Analytical Design of Inter-vehicular Communications for Collision Avoidance

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***Abstract*—We address the analytical design of a communications system that seeks to provide timely safety information for drivers who are unaware of an imminent collision. We develop a model to characterize the delay requirements needed to prevent such collisions. By simultaneously addressing the multiuser interference and propagation effects such as path loss and fading, we then analytically derive the optimal channel access probability and transmission range and rate of nodes that satisfy the delay requirements at a target success probability.**

***Index Terms*—Collision avoidance, Inter-Vehicular Communications (IVC), Vehicular Ad Hoc Network (VANET).**

I. INTRODUCTION

Communications between vehicles are considered to augment the safety and efficiency level of tomorrow's transportation systems. To this end, the FCC has dedicated 75 MHz of bandwidth in the 5.9 GHz band for Dedicated Short Range Communications (DSRC) [1]. The US Department of Transportation furthered this effort by introducing initiatives such as the recent Connected Vehicle Technology Challenge[1] that promote the use of DSRC to establish vehicle-to-vehicle and vehicle-to-roadside communications to deliver timely information to save lives, reduce congestion, and improve quality of life.

It has previously been seen by the authors that vehicle-to-vehicle communications through which vehicles gain information on each other's status, greatly benefits the efficiency of transportation networks [2]. In the current paper we develop an analytical framework that adjusts the communications parameters in order to ensure safety.

There has been a plethora of studies considering the exchange of safety related information between vehicles in the literature [3, 4]. The prevalent mode of vehicular communications in those papers is *geocast* where each vehicle periodically broadcasts its status packets to all other vehicles that reside within a predefined range of its vicinity. The main purpose of such studies is to propose efficient algorithms that achieve high packet success probabilities. This usually goes without accounting for the safety requirements of the system. Recently some studies addressed the communications requirements needed for safety applications. Bai et al. [5] utilize an experimental set-up to analyze the link-level behavior of DSRC vehicle-to-vehicle communication in a wide variety of traffic environments. They also characterize the application-level reliability of DSRC for vehicle safety communications. Finally they provide a simple formula to analytically relate the link-level reliability to the application-level reliability. In [6], Haas et al. develop a scalable simulation environment that simulates actual crashes and address the communications requirements for crash avoidance. To the best of our knowledge, the first paper that analytically studies the communications requirements for a safety application is [7]. There, a car following model is developed to address the acceptable message delivery latency to avoid rear-end collisions. Based on this, the minimum required transmission frequency which meets the delay requirement is deriven. However this paper lacks realisticity in that it only analytically addresses packet losses due to path loss and shadowing, and MAC layer losses i.e. losses due to interference are assumed given as a parameter of the model. Moreover the feedback effect of packet retransmissions on the packet loss probability has not been accounted for.

In this paper we study how a rear-end crash situation could be prevented by the use of inter-vehicular communications. Here when a vehicle applies a sudden brake, the following vehicle's driver sees this event and also applies the brake after going through a perception-reaction time. With no inter-vehicle communications, the vehicle following this latter vehicle is not able to take preventive action up until the time he perceives its brake lights. This cumulative effect of response time could lead to possible collisions that could be prevented by giving drivers beforehand "virtual" visions by utilizing inter-vehicle communications.

With the above model we formulate the exact delay requirements needed by the safety application for different traffic scenarios. Using a novel analytical approach, we derive the communications parameters (media access probability, p, transmission range, r, and rate, R) that meet the delay requirements with a given success probability. The framework allows us to simultaneously account for collisions due to interference, path loss and fading. The effect of transmission frequency on packet loss is also considered. Our design methodology allows for the adjustment of communications parameters for a whole range of traffic scenarios and driver characteristics.

II. TRAFFIC FUNDAMENTALS AND COMMUNICATIONS SYSTEM MODEL

A. *Traffic Model and Accident scenario*

In this section we elaborate on the crash scenario. Assume a string of vehicles in a traffic stream. Here if one of the vehicles applies a sudden brake for any apparent reason, all those trailing drivers would also slow down but not before observing the brake lights of their immediate leader vehicle. The braking decisions being based on the visual information,

[1] http://connectedvehicle.challenge.gov/

leads to a cumulative response time for downstream vehicles and increases the chance of collisions. However, wireless communications between vehicles typically reduces the response time of the downstream vehicles by rendering them with warning messages well in advance of the shock wave of braking vehicles reaching them.

Consider a traffic stream that is locally in equilibrium before the sudden brake. In this scenario all vehicles move with the same speed v and inter-vehicle spacing s [2]. Now assume that vehicle i suddenly applies a sudden brake with deceleration rate b at time $t = 0$. The driver of the following vehicle $i + 1$ observes this phenomenon and applies a brake after going through an initial perception-reaction time of τ seconds. Hence, the motion of vehicle i is a deceleration from the beginning whereas that of vehicle $i+1$ is with constant speed v for the first τ seconds and then a deceleration. In Figure 1 we have demonstrated the four possible collision scenarios between the two vehicles. These result from analyzing the vehicles' equations of motion. However, the derivation details are omitted here to enhance the flow of the paper. For any given set of values for v, s, b, and τ, Figure 1 tells you whether there is going to be a collision between vehicles i and $i + 1$, and if yes, when and where it will happen.

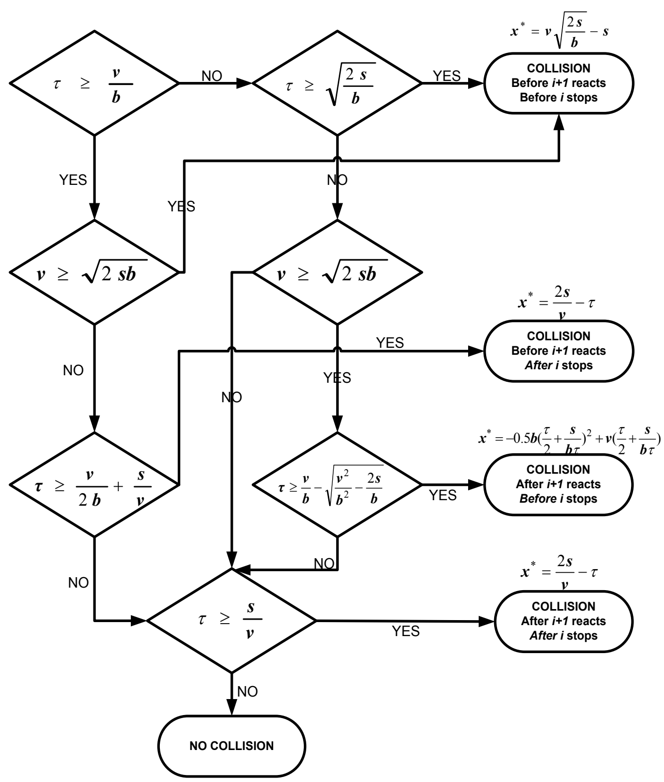

Fig. 1. Various crash scenarios between vehicles i and $i + 1$. x^* denotes the location of the crash site.

[2]This is a good approximation when the highway is not operating at the free flow regime which is where the study of safety is of special importance.

The sudden brake by vehicle i also sets of an alarm that is disseminated by its wireless radio to all trailing vehicles. Hence vehicle $i + 2$ is able to start slowing down after $\tau + d$ seconds where d denotes the delay of communications. Note that the safety system is effective only if $d < \tau$, that is if driver $i + 2$ is informed of the traffic anomaly via means of communications before himself observing the brake lights of vehicle $i + 1$.

Now based on the specific crash scenario and the distance of the crash site from vehicle $i+2$'s initial position, the following delay requirements should be met in order for vehicle $i+2$ to decelerate soon enough to avoid the site of the accident. When the crash happens before vehicle $i + 1$ reacts and i comes to a complete stop, then:

$$d < \sqrt{\frac{2s}{b}} - \tau + \frac{s}{v} - \frac{v}{2b} \tag{1}$$

If the crash happens after $i + 1$ reacts but before i stops, then:

$$d < \frac{2s}{v} - \frac{b}{2v}\left(\frac{\tau}{2} + \frac{s}{b\tau}\right)^2 - \frac{\tau}{2} + \frac{s}{b\tau} - \frac{v}{2b} \tag{2}$$

For the other two cases we have:

$$d < \frac{2s}{v} - \tau \tag{3}$$

Also when no collision occurs between i and $i+1$, there is essentially no collision between $i+1$ and $i+2$ since we have $d < \tau$.

B. Communications System Model

Here we elaborate on the communications model of the VANET. The Media Access Control (MAC) scheme of the network is slotted ALOHA. This is a justified scheme for the dissemination of delay-critical safety messages, since for such applications using carrier sensing strategies like CSMA/CA would incur much unwanted delay on message delivery [4].

Based on the traffic scenario, nodes are equidistantly placed on an infinite line, where the spacing between adjacent nodes is s and nodes move with a constant speed v. At each time slot, a node transmits with probability p and is a potential receiver with probability $1 - p$, independent of all others.

We consider the network to be interference limited. This means that nodes can arbitrarily increase their transmit power to overcome the power of noise. This is a realistic assumption for VANETs as vehicles are not usually faced with power constraints. As we are considering the Signal to Interference Ratio (SIR), we can assume that all nodes transmit with the same (unit) power as only relative received powers matter. Signal propagation characteristics are formalized by path loss and fading. Hence the received power at distance r is $hr^{-\alpha}$, where $\alpha > 1$ is the path loss exponent and h is the fading coefficient. All fading coefficients are independently and identically distributed across space and time.

We also consider a *fixed* coding scheme that requires the SIR to be larger than some threshold β to have successful transmission at a given bit-rate. Given that a node transmits and the intended receiver distance r away listens, the outage probability is:

$$P_s = \mathbb{P}(\frac{S}{I} > \beta) = \mathbb{P}(\frac{hr^{-\alpha}}{\sum_{i\in\Phi} b_i h_i r_i^{-\alpha}} > \beta) \tag{4}$$

Where Φ is the set of possible interferers, b_i is a bernoulli random variable with parameter p, and h_i and r_i are the fading coefficient and distance from the i^{th} interferer to the receiver.

In our study we consider the case where $\beta > 1$[3]. For this case it is shown in [8] that when fading is not present and in a one-dimensional setting, the successful reception at a receiver distance r away, ensures successful reception at all nodes in between as well. The above also holds true with negligible error in the presence of fading. Based on this, the communications model developed in this section can be used to analyze our geocast scenario where all vehicles within distance r of the transmitter are supposed to successfully receive its packets.

III. Analysis and Design

In this section we develop a framework to design the PHY and MAC parameters of the VANET (namely the transmission rate R, transmission range r, and the access probability p) in order to fulfil the delay requirements of the collision avoidance application.

Conditioned on the fact that a transmitter and receiver distance r apart pair up, the success probability is:

$$\begin{aligned} P_s &= \mathbb{P}(h > \beta r^\alpha I) \\ &= \mathbb{E}[e^{-\beta r^\alpha I}] \end{aligned} \tag{5}$$

Where the last equality is due to the Rayleigh fading assumption. Further,

$$\begin{aligned} P_s = \mathbb{E}[e^{-\beta r^\alpha I}] &= \mathbb{E}[e^{-\beta r^\alpha \sum_{i\in\Phi} b_i h_i r_i^{-\alpha}}] \\ &= \prod_{i\in\Phi} \mathbb{E}[e^{-\beta r^\alpha b_i h_i r_i^{-\alpha}}] \\ &= \prod_{i\in\Phi} [1 - p + \frac{p}{1+\beta r^\alpha r_i^{-\alpha}}] \end{aligned} \tag{6}$$

Where the last inequality follows by taking the expectation with respect to the bernoulli variable b_i and Rayleigh fading h_i.

In our scenario of an infinite line, a transmitter is communicating with another node χ hops away, hence $r = \chi s$. Moreover, the interferers are symmetrically located about the receiver (with distances r_i to it) except that there is no interferer at the location of the transmitter (see Fig 2). Hence:

$$P_s = \frac{1+\beta}{1+(1-p)\beta}[\prod_{i=1}^{\infty} 1 - p + \frac{p}{1+\beta r^\alpha r_i^{-\alpha}}]^2 \tag{7}$$

[3] $\beta > 1$ holds for most physical layer modulation schemes including OFDM which is standardized for inter-vehicular communications. Typical values for β anticipated in DSRC are above 5 (see Table I).

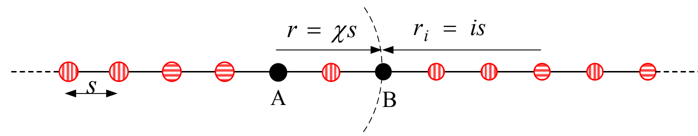

Fig. 2. Node A transmitting to node B. The set of potential interferers (dashed) are symmetrical about the receiving node B except that there is essentially no interferer where the transmitter is. In this specific realization the active interferers are dashed horizontally and the silent ones are dashed vertically.

Now, for $r = \chi s$, $r_i = is$ and $\alpha = 2$ we have:

$$P_s = \frac{1+\beta}{1+(1-p)\beta}[\prod_{i=1}^{\infty} \frac{1+(1-p)\beta\frac{\chi^2}{i^2}}{1+\beta\frac{\chi^2}{i^2}}]^2 \tag{8}$$

Now using the inequality $\sin(\pi z) = \pi z \prod_{i=1}^{\infty} 1 - \frac{z^2}{i^2}$, we have:

$$P_s = \frac{1+\beta}{1+(1-p)\beta}[\frac{\sinh(\pi\chi\sqrt{\beta(1-p)})}{\sqrt{1-p}\sinh(\pi\chi\sqrt{\beta})}]^2 \tag{9}$$

We can further show that:

$$P_s \lessapprox \frac{1+\beta}{1+(1-p)\beta} e^{-p\gamma} \tag{10}$$

Where $\gamma = \pi\chi\sqrt{\beta}\coth(\pi\chi\sqrt{\beta}) - 1$. The bound is tight for small p which as we shall see holds true in our study. For $\alpha = 4$ we again replace $r = \chi s$, $r_i = is$ but this time $\alpha = 4$ in (7). Then carrying out the same procedure as above and some manipulations, we can see that (10) holds true for $\alpha = 4$ as well but this time $\gamma = \frac{\pi\chi\beta^{\frac{1}{4}}}{\sqrt{2}} - 1$. For the rest of this section we consider $\alpha = 2$ noting that the analysis for $\alpha = 4$ is similar when its corresponding γ value is used.

For our safety application it is important for each vehicle to successfully deliver at least one packet within the delay requirement window, d. Let R be the rate of transmission and l be the packet length. This way, the allowable number of transmission opportunities is:

$$D = \lfloor \frac{dR}{l} \rfloor \tag{11}$$

Where d is the delay constraint that is determined based on the mobility and driver characteristics v, s, b, τ as per the discussion in section II-A.

The safety criterion of the collision avoidance system is to achieve a *target* success probability of $1-\epsilon$ in delivering a packet within the period of the D time slots. Hence ϵ is the vehicle collision probability since vehicle $i+2$ collides with the site of the accident if it is not informed about the collision within the D slots. Now denoting by P_s^D the success probability after D transmission trials (hence P_s^D shall be referred to as the delay-bounded success probability hereafter), we have:

TABLE I

IEEE 802.11P DATA RATES AND CORRESPONDING SIR DECODING THRESHOLDS FOR A 10 MHZ WIDE CHANNEL

R (Mbps)	3	4.5	6	9	12	18	24
β (db)	5	6	8	11	15	20	25

$$P_s^D \approxeq 1-(1-p(1-p)P_s)^D \tag{12}$$

$$\lessapprox 1-(1-p(1-p)\frac{1+\beta}{1+(1-p)\beta}e^{-p\gamma})^D \tag{13}$$

The approximation in (12) is due to the fact that success events across time slots are assumed to be independent. This assumption is true for small enough densities of transmitters [9]. Usually $k \ll 1$ in vehicular networks and as we shall later see $p \ll 1$ for typical values of ϵ, hence the approximation renders negligible error.

Equation (13) demonstrates the dependence of P_s^D on p and β. Note that in (13), γ and D are both a function of β (to see the dependence of the latter see (11) and notice that the transmission rate R is directly related to β via Table I).

The design goal is to select the media access probability p, and the transmission rate R such that $P_s^D = 1-\epsilon$ is guaranteed for the largest possible transmission range r greater than $2s$ (the spacing between vehicles i and $i+2$)[4]. In case such a range is not feasible, we need to adjust the parameters such that the collision probability for vehicle $i+2$ is as low as possible. In both cases $1-\epsilon'$ denotes the *achievable* success probability where essentially $\epsilon' \geq \epsilon$.

Note that there is always an optimal value of the media access probability p that maximizes the success probability in a given time slot. A large p typically causes excessive multi-user interference and reduces the success probability. On the other hand too small of a value for p although reduces the interference, but again leads to reduced success probability as the probability of the desired transmission taking place is low itself.

Any increase in the SIR threshold value β also has a double-fold effect on the success probability. A large value for β translates to a higher transmission rate R which means lower packet transmission time and hence a larger D. This tends to improve the success probability since it allows for more transmission opportunities within the delay requirement window. At the same time, larger β means less interference is tolerable which results in a reduction in the success probability. This way, the existence of an optimal value for β is also predictable.

Differentiating (13) with respect to p renders the optimal access probability. For $\beta(1-p) \gg 1$ which holds here, we can maximize $P_s^D = 1-(1-pe^{-p\gamma})^D$ instead. This yields:

[4]This provides a fixed level of QOS for vehicle $i+2$, while allowing for the largest population of vehicles beyond it being informed as a bonus.

$$p^* = \frac{1}{\gamma} \tag{14}$$

$$P_s^{D^*} = 1-(1-\frac{1+\beta}{\gamma e\beta})^D \tag{15}$$

We can now investigate the value of β (and the corresponding rate, R) which maximizes $P_s^{D^*}$. For this we use the values in Table I. Interestingly we observed that the maximizing transmission rate was only a function of the path loss exponent α. That is for $\alpha = 2$, $R^* = 9$ Mbps, and for $\alpha = 4$, $R^* = 18$ Mbps. As evident, the optimum transmission rate increases with the path loss exponent. The intuition behind this fact is as follows. As previously mentioned, the existence of an optimal transmission rate is due to the tradeoff between packet retransmission time and SIR decoding threshold. At lower path loss exponents the signal of the interferers decays less rapidly and hence the negative effect of the latter starts dominating the positive effect of the former at lower rates. However as α increases the interference signal becomes more "local" and a typical receiver can still decode at a higher SIR threshold, and hence the optimal rate is higher.

To guarantee a target delay-bounded success probability, we set $P_s^{D^*} = 1-\epsilon$ where $P_s^{D^*}$ is as in (15) but with β^* as the SIR decoding threshold corresponding to the optimal rate. Doing this, the corresponding optimal access probability then follows from (14):

$$p^* = e(1-\epsilon^{\frac{1}{D}})\frac{\beta^*}{\beta^*+1} \tag{16}$$

Also the maximum transmission range is $r^* = \chi^* s$ where:

$$\chi^* = \lfloor \frac{1}{\pi\sqrt{\beta^*}}(1+\frac{\beta+1}{\beta e(1-\epsilon^{\frac{1}{D}})}) \rfloor \tag{17}$$

In deriving (17) we have used the fact that $\gamma \approxeq \pi\chi\sqrt{\beta}-1$, with the approximation being very tight as soon as $\beta > 1$.

Note that it is possible that for a given scenario, there exists no transmission range larger than $2s$ that achieves the target delay-bounded success probability of $1-\epsilon$. In this case we look for a set of parameters that achieves a collision probability ϵ' for vehicle $i+2$ that is as low as possible. For that we let $\chi^* = 2$ which according to (15) achieves a success probability of:

$$1-\epsilon' = 1-(1-\frac{1+\beta}{e\beta(2\pi\sqrt{\beta}\coth(2\pi\sqrt{\beta})-1)})^D \tag{18}$$

From which the new p^* can be obtained from (16) using ϵ' as the new collision probability. Note that in all cases the access probability that maximizes P_s^D could be greater than one in which case we let $p^* = 1$ and the new success probability is calculated through (13).

IV. NUMERICAL RESULTS

In this section we numerically investigate our design methodology for the collision avoidance system. We choose the typical values $\tau = 1.8s$ and $b = 6\frac{m}{s^2}$. For this set of values, we use the decision diagram of Figure 1 to determine the crash scenario over a range of inter-vehicle spacings at a given speed. The corresponding delay requirement is then calculated through (1)-(3). For $\alpha = 2$, as previously shown, the optimal rate is $R = 9$ Mbps. With the assumption of $l = 250$ byte packets, the allowable number of transmission opportunities, D, then results from (11).

The optimal transmission hop number χ^* and access probability p^* are derived using our design methodology for $\epsilon = 10^{-3}, 10^{-4}, 10^{-5}$. See Figures 3 and 4. The two figures suggest that the range and the media access probability of the nodes need to be dynamically adopted to the traffic conditions, whereas the transmission rate is fixed for the specific path loss exponent of the environment. From Figure 3 it is evident that for each value of ϵ there is a threshold inter-vehicle spacing beyond which the target ϵ is actually achievable. For example in our scenario where $v = 13.5\frac{m}{s}$, $\epsilon = 10^{-5}$ is achievable for inter-vehicle spacings of 13.1 meters and above. Moreover in this range of inter-vehicle spacings, all ϵ values larger than 10^{-5} are also achievable with the appropriate setting of transmission range and access probability.

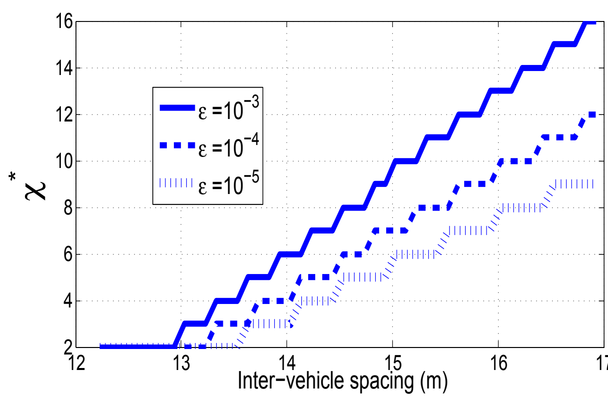

Fig. 3. Maximum transmission hop number as a function of inter-vehicle spacing. $v = 13.5\frac{m}{s}$.

V. Conclusion and Future Work

In this paper we studied the communication requirements of a safety system deployed to inform the trailing vehicles about an obscure crash site. We initially developed a model to determine the delay requirements of the safety message delivery system for arbitrary speed, inter-vehicle spacing, and driver and vehicle characteristics. We then addressed the packet success probability under interference, path loss and fading. We derived analytical expressions for the channel access probability and transmission range that guaranteed a target delivery success rate satisfying the calculated delay requirement. Although in this study we assumed that all vehicles are equipped with communication devices, for future work we plan to study the optimal parameter adjustment problem in the early phases of the technology market penetration.

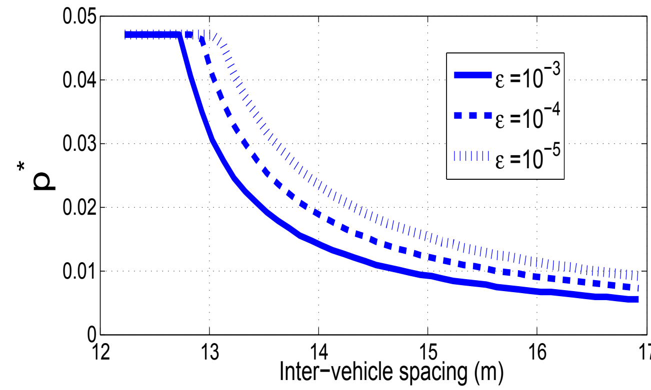

Fig. 4. Optimal access probability as a function of inter-vehicle spacing. $v = 13.5\frac{m}{s}$.

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