Artificial Intersymbol Interference (ISI) to Exploit Receiver Imperfections for Secrecy

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Abstract—Secure communication over a wireless channel in the presence of a passive eavesdropper is considered. We present a method to exploit the eavesdropper's inherent receiver vulnerabilities to obtain everlasting secrecy. An ephemeral cryptographic key is pre-shared between the transmitter and the legitimate receiver and is utilized to induce intentional intersymbol interference (ISI). The legitimate receiver uses the key to cancel the ISI while the eavesdropper, since it does not have the key, cannot do such. It is shown that although ISI reduces the capacity of the main channel, it can lead to a net gain in secrecy rate. The achievable secrecy rates for different ISI filter settings are evaluated and the proposed method is compared with other information-theoretic security schemes.

I. INTRODUCTION

The messages sent over a wireless network are vulnerable to being overheard by any malicious party in the coverage range of the transmitter. The traditional way to prevent an eavesdropper from obtaining a secret message is to encrypt the message such that decoding the cypher without having the key is beyond the eavesdropper's computational capability [1]. However, in many applications, everlasting security is desired, and hence one must be concerned not only with the current capabilities of the eavesdropper but also with the eavesdropper's future capabilities both in computation and in ability to exploit vulnerabilities of the implementation. In particular, when a cryptographic scheme is employed, the adversary can record the clean cypher and recover it later when the cryptographic algorithm is broken [2] or when the eavesdropper obtains the key.

The desire for such everlasting security motivates considering information-theoretic approaches, where the eavesdropper is unable to extract any information about the secret message from the received signal. The feasibility of informationtheoretic security was demonstrated by the seminal work of Wyner [3], where he showed for discrete memoryless wiretap channel, that, if the eavesdropper's channel is degraded with respect to the main channel, adding randomness to the codebook allows achievement of a positive secrecy rate. Later, the idea was extended to the more general case where the eavesdropper's channel is not necessarily degraded, but "more noisy" or "less capable" with respect to the main channel [4]. However, in wireless systems it can be difficult to guarantee that the eavesdropper is at a disadvantage relative to the

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intended receiver, as the eavesdropper can be very close to the transmitter or can use a directional antenna to improve its received signal, while the eavesdropper's location and its channel state information is not known to the legitimate nodes. When such an advantage does not exist, approaches based on "public discussion" [5], [6] can be employed. However, these approaches, while they could be used to generate an information-theoretically secure one-time pad, are basically designed for secret key agreement by performing multiple twoway transmissions and utilizing a public authenticated channel [7, Chapter 7.4] rather than one-way secret communication. Recently, approaches based on the cooperative jamming approach of [8] and [9] have been considered. However, all of these approaches require either multiple antennas, helper nodes, and/or fading and thus are not robust across all operating environments envisioned for wireless networks.

We will consider exploiting current hardware limitations of the eavesdropper to achieve everlasting security. Prior work in this area includes that of Cachin and Maurer [10], who introduced the "bounded memory model" in such a way that the eavesdropper is not able to store the information it would need to eventually break the cypher. However, it is difficult to plan on memory size limitations at the eavesdropper, since not only do memories improve rapidly as described by the wellknown Moore's Law [11], but, more importantly, memories can be stacked arbitrarily subject only to (very) large space limitations. Our approach to provide everlasting security is that, instead of attacking the memory in the receiver back-end, we attack the receiver front-end. In particular, the technology of analog-to-digital (A/D) converters progresses slowly and unlike memory, they cannot be stacked arbitrarily due to jitter considerations. Also, importantly from a long-term perspective, there is a fundamental bound on the ability to perform A/D conversion [12], [13]. Hence, our goal is to exploit the receiver analog-to-digital conversion processing effect for security. The transmitter (Alice) and the intended receiver (Bob) pre-share a cryptographic key that only needs to be kept secret for the duration of the transmission (i.e. it can be given to the eavesdropper immediately afterward). Using this key, we insert intentional distortion on the transmitted signal. Since Bob knows the distortion, he can undo its effect before his A/D, whereas Eve must store the signal and try to compensate for the distortion after her A/D; however, she already lost the information she need to recover the message. We considered a



Fig. 1. The transmitter perform fast power modulation to obtain secrecy by utilizing a cryptographic key pre-shared between Alice and Bob.

rapid power modulation instance of this approach in [14] and [15], where the transmitted signal is modulated by two vastly different power levels at the transmitter (Figure 1). Since Bob knows the key, he can cancel the effect of power modulator before his A/D, putting his signal in the appropriate range for A/D conversion. On the other hand, Eve must compromise between larger quantization noise and more A/D overflows. Consequently, she will lose information she needs to recover the message and information-theoretic security is obtained. However, a clear risk of the approach of [14], [15] is an eavesdropper with multiple A/Ds.

Motivated by the fact that an additive white Gaussian noise (AWGN) channel will have a higher capacity than an intersymbol interference channel under the same output power constraint, we seek to induce an ISI channel for Eve while preserving an AWGN channel at Bob. The transmitter is equipped with a linear filter with random coefficients that are taken based on a pre-shared key between Alice and Bob. The secret message is broken into chunks of data with a guard interval between every two chunks and is transmitted over the channel after going through the ISI filter. Bob, since he knows the key, places a filter before his A/D in concert with the ISI filter to cancel the ISI. In order to prevent Eve from performing any kind of adaptive equalization to cancel the ISI, the coefficients of the ISI filter are changed based on the key during each chunk. By exploiting the resulting distortion, information-theoretic secrecy can be obtained, even if the key is given to Eve immediately after message transmission.

The rest of paper is as follows. Section II describes the system model, metrics, and the proposed idea in detail. In Section III, the achievable secrecy rates for the proposed method are characterized. In Section IV, the results of numerical examples for various realizations of the system and comparison of the proposed method to the conventional Gaussian wiretap channel [16] are presented. Conclusions and ideas for future work are discussed in Section V.

II. SYSTEM MODEL AND APPROACH

A. System Model

A simple wiretap channel is considered, which consists of a transmitter, Alice, an intended receiver, Bob, and an eavesdropper, Eve. The eavesdropper is assumed to be passive, i.e. it does not attempt to actively thwart (i.e. via jamming, signal insertion) the legitimate nodes. Thus, the location and channel state information of the eavesdropper is assumed to be unknown to the legitimate nodes.

Alice and Bob either pre-share a cryptographic key or use a standard key agreement scheme (e.g. Diffie-Hellman [17]) to generate a shared key. This initial key will be used to generate a very long key-sequence by using a standard cryptographic method such as AES in counter mode (CTR), or by using standard methods that are specifically designed for generating stream-ciphers, such as Trivium, and the rate overhead that these algorithms place on our scheme is negligible [15]. We assume that Eve cannot recover the initial key before the key renewal and during the transmission period, i.e. we assume that the computational power of Eve during the time of transmission is not unlimited. However, we assume (pessimistically) that Eve is handed the full key (and not just the initial key) as soon as transmission is complete. Hence, unlike cryptography, even if the encryption system is broken later, the eavesdropper obtains access to an unlimited computational power, or other forms of computation such as quantum computers are implemented, Eve will not have enough information to recover the secret message (using the method described in Section II.B).

We consider a one-way communication system, and assume that both Bob and Eve are at a unit distance from the transmitter by including variations of the path-loss in the noise variance; thus, the channel gain of both channels is unity. Both channels experience additive white Gaussian noise (AWGN). Let n_B and n_E denote the zero-mean noise processes at Bob's and Eve's receivers with flat power spectrum $N_B(f) = N_B/2$ and $N_E(f) = N_E/2$, respectively. Let \hat{X} denote the input of both channels, \hat{Y} denote the received signal at Bob's receiver, and \hat{Z} denote the received signal at Eve's receiver (Figure 2).

Both Bob and Eve employ high precision uniform analogto-digital converters and the effect of the A/D on the received signal (quantization noise) is modeled by an additive Gaussian noise with variance $\delta^2/12$, where δ is the length of one quantization level. The assumption that quantization noise follows a Gaussian distribution is not accurate; however, it enables us to use results of Gaussian channels to obtain some insight about the actual system. The equivalent continuoustime power spectrum of the quantization noise of Bob's A/D, n_{QB} , and quantization noise of Eve's A/D, n_{QE} , are assumed to be flat, i.e. $N_{QB}(f) = N_{QB}/2$ and $N_{QE}(f) = N_{QE}/2$, respectively. Let X denote the current code symbol; since all noises are assumed to be Gaussian processes, we assume that X is taken from a standard Gaussian codebook where each entry has variance P, i.e. $X \sim \mathcal{N}(0, P)$.

B. Approach

Our goal is to study how Alice and Bob can employ bits of the shared cryptographic key to modify their radios to gain an information theoretic advantage. Assume that Alice applies a linear filter after her D/A with spectral density $G_{\underline{k}}(f)$ (as shown in Figure 2). The spectral density of this filter is chosen



Fig. 2. Alice sends the message through an ISI filter that is determined by the key sequence, pre-shared between Alice and Bob. Bob uses the key sequence to cancel the effect of ISI on his signal before the analog-to-digital conversion process.

based on the pre-shared key between Alice and Bob. Since Bob shares the (long) key with Alice, he easily cancels the effect of this filter before his A/D properly, whereas Eve will struggle with such. In essence, we are inducing an ISI channel that Bob is able to equalize before his A/D, while Eve cannot. Thus, Eve will suffer from the channel degradation due to ISI and information-theoretic security is obtained. Further, Alice changes the weights of ISI filter taps frequently (based on the shared key) to ensure that Eve is not able to perform any kind of adaptive ISI cancellation.

Here we use a filter similar to a *n*-tap ISI channel,

$$\hat{x}(j) = \frac{1}{\sqrt{n}} \sum_{j=0}^{n-1} g_{\underline{k}}(i) x(j-i)$$

where x(j) is the channel input, $\hat{x}(j)$ is the channel output, and $g_{\underline{k}}(i), i = 0, \dots, n-1$ are filter coefficients. Hence, the spectrum of the ISI filter is,

$$G_{\underline{k}}(f) = \begin{cases} \frac{1}{\sqrt{n}} \sum_{i=0}^{n-1} g_{\underline{k}}(i) e^{-ji\pi f/W}, & -W \le f \le W\\ 0, & \text{else} \end{cases}$$
(1)

In order to confuse Eve, the coefficient vector $g_{\underline{k}} = [g_{\underline{k}}(0) \ g_{\underline{k}}(1) \cdots g_{\underline{k}}(n-1)]$ is chosen randomly from an i.i.d ensemble that follows $\mathcal{N}(\mu, \sigma^2)$ according to the key sequence, \underline{k} , pre-shared between Alice and Bob. These coefficients are chosen such that the ISI filter does not change the average output transmit power, i.e. $E[g_{\underline{k}}(i)^2] = 1, \ i = 0, \cdots, n-1$. To cancel the ISI at Bob's receiver, we put a filter with power spectrum $H_{\underline{k}}(f) = 1/G_{\underline{k}}(f)$ at the input of Bob's receiver. Note that the assumption that Bob applies a $H_{\underline{k}}(f) = 1/G_{\underline{k}}(f)$ is for ease of proof of Theorem 1 and establishes an achievable secrecy rate of the proposed approach. The optimization of $H_{\underline{k}}(f)$ to maximize the provable secrecy rate is a topic of ongoing work. In the sequel, the average secrecy rates that can be obtained using the proposed method will be investigated.

III. ARTIFICIAL ISI FOR SECRECY

To the best of our knowledge the secrecy capacity of the wiretap ISI channel has not yet been established. Therefore, we present a theorem for the achievable secrecy rates of the Gaussian band-limited wiretap channel that is shown in Figure



Fig. 3. Equivalent wiretap channel.

2. Note that we do no adaptive loading on the signal stream X to match the filter $(G_k(f))$ or (unknown) channel conditions.

Theorem 1. The average secrecy rate of the wiretap channel shown in Figure 2 without CSI of both the main and eavesdropper channels for a given key sequence, \underline{k} , is:

$$R_{s} = \int_{-W}^{W} \frac{1}{2} \log \left(1 + \frac{P|G_{\underline{k}}(f)|^{2}}{W(N_{QB}|G_{\underline{k}}(f)|^{2} + N_{B})} \right) - \frac{1}{2} \log \left(1 + \frac{P|G_{\underline{k}}(f)|^{2}}{W(N_{QE} + N_{E})} \right) df.$$
(2)

Proof: The equivalent wiretap channel is shown in Figure 3. The channel noise of the main channel can be modeled by $n'_B(t)$ with spectrum $N'_B(f) = N_B(f)/|G_{\underline{k}}(f)|^2 = N_B/2|G_{\underline{k}}(f)|^2$, and the quantization noise can be modeled by $n'_{QB}(t)$ with spectrum $N_{QB}(f)H_{\underline{k}}(f)G_{\underline{k}}(f) = N_{QB}/2$. Similarly, the total noise of the eavesdropper channel can be modeled by $n'_E(t)$ with spectrum $N'_E(f) = (N_E(f) + N_{QE}(f))/|G_{\underline{k}}(f)|^2 = (N_E + N_{QE})/2|G_{\underline{k}}(f)|^2$. Since the term $1/|G_{\underline{k}}(f)|^2$ is common in both $N'_B(f)$ and $N'_E(f)$, let us define a Gaussian random process n(t) with autocorrelation function $R_n(t,s)$ associated with power spectrum $1/|G_{\underline{k}}(f)|^2$. In a finite time interval of length T, by expanding $R_n(t,s)$ using the Karhunen-Loeve expansion,

$$n(t) = \sum_{l=1}^{\infty} n_l \phi_l(t),$$

where $\{\phi_l(t)\}\$ are the orthonormal eigenfunctions generated by kernel $R_n(t,s)$ and the coefficients n_l are independent Gaussian random variables of variance λ_l . Hence, we can represent $n'_B(t)$ and $n'_E(t)$ in terms of $\phi_l(t)$ s,

$$n'_B(t) = \sum_{l=1}^{\infty} n'_{Bl} \phi_l(t),$$

and,

$$n'_E(t) = \sum_{l=1}^{\infty} n'_{El} \phi_l(t),$$

where $\{n'_{Bl}\}\$ and $\{n'_{El}\}\$ are independent Gaussian random variables with variances $\lambda_l N_B/2$ and $\lambda_l (N_E + N_{QE})/2$, respectively. Thus, our wiretap channel is decomposed into an infinite number of independent parallel wiretap sub-channels. The transmitter does not perform spectral-loading (e.g. waterfilling) at the transmitter and thus the power spectral density of

the input of the wiretap channel over all sub-channels is fixed and equal to P/2W. Also, the spectrum of the quantization noise of the main channel is uniform over all sub-channels, and its power spectral density is equal to $N_{QB}/2$. Thus,

$$R_s = \lim_{T \to \infty} \frac{1}{T} R_s(T)$$

=
$$\lim_{T \to \infty} \sum_{l=0}^{\infty} \frac{1}{2} \log \left(1 + \frac{P/2W}{(N_{QB}/2 + \lambda_l N_B/2)} \right)$$

$$- \frac{1}{2} \log \left(1 + \frac{P/2W}{\lambda_l (N_{QE} + N_E)/2} \right).$$

By applying the Toeplitz distribution theorem for continuous random variables [18] and since the spectrum of the output filter is limited to [-W, W], R_s in (2) is obtained.

The secrecy rate averaged over all key sequences is,

$$R_{s} = E_{\underline{k}} \bigg[\int_{-W}^{W} \frac{1}{2} \log \left(1 + \frac{P|G_{\underline{k}}(f)|^{2}}{W(N_{QB}|G_{\underline{k}}(f)|^{2} + N_{B})} \right) \\ - \frac{1}{2} \log \left(1 + \frac{P|G_{\underline{k}}(f)|^{2}}{W(N_{QE} + N_{E})} \right) df \bigg].$$
(3)

Equation (1) for a given $G_{\underline{k}}(f)$ is complicated, and thus it is not possible to obtain a closed form for the average achievable secrecy rates. However, in the high SNR regime, it can be shown that R_s in (3) is always greater than the secrecy capacity of the corresponding wiretap channel without applying the ISI filter, which is,

$$C_s =$$

$$W \log \left(1 + \frac{P}{W(N_B + N_{QB})}\right) - W \log \left(1 + \frac{P}{W(N_E + N_{QE})}\right)$$

Suppose that $C_s > 0$. Since we are working in the high SNR

Suppose that $C_s > 0$. Since we are working in the high SNR regime,

 C_s

$$\approx W \log \left(\frac{P}{W(N_B + N_{QB})}\right) - W \log \left(\frac{P}{W(N_E + N_{QE})}\right)$$

$$= -W \log(N_B + N_{QB}) + W \log(N_E + N_{QE})$$

$$(a) \frac{1}{2} \int_{-W}^{W} \log (N_E + N_{QE})$$

$$- \log \left(N_B E_{\underline{k}}[|G_{\underline{k}}(f)|^2] + N_{QB}\right) df$$

$$(b) \frac{1}{2} \int_{-W}^{W} \log (N_E + N_{QE})$$

$$- E_{\underline{k}} \left[\log \left(N_B |G_{\underline{k}}(f)|^2 + N_{QB}\right) \right] df$$

$$(c) E_{\underline{k}} \left[\int_{-W}^{W} \frac{1}{2} \log \left(\frac{P |G_{\underline{k}}(f)|^2}{W(N_B |G_{\underline{k}}(f)|^2 + N_{QB})} \right)$$

$$- \frac{1}{2} \log \left(\frac{P |G_{\underline{k}}(f)|^2}{W(N_E + N_{QE})} \right) df$$

$$\approx E_{\underline{k}} \left[\int_{-W}^{W} \frac{1}{2} \log \left(1 + \frac{P |G_{\underline{k}}(f)|^2}{W(N_B |G_{\underline{k}}(f)|^2 + N_{QB})} \right)$$

$$- \frac{1}{2} \log \left(1 + \frac{P |G_{\underline{k}}(f)|^2}{W(N_B |G_{\underline{k}}(f)|^2 + N_{QB})} \right)$$

 $\leq R_s$



Fig. 4. The frequency spectrum of 10-tap ISI filters for various values of the variance of the filter tap coefficients, $\sigma^2 = 0.5, 0.7$, and 0.9, and three realizations of filter coefficients in each case.

where (a) is from the fact that $E_{\underline{k}}[|G_{\underline{k}}(f)|^2] = 1$, (b) is Jensen's inequality, and in (c) we use the fact that $|G_{\underline{k}}(f)| = 0$ only on a set of measure zero. This shows that inducing the ISI, which lowers the capacity of the main channel, provides a net gain in secrecy capacity at high SNRs.

IV. NUMERICAL RESULTS AND COMPARISON TO GAUSSIAN WTC

In this section we study the achievable secrecy rates of the proposed method for various scenarios. Also, we compare the proposed method to the conventional Gaussian wiretap channel (WTC) [16].

In the proposed method, suppose that the bandwidth of the transmit filter is normalized as [-W, W] = [-1, 1] and its coefficients are taken from a normal distribution such that the ISI filter does not change the average transmit power, i.e. $g_{\underline{k}}(i) \sim \mathcal{N}(\mu, \sigma^2), \ i = 0, \cdots, n-1$ such that $E[g_{\underline{k}}(i)^2] = \mu^2 + \sigma^2 = 1$. The frequency spectrum of 10-tap ISI filter for various values of σ^2 for three realizations of the filter coefficients are shown in Figure 4. Observe that as the variance of the filter coefficients becomes smaller, the uncertainty of the shape of the frequency response of the ISI filter lessens and the eavesdropper might be able to increase the information leakage by using this information. However, by applying a random phase shift to the transmitted signal based on the key, we can prevent the eavesdropper from doing such while the achievable secrecy rates remain unchanged.

First we look at the extreme case that Eve is able to receive exactly what Alice transmits (e.g. the adversary is able to pick up the transmitter's radio and hook directly to the antenna), but the channel between Alice and Bob is noisy and hence the conventional wiretap channel has zero secrecy capacity. In



Fig. 5. Achievable secrecy rate of the proposed method and conventional wiretap channel (WTC) vs. SNR of channel between Alice and Bob while the channel between Alice and Eve is noiseless (Eve has perfect access to the transmitted signal). σ^2 is the variance of each ISI filter coefficient. Note that the secrecy rate of the wiretap channel is zero.



Fig. 6. Achievable secure rate of the proposed method and conventional wiretap channel (WTC) vs. SNR of channel between Alice and Bob when SNR of channel between Alice and Eve is 60 dB. σ^2 is the variance of each ISI filter coefficient.

other words, the channel between Alice and Bob experiences additive white Gaussian noise, while Eve's channel is noiseless $(n_E = 0)$. Figure 5 shows the achievable secrecy rate versus signal-to-noise ratio at Bob's receiver when Eve's receiver is noiseless. The average transmit power P = 1 and both Bob and Eve use 10-bit A/Ds. It can be seen that although the eavesdropper's channel is much better than the main channel, when the SNR at Bob's receiver is greater than 55 dB positive secrecy rates are obtained.

Another observation is that as the ISI channel gets further from the flat channel, higher secrecy rates are achievable due to greater variation of the channel. In the current construction of the ISI filter, this occurs when σ^2 gets smaller.

In Figure 6, the achievable secrecy rate versus SNR at Bob's receiver when SNR of channel between Alice and Eve is 60 dB is shown. Again, the average transmit power P = 1 and both Bob and Eve use 10-bit A/Ds. Similar to the previous case, as expected, channels with greater variations lead to higher

secrecy rates.

V. CONCLUSION

In this paper, a new method that utilizes an ephemeral cryptographic key to achieve secrecy is introduced. The secret message goes through a time-varying ISI filter with filter coefficients determined by the shared key. The intended receiver uses the key sequence to cancel the effect of ISI on its signal, while the eavesdropper cannot. The coefficients of the filter are changed frequently and thus it is assumed that the eavesdropper is not able to perform adaptive ISI cancellation. The results suggest that this method can substantially improve the achievable secrecy rate of the corresponding wiretap channel and provide secrecy even in the case that the eavesdropper has perfect access to the output of the transmitter's radio.

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