Improving Safety on Highways by Customizing Vehicular Ad Hoc Networks

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***Abstract*— This paper studies the need for individualizing vehicular communications in order to improve safety for a highway scenario. Adapting a vehicular ad hoc network to both its individual driver's characteristic and traffic conditions enables it to transmit in a smart manner to other vehicles. This radical improvement is now possible due to the progress that is being made in vehicular ad hoc networks (VANET). In this paper, we first derive the packet success probability for a chain of vehicles by taking multi-user interference, path loss, and fading into account. Then, by considering the delay constraints and types of potential collisions, we approximate the optimal channel access probabilities. Lastly, we propose an algorithm for customizing channel access probabilities in VANET. Our Monte Carlo simulation results show that this approach achieves more than 25% reduction in traffic collision probability compared with the case with equal channel access probabilities in its optimal range. Therefore, it has a huge advantage over other non-optimal systems.**

***Index Terms*— Ad Hoc networks, vehicular networks, medium access control, safety, packet success probability, wireless networks.**

I. Introduction

DURING the past decade, the automobile industry has seen a surge in the use of advanced technologies, such as state-of-the-art electronic devices, in order to improve automobile safety. Sadly, however, the fatalities and injuries caused due to automobile accidents have remained at an alarming level. In particular, statistics from 2013 [1] report over five million crashes in the U.S., causing over two million injuries and more than 30,000 fatalities.

A major cause of accidents is the slow response time of drivers to stopped traffic, i.e., the average time a driver takes to hit the brake after a preceding car has stopped. The cumulative response times for the leading vehicles play the main role in the collision probability[1] of the upstream vehicles, potentially resulting in domino-style collisions. To reduce the drivers' response time to accidents, recent research and development in the automobile industry has introduced collision warning systems to be installed on modern automobiles. Collision warning systems are capable of cautioning about critical, time-sensitive incidents such as crashes or traffic jams.

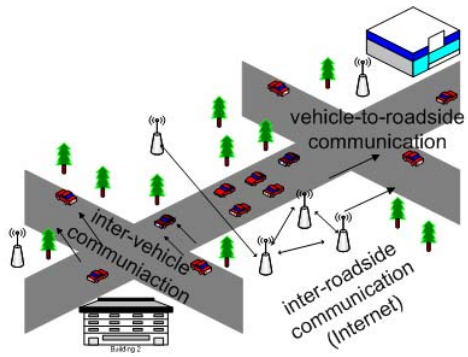

Fig. 1. VANET: Vehicular Ad-hoc NETwork.

With the advancements in Vehicular Ad Hoc Networks (VANET)(Fig. 1), recent research [2] suggests the use of VANETs to improve the effectiveness of collision warning systems. VANETs allow for cross-communication between cars within a close proximity of each other, which can enable them to efficiently and reliably communicate sensitive traffic messages such as crash-relevant information. The Federal Communications Commission has allocated 75 MHz of spectrum in the 5.9 GHz band for Dedicated Short Range Communications (DSRC). To serve as the groundwork for DSRC, the IEEE 802.11p standard was published in the year 2010 for Wireless Access in Vehicular Environments (WAVE) [3].

The 75 MHz spectrum of DSRC is divided into seven 10 MHz-wide channels. One channel is called the control channel (CCH) and serves exclusively for safety messages. The other channels are called service channels (SCH) and are reserved for commercial applications. Safety messages are either event-driven or periodic. Each vehicle sends periodic messages in a single hop regularly in order to inform other vehicles inside its given neighborhood of important information such as location, speed, and acceleration while it sends event-driven messages to warn other vehicles of a collision. A portion of CCH is usually left unused for event-driven messages which wastes bandwidth. In this paper, we focus on exploiting periodic messages intelligently for collision avoidance.

In order to ameliorate drivers' safety using the personalized vehicular communications, first we need to know the delay requirements of the safety applications. In general, the difference between the communication delay and the sum of perception reaction times of drivers in a chain plays the

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[1]Hereafter, the term collision shall refer to vehicle collisions unless explicitly stated to denote packet collisions.

main role in reducing the average collision probability of the vehicles. Perception reaction time (PRT) is the time needed for a driver to perceive that something has happened and react to it.

Next, we need to know about the uncertainty of the packet delivery between two specific vehicles while other vehicles might also transmit simultaneously, thus interfering with the selected packet transmission. Deriving this probability helps us with finding the communication delay to inform each vehicle in a chain while employing vehicular communications. It is desirable to reduce this delay as much as possible by lessening the interference caused by other vehicles. Our proposed algorithm tunes this transmission probability of each vehicle based on the individual characteristics of drivers and the traffic conditions around the vehicles.

Our main contributions in this paper are as follows:

1) We propose a customized MAC layer design in order to reduce the number of collisions on highways.
2) We find the expression of packet success probability for two specific scenarios regarding a chain of vehicles on a highway.
3) We find the approximated optimal channel access probabilities equations.
4) We illustrate the collision probability reduction for the specified models using Monte Carlo simulations.

The remainder of this paper is organized as follows. Section II summarizes the related work that has been done in the field of vehicular communications regarding improving drivers' safety and driver behaviour detection. We propose our novel MAC level design with respect to personalized vehicular communications to avoid vehicle collisions in section III. Section IV presents the algorithm for customizing channel access probabilities in VANET. In section V, the simulation and numerical results are demonstrated to verify the effectiveness of the proposed scheme. Conclusion is given in section VI.

II. Background and Literature Review

We aim at customizing VANET by changing the communications parameters in a smart way. None of the related works actually have proposed a MAC level design to tune the VANETs to drivers and traffic conditions for safety applications. We proposed a regression method to estimate drivers' PRTs distributions using VANET [4]. Also, Al-Sultan *et al.* [5] utilized Bayesian graphical models to detect drivers' behaviors and categorize them. However, these two papers only focused on estimating the PRT of drivers and deriving an index for a driver, respectively. They didn't propose any algorithm to use the driver's index or any other factor in individualizing vehicular communications. Also, we showed in [6] how using an individual driver's PRT distribution in order to individualize warnings results in an impactful reduction in the probability of the driver not being able to brake in time. However, that paper only took the PRTs distribution of drivers into account to customize warnings to the drivers. In other words, it employed the estimated perception reaction time after the vehicle receives the safety messages. It doesn't analyze how channel access probabilities of vehicles and vehicular communications can be adapted to drivers' characteristics and traffic conditions. Besides, the vehicles' collision probability were assumed to vary in a specific range in [6] because only the trade-off between vehicles' collision probabilities and the false alarm rates were discussed for two types of collision warning systems. Wan *et al.* [7] presented context-aware vehicular clouds. However, they didn't propose any solid framework to improve safety. Moreover, they discussed their proposed methods only in theory. In contrast, our context-aware approach takes advantage of tuning MAC-level communication parameters to lower the vehicles' collision probability. Haas and Hu [8] simulated two vehicular safety applications and determined the effect of various communication parameters on vehicle crash avoidance through simulations. However, they didn't develop any mathematical framework for safety requirements of VANET. Also, they neglected the fact that different drivers face different needs. Therefore, their simulation-based study both couldn't achieve the potential decrease in the number of collisions and waste the communication resources. Qian *et al.* [9] proposed a MAC protocol for vehicular communications with different message priorities. However, their study was only focused on security aspects of safety applications and does not attempt to reduce the number of collisions. Mughal *et al.* [10] evaluated transmission rate or power control techniques which were employed to control congestion in dense traffic. There is no mathematical framework presented in [10]. It suggested the combination of transmission rate and transmission power control methods would be more efficient as a congestion control mechanism only in theory. Chang *et al.* [11] proposed a series of repetition-based Media Access Control (MAC) protocols to deliver periodic status updates within their useful lifetime to within a specified range. For a scheme in which nodes transmit with a given probability in each slot, Chang *et al.* [11] derived the Probability of Reception Failure (PRF) at the border of the range of interest. However, they only considered the strongest interferer in their derivation and neglect fading. The authors did not mention how their design meets the specific packet reception probabilities and delay requirements that are associated with the different driver safety characteristics in general or the specific safety information demands of a given situation in which a collision may be imminent. Garcia-Costa *et al.* [12] developed a stochastic model in which they derived the average number of collisions (when the leading vehicle stops instantly) in a chain of vehicles that are equipped with a collision warning system. The operation of the communications system was abstracted by a message delay variable whose distribution was assumed given for any specific MAC scheme. Moreover, it was assumed that all vehicles in the chain receive the warning message at the same time. Neither of these assumptions seems realistic. Carbaugh *et al.* [13] compared the safety of automated and manual highway systems with respect to rear-end collision frequency and severity. Yet, they assumed a fixed communications delay of 300, 150, and 120 milliseconds for autonomous, low-cooperative, and high-cooperative vehicles, respectively, an assumption which might not be realistic. Furthermore, Darus [26], [27] both categorized and proposed different

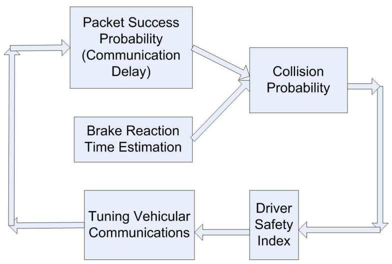

Fig. 2. Optimizing loop of VANET for Collision Warning Systems.

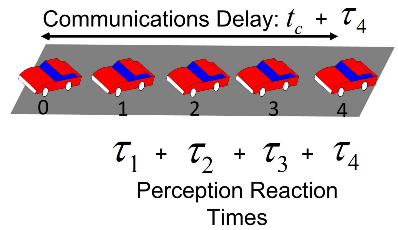

Fig. 3. Communications delay versus sum of PRTs. This figure illustrates the time before a driver in a chain applies the brake.

congestion control algorithms. Most of these algorithms were efficient based on message priorities. They nevertheless ignored the drivers' characteristics completely. In addition, it is not well-specified how these priorities are defined. To wrap this section up, none of the previous studies have actually proposed a MAC level design for employing both the drivers' behaviour and traffic information in order to improve safety (also see [19]).

In the next sections, we will show that by taking the estimate of drivers' behaviours and traffic data into account, vehicular communications can be tailored to the needs of drivers and the network (Fig. 2). Therefore, while each vehicle increases its level of safety by obtaining additional information from the network, it transmits valuable data to other vehicles especially before it causes danger to others. As a vital result, the number of fatalities on highways will be decreased drastically.

III. Driver-Based Adaptation of Vehicular Communications

Communications between vehicles can help decrease collisions in an N-lane highway. It can help drivers with making proper reactions to the deceleration events especially when a driver cannot either observe or perceive the deceleration of other vehicles due to low visibility, high unexpectedness of the incident, defected brake lights, and many distractions that nowadays exist on the roads. In a network of vehicles, each vehicle transmits with a specific probability in the transmission medium. Large channel access probabilities lead the system to excessive interferences and consequently low probability of packets being successfully received (success probability) while very small values reduces the success probabilities since the probability of the favorite transmission is low itself. Therefore, there is an optimal value given both the physical data (distances, velocities, and deceleration rates) obtained by vehicular networks and the communications protocol requirements, which results in lower collision probability of vehicles. Now, can we achieve even lower collision probabilities? In section V, it is shown that there could be individualized channel access probabilities for different vehicles leading to even lower collision probability. Our main idea is that unsafe vehicles need to inform other vehicles of their perilous situation more frequently than safer vehicles, i.e, with higher channel access probability. Our simulation results confirm this assumption which will be discussed in the following section.

A. Delay Requirements of the Safety Application

Consider a traffic stream where a chain of vehicles move with constant speed v and randomly chosen inter-vehicle spacing. When V_0 (the first vehicle in the chain) brakes, the driver of V_1 (the following vehicle), after her PRT, τ_1, applies the brake. Having no inter-vehicle communications employed, vehicle $V_i \quad (i > 1)$ applies the brake after $\sum_{j=1}^{i} \tau_j$, the sum of PRTs up to the driver i. With the communications, this time will change to $\tau_i + t_c$ in which t_c is the communications delay to inform vehicle V_i. Note that t_c can be a result of direct communications from V_0 to V_i or the retransmission of V_0's signal by one of the vehicles in the middle. Understandably, when $t_c < \sum_{j=1}^{i-1} \tau_j$, which is almost always the case especially in critical scenarios, V_i has more time to react and as a result the collision probability is reduced (Fig. 3).

B. Analysis and Design

May [14] states that vehicles traffic are more likely to follow Poisson distribution under low flow conditions. Under near-capacity conditions, however, the equal distance assumption between vehicles is justified. It is noted in [14] (CH2) that in a dense and non-free traffic flow regime all drivers tend to maintain a constant spacing with their leader. Therefore, our design is divided into two cases: 1. Equal distance model 2. Poisson distribution model. We believe examining these two scenarios gives us a thorough picture of how vehicular communications can affect collision probability in general. The traffic model does not ignore congestion from intersecting roadways, however, we assume as new vehicles enter a highway our model is still preserved.

Although the Media Access Control (MAC) protocol for DSRC communications is a variation of the conventional CSMA/CA scheme, because of the short length of the packet payload and the broadcast nature of communications, the 4-way handshake anticipated by the standard is not efficient for the dissemination of periodic safety messages. RTS/CTS and ACK message exchanges increase the hidden node problem thus resulting in higher probability of packet collisions [16]. Since the topology of VANETs is highly dynamic, we need protocols which do not need a detailed description of the network topology to schedule packet transmissions. Repetition-based protocols not only reveal this property, but also fight packet collisions due to the problem of hidden nodes. Hence, in this section, we make use of repetition-based protocols for the dissemination of periodic safety messages. A similar approach has been used in other papers, e.g. in [16] and [17].

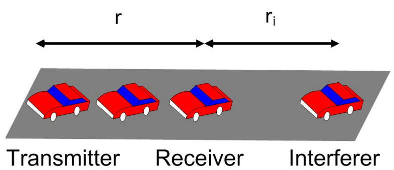

Fig. 4. A chain of vehicles. Distance between the transmitter and the desired receiver $= r$. Distance between interferer i and desired receiver $= r_i$.

1) Equal distance:
The MAC scheme that we consider is SSP (Slotted Synchronous P-persistent) where at each slot a node (vehicle) transmits with probability p and receives with probability $1-p$ independent of others. The important assumption is that the slots are synchronized because of the on-board GPS devices. Moreover, since the vehicles are not faced with power constraints, the nodes can increase the transmission power to overcome the interference (noise can be ignored). In this paper, we consider path loss and Rayleigh fading for formalizing the signal propagation characteristics. If we assume that the nodes transmit with unit power, the received power at distance r is $hr^{-\alpha}$, where $\alpha(>1)$ is the path loss exponent and h is the fading coefficient. Assuming that a node transmits a packet, the probability that a receiver at distance r receives the packet successfully is ($E(h) = E(h_i) = 1$):

$$\begin{aligned} P_s &= P\left(\frac{S}{I} > \beta\right) \\ &= P\left(\frac{hr^{-\alpha}}{\sum_{i=-\infty}^{\infty} b_i h_i r_i^{-\alpha}} > \beta\right) \\ &= \frac{1+\beta}{1+(1-p_{tr})\beta} \cdot \\ &\quad \prod_{i=-\infty-\{0\}}^{+\infty} \frac{1+(1-p_i)\beta\left(\frac{m}{i}\right)^{\alpha}}{\left(1+\beta\left(\frac{m}{i}\right)^{\alpha}\right)} \end{aligned} \tag{1}$$

Proof: see Appendix A.

where β is the SIR decoding threshold, b_i is a Bernoulli random variable with parameter p_i, node i transmits with probability p_i (the specified transmitter transmits with probability p_{tr}), r_i denotes the distance from the interferer i to the receiver (Fig. 4), and i and m denote the index of interferer i and receiver, respectively. Also, S and I denote the transmitter signal and interference power at the receiver, sequentially. Our assumption is that vehicles (interferers) are located around the receiver to infinity symmetrically. In other words, we are considering the worst case scenario to deal with the highest expected collision probability for our customized approach. If the channel access probabilities (ps) are constant, the closed-form packet success probability is ($\alpha = 2$):

$$\begin{aligned} P_s = &\frac{1+\beta}{(1-p)[1+(1-p)\beta]} \\ &\cdot \frac{\left[\sinh \pi\sqrt{(1-p)\beta m}\right]^2}{\left(\sinh\sqrt{\beta}m\right)^2} \end{aligned} \tag{2}$$

Proof: see Appendix B.

If x denotes the distance between two adjacent nodes, mx represents the distance between receiver and transmitter. It is noteworthy to mention that Equations 1 and 2 do not depend on the inter-vehicle distance.

There are two approaches for an N-lane highway. The first approach is called the Single Lane Abstraction (SLA) model. In this model, all the traffic lanes are mapped into one lane with the aggregated traffic intensity. Using this model, Equations 1 and 2 can still be employed to obtain packet success probability. SLA model can be used only if $d^2 \ll mx^2$ in which d shows the distance between two adjacent lanes (see Appendix C). If this condition is not satisfied, we cannot ignore d. Therefore, packet success probability can be obtained using:

$$\begin{aligned} P_s = &\frac{1+\beta}{1+(1-p_{tr})\beta} \\ &\cdot \prod_{i\in-\infty-\{0\}}^{+\infty} \frac{1+(1-p_i)\beta\left(\frac{mx}{ix+\frac{d^2}{2ix}}\right)^2}{1+\beta\left(\frac{mx}{ix+\frac{d^2}{2ix}}\right)^2} \end{aligned} \tag{3}$$

Proof: see Appendix D.

If the time slots in which nodes transmit are not synchronized, this scheme is named *Slotted Asynchronous P-persistent* (SAP). In this case, an interferer can potentially interfere with *at most* two time slots of another transmission. Hence, the transmission probability for the interferers is:

$$p_i' = p_i + p_i - p_i \cdot p_i \simeq 2p_i \tag{4}$$

Since the probabilities are small, this approximation is tight.

2) Poisson Distribution:
Poisson point processes have been widely employed as a model for wireless networks [21]–[23]. In this case, the nodes are distributed on a highway according to a Poisson point process. The packet success probability can be obtained by considering the fact that the transmitter-receiver distance is a random variable, not a constant value.

$$\begin{aligned} P_S &= P(SIR > \beta) \qquad (5) \\ &= \int_r P\left(\frac{P_1 hr^{-\alpha}}{k+I} > \beta\right) f_R(r)dr \qquad (6) \\ &= \int_r P\left(h > \frac{\beta(k+I)r^{\alpha}}{P_1}\right) f_R(r)dr \qquad (7) \\ &= \int_r e^{\frac{-\beta k r^{\alpha}}{P_1}} \cdot E_I\left[e^{\left(\frac{\beta r^{\alpha}}{P_1}\right)}\right] f_R(r)dr \qquad (8) \\ &= \int_r e^{\frac{-\beta k r^{\alpha}}{P_1}} \cdot L_I\left(\frac{\beta r^{\alpha}}{P_1}\right) f_R(r)dr \qquad (9) \\ &\text{if} \quad (k=0, \alpha=4, P_1=1) \\ &= \int_r L_I\left(\beta r^4\right) f_R(r)dr \qquad (10) \end{aligned}$$

where P_1, h, r, α, k, and I represent the transmitter signal power, the channel fading, the distance between transmitter and receiver, the path loss exponent, the noise variance, and the interference, respectively. Assuming the transmitter and receiver are located in the same lane, the distribution of the distance between transmitter and receiver is Erlang.

$$f_R(r) = \frac{\lambda_p^n r^{n-1} e^{-\lambda_p r}}{(n-1)!} \tag{11}$$

in which λ_p represents the intensity of vehicles in a lane. Also, n denotes the number of nodes between transmitter and receiver plus one. [15] obtains closed-form expressions for the Laplace transform of the approximate aggregate interference. For this specific scenario, this Laplace transform is equal to:

$$L_I\left(\beta r^4\right) = e^{-\pi\lambda_M\left[b^2(1-e^{-\lambda_p\beta K(\frac{r}{b})^4})+(\lambda_p\beta r^4 K)^{\frac{1}{2}}\Gamma(0.5,0.5Kb^{-4})\right]} \tag{12}$$

in which

$$\Gamma(s,x) = \int_x^\infty t^{s-1}\cdot e^{-t}dt \tag{13}$$

$$K = \left(\frac{c}{4\pi f_c}\right)^2 \tag{14}$$

Also, b, c, f_c represent the radius from the receiver node in which the aggregate interference is considered, the speed of radio propagation, and the carrier frequency.

Let's assume $\Phi = \{x_i; i = 1, 2, 3, \cdots\}$ are the nodes in the network. The probability of having n points in a part of a lane (length = L) is

$$P(|\Phi| = n) = \frac{(\lambda_p L)^n e^{-\lambda_p L}}{n!}$$

Now, we employ the concept of marked point processes [20] since we want to include additional information about the points in the model. A marked point (n_i, m_i) is selected to be retained if and only if it has the lowest mark m_i in a circle of radius L centered at x_i (HCPP-II model). L denotes the minimum distance between any two simultaneously active transmitters. If we assume that the distribution of the marks in one circle is Uniform, then the probability of retaining a random point can be written as:

$$= \sum_{n=1}^{\infty}\frac{1}{n+1}P(\text{having n points in the lane}) \tag{15}$$

$$= \sum_{n=1}^{\infty}\frac{1}{n+1}\frac{(\lambda_p L)^n e^{-\lambda_p L}}{n!} \tag{16}$$

$$= \frac{1-e^{-\lambda_p L}}{\lambda_p L} \tag{17}$$

λ_M denotes the intensity of the simultaneously active nodes from the parent Poisson point process which is equal to:

$$\lambda_M = \frac{1-e^{-\lambda_p L}}{L} \tag{18}$$

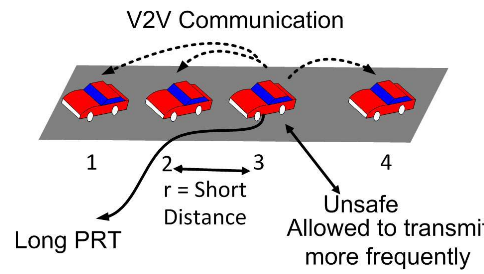

Fig. 5. Vehicle 3 needs to transmit more frequently than other vehicles because it has higher collision probability.

It is often useful to include additional information about the points in the model. Thus, in marked point processes each point x_i is assigned a random variable, the mark m_i. It is necessary to choose m_i in a smart way in order to model the spatial distribution of the active set of interferers. We define m_i as the safety index of vehicle i which means the lowest mark represents the most unsafe vehicle.

C. Indexing

A number of general indices of driver safety have been suggested or developed with the advent of relatively inexpensive in-vehicle sensors that can record, among other things velocity, acceleration, and lane position. We need to make it clear at the outset that we do not expect to have access to information on the age of a driver or any other demographic information. We assume that the less safe a driver is, the more frequently the driver needs to transmit information to the network. Moreover, the driver safety index could be changed in real time. As an example, if a driver's brake reaction time is relatively long, the driver's safety index will be relatively low, so more data will be put on the air from the corresponding vehicle. In this paper, vehicles are simply divided into two categories: 1. unsafe vehicles, 2. safe vehicles. Unsafe vehicles are the ones in which their drivers have long PRT and low distance to the vehicle in front (Fig. 5). To put it differently, unsafe vehicles have higher collision probability.

Providing the unsafe drivers with more access to the channel actually make other vehicles safer. In other words, the unsafe vehicles should transmit more frequently to other vehicles. Since these messages helps other vehicles avoid collisions, this design awards every vehicle with additional crash avoidance probabilities.

Despite neither disclosing any private information to other vehicles nor imposing a burdensome overhead, sharing safety indices with other vehicles will be of vital importance in improving the safety of the network.

IV. Customizing Channel Access Probabilities

This section proposes a new algorithm to individualize vehicular communications. Algorithm 1 (see appendix E) is a recursive algorithm which adapts channel access probabilities of all vehicles to the safety needs of drivers in the network. From a safety point of view, three factors are of vital

TABLE I
FOUR CLASSES OF VEHICULAR COMMUNICATIONS

Transmitter	Receiver	Percentage of the class
Safe	Safe	$\alpha_1 = \frac{S(S-1)}{N(N-1)}$
Safe	Unsafe	$\alpha_2 = \frac{2S\times(N-S)}{N(N-1)}$
Unsafe	Safe	$\alpha_3 = \frac{2S\times(N-S)}{N(N-1)}$
Unsafe	Unsafe	$\alpha_4 = \frac{(N-S)(N-S-1)}{N(N-1)}$

TABLE II
COLLISION SCENARIOS BETWEEN V_0 AND V_1

Collision 1	Collision 2
Before V_0 stops Before V_1 Reacts	After V_0 stops Before V_1 Reacts
Collision 3	Collision 4
Before V_0 stops After V_1 Reacts	After V_0 stops After V_1 Reacts

importance for a vehicle: 1. the PRT of the driver, 2. traffic conditions, and 3. communications delay. In one iteration of Algorithm 1, these factors play roles in assigning channel access probabilities to vehicles while the probabilities are being used in the next iteration to compute the new delay of reception at vehicle V_i. Algorithm 1 is robust and of polynomial time. The most time-consuming part of the algorithm is the response time estimation line. As we proposed in [4], the whole estimation computation has complexity $O(n_d^3)$ in which n_d is the number of observations for driver d. Moreover, as the simulation results have shown us, computing the optimal channel access probabilities (p_1 and p_2) does not need to be real time since those values are not of high sensitivity to (only) distances. Therefore, we can use the approximated p_1 and p_2 for a sufficient number of iterations in the algorithm. When new vehicles arrive in the transmission range, those are labeled as safe until the algorithm verifies whether they are causing any peril to other vehicles. Let's assume there are N vehicles on a highway and S vehicles among them are recognized as safe vehicles. A vehicle identifies itself as safe with the probability $\frac{S}{N}$. Clearly, this ratio can vary from time to time. Furthermore, after a while, any vehicle can move from one category to the other one.

The design goal is to choose p_is such that P_s is guaranteed for the vehicles and as a result the expected collision probability is minimized. Four classes of communications can be established between any two vehicles (Table I). Thus, packet success probability for the network is stated in equation:

$$P_s = \alpha_1 P_1 + \alpha_2 P_2 + \alpha_3 P_3 + \alpha_4 P_4 \tag{19}$$

P_is denote the packet success probability for the class i and are obtained by substituting $p_i = p_1$ (channel access probability), when the vehicle is safe, and otherwise $p_i = p_2$, into a packet success probability equation (e.g. Equation 1). Now, we try to find the optimal value for p_1 and p_2. Using the first-order Taylor approximation, taking derivative leads us to two quadratic equations. The intersection point of the two ellipsis, described by the following equations, in range [0,1] shows the optimal values in the network.

$$\begin{aligned}
&p_1^2\,(-\alpha_1 C_{N-S} B_{S-2} D_{S-2})\\
&\quad + p_1\,(\alpha_1 C_{N-S} B_{S-2}(D_{S-2}-2) + \alpha_2 C_{N-S-1} B_{S-1} D_{S-1})\\
&\quad + p_2^2\,(-\alpha_4 C_{N-S-2} B_S D_S)\\
&\quad + p_2\,(\alpha_2 C_{N-S-1} B_{S-1}(D_{S-1}-2) + \alpha_4 C_{N-S-2} B_S D_S)\\
&\quad + p_1 p_2\,(-2\alpha_2 C_{N-S-1} B_{S-1} D_{S-1})\\
&\quad + (\alpha_1 C_{N-S} B_{S-2} + \alpha_2 C_{N-S-1} B_{S-1}) = 0
\end{aligned} \tag{20}$$

$$\begin{aligned}
&p_1^2\,(-\alpha_1 B_{S-2} C_{N-S} D_{N-S})\\
&\quad + p_1(\alpha_1 B_{S-2} C_{N-S} D_{N-S}\\
&\quad + \alpha_2 B_{S-1} C_{N-S-1}(D_{N-S-1}-2))\\
&\quad + p_2^2(-\alpha_4 B_S C_{N-S-2} D_{N-S-2})\\
&\quad + p_2(\alpha_2 B_{S-1} C_{N-S-1} D_{N-S-1}\\
&\quad + \alpha_4 B_S C_{N-S-2}(D_{N-S-2}-2))\\
&\quad + p_1 p_2(-2\alpha_2 B_{S-1} C_{N-S-1} D_{N-S-1})\\
&\quad + (\alpha_2 B_{S-1} C_{N-S-1} + \alpha_4 B_S C_{N-S-2}) = 0
\end{aligned} \tag{21}$$

in which

$$B_S^{(j)} = \prod_{i=1}^{S}\left[\frac{p_1^{(j-1)}}{1+\beta r^{\alpha} r_i^{-\alpha}} + (1-p_1^{(j-1)})\right] \tag{22}$$

$$C_{N-S}^{(j)} = \prod_{i=1}^{N-S}\left[\frac{p_2^{(j-1)}}{1+\beta r^{\alpha} r_i^{-\alpha}} + (1-p_2^{(j-1)})\right] \tag{23}$$

$$D_S = \sum_{i=1}^{S}\left[\frac{1}{1+\beta r^{\alpha} r_i^{-\alpha}}\right] - S \tag{24}$$

The coefficients have to be computed very carefully since it is important to know which vehicles are included in the multiplications. After B_S, C_{N-S}, and D_S are obtained in iteration j, p_1 and p_2 will be computed in the next one. Our Monte Carlo simulation results show that, on average, the optimal probabilities found by brute-force search algorithm results in the expected collision probability, only $<1\%$ less than that obtained from using our optimal values. This means the approximated values are sufficiently tight.

V. NUMERICAL AND SIMULATION EVALUATION OF DESIGN

When vehicular communications are employed, communications delay is a main factor that influences the vehicle collision probability on highways. Also, we know that some of the vehicles are too far from the vehicle V_0 (the leading vehicle) to be able to receive the messages directly from it. Thus, when one of the vehicles in the middle gets informed and reacts to the event, the message will be forwarded to the vehicles at a greater distance from the leading vehicle. In other words, after a vehicle in the middle starts decelerating, the new status will be included in the new messages from this vehicle to further upstream vehicles. Therefore, we need to compute the time it takes for a message to be received by vehicle i. It is sufficient that the message be received successfully only one time. As a result the successful reception at vehicle V_i

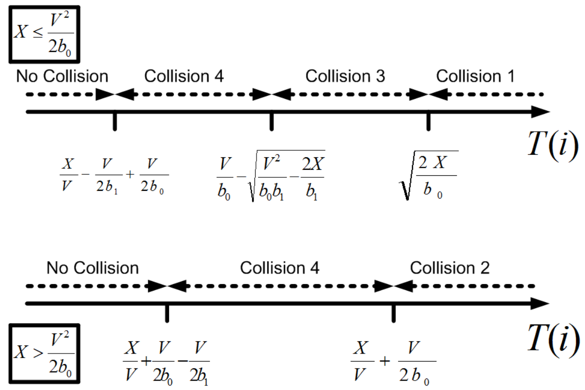

Fig. 6. An example of collision scenarios (Table II) between vehicles V_0 and V_1 in a very dense traffic. X, V and b represent inter-vehicle spacing, velocity, and deceleration rate respectively.

has a geometric distribution with parameter: (if it remains the same for each iteration and packet deliveries are independent)

$$P_s(i) \cdot p_{tr} \cdot (1 - p_i) \tag{25}$$

where $P_s(i)$ is given in Equations 1,2,3 and 10. Also, p_{tr} and p_i represent the channel access probability for the transmitter and the desired receiver (i^{th} vehicle) respectively. This parameter demonstrates the probability that the transmitter is sending packets, the desired receiver is obtaining them, and the safety packets are successfully delivered, all simultaneously. This gives us the number of required slots on average for vehicle V_i to receive vehicle V_0's messages:

$$s(i) = \frac{1}{P_s(i) \cdot p_{tr} \cdot (1 - p_i)} \tag{26}$$

If SAP scheme is employed, we need to alter the equation:

$$s(i) = \frac{1}{P'_s(i) \cdot p_{tr} \cdot (1 - p'_i)} \tag{27}$$

in which p'_i represents the channel access probability when the time slots are not synchronized and $P'_s(i)$ denotes Equation 1 using the new channel access probabilities.

The allowable number of transmission opportunities within the tolerable delay period is:

$$D = \lfloor \frac{T(i)R}{L} \rfloor \tag{28}$$

R represents the data rate which is chosen from Table III while L denotes the packet length. $T(i)$ denotes the maximum tolerable delay to inform vehicle V_i which can be obtained from Fig. 6 and Table III. Let P_s^D denote the success probability at V_j after D transmission opportunities:

$$P_s^D = 1 - (1 - s(j)^{-1})^D$$

This equation demonstrates the dependence of packet success probability on channel access probabilities and inter-vehicle distances (Fig. 7). Clearly, it takes longer time for the vehicles far away from V_0 to receive the packets due to delay. However, those far vehicles (for example V_j) receive the messages about the deceleration of V_0 from the vehicles $V_1 \cdots V_{j-2}$ as well. V_{j-1} is not included since V_j can see the brake lights of V_{j-1}

TABLE III
IEEE 802.11P DATA RATES AND CORRESPONDING SIR DECODING THRESHOLDS

R (Mbps)	3	4.5	6	9	12	18	24
β (db)	5	6	8	11	15	20	25

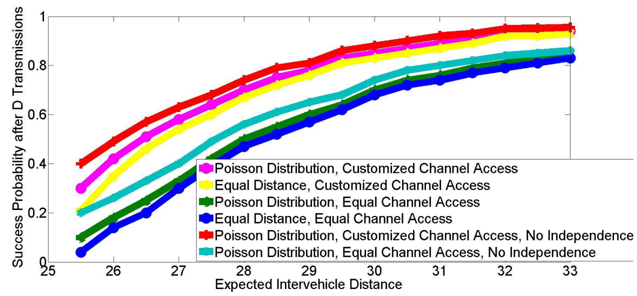

Fig. 7. Packet success probability after D transmissions at vehicle V_2 for different traffic models and different expected inter-vehicle distance.

with no need of vehicle-to-vehicle communications. Taking all of the above into account, the average delay of reception at vehicle V_i is:

$$D(i) = min(min_{(j\in 1,\cdots,i-2)} \frac{L}{R}s(j) + \tau_j + \frac{L}{R}s(i-j), \frac{L}{R}s(i), \frac{L}{R}s(i-1) + \tau_{i-1}), \quad i > 2 \tag{29}$$

where $s(1) = D(1) = 0$ since there is no need for communications between two adjacent vehicles. If the distance between a vehicle and the one ahead of it is short, and also the PRT of the following vehicle is long enough, we consider the vehicle unsafe. Otherwise, the vehicle is a safe one. In other words, if the collision probability calculated based on only physical/traffic parameters (without considering the vehicle-to-vehicle communication) is higher than a threshold, the vehicle is unsafe. We can run an algorithm recursively such that the channel access probability at a specific time depends on the collision probability at the previous time.

We run Monte Carlo simulations to study vehicle collisions within a chain. We consider the perception reaction time of vehicles being independently drawn from a log-normal distribution with parameters 1.31 and 0.61 [25]. Moreover, we assume that each vehicle decelerates with a rate chosen uniformly at random from the interval $[-4-8]\frac{m}{s^2}$. Next, different types of vehicle collision are defined based on Table II. Also, the distance between vehicles is a random variable for the second model (Poisson point process). Therefore, we can obtain the collision probability of vehicles in the chain by employing the motion equations of vehicles. In our model, the drivers can only avoid accidents by applying the brake. Furthermore, most of the drivers tend to keep a minimum distance with the lead vehicle which is ignored in our model because that is not always the case. In other words, we actually calculate an upper bound for the collision probability which shows us to a great extent how this probability really varies for the scenarios which lead to collisions.

TABLE IV
SIMULATION PARAMETERS. DATA RATE AND SIR DECODING THRESHOLD ARE CHOSEN BASED ON [17]

Distribution	Poisson Equal distance
Velocity	$20\frac{m}{s}$
Deceleration rate	$[-8, -4]\frac{m}{s^2}$
Average distance between adjacent vehicles	$25m$
Total number of vehicles	32
Total number of unsafe vehicles	4
SIR decoding threshold	11 dB
Data rate	9 Mbps
Packet length	250 Bytes

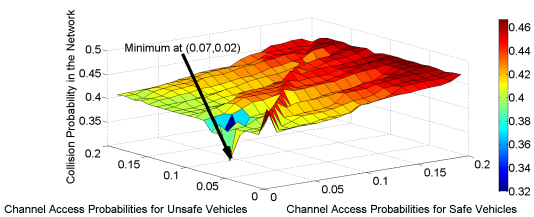

Fig. 8. The X, Y, and Z axes represent channel access probability for safe vehicles, channel access probability for unsafe vehicles, and collision probability average over all vehicles respectively. Vehicles' locations are randomly drawn from the Poisson distribution. The minimum collision probability in this case is 25% less than the scenario in which equal channel access probabilities are assigned to all vehicles. Therefore, we conclude that tailoring the channel access probabilities to unsafe and safe vehicles leads the network to reduction of collision probability.

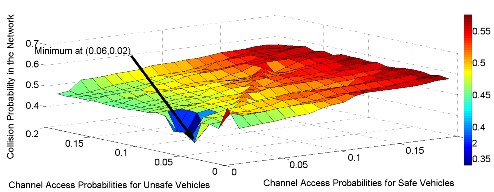

Fig. 9. Vehicle collision probability versus channel access probability for safe and unsafe vehicles. The inter-vehicle distance is assumed to be equal for all vehicles.

Using simulation parameters in Table IV, Fig. 8 illustrates the collision probabilities when different channel access probabilities are assigned to unsafe and safe vehicles. Obtained collision probability values are greater than what we usually expect based on our life experience since these probabilities are computed conditional on the scenarios in which a high number of collisions is expected.

In Fig. 8, X axis represents the channel access probabilities for safe vehicles, Y axis shows the channel access probabilities for unsafe vehicles, and Z axis denotes the collision probabilities. Assuming equal transmission probabilities (Fig. 10), the minimum number of collisions happens at around $p_0 \approx 0.05$. However, 25% reduction in collision probability can be achieved when unsafe and safe vehicles transmit with specific probabilities more and less than p_0 respectively. In other words, the minimum collision probability in Fig. 8 is located in a value greater than p_0 on Y axis and less than p_0 on X axis. Here, we are actually comparing these customized communications (Fig. 8) to the communications with equal channel access probabilities in its optimal range (Fig. 10). With this simulation, it becomes clear that using the driver-based adaptation of communications in warning systems has a noticeable advantage over these systems employing the same optimal channel access probabilities for all vehicles and therefore has a huge advantage over the currently used warning systems.

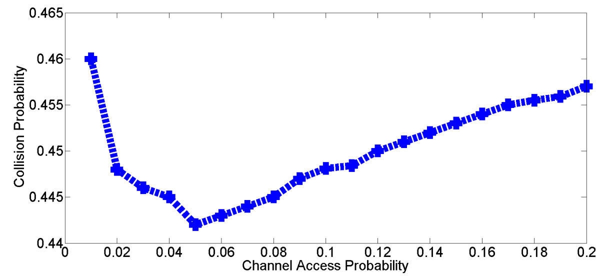

Fig. 10. Collision probability versus channel access probability. Channel access probability is assumed to be equal for all vehicles. Vehicles' locations values are generated from Poisson distribution.

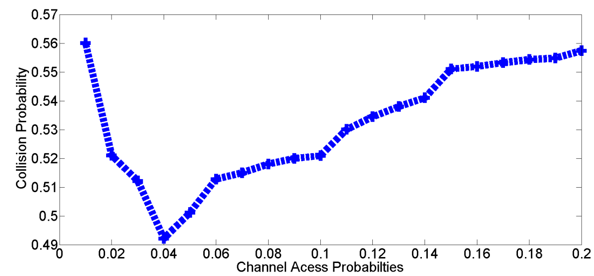

Fig. 11. Collision probability versus channel access probability. Channel access probability is assumed to be equal for all vehicles. Also, the vehicles' distances are assumed to be equal.

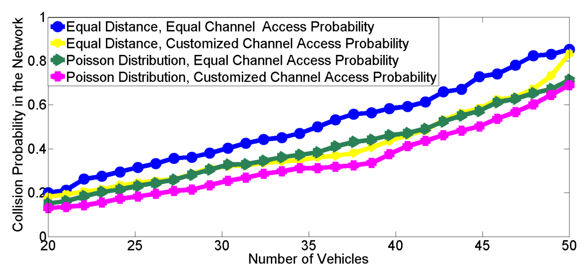

Fig. 12. Collision probability in the chain versus the number of vehicles. The length of the specified part of highway is assumed to be 500 meters. The other parameters are chosen from Table IV. The comparison is between four cases (Vehicle locations, Communications): 1. Equal distance, equal channel access probability. 2. Equal distance, customized channel access probability. 3. Poisson distribution, equal channel access probability. 4. Poisson distribution, customized channel access probability.

Fig. 12 illustrates the advantage of employing the customized communications in a 500-meter part of a highway, assuming a different number of vehicles are placed on that part. If we use the same simulation parameters for the equal-distance scenario, even greater reduction in collision probabilities are achieved. This seems to be justifiable because the equal-distance model represents the dense traffic, thus more collisions happen. The two extreme scenarios which we considered in this paper lead us to the conclusion that we may achieve a model-free approach to improve the performance of VANETs since both of the examined scenarios benefit the drivers following a similar pattern.

VI. Conclusion

There are methods available to estimate individual drivers' characteristics from vehicular ad hoc networks. In addition, we can obtain traffic information (such as distance between vehicles) using vehicular communications. In order to compute the collision probability, we derived the equations of packet success probability for two extreme cases. Furthermore, we derived optimal channel access probabilities for each category of vehicles which are tight approximations of the actual values. If a vehicle has high probability of collision, it needs to transmit more frequently in order to make other vehicles aware of its perilous situation. Next, we proposed an efficient algorithm to adjust transmission rates of vehicles to safety needs of drivers using the aforementioned data. By employing this algorithm in a network of vehicles, fatalities on highways will be drastically reduced.

Appendix A
Proof of Equation 1

If there is distance r between a transmitter and the desired receiver, the success probability is

$$\begin{aligned} P_s &= P(SIR > \beta) && (30)\\ &= P\left(\frac{hr^{-\alpha}}{I} > \beta\right) && (31)\\ &= \int P(h > \beta r^{\alpha} I | I = i) f_I(i) di && (32)\\ &= \int e^{-\beta r^{\alpha} i} f_I(i) di && (33)\\ &= E\left[e^{-\beta r^{\alpha} I}\right] && (34)\\ &= E\left[e^{-\beta r^{\alpha} \sum_{i\in\Phi} b_i h_i r_i^{-\alpha}}\right] && (35)\\ &= \prod_{i\in\Phi} [E\left[e^{-\beta r^{\alpha} h_i r_i^{-\alpha}}\right] p_i + 1 - p_i] && (36)\\ &= \prod_{i\in\Phi} \left[\frac{p_i}{1+\beta r^{\alpha} r_i^{-\alpha}} + (1-p_i)\right] && (37) \end{aligned}$$

Assuming,

$$r = mx \quad \text{and} \quad r_i = ix \tag{38}$$

Equation 1 is obtained.

Appendix B
Proof of Equation 2

Using Euler's product formula, we obtain the second equation.

$$sin(\pi z) \equiv \pi z \prod_{i=1}^{\infty} \left(1 - \frac{z^2}{i^2}\right) \tag{39}$$

Algorithm 1 Algorithm for Customizing Channel Access Probabilities in VANET

Input: Vehicles: $V_1, V_2, \cdots, V_N$, VANET data
Output: Customized channel access probabilities for all vehicles

Derive all physical parameters from VANET
(Distances between vehicles, deceleration rates, and velocities)
Divide vehicles into safe and unsafe categories (compute collision probabilities).
Compute the optimal channel access probabilities.
for $i = 1$ to N **do**
  Estimate the response time distribution (τ_i).
end for
for $i = 1$ to N **do**
  Determine if any type of collision can happen to vehicle i based on both equations of motion and the delay of receiving packets from other vehicles.
  if yes **then**
    $p_i = p_2$(channel access probability for unsafe vehicles)
  else
    $p_i = p_1$(channel access probability for safe vehicles)
  end if
end for
return $p_{1:N}$

Appendix C
Difference Between SLA Model and Multi-Lane Model

Assume d is the distance between two specific lanes, x denotes the distance between two adjacent vehicles, and the transmitter is located in the middle lane. Let's assume r specifies the distance between transmitter and receiver (which is in a lane with distance d from the middle lane).

$$\begin{aligned} r &= mx\sqrt{1+\left(\frac{d}{mx}\right)^2} && (40)\\ &\approx mx\left(1+\frac{\left(\frac{d}{mx}\right)^2}{2}\right) && (41)\\ &= mx + \frac{d^2}{2mx} && (42) \end{aligned}$$

Therefore, if $d^2 \ll mx^2$, $r \approx mx$. Hence, we can assume the receiver is in the middle lane too. If the inequality does not hold, that approximation cannot characterize the performance of vehicular networks on N-lane highways.

Appendix D
Proof of Equation 3

In appendix A, the last equation is modified with respect to the new assumption that the inter-lane distance cannot be overlooked:

$$P_s = \prod_{i\in\Phi} \left[\frac{p_i}{1+\beta\left(\frac{mx}{ix+\frac{d^2}{2ix}}\right)^{\alpha}} + (1-p_i)\right]$$

Then, Equation 3 will be obtained.

APPENDIX E
ALGORITHM FOR CUSTOMIZING CHANNEL ACCESS PROBABILITIES IN VANET

See Algorithm 1.

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