m}{m!}$$

Hence, the expected value of the waiting time at v_0 is,

$$E[W_0] = \sum_{k=0}^{\infty} k p(W_0 = k)$$

$$= \sum_{k=0}^{\infty} k \sum_{m=1}^{\infty} (1 - (1-p)^m)^k (1-p)^m \frac{e^{-\sigma A}(\sigma A)^m}{m!}$$

$$= \sum_{m=1}^{\infty} \frac{e^{-\sigma A}(\sigma A)^m}{m!} \sum_{k=0}^{\infty} k(1 - (1-p)^m)^k (1-p)^m$$

$$= \sum_{m=1}^{\infty} \left(\frac{1}{(1-p)^m} - 1 \right) \frac{e^{-\sigma A}(\sigma A)^m}{m!}$$

$$= \frac{e^{-\sigma A}}{e^{-\sigma A/(1-p)}}(1 - e^{-\sigma A/(1-p)}) - (1 - e^{-\sigma A})$$

$$= e^{\sigma A p/(1-p)} - 1$$

□

APPENDIX B

In this appendix proof of Theorem 1 is presented.

Proof. In this case since the transmission range of each jammer is large, each jammer can affect multiple nodes and thus the jamming regions around the system nodes overlap. This causes some dependency between the different link processes. Because of the linearity of the network, Lemma 2 can be applied, which states that in a linear network, the event that a link is open given that the last visited link is open is independent of the state of other previously visited links. Using this fact and the homogeneity of the network, $E[W_i] = E[W_1]$, for $i = 1, \ldots, n-1$, and since v_0 has no previous node, the expected waiting time at v_0 in this section is same as $E[W_0]$ in the previous section. The expected end-to-end latency is,

$$E[W] = E[W_0] + (n-1)E[W_1]. \tag{2}$$

Hence, our goal is to find $E[W_1]$.

Consider the link l_2 and assume v_1 wants to send a message to v_2. Suppose M_c denotes the number of jammers common

between v_1 and v_2 and M denotes the number of jammers that just affect v_2. We want to calculate the waiting time at v_1. The probability that waiting time is zero is given by the probability that all the jammers that just affect v_2 are off. The reason is that zero waiting time means that the message just arrived at v_1 and thus all the jammers around v_1 are off. This means that all the jammers common between v_1 and v_2 are also off and in order to be able to send the message to v_2 without delay (zero waiting time), we need the jammers that only affect v_2 to be off. Thus,

$$p(W_j = 0 | M_c = m_c, M = m) = (1 - p)^m$$

But if $W_1 \neq 0$, the message has to wait at v_1 until the next time slot that the states of jammers change. Since the states of jammers change independently from their previous states, the probability that waiting time at v_1 is greater than k time slots is:

$$p(W_1 > k | M_c = m_c, M = m)$$
$$= (1 - (1 - p)^m)(1 - (1 - p)^{m + m_c})^k$$

Let us denote the area of the region that contains the jammers that only affect v_2 by A and the area of the region that contains the jammers common between v_1 and v_2 by A_c. Using the law of total probability,

$$p(W_1 > k) = \sum_{m_c = 0}^{\infty} \sum_{m = 0}^{\infty} (1 - (1 - p)^m)(1 - (1 - p)^{m + m_c})^k$$
$$\frac{e^{-\sigma A_c}(\sigma A_c)^{m_c}}{m_c!} \frac{e^{-\sigma A}(\sigma A)^m}{m!}$$

Thus,

$$E[W_1] = \sum_{k=0}^{\infty} p(W_1 > k)$$
$$= \sum_{k=0}^{\infty} \sum_{m_c=0}^{\infty} \sum_{m=0}^{\infty} (1 - (1 - p)^m)(1 - (1 - p)^{m + m_c})^k$$
$$\frac{e^{-\sigma A_c}(\sigma A_c)^{m_c}}{m_c!} \frac{e^{-\sigma A}(\sigma A)^m}{m!} \quad (3)$$
$$= \sum_{m_c=0}^{\infty} \sum_{m=0}^{\infty} ((1 - p)^{-(m + m_c)} - (1 - p)^{-m_c})$$
$$\frac{e^{-\sigma A_c}(\sigma A_c)^{m_c}}{m_c!} \frac{e^{-\sigma A}(\sigma A)^m}{m!}$$
$$= e^{\frac{\sigma p(A + A_c)}{1 - p}} - e^{\frac{\sigma p A_c}{1 - p}} \quad (4)$$

where $A + A_c = \pi r^2$ and A_c is the common jamming area of two neighbor nodes,

$$A_c = 2r^2 \cos^{-1}(\frac{1}{\sqrt{2}r}) - \sqrt{2r^2 - 1}.$$

Hence, by substituting (4) in (2), the average end-to-end waiting time follows. $\square$

APPENDIX C

The proof of Theorem 2 is presented in this appendix.

Proof. The end-to-end latency of the network is,

$$E[W] = \sum_{j=0}^{n-1} E[W_j]$$

Since the network is homogeneous, and the jammers affecting each node are independent from the jammers affecting the other nodes, we have $E[W_0] = E[W_j]$, for $j = 1, \ldots, n - 1$, and thus

$$E[W] = nE[W_0]. \quad (5)$$

Hence, given that the message is at node v_0 at a random time t, our goal is to find the expected waiting time until l_1 is open (all the jammers around v_1 are silent).

Suppose that the number of jammers around v_1 is M and the process describing l_1 given that $M = m$ is denoted by E_1. Assuming that the jammers are running for a long time (from $-\infty$), the probability that E_1 is in state i is denoted by P_i, i.e. $P_i = \lim_{t \to \infty} P_{ji}(t)$. These probabilities can be obtained by substituting the mean sojourn times $\nu_i = \lambda_i + \mu_i$, and the transition probabilities (1) in the following balance equations,

$$\nu_i P_i = \sum_j \nu_j p_{ji} P_j, \text{ and } \sum_i P_i = 1.$$

Hence, the limiting probabilities are,

$$P_i = \frac{\binom{m}{i} \rho^i}{(1 + \rho)^m}, \ i = 0, \ldots, m \quad (6)$$

where $\rho = \frac{\lambda}{\mu}$. Given that the jammers are operating for a long time, using the law of total probability, the expected waiting time at v_0 is,

$$E[W_0 | M = m] = \sum_{i=0}^m E[W_0 | M = m, E_1 = i]P_i,$$

where $E[W_0 | l_1 = 0] = 0$, and $E[W_0 | M = m, E_1 = i]$ for $i > 0$ is equal to the mean first passage time from state i to state 0. Hence,

$$E[W_0 | M = m] = \sum_{i=1}^m E[T_{i0}]P_i \quad (7)$$

The mean first passage time from state i to state 0 for a birth-and-death process is given by [11],

$$E[T_{i0}] = \sum_{j=1}^i \frac{1}{\mu_j P_j} \sum_{k=j}^m P_k \quad (8)$$

By substituting (6) and (8) into (7) the average waiting time given $M = m$ is,

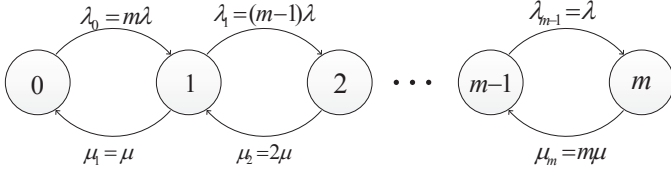$$E[W_0 | M = m] = \sum_{i=1}^m P_i \sum_{j=1}^i \frac{1}{\mu_j P_j} \sum_{k=j}^m P_k$$

Fig. 3. Equivalent birth-and-death process of the link l_1 in the presence of m independent on-off jammers. When the process is in state i, it means i jammers are on and the rest are off. Thus, the link is open if the process is in state 0 (all the jammers are off). The birth rate is $\lambda_i = (m-i)\lambda$ and the death rate is $\mu_i = i\mu$.

Hence,

$$
E[W_0] = \sum_{m=0}^{\infty} E[W_0|M=m]p(M=m)
$$

$$
= \sum_{m=0}^{\infty} \sum_{i=1}^{m} P_i \sum_{j=1}^{i} \frac{1}{\mu_j P_j} \sum_{k=j}^{m} P_k \frac{e^{-\sigma A}(\sigma A)^m}{m!} \quad (9)
$$

By substituting (9) in (5), the result follows. $\qquad \square$

APPENDIX D

Proof. In this case, again each jammer can affect multiple nodes and thus dependency between the process of different links exist.

From Lemma 2, since the network is linear, the probability of a link being open given that its previous link is open is independent from the state of the other previously visited links. Hence, except v_0, which is the source of the message, because of the homogeneity of the network, the waiting time at the other nodes is the same, i.e.

$$
E[W_1] = E[W_j], \text{ for } j = 2, \ldots, n-1
$$

and the waiting time at v_0 ($E[W_0]$) is equal to the waiting time of the previous case (when each jammer can just affect one node). Therefore,

$$
E[W] = E[W_0] + (n-1)E[W_1] \quad (10)
$$

Hence, we just consider the waiting time at v_1. Suppose that $M+M_c = m+m_c$ jammers are affecting v_2, where $M_c = m_c$ of them are common between v_1 and v_2, and $M = m$ of them are just affecting v_2. The process describing l_2 (the link that connects v_1 and v_2) given that $M = m$ and $M_c = m_c$ is denoted by E_2. Hence, E_2 is an Ehrenfest process with $m+m_c$ states $\{0, 1, \ldots, m+m_c\}$. We need to find the expected time from when a message arrives at v_1 until it can be sent to v_2 (all the jammers affecting v_2 are off). When a message just arrives at v_1, it means that at the time of arrival to v_1 all the jammers around v_1 are silent. Therefore, at least M_c of the jammers affecting v_2 are silent and thus the birth-and-death

process E_2 is in one of the states $\{0, 1, \ldots, m\}$. Using the law of total probability, the waiting time is:

$$
E[W|M=m, M_c=m_c]
$$

$$
= \sum_{i=0}^{m} E[W|M=m, M_c=m_c, E_2=i]P_i
$$

$$
= \sum_{i=1}^{m} E[T_{i0}|M=m, M_c=m_c]P_i
$$

$$
= \sum_{i=1}^{m} P_i \sum_{j=1}^{i} \frac{1}{\mu_j P_j} \sum_{k=j}^{m+m_c} P_k
$$

Hence, the expected waiting time at v_1 is,

$$
E[W_1]
$$

$$
= \sum_{m_c=0}^{\infty} \sum_{m=0}^{\infty} E[W_1|M=m, M_c=m_c]p(M=m, M_c=m_c)
$$

$$
= \sum_{m_c=0}^{\infty} \sum_{m=0}^{\infty} E[W_1|M=m, M_c=m_c]p(M=m)p(M_c=m_c)
$$

$$
= \sum_{m_c=0}^{\infty} \sum_{m=0}^{\infty} \sum_{i=1}^{m} P_i \sum_{j=1}^{i} \frac{1}{\mu_j P_j} \sum_{k=j}^{m+m_c} P_k \frac{e^{-\sigma A}(\sigma A)^m}{m!} \frac{e^{-\sigma A_c}(\sigma A_c)^{m_c}}{m_c!}
$$

$$
= \sum_{m_c=0}^{\infty} \sum_{m=0}^{\infty} \sum_{j=1}^{m} \sum_{i=j}^{m} \sum_{k=j}^{m+m_c} \frac{P_i P_k}{j\mu P_j} \frac{e^{-\sigma(A+A_c)}(\sigma A)^m (\sigma A_c)^{m_c}}{m! m_c!}
$$

$$
\quad (11)
$$

By substituting (9) and (11) in (10), the result follows. $\quad \square$

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