

# Stochastic Geometry Based Pricing for Infrastructure Sharing in IoT Networks

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**Abstract**—In this letter, we propose a stochastic geometry based pricing for infrastructure sharing in Internet of Things (IoT) networks. We consider a game consisting of a Network Operator (NO) as the seller and an IoT Device Owner (DO) as the buyer in which the seller owns an infrastructure that can address the communication needs of the DO. Using the proposed scheme, we show that DO and NO can reach a win-win deal in which a reasonable cost is imposed to DO in exchange of providing an acceptable coverage by the NO. In particular, we show that the DO can achieve a coverage probability of interest at a lower cost compared to the case in which the proposed pricing model is absent. The proposed idea provides a transparent pricing model between NOs and DOs and paves the road for IoT applications to become more widespread.

## I. INTRODUCTION

Internet of Things (IoT) is a novel framework that allows billions of smart devices to be connected to the Internet to transmit their data. The data can then be used to improve quality of life in different aspects such as healthcare, transportation, and manufacturing [1].

In the core of most IoT systems lay smart wireless sensors that can collect data from the environment and convey such data to the central controllers for further processing [2]. The entity that owns such sensors is referred to as the Device Owner (DO). Due to the distributed nature of IoT, a DO needs a cost effective network infrastructure to exchange the data within itself and/or transmit it to another interested entity. However, such a network generally cannot be owned and operated by the DO itself and most DOs have to deal with already established Network Operators (NOs) to use their existing resources and infrastructures.

To compensate the NOs for the use of their resources, the DO can either pay the NO directly or exchange its own resources. Examples of such resources include valuable anonymized data obtained by the DO or DO buildings that can be shared with the NO to provide additional network services for its clients.

Reaching a financial resource sharing agreement between any two entities is usually a challenging task. As far as infrastructure sharing is concerned, this is already a common practice between large NO companies to reduce their operational costs. This can include sharing the spectrum, base stations, or site locations. Yet, the agreements are usually achieved by traditional negotiations without the use of a pricing model. However, conventional negotiations between a large NO and a small DO that does not specialize in communication

networks may result in an unfair deal for the DO and it may be taken advantage of by the NO's unreasonable rates. In fact, lack of a transparent pricing model has been one of the important barriers in preventing the IoT applications from becoming more widespread. To take care of this issue, a solid trading/pricing model is necessary to regulate such deals between DOs and NOs.

In this paper, we focus on finding game theoretic pricing models for infrastructure sharing between DOs and NOs. As the first step in this important framework, we propose a pricing model in which we assume that the DO only wants to share the base stations of the NO in exchange for money. In doing so, we deploy stochastic geometry as a very efficient and powerful network planning tool [3]–[5]. By proposing a stochastic geometry based infrastructure pricing model and finding the Nash equilibrium, we demonstrate how the NOs and DOs can reach a win-win deal. In particular, we show that the DO can achieve a coverage probability of interest at a lower cost compared to the case in which the proposed pricing model is absent. The proposed model can be seen as a basis for more advanced pricing models that if developed, will facilitate the IoT to become more pervasive.

There are a number of works in wireless networking literature that use pricing methods to model the trade-offs among different entities. Examples include secondary and primary operators in cognitive radio networks [6]–[10], device to device communications [11]–[13], and heterogeneous networks [14], [15]. The focus of all these works is on the spectrum as the resource to be traded and they are established on a completely different pricing framework than the one presented in this paper. There are a few papers that consider different aspects of infrastructure pricing between network operators [16]–[18]. However, they are not in the context of IoT and do not use a stochastic geometry framework in contrast to our work.

As far as pricing in IoT is concerned, only a few works exist in the literature [19]–[21]. As a simple market, [19] investigates the pricing scheme in a business model of IoT with three participants: multiple sensing data owners, service providers, and users. With similar market components, [20] proposes an economic model in big data and IoT in which the authors use the classification-based machine learning algorithms to define the generic utility function of data. Then using a Stackelberg game, optimal raw data selling price is obtained. Service management of an IoT device has been investigated in [21] where the Markov decision process (MDP) is used to model an optimization framework in order to obtain an optimal policy

for the device owner. As can be seen, despite its absolute necessity, there is no pricing platform in the IoT literature that can address the infrastructure sharing and in the context of stochastic geometry.

The organization of this paper is as follows. In the next section, we propose the system model and in Section III, the formulated problem and its solution are presented, Section IV demonstrates the simulation results, and Section V concludes the paper.

## II. SYSTEM MODEL

We assume a realistic case that the DO does not own any BS infrastructure and uses the NO's BSs. However, we assume that the DO uses its own spectrum and so there is no spectrum sharing.<sup>1</sup>The considered NO system model is a cellular network with BSs located according to a homogeneous spatial Poisson Point Process (PPP)  $\Phi$  of density  $\lambda$ . Each BS has a single transmit antenna and the transmit power of each BS is denoted by  $P$ . Mobile users are equipped with single receive antennas, and they are located according to an arbitrary configuration over the plane. It is assumed that there is a test user located at the origin and we focus on the coverage probability of this user.

The SNR-based coverage probability can be written as

$$\mathcal{C} = \Pr\{SNR_{x^*} > \tau\}, \quad (1)$$

where  $x^*$  is the nearest BS to the user and  $\tau$  is a defined threshold for SNR. After some mathematical calculations, the SNR-based coverage probability can be obtained as [22]

$$\mathcal{C} = 1 - \exp\left(-\lambda\pi\left(g^{-1}\left(\frac{\tau}{P}\right)\right)^2\right), \quad (2)$$

where  $g$  is the path loss function shown by  $g(r) = (1 + r^\alpha)^{-1}$  and  $\alpha$  is the path loss exponent. Note that for a fixed coverage probability, the transmit power  $P$  is conversely proportional to the BS density  $\lambda$ , i.e.,  $P = \rho/\lambda$  where  $\rho$  is a constant [22].

Let  $\mu$  be the cost per unit BS density. In order to include the price, we should write  $P$  as a function of  $\lambda$  and  $\mu$ , i.e.,  $P = h(\lambda, \mu)$ , where  $h$  is a decreasing function of  $\lambda$  and an increasing function of  $\mu$ , as the DO should pay more if it requires more power from NO side. In the absence of  $\mu$ , we already know that  $P = h(\lambda) = \rho/\lambda$  and therefore, it is reasonable to assume  $P = \frac{\rho}{\lambda} + \zeta\mu$  where  $\zeta$  is a constant that is set to 1 for notational convenience. Based on this setting we have

$$\lambda = \rho/(P - \mu) \quad (3)$$

This choice of function  $h$  might not be unique but has shown to fit well our current pricing model. Now we replace  $P$  in

<sup>1</sup>This is the case for certain entities (e.g. police departments or military) that already own the spectrum for older applications (e.g. communication using old fashioned analog devices) and may now want to exploit it in an IoT framework without investing in infrastructure.

(2) and obtain

$$\mathcal{C} = 1 - \exp\left(-\lambda\pi\left(g^{-1}\left(\frac{\tau\lambda}{\rho + \mu\lambda}\right)\right)^2\right). \quad (4)$$

## III. PROBLEM FORMULATION

In this section, we first propose optimization problems for the NO and DO and then develop an algorithm by exploiting the game theory approaches. Since there is conflict of interests in the objectives of the game problem, the adopted game theory is non-cooperative [23]. Hence, in our game theoretic infrastructure sharing system, the NO first determines the density of the BSs to be shared and the DO optimizes the price to be paid.

### A. NO Problem

The NO side is obligated to maximize its coverage probability for the DO but wants to achieve it by maximizing  $\lambda$ , the density of BSs to obtain higher revenue from the DO. The optimization problem is then written as

$$\mathcal{O}^{\text{NO}} : \max_{\lambda} U^{\text{NO}} \quad (5)$$

$$\text{s.t. } \max\left\{\lambda_{\min}, \frac{\rho}{P_{\max} - \mu}\right\} \leq \lambda \leq \min\left\{\lambda_{\max}, \frac{\rho}{P_{\min} - \mu}\right\}, \quad (6)$$

where

$$U^{\text{NO}} = \mathcal{C} \quad (7)$$

In the above equations,  $P_{\min}$  and  $P_{\max}$  are the minimum and maximum allowable transmit power of each BS and  $\lambda_{\min}$  and  $\lambda_{\max}$  are the minimum and maximum range of BS's density. Note that the constraint actually represents the feasible set of the  $\lambda$  as obtained according to (3).

To maximize the utility, we can equivalently maximize the following function:

$$f(\lambda, \mu) = \lambda\left(g^{-1}\left(\frac{\tau\lambda}{\rho + \mu\lambda}\right)\right)^2. \quad (8)$$

To find the optimal density denoted by  $\lambda^*$ , we set the derivative to zero as follows where  $\mu_0$  is the initial price that we choose for the stochastic pricing problem

$$\frac{d(f(\lambda, \mu_0))}{d\lambda} = 0. \quad (9)$$

By solving the above equation, the optimal density denoted by  $\lambda^*$  is obtained as

$$\lambda^* = \frac{\rho(\alpha - 2)}{\alpha(\tau - \mu^*)}, \quad (10)$$

where  $\mu^*$  is the price which will be obtained through the DO problem in the next subsection. Furthermore, to globally maximize  $\mathcal{C}$ , we should check that  $f''(\lambda^*) < 0$ .

### B. DO problem

The DO side plans to reduce the cost, i.e.,  $\mu\lambda$ , while maintaining the required probability of coverage. Therefore, the optimization problem is written as

$$\mathcal{O}^{DO} : \min_{\mu} U^{DO} = \min_{\mu} \{\mu\lambda\}, \text{ s.t. } \mathcal{C} \geq C_0, \quad (11)$$

where  $C_0$  is a predefined threshold for coverage probability. As the objective of this optimization problem is a monotonic function versus the parameter  $\mu$ , to obtain it, we first use Eq. (4) to obtain the value of  $C$ , then by applying this  $C$  to the constraint of problem (11), we can verify the value of  $\mu$  which minimizes the overall cost of DO, called  $\mu^*$ . This  $\mu^*$  then is used to obtain the  $\lambda^*$  from (10) as described later in Algorithm 1. Finally, the optimal price denoted by  $\mu^*$  is obtained as

$$\mu^* = \tau \left( \ln \left( \frac{1}{1-C_0} \right) \times \frac{1}{\pi\lambda} \right)^{\frac{\alpha}{2}} + \tau - \frac{\rho}{\lambda}. \quad (12)$$

Note that the obtained  $\mu^*$  should satisfy the inequality  $\mu \leq \tau$  which is the feasible set of  $\mu$ .

### C. Nash Equilibrium of NO and DO problems

Algorithm 1 illustrates our iterative method to find the optimal solution of the proposed stochastic pricing problem.

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#### Algorithm 1

- s1:** Choose randomly  $\mu_0$  from the feasible set of  $\mu$  (i.e.  $0 < \mu < \tau$ ).
- s2:** Calculate  $\lambda^*$  from (10).
- s3:** **While**  $(\lambda, \mu) \notin$  feasible set, calculate  $\lambda^*$  and  $\mu^*$  from (12) and (10).
- s4:**  $(\lambda^*, \mu^*)$  is the optimal solution.
- end**
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To see if the iterative method converges for a given set of parameters, i.e., to reach a Nash equilibrium, we act as follows. We substitute (10) in (12), which results in the following equation for  $\mu^*$ :

$$\mu^* = \tau \left( \ln \left( \frac{1}{1-C_0} \right) \frac{\alpha(\tau - \mu_0)}{\rho\pi(\alpha-2)} \right)^{\frac{\alpha}{2}} + \mu_0 \left( \frac{\alpha}{\alpha-2} \right) - \tau \left( \frac{2}{\alpha-2} \right) \quad (13)$$

For convenience, in (13), we introduce the coefficient  $k$  as

$$k = \tau \left( \ln \left( \frac{1}{1-C_0} \right) \times \frac{\alpha}{\rho\pi(\alpha-2)} \right)^{\frac{\alpha}{2}}. \quad (14)$$

In the next step, we can show (13) in the form of a recurrence equation as

$$\mu_{n+1} = k(\tau - \mu_n)^{\frac{\alpha}{2}} + \mu_n \left( \frac{\alpha}{\alpha-2} \right) - \tau \left( \frac{2}{\alpha-2} \right). \quad (15)$$

To be able to evaluate this non-homogeneous recurrence equation, we first have to transform it to its homogeneous counterpart by getting rid of the constant term  $\tau$ . This can be achieved by replacing index  $n$  with  $n+1$  and subtracting the resulting equation from (13). This will result in the following equation:

$$\mu_{n+2} - \mu_{n+1} = k \left( (\tau - \mu_{n+1})^{\frac{\alpha}{2}} - (\tau - \mu_n)^{\frac{\alpha}{2}} \right) + \frac{\alpha}{\alpha-2} (\mu_{n+1} - \mu_n). \quad (16)$$

By letting  $\mu_n = r^n$ , and consequently  $\mu_{n+1} = r^{n+1}$  and  $\mu_{n+2} = r^{n+2}$  and replacing them in (16) we obtain the so-called polynomial characteristic equation as [24]

$$k \left( (\tau - r^{n+1})^{\frac{\alpha}{2}} - (\tau - r^n)^{\frac{\alpha}{2}} \right) = r^{n+2} - \left( 1 + \frac{\alpha}{\alpha-2} \right) r^{n+1} + \frac{\alpha}{\alpha-2} r^n. \quad (17)$$

Now if for the given values of  $\tau, C_0, \alpha$  and  $\rho$ , the equation has a root within the feasible set, we can state that the proposed game has a Nash equilibrium. For example, by setting  $\tau = 1.5, C_0 = 0.8, \rho = 0.5, \alpha = 4$  we plot the two sides of (17). As can be seen in Fig. 1, the equation does have a root in the feasible set.

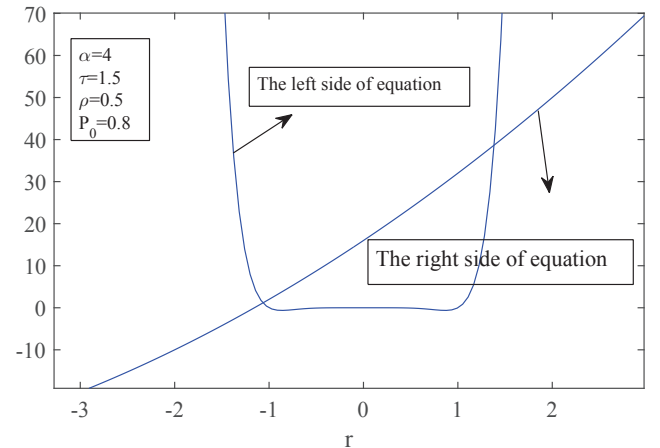


Fig. 1. Obtaining the intersection of two sides of (17).

#### IV. SIMULATION RESULTS

Throughout this section, we set the values of  $\tau$  and  $\rho$  as before. For the first part, we set  $\alpha = 3$  and  $C_0 = 0.9$  and apply Algorithm 1. Doing so, this algorithm converges to  $\lambda^* = 2.7227$  and  $\mu^* = 1.4388$ . To have a better understanding of converging to the Nash equilibrium, in Fig. 2 we have plotted two curves which correspond to NO and DO problems. To obtain the NO curve, for different values of  $\mu$  we solve the NO problem and obtain the optimal value of  $\lambda$  and plot them versus  $\mu$ . To obtain the DO curve, for different values of  $\lambda$ , we obtain optimal values of  $\mu$  and plot them versus  $\lambda$ . To be able to see both curves in one figure, we have to plot the inverse of the NO curve. The intersection of the two curves represents the Nash equilibrium obtained by Algorithm 1. The trajectories on the figure also represent the iterative process leading to convergence in Algorithm 1.

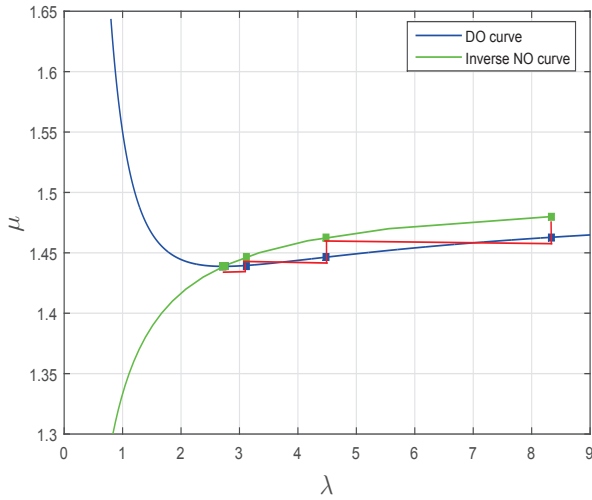


Fig. 2. The graphical representation of convergence in Algorithm 1.

We now change the value of  $C_0$  and apply Algorithm 1 for  $\alpha = 3, 4$  to obtain optimal values of  $\lambda$  and  $\mu$ . In Fig. 3, we have plotted the achieved coverage probability for different values of  $C_0$ , the minimum guaranteed coverage probability threshold. As can be predicted, larger values of  $C_0$  results in better coverage. However, this comes at the expense of higher cost for DO, i.e., larger  $\mu\lambda$  as shown in Fig. 4 where the cost  $\mu\lambda$  has been plotted versus  $C_0$ . As can be seen, the larger we set the  $C_0$ , the larger the cost becomes and we eventually achieve larger coverage probabilities than necessary. Therefore, it is to the benefit of the DO to choose a smaller value for  $C_0$ . For example, for  $\alpha = 3$ , if the DO needs a 0.95 coverage probability, the NO can suggest it to blindly set  $C_0$  to 0.95 to charge the DO more money, i.e.,  $\mu\lambda = 26$ . However, using the proposed pricing model, the DO can decrease  $C_0$  to 0.77 to pay less money, i.e.  $\mu\lambda = 4$ , while being assured in obtaining the necessary coverage probability, i.e., 0.95.

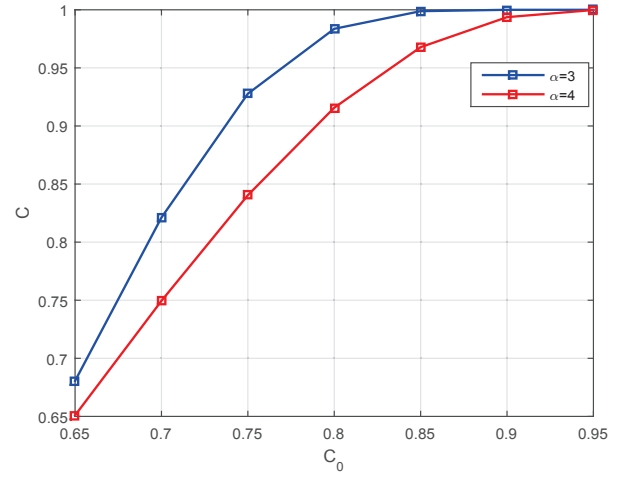


Fig. 3. Resulting coverage probability vs minimum guaranteed coverage threshold

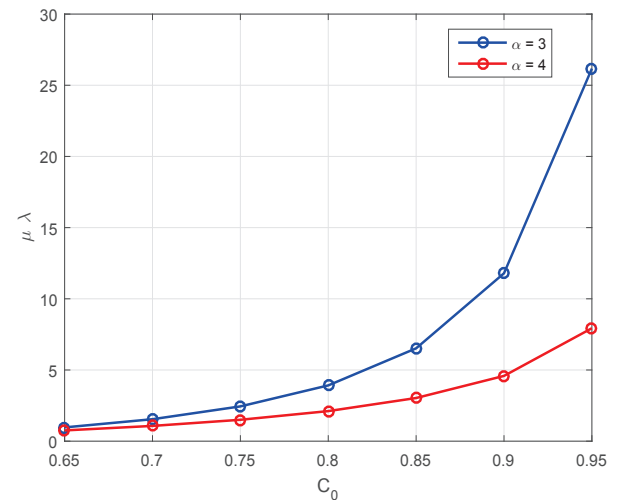


Fig. 4. The cost imposed to the DO vs minimum guaranteed coverage threshold

#### V. CONCLUSION

In this letter, we proposed a stochastic geometry based pricing for infrastructure sharing in IoT networks. Using this scheme within a simplified framework, we showed that DO and NO can reach a win-win deal in which a reasonable cost is imposed to DO in exchange of providing an acceptable coverage by the NO. The proposed idea provides a transparent pricing model between NOs and DOs and paves the road for IoT applications to become more widespread and introduces an economic incentive for the NOs involvement. More practical cases, such as concurrent consideration of both spectrum and infrastructure sharing, multiple NOs-multiple DOs infrastructure sharing and intra-cell or inter-cell interference consideration are left for our future work.

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