Analytic Design of Active Safety Systems for Vehicular Ad hoc Networks

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***Abstract*—We analytically study parameter design for safety applications in Vehicular Ad hoc Networks. Our goal is to fill in the current gap between purely traffic-based studies that fail to account for the non-idealities of communications, and communications-based ones which neglect the application needs of the system. Initially and by studying the dynamic behavior of vehicles we address the delay requirement of the safety application. We then derive the delay-bounded packet success probability of three Media Access Control (MAC) schemes proposed for the dissemination of periodic safety messages. By simultaneously accounting for multi-user interference and propagation effects such as path loss and fading, we determine the optimal transmission rate, channel access probability, and when pertinent, carrier sensing range of those schemes that satisfy the delay requirement at a target success probability. Of our findings is that the optimal transmission rate varies only with the path loss exponent, irrespective of the MAC scheme used. Also, the optimal communications parameters that minimize the maximum expected collision probability in a chain of vehicles is only dependent on the human factors (perception-reaction time and deceleration behavior) and highway characteristics (number of lanes and path loss exponent) and hence do not have to vary with traffic conditions.**

***Index Terms*—Active safety, inter-vehicular communications (IVC), vehicular ad hoc network (VANET).**

I. Introduction

EVERY year, more than $40,000$ people are killed and 3 million injured in highway traffic accidents in the United States alone [1]. This places motor vehicle accidents as the leading cause of death for the $3-6$ and $8-34$ age group [2]. In order to reduce such casualties, the IEEE and numerous federal agencies are actively promoting the deployment of Intelligent Transportation Systems (ITS). The Federal Communications Commission has allocated 75 MHz of spectrum in the 5.9 GHz band for Dedicated Short Range Communications (DSRC). To serve as the groundwork for DSRC, the IEEE 802.11p standard was published in the year 2010 for Wireless Access in Vehicular Environments (WAVE) [3]. During recent years, the US Department of Transportation has also hosted numerous initiatives to promote ITS [4].

The National Highway Traffic and Safety Administration reports that in the year 2000, rear-end collisions accounted for about 28 percent of the total $6,318,000$ police-reported crashes in the United States [5]. A rear-end collision (hereafter, the term collision shall refer to vehicle collisions unless explicitly stated to denote packet collisions) is typically due to the delay in a driver's response time when its leader suddenly decelerates. This delay could be due to the driver's inattentiveness or due to the ever-present Perception-Reaction (P-R) time even for vigilant drivers. Within a chain, the problem could be exacerbated with such a collision leading into a pile-up. Here a trailing vehicle in the chain cannot respond to the event until after observing the brake light of its immediate leader. This dependence of braking decisions on the state of the leading vehicle results in a cumulative response time for upstream vehicles that could lead into domino-style collisions in the chain.

Communications between vehicles can significantly help decrease collisions. For example in the previous scenario, inter-vehicle communications can be used to warn the following driver if he is distracted or cannot observe the deceleration event due to factors such as low visibility or defected brake lights. However, the spacing between the two vehicles might be so small (or the P-R time of the following driver so large) that no matter how early the follower is informed, it cannot avoid a collision with the fast braking one in front. In such scenarios it might be meaningful to invest some of our resources on notifying other subsequent drivers in the chain that cannot typically see the initial deceleration event but could avoid an accident if informed soon enough.

Current efforts in the realm of VANET safety communications generally fall under one of the following two broad categories. One that is mostly led by the ad hoc networking community, seeks to propose efficient algorithms to enhance the networking metrics (such as throughput and/or delay, based on the specific application) in VANETs [6]–[10]. The main challenge there is to come up with schemes that can cope with the specific networking restrictions of such environments such as the fast changing topology and the high mobility of nodes. These studies, however, do not mostly address the implication of their design on safety applications that are the most important factor to the end drivers. The other major trend is led by transportation engineers whose primary concern is to study vehicle collisions and ways to reduce them via ITS. However, the wireless networking restrictions are either not accounted for (ideal communications assumed between vehicle) or considered in an abstract manner [11], [12]. The current paper seeks to fill in this gap by seeking to analytically study the communications parameters of a VANET designed to fulfil the safety application needs of driving.

The 75 MHz spectrum of DSRC is divided into seven 10 MHz wide channels. From among these channels, one called

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the *control channel* (CCH) is reserved exclusively for safety messages. The other channels are called *service channels* (SCH) and serve commercial and convenience applications. Moreover, safety messages are either *event-driven* or *periodic*. Periodic messages are sent by each vehicle on a regular basis and in a single hop to inform all others inside a given neighborhood on its current status such as location, speed and acceleration. Yet event-driven packets are sent in an emergency to warn upstream vehicles of a crash site ahead. Since the latter type of messages are rare in their occurrence, they do not pose a capacity issue in the design of VANETs. Yet due to their importance, it is usually the case that a portion of CCH is left unused for when event-driven messages need to be propagated. This by itself could lead to inefficient use of bandwidth. In this paper we analytically study how periodic status packets can be sent in an intelligent manner to actively attain proven collision avoidance. Note that the use of periodic messages for collision avoidance has recently been studied in [13] but only through simulations.

The Media Access Control (MAC) protocol anticipated for DSRC communications is a variation of the conventional CSMA/CA scheme proposed for IEEE 802.11 standards [3]. However, due to the short length of the safety packet payload and the broadcast nature of communications, the 4-way handshake anticipated by the standard is not efficient for the dissemination of periodic safety messages. Forgoing RTS/CTS and ACK message exchanges gives rise to the *hidden node* problem, increasing the probability of packet collisions. On the other hand, the highly dynamic topology of VANETs requires appropriate topology-transparent protocols [14]. Topology-transparent protocols are ones which do not need a detailed description of the network topology in order to schedule packet transmissions. Repetition-based protocols not only exhibit topology-transparent properties, but also seek to combat packet collisions with that of the hidden nodes' [7]. Hence in our study we make use of repetition-based protocols for the dissemination of periodic safety messages (a similar approach has been used in [6], [7]). Here a message is repeated within its useful lifetime according to some scheme that we shall later discuss.

In this paper, by first analyzing the vehicles' equations of motion during a deceleration event, we find the tolerable delay of a collision avoidance safety application. This delay corresponds to the closest vehicle in chain with a blocked eyesight to the one decelerating and is shown to depend on the traffic conditions (vehicle speed and spacings), vehicle characteristics (deceleration rate), and the human factors (P-R time). We then analytically study the performance of three repetition-based MAC schemes for sending periodic safety messages: Slotted Synchronous P-persistent (*SSP*), Slotted Asynchronous P-persistent (*SAP*), and Slotted Asynchronous P-persistent with Carrier Sensing (*SAP/CS*). We account for the influential radio propagation factors in a VANET such as path loss, fading and multi-user interference. Our analysis helps us determine, for each scheme, the set of parameters i.e. transmission rate, channel access probability and for the case of the *SAP/CS* scheme, carrier sensing range, that satisfy the tolerable delay of the application at a target QoS level. We also study the optimal parameter setting to mitigate the effect of subsequent upstream collisions within a chain. Although here we focus more on a collision avoidance application, our general analytic framework allows addressing other safety applications in which the delay of communications is a critical factor.

Our main contributions in this paper are as follows:

1) For both the *SSP* and *SAP* schemes we find closed-form expressions for the channel access probability that targets a QoS level in satisfying the safety application demands. Moreover, our analysis reveals that the optimal transmission rate is only dependent on the path-loss exponent of the environment, hence eliminating the need for its adaption to the varying traffic conditions. This result, which we validated through NS-2 simulations, is in accordance with a similar conclusion made from simulations in [6], though the dependence on the path-loss exponent is not mentioned there.
2) We analytically study the *SAP/CS* scheme. In general the spatial and temporal dependence of transmissions in carrier sensing schemes makes them hard to study analytically. Yet by making some simplifying assumptions, we are able to derive a closed form expression for the packet success probability, which we use to find the optimal channel access probability, transmission rate, and carrier sensing range that suit our safety application needs. We further verify our analytic results via NS-2 simulations. We believe that this result has standalone significance in addressing the performance of a broadcast CSMA scheme (which does not entail the RTS/CTS and ACK handshake) even outside the context of a VANET.
3) To alleviate the effect of upstream collisions, we propose a choice of communications parameters with a goal of minimizing the maximum expected collision probability for vehicles in a chain. Interestingly, we find that such design depends only on the human factors (P-R time and deceleration behavior) and the characteristics of the highway (number of lanes and path-loss exponent); hence again eliminating the need to adapt the parameters to traffic conditions when such a goal is set.

The rest of the paper is organized as follows. Section II studies where the current paper stands with respect to related work in this area. In Section III-A we address the delay requirement of the safety application. Analytical study of the proposed MAC schemes and their application-based parameter design are presented in Section III-B. Numerical evaluation of our analysis and its verification against NS-2 simulations are presented in Section III-C. Section III-D further studies the design choices to combat chain collisions. The paper is finally concluded in Section IV. Note that the current paper builds upon our prior work [15] which handles the analysis and numerical evaluation of only the *SSP* scheme and which does not consider design implications for chain collisions.

II. Related Work

As previously mentioned, current efforts in this field are either focused on enhancing the communications metrics of a VANET, or abstractly account for the communications parameters to focus more on the safety aspects of the network. Here we first point to the main studies in each of the above two categories and then look at some recent simulation-based and experimental studies that try to pull the two sides together.

A. Ad hoc Networking-based Studies

There has been a plethora of studies considering the exchange of safety related information between vehicles in the literature [6]–[10]. In [6] the authors propose a series of repetition-based MAC protocols to deliver periodic beacon messages within their useful lifetime to within a specified range. For a scheme in which nodes transmit with a given probability in each slot, they derive the Probability of Reception Failure (PRF) at the border of the range of interest. However, they only consider the strongest interferer in their derivation and neglect fading. They also study (only through simulations) a similar scheme where nodes perform carrier sensing prior to transmission. The authors do not however mention how their design suits specific packet reception probability and delay requirements affiliated with different safety demands of driving. With the mind-set of satisfying the requirements of a collision avoidance application, in this paper we introduce and derive the *delay-bounded packet success probability*. Our derivation accounts for the cumulative interference from all simultaneously transmitting nodes and also considers fading. Moreover, we analytically study the MAC scheme with carrier sensing. Furthermore, we show that the right choice of the carrier sensing range (assumed fixed in [6]) has a considerable effect on increasing the reception probability.

As opposed to [6] which is based on random transmission decisions at each slot, [7] proposes predetermined transmission patterns for each node. Here, nodes are allocated minimally correlated codewords that indicate the specific time slots assigned to each as their transmission opportunities. Although the latter algorithm is shown to perform better in case of lower traffic loads, the two more or less attain the same performance measures in saturated traffic load scenarios which is the case for periodic safety messages. [8] also addresses the efficiency and reliability of one-hop broadcast considering only the strongest interferer.

Although the above schemes have PRF as their main performance metric, the Channel Busy Time (CBT) should also be controlled when disseminating periodic beacon messages in order to leave enough room for the timely delivery of event-driven emergency messages (which need to be propagated after an accident happens). [16] proposes a distributed transmit power control algorithm to avoid saturated channel conditions and ensure the best use of the channel for safety-related purposes. Along the same lines, [17] proposes Adaptive Traffic Beacon (ATB) to ensure an uncongested channel for the delay-sensitive transmission of emergency messages. There the beacon interval is dynamically adapted to the channel quality and message priority. [18] adapts a position-based message forwarding strategy to disseminate time-critical safety information when the beaconing load is controlled by [16]. [19] shows that the bipolar traffic behavior which could lead into disconnected VANETs, mandates new routing schemes for the dissemination of non-periodic messages. They also report a multi-hop broadcast protocol called DV-CAST that copes with such diverse traffic regimes in VANETs [20]. Again our study differs from the above in that we analytically design the dissemination of periodic beacon packets in order to *prevent* collisions.

B. Traffic Based Studies

In [11] a stochastic model is developed to derive the average number of collisions (when the leading vehicle stops instantly) in a chain of vehicles that are equipped with a collision warning system. The operation of the communications system is abstracted by a message delay variable whose distribution is assumed given for any specific MAC scheme. Moreover, it is assumed that all vehicles in the chain receive the warning message at the same time. In the current paper we account for the dynamics of the suddenly decelerating vehicle and drop the strong assumption that it stops instantly. We also analytically derive the delay of communications for the proposed MAC schemes. We observed that assuming simultaneous delivery to different vehicles within the chain is far from realistic and that the average delay grows exponentially large as we move further upstream when deploying a slotted MAC scheme. We show that this discrepancy between delivery times is itself one of the main causes of secondary collisions in the chain. In [12] the authors compare the safety of automated and manual highway systems with respect to rear-end collision frequency and severity. Yet, they assume a fixed communications delay of 300, 150, and 120 milliseconds for autonomous, low-cooperative, and high-cooperative vehicles, respectively.

C. Simulation-based and Experimental Studies

Recently some studies addressed the communications requirements needed for safety applications. In [21], the authors present a class of packet forwarding protocols for cooperative collision avoidance and study their effect on the frequency and severity of accidents through simulations. Bai et al. [22] utilize an experimental set-up to analyze the link-level behavior of DSRC vehicle-to-vehicle communications in a wide variety of traffic environments. They also characterize the application-level reliability of DSRC for vehicle safety communications. Finally they provide a simple formula to relate the link-level reliability to the application-level reliability. In [13], Haas et al. develop a scalable simulation environment that simulates actual crashes and addresses the communications requirements for crash avoidance. [23] utilizes *awareness* in order to make a connection between network performance (e.g. PRF) and safety application performance (e.g. vehicle collision probability). However they extract the PRF values from an empirical model and a simulation study. In [24] the authors demonstrate the beneficial impacts of vehicular ad hoc networks on traffic safety via simulations. They design an Accident Prevention Application (APA) which shall be used in the future to assess the communication system's effect on traffic safety.

To the best of our knowledge, the first paper to study the communications requirements of a safety application through a simple analytic approach is [25]. There, the minimum required

TABLE I
MAXIMUM TOLERABLE DELAY TO INFORM VEHICLE V_2

	$\tau_c(2)$
Collision 1	$\frac{x}{v} + \sqrt{\frac{2x}{b}} - \frac{v}{2b} - \tau_{\max}$
Collision 3	$\frac{2x}{v} - \frac{b}{2v}(\frac{\tau_{\max}}{2} + \frac{x}{b\tau_{\max}})^2 + \frac{x}{b\tau_{\max}} - \frac{v}{2b} - \frac{\tau_{\max}}{2}$
Collision 2, 4	$\frac{2x}{v} - \tau_{\max}$

transmission frequency to avoid rear-end collisions is obtained. However this paper only considers packet losses due to path loss and shadowing, and MAC layer losses i.e. losses due to interference from other nodes are assumed given as a parameter of the model. Moreover, the feedback effect of transmission frequency on MAC layer losses has not been accounted for.

III. Application-based VANET Design

A. Delay Requirements of the Safety Application

Here we account for the vehicles' dynamics during a sudden deceleration to find the delay requirement of the safety application. We initially study traffic flow in the non-free flow regime where safety of driving typically gains higher importance. It is noted in [26, Ch. 2] that in this car-following regime all drivers tend to maintain a constant spacing with their leader. Yet in reality, driver error gives rise to some variation about the mean. As the small variation does not have a critical effect on our results, we neglect it here to make analysis tractable. With this introduction consider a homogenous traffic stream in local equilibrium (where a chain of vehicles move with speed v and inter-vehicle spacing x), when at time $t = 0$ a vehicle (denoted hereafter by vehicle V_0) suddenly brakes with deceleration rate b. The vigilant driver of the following vehicle V_1 observes this phenomenon and applies the brake after going through an initial P-R time of $\tau_{pr}(1)$ seconds. According to the available literature on human factors, the log-normal distribution (which has a positive domain and rare extreme values) is a good representation of P-R time [27]. For unexpected though common events, such as the braking of a leading vehicle, the distribution has a mean of 1.31 and standard deviation of 0.61 seconds[1].

With no inter-vehicle communications, vehicle V_i $(i > 1)$ starts decelerating after $\sum_{j=1}^{i} \tau_{pr}(j)$ seconds. However if vehicles are able to communicate, a trailing vehicle V_i applies the brakes only after $\tau_{pr}(i) + \tau_c(i)$ seconds after the initial braking of vehicle V_0 where $\tau_c(i)$ is the incurred delay of communications to inform vehicle V_i. Hence as it can react well before observing the brake lights of its immediate leader, the probability of a collision is reduced. A similar assumption where vehicles react as soon as they receive a warning message has been made in [11].

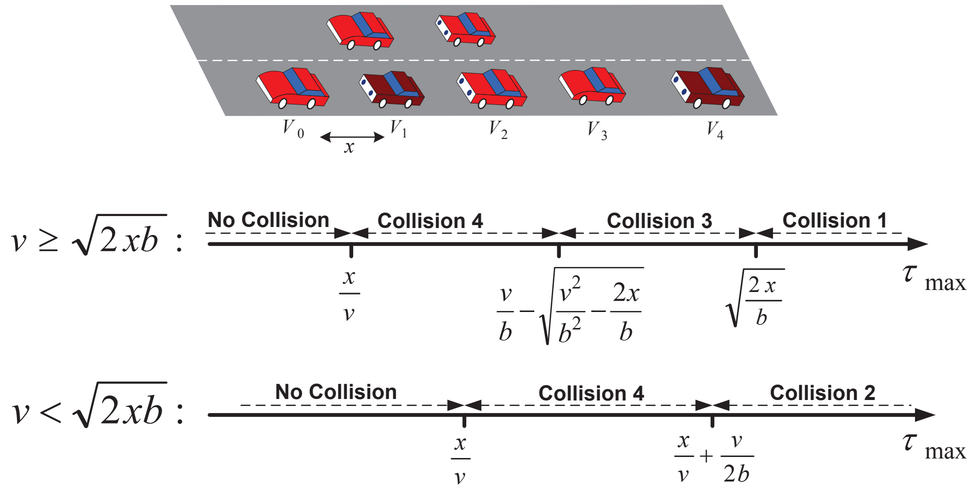
Our strategy to determine the delay requirement is dependent on the traffic condition. We initially assume that the driver of vehicle V_1 which has line-of-sight to its leader vehicle V_0 is vigilant and does not need the assistance of the safety application in order to respond to a deceleration event. Fig. 1 shows the possible collision scenarios between the two vehicles. To account for the worst case in determining whether an accident happens we consider $\tau_{pr}(1) = \tau_{\max}$. When a collision between vehicles V_0 and V_1 is inevitable, the safety application should be designed to warn the next vehicle in chain (vehicle V_2). This is because vehicle V_2 which is the closest vehicle among others in the chain to the site of the accident and hence the most endangered, is initially due to blocked line-of-sight, oblivious to the declaration event until informed by the application. Here we address the tolerable delay to inform vehicle V_2. Note that as the P-R times of the vehicle are random, collisions (with vehicle V_2 involved) occur in one of the following two fashions. It is either that vehicles V_0 and V_1 collide first, with vehicle V_2 hitting the pile-up, or that vehicles V_1 and V_2 collide before hitting vehicle V_0. Not surprisingly, our simulations confirmed that over 80 percent of the accidents are of the first category. Hence we base our design on this category of collisions. Using the vehicles' dynamic equations, we present in Table I the tolerable delay before which vehicle V_2 should be notified in order to prevent hitting the pile-up of vehicles V_0 and V_1. Note that we let $\tau_{pr}(2) = \tau_{\max}$ in Table I. It can easily be verified that this setting shall lead to an upper bound on the tolerable delay for other (smaller) values of driver P-R time.

On the other hand if Fig. 1 predicts "No collision" between the vehicles V_0 and V_1, we can relax the initial assumption and consider the scenario where the driver of vehicle V_1 does not immediately observe the brake lights of vehicle V_0 when it decelerates (even though it has line-of-sight to it). This could happen for a number of reasons such as low visibility in extreme weather conditions, distracted driver attention, or defected brake lights. Hence vehicle V_1 could use the assistance of the safety application to get to know about the deceleration event. Here, equations of motion of the two vehicles dictate that the maximum tolerable delay is $\tau_c(1) = \frac{x}{v} - \tau_{\max}$.

B. Analysis and Design

We now analytically obtain the communications parameters that fulfil the delay requirements of the collision avoidance application. To this end, we first analytically study the performance of three schemes for the dissemination of periodic messages. Note that the generation rate of periodic messages at each vehicle is equal to its GPS update rate. The time interval between the generation of two subsequent packets is a packet's useful lifetime during which it is transmitted via one of the following repetitive-based schemes.

[1]More specifically, "perception" is composed of presumably uncorrelated elements of latency, eye movement, fixation, and recognition. Since it is doubtful that any driver would produce 95th percentile values for each of the individual elements, it is noted that $\tau_{\max} = 2.5$ seconds probably represents an extreme upper limit for a driver's P-R time.



Collision 1	Collision 2	Collision 3	Collision 4
Before V_0 stops, Before V_1 reacts.	After V_0 stops, Before V_1 reacts.	Before V_0 stops, After V_1 reacts.	After V_0 stops, After V_1 reacts.

Fig. 1. Collision scenarios between vehicles V_0 and V_1.

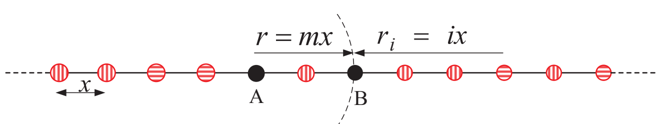


Fig. 2. Schematic used to derive the *SSP* scheme success probability (see proof of Theorem 1). In this realization the active interferers are dashed horizontally and the silent ones are dashed vertically.

1) Slotted Synchronous P-persistent: The MAC scheme that we initially consider is a slotted scheme where at each time slot a status packet is transmitted with a given probability p (we assume that slots are synchronized among vehicles). This shall be referred to as the *Slotted Synchronous P-persistent (SSP)* scheme hereafter. The synchronization issue which hinders practical deployment of such MAC schemes in conventional ad hoc networks is not as much of a problem in vehicular networks due to the presence of on-board GPS devices. Later we shall study the performance of asynchronous transmitters and also the case where nodes perform carrier sensing prior to transmission at each time slot.

As before, nodes are placed on a line with a spacing of x meters. At each time slot, a node transmits with probability p and is a potential receiver with probability $1-p$, independent of all others (See Fig. 2).

We consider the network to be interference limited. This means that nodes can increase their transmit power to overcome the power of noise. This is a realistic assumption for VANETs as vehicles are not usually faced with power constraints. As we are considering the Signal to Interference Ratio (SIR), we can assume that all nodes transmit with the same (unit) power as only relative received powers matter. Signal propagation characteristics are formalized by path loss and Rayleigh fading. Hence the received power at distance r is $hr^{-\alpha}$, where $\alpha > 1$ is the path loss exponent and h is the fading coefficient. All fading coefficients are independently and identically distributed across space and time. We also consider a fixed coding scheme that requires the SIR to be larger than some threshold β to have successful transmission at a given bit-rate R. Given that a node transmits and the intended receiver distance r away listens, the outage probability is:

$$P_s = \mathbb{P}(\frac{S}{I} > \beta) = \mathbb{P}(\frac{hr^{-\alpha}}{\sum_{i \in \Phi} b_i h_i r_i^{-\alpha}} > \beta) \qquad (1)$$

Where Φ is the set of possible interferers, b_i is a bernoulli random variable with parameter p denoting whether the i^{th} interferer transmits or not, and h_i and r_i are the fading coefficient and distance from the i^{th} interferer to the receiver.

Theorem 1. *For homogenous traffic in a single-lane highway, the success probability of a source's transmission using the SSP scheme at a destination m hops away (distance $r = mx$ away) satisfies:*

$$P_s \lessapprox \frac{1+\beta}{1+(1-p)\beta} e^{-p\gamma_m} \qquad (2)$$

Where $\gamma_m \approx m\pi\sqrt{\beta} - 1$ for $\alpha = 2$, and $\gamma_m \approx \frac{m\pi\beta^{\frac{1}{4}}}{\sqrt{2}} - 1$ for $\alpha = 4$, with both approximations being tight for $\beta > 1$.

Proof: see Appendix. ■

Two points are worth mentioning here. First note that the success probability is independent of the actual value of the inter-vehicle spacings x. However as we shall see, x is an important factor in determining the delay requirement and hence the allowable number of transmission attempts. Secondly (2) shows that success probability decreases *exponentially* for

TABLE II
IEEE 802.11P DATA RATES AND CORRESPONDING SIR DECODING THRESHOLDS

R (Mbps)	3	4.5	6	9	12	18	24
β (db)	5	6	8	11	15	20	25

distant vehicles as we move further away from the source. For the rest of this section we consider $\alpha = 2$ noting that the analysis for $\alpha = 4$ is similar.

Corollary 1. *For an N-lane highway:*

$$P_s \approx \frac{1+\beta}{1+(1-p)\beta} e^{-Np\gamma_m} \tag{3}$$

The result follows by neglecting the distance between lanes. Note that we no longer have a tight upper bound on the success probability, but rather an approximation. This is because accounting for the physical distance between lanes would have resulted in a lower interference power and hence a higher success probability. For the remainder of this section we consider a single lane highway which we shall later generalize to account for N-lane highways.

We can now proceed to find the optimal parameters of the *SSP* scheme that satisfy the delay requirement derived in Section III-A. Let R be the transmission rate during each slot and l be the packet length. The allowable number of transmission opportunities within the tolerable delay period is:

$$D = \lfloor \frac{\tau_c(2)R}{l} \rfloor \tag{4}$$

Where again $\tau_c(2)$ is determined by Table I. The safety criterion of the collision avoidance application is to achieve a *target* success probability of $1-\epsilon$ in delivering a packet to vehicle V_2 within the period of the D time slots. Let P_s^D denote the success probability at V_2 after D transmission opportunities. We have:

$$P_s^D = 1 - (1 - p(1-p)P_s)^D \tag{5}$$

$$\lessapprox 1 - (1 - p(1-p)\frac{1+\beta}{1+(1-p)\beta} e^{-p\gamma_2})^D \tag{6}$$

Where (6) results by replacing (2) in (5) for $m = 2$. We shall refer to P_s^D as the *delay-bounded packet success probability* hereafter. Equation (6) demonstrates the dependence of P_s^D on p and β. Note that in (6), γ_2 and D are both a function of β. Table II shows the corresponding transmission rate R for each value of β as suggested by the IEEE 802.11p standard.

The design goal is to select the channel access probability p, and the transmission rate R such that $P_s^D = 1-\epsilon$ is guaranteed for vehicle V_2. In case such success probability is not achievable, we need to adjust the parameters such that the lowest possible ϵ is achieved. In both cases $1-\epsilon'$ denotes the *achievable* success probability where essentially $\epsilon' \geq \epsilon$.

Note that there is always an optimal value of the channel access probability p that maximizes the success probability in a given time slot. A large p typically causes excessive multi-user interference and reduces the success probability. On the other hand too small of a value for p although reduces the interference, but again leads to reduced success probability as the probability of the desired transmission taking place is low itself. Any increase in the SIR threshold value β also has a double-fold effect on the success probability. A large value for β translates into a higher transmission rate R which means lower packet transmission time and hence a larger D. This tends to improve the success probability since it allows for more transmission opportunities within the tolerable delay period. At the same time, larger β means less interference is tolerable which results in a reduction in the success probability. This way, the existence of an optimal value for β is also predictable.

Numerical study of (6) reveals that the optimal data rate is only a function of the path loss exponent α. That is, for $\alpha = 2$, the optimal data rate is $R^* = 9$ Mbps and for $\alpha = 4$, $R^* = 18$ Mbps. This fact was also confirmed through our NS-2 simulations. As evident, the optimum transmission rate increases with the path loss exponent. The intuition behind this fact is as follows. As discussed above, the existence of an optimal transmission rate is due to the tradeoff between tolerable delay for successful packet delivery and the decoding threshold. At lower path loss exponents the signal of the interferers decays less rapidly and hence the negative effect of the latter starts dominating the positive effect of the former at lower rates. However as α increases the interference signal becomes more "local" and a typical receiver can still decode at a higher SIR threshold, and hence the optimal rate is higher. For $\beta(1-p) \gg 1$, (6) reduces to:

$$P_s^D = 1 - (1 - p\frac{1+\beta}{\beta} e^{-p\gamma_2})^D. \tag{7}$$

To guarantee a target delay-bounded packet success probability, we set $P_s^{D^*} = 1-\epsilon$ where $P_s^{D^*}$ is as in (7) with β^* which corresponds to the optimal rate R^* (see Table II). Solving this we have:

$$p^* = \frac{-1}{\gamma_2} \mathcal{W}_0\left(\frac{\beta\gamma_2}{\beta+1}(\epsilon^{\frac{1}{D}} - 1)\right) \tag{8}$$

Where $\mathcal{W}_0(.)$ is the zeroth branch of the *Lambert W* function. This function is real-valued when its argument is larger than $-\frac{1}{e}$, hence $P_s^{D^*} = 1-\epsilon$ is achievable only when:

$$\epsilon > (1 - \frac{1+\beta}{\beta e \gamma_2})^D \tag{9}$$

Note that $P_s^{D^*} = 1-\epsilon$ has another solutions for p over the range $[-\frac{1}{e}, 0]$ which is expressible in terms of the first branch of the Lambert W function. However, this latter value for p^*, which is the larger of the two, leads to a smaller success probability for vehicles that are further away from the source than vehicle V_2. Hence we use (8) hereafter which leads to a better reception probability at the upstream vehicles. As we shall later see this is important in reducing the severity of secondary accidents in a chain.

In case (9) is not satisfied, $1-\epsilon' = 1 - (1 - \frac{1+\beta}{\beta e\gamma_2})^D$ is the highest achievable P_s^D. The access probability that achieves this, results from differentiating (7) with respect to p:

$$p^* = \frac{1}{\gamma_2} \tag{10}$$

For an N-lane highway, the delay-bounded packet success probability (7), generalizes to $P_s^D = 1 - (1 - p\frac{1+\beta}{\beta} e^{-pN\gamma})^D$.

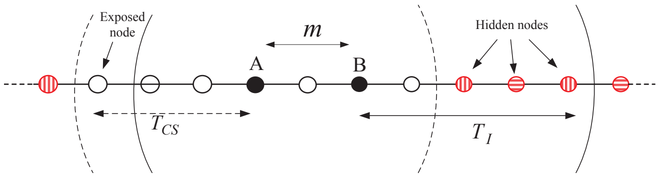


Fig. 3. Schematic used to prove the *SAP/CS* scheme success probability. Node A transmits only when no other inside its T_{CS} hop distance transmits. Interferers not more than T_I hops away from node B can individually cause outage at B. Distances shown in the number of hops.

Hence (8), (9), and (10) result similarly for the N-lane case by replacing γ_2 by $N\gamma_2$.

2) Slotted Asynchronous P-persistent: We now consider a scheme in which nodes again transmit with probability p at each time slot, though the slots at different nodes are not necessarily synchronized. We shall refer to this scheme as the *Slotted Asynchronous P-persistent (SAP)* scheme. Here a node could potentially interfere with an ongoing transmission of another node over at most two consecutive time slots. Hence we can assume an equivalent transmission probability of $p' = p + p - p^2 \simeq 2p$ for the interferers. The approximation is specially tight for small p values. This way, we can obtain (similar to the argument for the synchronous case):

$$P_s^D \lessapprox 1 - (1 - p(1-p')\frac{1+\beta}{1+(1-p')\beta}e^{-p'\gamma_2})^D \tag{11}$$

For $\beta(1-p') \gg 1$, $P_s^D = 1 - (1 - p\frac{1+\beta}{\beta}e^{-p'\gamma_2})^D$. Through similar procedures we can see that (8), (9), and (10) all hold in the asynchronous case by replacing γ_2 by $2\gamma_2$. Moreover the optimal rate is still $R^* = 9$ Mbps for $\alpha = 2$ and $R^* = 18$ Mbps for $\alpha = 4$ as for the synchronous case.

3) Slotted Asynchronous P-persistent with Carrier Sensing: Up until now we have assumed that each node in the network transmits independent of all others with a given probability during each slot. However, as a possible improvement, here we study a scheme where each node senses the channel at the beginning of each slot and transmits with probability p only when finds it idle. We shall refer to this scheme as the *Slotted Asynchronous P-persistent with Carrier Sensing (SAP/CS)* scheme. Here the carrier sensing threshold is introduced as another parameter to be optimized alongside the channel access probability and transmission rate. Too large of a sensing distance inhibits spatial reuse whereas too small of a value causes excessive interference. Our goal here is to find the success probability (P_s) of a typical transmission from node A to node B using the *SAP/CS* scheme (see Fig. 3). Here we can no longer use the steps used in the proof of Theorem 1 as transmissions are no longer spatially independent. Hence to make analysis feasible, we make some simplifying assumptions that we shall point to in the process of our derivation. To start, using the chain rule we have:

$$P_s = P_a P_{s|a} \tag{12}$$

Were P_a denotes the probability that node A accesses the channel i.e. finds it idle and transmits, and $P_{s|a}$ is the success probability at node B given that node A has accessed the channel. We first find P_a. Note that carrier sensing involves nodes monitoring the received power to determine when the channel is idle. The channel is declared idle if the received power is below P_{CS} Watts. Instead and to make analysis feasible, here we define and address the *carrier sensing hop distance*, T_{CS} : a node can transmit when no other is transmitting within T_{CS} hops of its vicinity. Later in this section we shall study the relation between the two metrics. Since node A can transmit only when no other is transmitting within its T_{CS} hop neighborhood we have:

$$P_a \approx p(1-p)^{2T_{CS}} \tag{13}$$

Where the approximation is because transmission probabilities of nodes within node A's neighborhood are not spatially independent (yet the approximation renders a good enough result for $p \ll 1$ which is the case for typical ϵ values). We now find $P_{s|a}$, the success probability at B knowing that a transmission has taken place by A. We consider the nodes that can individually cause outage at B (assumption 1)[2]. That is, we seek to find the radius in hops of a disk centered at B such that any active node inside it would by itself cause outage at B. Due to the presence of fading we call this radius the *effective interference hop distance*, T_I. According to the SIR-based reception model, to have successful reception at B we must have $\frac{hr^{-\alpha}}{h_1 r_1^{-\alpha}} > \beta$ where h and h_1 are the respective fading terms, and r, r_1 the respective distances of node A and the sole interferer that can cause outage at node B. Based on this we have:

$$\begin{aligned}\mathrm{T}_I = \lfloor m\beta^{\frac{1}{\alpha}}\mathbb{E}[h^{\frac{1}{\alpha}}]\mathbb{E}[h_1^{-\frac{1}{\alpha}}]\rfloor &= \lfloor m\beta^{\frac{1}{\alpha}}\Gamma(1-\frac{1}{\alpha})\Gamma(1+\frac{1}{\alpha})\rfloor \\ &= \lfloor m\beta^{\frac{1}{\alpha}}\frac{\pi}{\alpha}\csc(\frac{\pi}{\alpha})\rfloor\end{aligned} \tag{14}$$

Note that in the absence of fading, we simply have $\mathrm{T}_I = \lfloor m\beta^{\frac{1}{\alpha}}\rfloor$. Based on (14) for both $\alpha = 2$ and $\alpha = 4$, the effective interference hop distance is larger than when there is no fading. Now when A transmits, only the hidden nodes whose activities are not sensed by node A, can cause outage at node B (see Fig. 3). Assume there are t hidden nodes and let E_i denote the event that the i^{th} hidden node $1 \le i \le t$ does not transmit. Then:

$$P_{s|a} = \mathbb{P}(\bigcap_{i=1}^{t} E_i) = 1 - \mathbb{P}(\bigcup_{i=1}^{t} E_i^c) = 1 - \sum_{i=1}^{t}\mathbb{P}(E_i^c) \tag{15}$$

The last equality is true when $E_i^c \cap E_j^c = \emptyset$. As we shall later see, for practical values of T_{CS}, this condition holds true since the MAC scheme prohibits the hidden nodes from simultaneous transmissions. We make another assumption here that the strongest signal sensed by the hidden nodes is that of the transmitting node A (assumption 2). Hence given that node A has accessed the channel, the hidden nodes each transmit independently with probability p (since they are hidden to node A and cannot sense its transmission). Based on the above discussion when generalized to the N-lane case, (16) results using (12), (13), and (15). Numerical evaluation of (16) suggests the optimal carrier sensing hop distance of $T_{CS}^* \approx m + T_I$. Hence for clarity we have presented the result for $T_{CS} \ge \max(T_I - m, \lceil\frac{m+T_I}{2}\rceil)$ in (16). In the first region

[2]A similar assumption has been made in [28] which results in a tight lower bound on outage at B.

$$P_s \approx \begin{cases} p(1-p)^{2NT_{CS}}(1-N[m+T_I-T_{CS}]p') & \max(T_I-m, \lceil\frac{m+T_I}{2}\rceil) \le T_{CS} < m+T_I \\ p(1-p)^{2NT_{CS}} & T_{CS} \ge m+T_I \end{cases} \quad (16)$$

we have $t = m + T_I - T_{CS}$. Here $T_{CS} \ge T_I - m$ represents the case where there are no hidden nodes to the left of node A and $T_{CS} \ge \frac{m+T_I}{2}$ is to make sure that the maximum distance between the hidden nodes is less than T_{CS} such that they can not be active together and hence the intersection terms are all zero and (15) holds. Finally, note that the hidden nodes can cause outage at node B over at most two consecutive time slots, hence the use of the equivalent transmission probability, $p' = 2p - p^2$. Now, assuming independent transmissions across time slots (assumption 3), the delay-bounded packet success probability can now be computed as $P_s^D = 1 - (1 - P_s)^D$. Our results confirm that for this MAC scheme, the optimal data rate is still $R^* = 9$ Mbps when $\alpha = 2$ and $R^* = 18$ Mbps when $\alpha = 4$.

As for the relation between the two metrics for channel sensing, we have $P_{CS} = (xT_{CS})^{-\alpha}$ in the absence of fading. Note that we have accounted for only the closest interferer, and neglected the case where the cumulative interference from many interferers, that are all more than T_{CS} hops away, adds up to P_{CS} watts (assumption 4). Furthermore when Rayleigh fading is present, it can be shown that the *effective* P_{CS} is emitted from a node that is $\Gamma(1 + \frac{1}{\alpha})$ times closer to the transmitting node than that predicted by T_{CS}.

Fig. 4 numerically depicts the variation of delay-bounded packet success probability with channel access probability and carrier sensing hop distance using the *SAP/CS* scheme. As discussed before, the optimal rate is fixed for a given path loss exponent. To validate our analytic result, we performed simulations using NS-2 version 2.34 [29]. This version has the added capability of accounting for the capture effect and cumulative interference in calculating the SIR. We exploit the capture capability to have a fair comparison with our analytical result. The already implemented MAC scheme is the standard 802.11 CSMA/CA which we modified into a slotted version that performs carrier sensing at the beginning of each slot without performing back-offs. We also used the suggested 802.11p parameters in our simulations (see Table III). Fig. 5 shows the simulation results for a single lane highway in which the path loss exponent is $\alpha = 2$. As evident from the figure, the analytic result gives the precise carrier sensing range. We also observed that it correctly renders the optimal transmission rate. However, the channel access probability proposed by analysis is smaller than the optimal value, though our simulations revealed that using the access probability proposed by analysis, results in a success probability which is always within 5% of that achieved using the optimal access probability from simulations. Intuitively, the reason why the success probability is not too sensitive to the channel access probability in the *SAP/CS* scheme lies in making "intelligent" transmissions (and not just random ones as in the *SAP* or *SSP* schemes) by performing carrier sensing prior to each transmission.

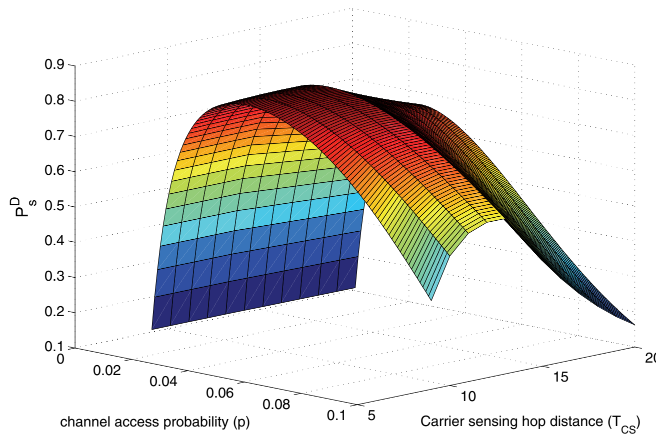


Fig. 4. Numerical evaluation of $P_s^D = 1 - (1 - P_s)^D$ for the *SAP/CS* scheme where P_s is given by (16). Here P_s^D is shown as a function of p and T_{CS}, for the optimal rate of $R^* = 9$ Mbps when $\alpha = 2$. Also in this scenario $\tau_c = 0.03$ seconds and $l = 200$ Bytes.

TABLE III
SIMULATION PARAMETERS

Channel frequency	5.9 G.Hz
Bandwidth	10 M.Hz
Header duration	40 μs
Symbol duration	8 μs
Noise floor	-99 dBm
Antenna gain	1

C. *Numerical and Simulation Evaluation of Design*

We now numerically investigate our analytic design for the collision avoidance application and verify it against simulation results. We shall study the performance of all three proposed MAC schemes in a highway consisting of N lanes. We initially fix the equilibrium traffic speed at $v = 20\frac{m}{s}$ and vary the inter-vehicle spacing. We initially assume that the emergency deceleration rate is $b = -6\frac{m}{s^2}$ for all vehicles. We shall later consider a more general case where vehicles brake with different deceleration rates in the next section. We consider a 4-lane and 8-lane highway with nodes transmitting according to the *SSP* scheme. We also study a 4-lane highway where nodes deploy the *SAP* and *SAP/CS* schemes.

First consider the range of inter-vehicle spacings where Fig. 1 predicts a collision between vehicles V_0 and V_1 following a deceleration by the former. The corresponding tolerable delay to inform V_2 is calculated through Table I after the kind of collision between vehicles V_0 and V_1 has been determined from Fig. 1. Fig. 6 shows the achievable delay-bounded packet success probability $(1 - \epsilon')$ at V_2 for this range of spacings and for a target $\epsilon = 10^{-2}$.

Note that for inter-vehicle spacings below 25.3 meters we have $\tau_c(2) \le 0$ and hence $P_s^D = 0$ in Fig. 6. This means that vehicle V_2 shall collide with the pile-up no matter what

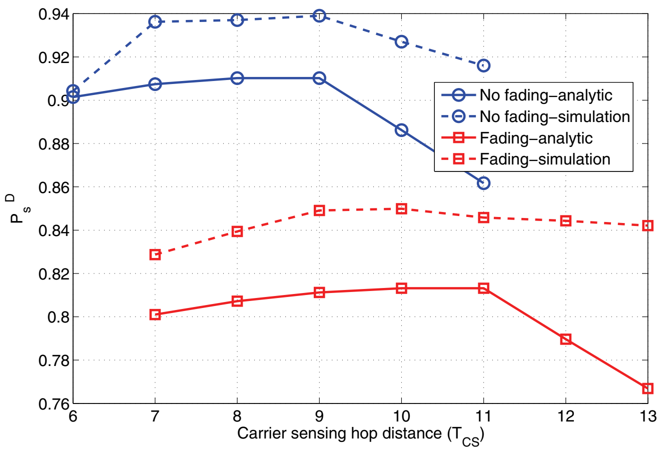


Fig. 5. Demonstrating the optimal carrier sensing hop distance of the *SAP/CS* scheme. Here again $P_s^D = 1 - (1 - P_s)^D$ where P_s is given by (16). Both analytic and simulation results are for optimal channel access probability and transmission rate. Here $\tau_c = 0.03$ seconds and $l = 200$ Bytes.

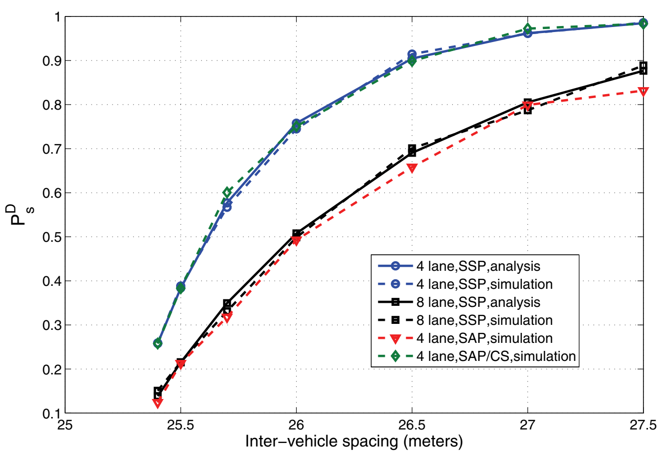


Fig. 6. $P_s^D (= 1 - \epsilon')$ at vehicle V_2 for different number of lanes and different transmission schemes. Here $\epsilon = 10^{-2}$.

communications parameters we choose. Though to reduce the severity of the accident, we find the optimal p that achieves maximum success probability at vehicle V_2. Based on prior discussions, for the *SSP* scheme this is $p^* = 1/(N\gamma_2)$ where $\gamma_2 = 2\pi\sqrt{\beta^*} - 1$. For larger inter-vehicle spacings where $\tau_c(2) > 0$ which happens for $x > 25.3$ meters, p^* for the *SSP* scheme is set according to (8) when (9) is satisfied and according to (10) otherwise, where in all cases γ_2 is replaced by $N\gamma_2$. Note that when the latter case happens the target ϵ is not achievable (which is the case for the range of spacings shown in Fig. 6). We also performed NS-2 simulations which as evident from the figure, validates our analytic result for the *SSP* scheme. Simulation results also confirm our previous analytic finding that a network whose nodes transmit asynchronously, performs similar to a network which has double the amount of lanes but deploys synchronous transmitters (compare 8-lane *SSP* and 4-lane *SAP*). We can also see that the *SAP/CS* scheme performs as well as the *SSP* scheme. The question of which one of the two schemes to use is really a question of which of the two factors of synchronization (for the *SSP* scheme) or adding the extra functionality of carrier sensing (for the *SAP/CS* scheme) is a greater burden to the network designer.

Now consider the range of inter-vehicle spacings in which Fig. 1 predicts "No collision" between vehicles V_0 and V_1. This happens for $x > v\tau_{\max} = 50$ meters which is not in the range covered by Fig. 6. In this case we can relax the vigilance condition and consider a distracted driver for vehicle V_1. We attain this by addressing P_s^D at vehicle V_1 instead of V_2. Remember that for this case $\tau_c(1) = \frac{x}{v} - \tau_{\max}$. Again for the *SSP* scheme p^* is set according to (8) when (9) is satisfied and according to (10) otherwise, where in both cases $N\gamma_1 = N(\pi\sqrt{\beta^*} - 1)$ replaces γ_2. Note that in this range of inter-vehicle spacings, and as we have considered homogenous traffic and worst case P-R time, if vehicle V_1 can avoid the accident, so can other vehicles in the chain.

D. Design Implications for Chain Collisions

In the previous section we studied the design of the collision avoidance application to address the primary collision; that is to achieve a target success probability of ϵ at vehicle V_2 (or vehicle V_1 based on the traffic conditions) upon vehicle V_0's sudden deceleration. In this section we study the effect of such parameter design on secondary collisions of subsequent vehicles in the chain. Note that with enabled communications between vehicles, an upstream vehicle can start slowing down as soon as it gets informed about the deceleration, irrespective of the state of its leader. In this sense, communications delay is the main factor that influences the collision probability and severity of upstream vehicles. We shall see that with the goal of minimizing the maximum expected collision probability in a chain of vehicles, one might have to compromise on the achievable ϵ for the primary collision. Equivalently, increased safety in terms of primary collisions comes at the expense of reduced safety for upstream vehicles in danger of secondary collisions. However as we shall see substitute solutions such as lane change maneuvers or cooperative braking can be undertaken to compensate for the increased probability of such secondary collisions.

Note that for periodic safety messages, multi-hop routing of packets is not relevant. Here, a vehicle which is too far to directly receive a message from the fast braking vehicle V_0, typically gets informed about the event via one of the closer vehicles' transmissions after the latter has reacted to the event. Moreover from the time it is informed, it takes vehicle V_i, $\tau_{pr}(i)$ seconds to start decelerating, after which the deceleration information shall reflect in its status messages and hence help inform further upstream vehicles of the event. To study the occurrence and severity of collisions in a chain, we need to find the average time it takes to inform an upstream vehicle. Let p be the chosen channel access probability of the *SSP* scheme. Due to prior discussion in Section III-B, successful reception at vehicle V_i has a distribution of a geometric random variable with parameter $\frac{1+\beta}{\beta} p e^{-Np\gamma_i}$ for an N-lane highway where $\gamma_i = \pi i\sqrt{\beta} - 1$. Hence it takes on average $\mu(i) = \frac{\beta e^{Np\gamma_i}}{p(1+\beta)}$ slots for vehicle V_i to receive vehicle V_0's packets. The discrepancy between reception times at subsequent vehicles is the main cause behind secondary collisions. Furthermore, since it takes exponentially longer to

$$Delay(i) = \min\Big(\min_{j\in\{1,\cdots,i-2\}} \mu(j) + \tau_{pr}(j) + \mu(i-j),\quad \mu(i),\quad \mu(i-1) + \tau_{pr}(i-1)\Big),\qquad i > 2 \tag{17}$$

receive a packet as we move further away from vehicle V_0, one might expect to see such vehicles being more involved in collisions. However, notice that vehicle V_i not only receives from vehicle V_0, but also from other vehicles that have already received the message and reacted to the sudden deceleration. With this in mind the average delay of reception at vehicle V_i can be found from (17).

In (17), $\mu(1) = Delay(1) = 0$ since there is no need for communications between two adjacent vehicles that have direct line-of-sight. In deriving (17) we have considered the fact that vehicle V_i either first receives the deceleration information directly from vehicle V_0, or via one of the leading vehicles that have already reacted to the event, or through its immediate leader after it has reacted to the event.

We run Monte Carlo simulations to study vehicle collisions within the chain. We consider P-R times of different vehicles being independently drawn from a log-normal distribution as in Section III-A [27]. Moreover we assume that each vehicle decelerates as soon as it is informed with a rate chosen uniformly at random from the interval $[-4 \quad -8]\frac{m}{s^2}$ [11]. Fig. 7 demonstrates the individual vehicle collision probabilities in a chain of 20 vehicles. As can be seen from the figure, for large ϵ ($\epsilon = 10^{-0.1}$), probability of collision at vehicle V_2 is large (about one third), and the highest amongst all other vehicles in the chain. Decreasing ϵ to a moderate value of $10^{-1.5} = 0.0316$, reduces the maximum collision probability of a vehicle in chain (which still corresponds to V_2) by about 82%. For any further reduction of ϵ beyond this threshold value, the maximum collision probability shall start rising but no longer correspond to V_2. That is, for $\epsilon = 10^{-3}$ this probability is about 0.1 and corresponds to V_8. Notice the tradeoff here: guaranteeing a better packet reception probability at V_2 comes at the expense of increased expected collision probability at the upstream vehicle V_8.

Fig. 8, better demonstrates the above observation over a range of ϵ values for two different inter-vehicle spacings in a 4-lane and 8-lane highway. Here, to get an understanding of the severity of the accidents, the y-axis shows the maximum expected relative speed at the time of impact (instead of the expected collision probability shown in Fig. 7). The numbers on the markers show the vehicle number which is involved in the maximum severity collision. Looking at Fig. 8(a), for $\epsilon = 10^{-1.5}$ the maximum accident severity is minimal and corresponds to vehicle V_2. Slightly decreasing ϵ shall increase the maximum collision severity which now corresponds to the last vehicle in chain (vehicle V_{20}). Hereafter, the maximum collision severity shall constantly increase, and the vehicle which suffers it be closer to vehicle V_0, as we further decrease ϵ (for $\epsilon = 10^{-3}$ the maximum severity collision shall correspond to vehicle V_8). For smaller ϵ values i.e. $\epsilon < (1 - \frac{1+\beta}{N\beta e\gamma_2})^D$, the maximum collision probability of a chain of vehicles remains fixed at a specific upstream vehicle (vehicle V_8 for the 4-lane and vehicle V_7 for the 8-lane highway). Note that the position of this vehicle in chain

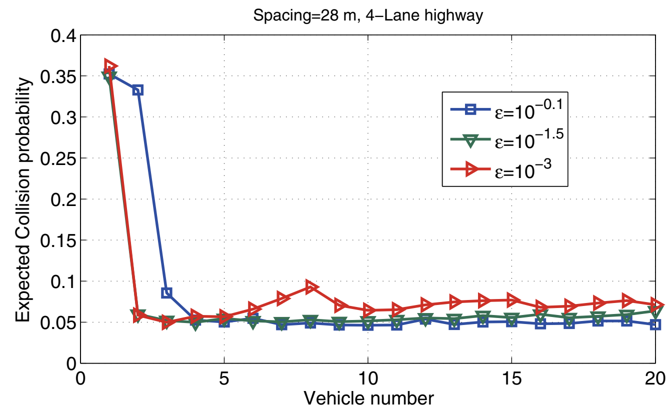


Fig. 7. Expected collision probability in a chain of 20 vehicles using the *SSP* transmission scheme. Notice the "ripples" in collision probability especially observable for $\epsilon = 10^{-3}$. The reason behind this effect is that the P-R time of vehicles before vehicle V_8 is comparable to the incurred reception delay of V_8 from V_0. Hence such vehicles cannot help by notifying V_8 of the event. For vehicles beyond V_8, however, such intermediate vehicles can help reduce the reception delay since they have had enough time to react themselves and notify other vehicles.

does not depend on the inter-vehicle spacing and is closer to vehicle V_0 when the number of lanes increases. This should not come as a surprise since we showed at the beginning of this section that the average delay to inform a vehicle does not depend on the inter-vehicle spacing x (but rather on its hop count to the source) and also increases exponentially with the number of lanes N. Furthermore, note again the tradeoff here where the higher packet reception probability for vehicle V_2 (smaller ϵ) comes at the expense of increased expected collision severity for an upstream vehicle. This could be a design option especially in scenarios where upstream vehicles have other means of avoiding the accident (e.g. using cooperative braking[3], maneuvering to adjacent lanes, etc). However our goal here is to minimize the maximum expected collision probability (or equivalently severity) in a chain when vehicles decelerate as soon as are informed about the event.

Fig. 9 demonstrates the individual vehicle collision probabilities for a single lane, 2-lane, 4-lane, and 8-lane highway when $\epsilon < (1 - \frac{1+\beta}{N\beta e\gamma_2})^D$. We can observe from the figure that for such ϵ values, the maximum collision probability in the chain corresponds to the 10^{th}, 9^{th}, 8^{th}, and 7^{th} upstream vehicle for the single lane, 2-lane, 4-lane, and 8-lane highway, respectively. Based on this, in order to minimize the maximum expected collision probability of vehicles in a chain, packet

[3]Up until now we have assumed that all vehicles react immediately after being informed about V_0's deceleration. This by itself could cause secondary collisions due to the discrepancy of reception times at adjacent upstream vehicles. As an example of cooperative braking, V_3 can hold off its deceleration to the extent possible such that the time difference between its deceleration with that of V_4 is smaller (which means it could tolerate larger reception delays as a result of smaller ϵ at V_2). This would reduce the collision probabilities between adjacent upstream vehicles. Actual deployment of such scheme demands cooperation between vehicles whose details are left for future research.

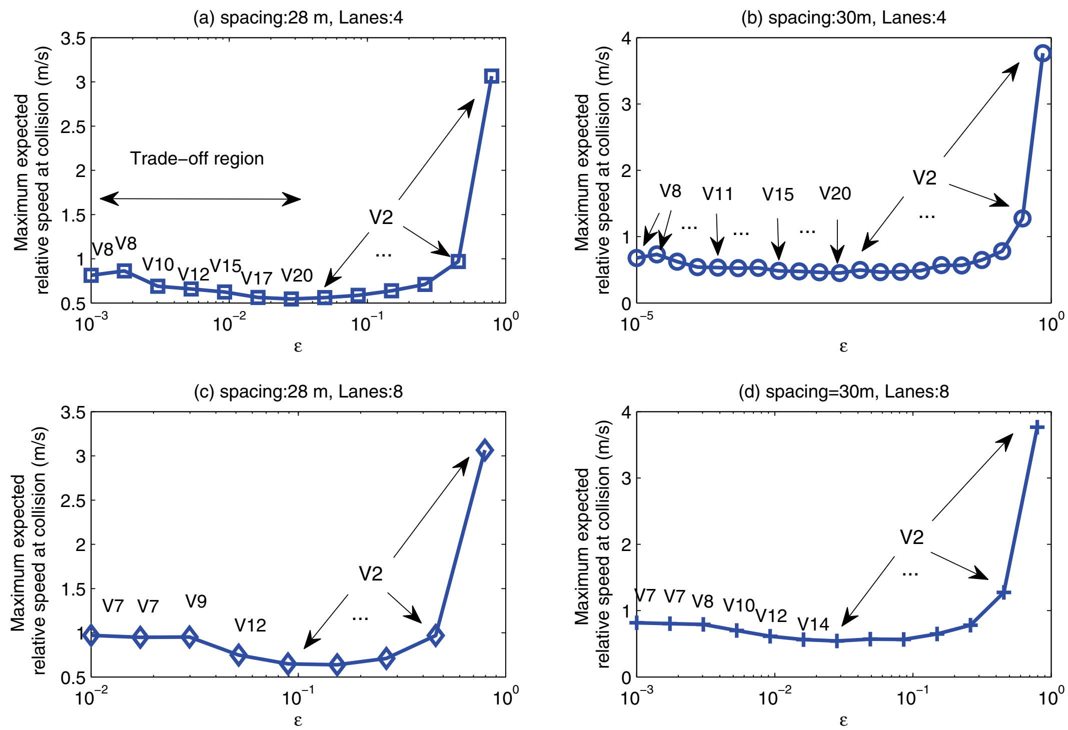


Fig. 8. Maximum expected relative speed at the time of collision in a chain of 20 vehicles using the *SSP* transmission scheme.

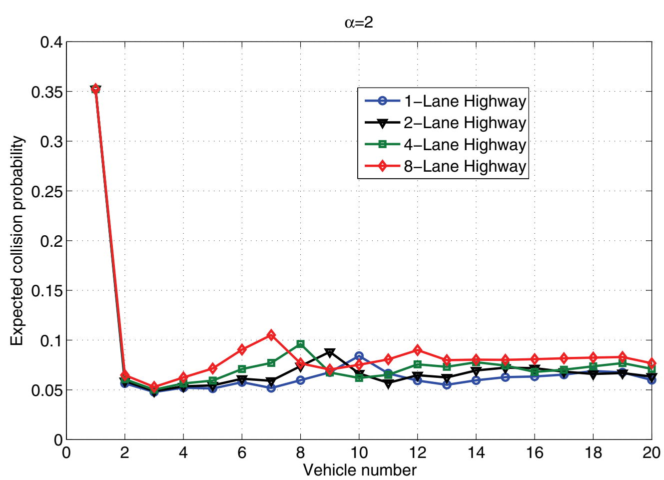


Fig. 9. Expected collision probability of vehicles in a chain of 20 vehicles for $\epsilon < (1 - \frac{1+\beta}{N\beta e\gamma_2})^D$ using the *SSP* transmission scheme. For the 3-lane and 5-lane highway (not shown), V_8; and for the 6-lane and 7-lane highway (not shown), V_7 suffers the highest collision probability.

TABLE IV
γ_{opt}

	$\alpha = 2$	$\alpha = 4$
1-Lane	$10\pi\sqrt{\beta_1} - 1$	$\frac{11}{\sqrt{2}}\pi\beta_2^{\frac{1}{4}} - 1$
2-Lane	$2(9\pi\sqrt{\beta_1} - 1)$	$2(\frac{10}{\sqrt{2}}\pi\beta_2^{\frac{1}{4}} - 1)$
4-Lane	$4(8\pi\sqrt{\beta_1} - 1)$	$4(\frac{9}{\sqrt{2}}\pi\beta_2^{\frac{1}{4}} - 1)$
8-Lane	$8(7\pi\sqrt{\beta_1} - 1)$	$8(\frac{8}{\sqrt{2}}\pi\beta_2^{\frac{1}{4}} - 1)$

success probability should be maximized at the vehicle which suffers the highest collision probability. Based on this we propose the following:

Proposition 1. *The optimal channel access probability of the SSP scheme that minimizes the maximum expected collision probability (or severity) in a chain is* $p_{\text{opt}} = \frac{1}{\gamma_{\text{opt}}}$ *where* γ_{opt} *is determined according to Table IV. Here as discussed before* $\beta_1 = 11$ *db and* $\beta_2 = 20$ *db.*

Note that Table IV also addresses the case for $\alpha = 4$ where the vehicles with the maximum collision probability are the 11^{th}, 10^{th}, 9^{th}, and 8^{th} vehicles, for the single lane, 2-lane, 4-lane, and 8-lane highways respectively. The significance of the above result is that the optimal channel access probability is only dependent on the human factors (lognormal distribution of perception-reaction time and the drivers' deceleration behaviors) and the highway characteristics (number of lanes, N, and path loss exponent, α), and does not rely on the speed and the spacing of the vehicles. Hence our design does not have to constantly adapt to the specific traffic conditions. The ϵ value attained at V_2 by this parameter selection, is essentially the one which resulted in the minimal maximum collision severity in Fig. 8. Based on prior discussions, for the *SAP* scheme all γ_{opt} values are multiplied by a factor of 2. As for the *SAP/CS* scheme, the optimal carrier sensing range and channel access probabilities are determined from (16) so as to maximize the reception probability at the corresponding vehicle mentioned above. We validated our proposition via extensive NS-2 simulations. In all cases, such parameter selection led to the minimized maximum collision probability in the chain. Fig. 10 illustrates an example scenario. The analytical result represents that obtained from (17) using the geometric distribution of

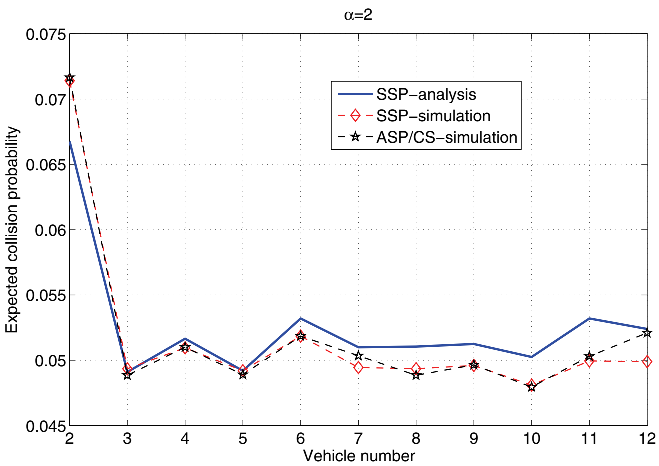


Fig. 10. Minimized maximum expected collision probability in a 4-lane highway. For *SSP*-analysis, the optimal channel access probability and rate are obtained from Proposition 1. The result then follows from (17) to obtain the expected collision probabilities.

success probability for the *SSP* scheme. As evident from the figure, as before the *SSP* and *SAP/CS* schemes perform equally well and close to what is proposed by analysis.

IV. Conclusions and Future Work

In this paper we analytically studied the performance of a collision avoidance application based on the dissemination of periodic safety messages in a dense vehicular ad-hoc network. We addressed the optimal channel access probability, transmission rate, and carrier sensing range of three synchronous and asynchronous MAC schemes with the goal of fulfilling the application needs of the safety system. We also studied parameter design for the goal of minimizing the maximum expected collision probability in a chain of vehicles. Although in this paper we focused more on a rear-end collision avoidance application, our general analytic framework also allows addressing other safety applications where delay is a critical factor. Note that an assumption of this paper was that all nodes where equipped with communications devices. In a future paper we shall study how market penetration rate of communications-enabled vehicles affects our design. We also seek to address traffic flow in the free-flow regime. According to the literature in traffic flow theory, nodes can be modeled as a Poisson Point Process in this regime. Hence accessorizing the use of stochastic geometry tools in our analysis.

Appendix

Proof of Theorem 1

Proof: Due to (1), conditioned on the presence of a transmitter and receiver distance r apart, the success probability is:

$$\begin{aligned} P_s &= \mathbb{P}(h > \beta r^\alpha I) \\ &= \mathbb{E}[e^{-\beta r^\alpha I}] \end{aligned} \tag{18}$$

Where the last equality is due to the Rayleigh fading assumption. Further,

$$\begin{aligned} P_s = \mathbb{E}[e^{-\beta r^\alpha I}] &= \mathbb{E}[e^{-\beta r^\alpha \sum_{i\in\Phi} b_i h_i r_i^{-\alpha}}] \\ &= \prod_{i\in\Phi} \mathbb{E}[e^{-\beta r^\alpha b_i h_i r_i^{-\alpha}}] \\ &= \prod_{i\in\Phi}[1-p+\frac{p}{1+\beta r^\alpha r_i^{-\alpha}}] \end{aligned} \tag{19}$$

Where the last equality follows by taking the expectation with respect to the bernoulli variable b_i and Rayleigh fading h_i. Here the interferers are symmetrically located about the receiver except that there is no interferer at the location of the transmitter (see Fig. 2). Hence:

$$P_s = \frac{1+\beta}{1+(1-p)\beta}[\prod_{i=1}^{\infty} 1-p+\frac{p}{1+\beta r^\alpha r_i^{-\alpha}}]^2 \tag{20}$$

Now, for $r = mx$, $r_i = ix$ and $\alpha = 2$ we have:

$$P_s = \frac{1+\beta}{1+(1-p)\beta}[\prod_{i=1}^{\infty} \frac{1+(1-p)\beta\frac{m^2}{i^2}}{1+\beta\frac{m^2}{i^2}}]^2 \tag{21}$$

Now using Euler's product formula $\sin(\pi z) \equiv \pi z \prod_{i=1}^{\infty}(1-\frac{z^2}{i^2})$, we have:

$$P_s = \frac{1+\beta}{1+(1-p)\beta}[\frac{\sinh(\pi m\sqrt{\beta(1-p)})}{\sqrt{1-p}\sinh(\pi m\sqrt{\beta})}]^2 \tag{22}$$

From which (2) follows with $\gamma_m = m\pi\sqrt{\beta}\coth(m\pi\sqrt{\beta}) - 1 > m\pi\sqrt{\beta} - 1$ where the latter inequality is tight for $\beta > 1$. For $\alpha = 4$ we replace $r = mx$, $r_i = ix$ but this time $\alpha = 4$ in (20). Here we can again use Euler's product formula after applying $1 - \frac{z^4}{i^4} = (1-\frac{z^2}{i^2})(1+\frac{z^2}{i^2})$. Now carrying out the same procedure as above and some manipulations, we can see that (2) holds true for $\alpha = 4$ when p is not too close to 1 and $\gamma_m \approx \frac{m\pi\beta^{\frac{1}{4}}}{\sqrt{2}} - 1$ which is tight for $\beta > 1$. ∎

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