Analytic Design of Active Vehicular Safety Systems in Sparse Traffic

Mohammad Nekoui and Hossein Pishro-Nik
University of Massachusetts, Amherst, Electrical and Computer Engineering Department
{nekoui, pishro}@ecs.umass.edu

ABSTRACT

We propose a design methodology to determine the optimal transmission parameters for delay-critical safety applications in vehicular ad hoc networks. We develop a model to characterize the delay requirements needed to prevent rear-end collisions. By adopting a stochastic geometry framework to simultaneously address multi-user interference, path loss, and fading, we then analytically derive the transmission rate, range, and channel access probability of the nodes that satisfy the delay requirement at a target success probability.

Categories and Subject Descriptors

C.2.1 [**Computer-Communication Networks**]: Network Architecture and Design-Wireless Communication.

General Terms

Design, Reliability, Theory.

Keywords

Safety, Stochastic geometry, VANET.

1. INTRODUCTION

VANETs are considered to augment the safety and efficiency level of tomorrow's transportation systems by utilizing Dedicated Short Range Communications (DSRC) to establish vehicle-to-vehicle and vehicle-to-roadside communications. There has been a plethora of studies considering the exchange of safety related information between vehicles (see for example [1]). The driving force behind most such studies is to propose efficient communications algorithms that achieve high packet success probabilities. This, however, usually goes without accounting for the safety requirements of the system. Only recently some experimental and simulation-based studies addressed the communications requirements needed for safety applications [2][3]. In this paper we take an analytic approach for adjusting the communications parameters of the VANET for such delay-critical safety applications.

In a typical crash scenario where a driver applies a sudden brake, the following vehicle's driver does not immediately observe this event due to a number of factors such as low visibility in extreme weather conditions, distracted driver attention, driver drowsiness, or even defected brake lights. Hence the need to use the assistance of a communications system which attracts the drivers attention just in time and avoids collisions is justified.

We study traffic flow in the sparse regime. In this regime it has been observed that at any given time, the positions of vehicles can be modeled based on a Poisson process on the road [4]. By analyzing the vehicle's equations of motion, we derive the delay requirements of a safety system that seeks to prevent collisions in case of emergency braking. With poisson distribution for nodes, stochastic geometry tools are utilized to address the communications parameters of the system (namely the optimal values for the channel access probability, transmission rate, and range) that fulfil the delay requirements of the collision avoidance application.

In a separate paper [5] we address parameter tuning for the non-sparse regime.

2. ANALYSIS AND DESIGN

The Media Access Control (MAC) scheme of the network is based on slotted ALOHA. Here, at every time slot, each node transmits with probability p independent of all other nodes. It has previously been shown that due to channel access delays, carrier sensing strategies such as CSMA/CA do not perform as well for delay-critical periodic broadcasts [1].

We assume that the distance x from the trailing vehicle $i+1$ is known to the leader vehicle i via prior communications between the two. By analyzing the equations of motion of the two vehicles, the communications delay τ_c should satisfy the following in order to prevent collision between the two:

$$\tau_c \leq \frac{x}{v_{i+1}} + \frac{v_i^2 - v_{i+1}^2}{2bv_{i+1}} - \tau_{PR} \qquad (1)$$

Where v_i and v_{i+1} are the speeds of the leading and trailing vehicles, respectively, b is the vehicle deceleration rate, and τ_{PR} is the driver Perception-Reaction time.

The safety application seeks to successfully deliver at least one packet within the τ_c seconds window to the trailing vehicle. Let R be the rate of transmission and l be the packet length. This way, the allowable number of transmission opportunities is:

$$D = \lfloor \frac{\tau_c R}{l} \rfloor \qquad (2)$$

We address P_s^D, which is the success probability after D transmission trials (hence P_s^D shall be referred to as the delay-bounded success probability hereafter). The safety criterion of the collision avoidance system is to achieve a *target* success probability of $1-\epsilon$ or higher in delivering a packet to vehicle $i+1$ within the period of the D time slots. This way ϵ is the vehicle collision probability. Assuming independent success probabilities across time slots, we have:

$$P_s^D = 1 - (1 - p(1-p)P_s)^D \qquad (3)$$

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Table 1: Optimum transmission rates (R) for various values of the path loss exponent (α).

α	R^* (Mbps)
$2 < \alpha \leq 3$	9
$3 < \alpha \leq 4$	18
$\alpha > 4$	24

Where P_s is the success probability given that a node transmits and the intended receiver distance r away listens. Given a fixed coding scheme that requires the SIR to be larger than some threshold β to have successful transmission at a given bit-rate, it can be shown that for a set of poisson distributed interferers and in the presence of Rayleigh fading [6]:

$$P_s = e^{-p\zeta} \tag{4}$$

In which:

$$\zeta = 2kr\beta^{\frac{1}{\alpha}}\frac{\pi}{\alpha}\csc(\frac{\pi}{\alpha}) \tag{5}$$

Where k is the node density and α is the path loss exponent. Note that since ζ and D are both a function of β, P_s^D depends on p and β.

In our design, we select the media access probability p, and the transmission rate R such that $P_s^D = 1 - \epsilon$ is guaranteed for the largest possible transmission range r greater than x. This provides a guaranteed QOS for vehicle $i+1$, while allowing for the largest population of vehicles beyond it being informed as a bonus. In case $P_s^D = 1 - \epsilon$ is not feasible at the trailing vehicle, we need to adjust the parameters such that the collision probability between vehicles i and $i+1$ stays the lowest possible.

Now, differentiating (3) with respect to p renders the optimal access probability:

$$p^* = \frac{1}{\zeta} + \frac{1}{2}(1 - \sqrt{1 + \frac{4}{\zeta^2}}) \gtrapprox \frac{1}{2+\zeta} \tag{6}$$

The latter approximation is because $p^*(0) = \frac{1}{2}$ and also that $p^* = \Theta(\frac{1}{\zeta})$ as $\zeta \to \infty$. Replacing (6) in (3) we have:

$$P_s^{D*} \gtrapprox 1 - (1 - \frac{1+\zeta}{(2+\zeta)^2}e^{-\frac{\zeta}{2+\zeta}})^D \tag{7}$$

We can now investigate the value of β (and the corresponding rate R) which maximizes P_s^{D*}. For this we use the set of transmission rates and the corresponding SIR decoding thresholds anticipated in IEEE 802.11p standard. Interestingly we observed that the maximizing transmission rate was only a function of the path loss exponent α. Table 1 shows the optimum transmission rates for some practical α values.

To guarantee a target delay-bounded success probability, we set:

$$P_s^{D*} = 1 - \epsilon \tag{8}$$

Where P_s^{D*} is as in (7), and the optimal data rate chosen from Table 1 is used to compute ζ. This equation needs to be solved numerically to find ζ^* which leads to the optimal access probability $p^* = \frac{1}{2+\zeta^*}$ and maximum tranmission range $r^* = \frac{\zeta^*}{2k\beta^{*\frac{1}{\alpha}}\frac{\pi}{\alpha}\csc(\frac{\pi}{\alpha})}$.

Note that in case (8) does not have a solution or when $r^* < x$, the target collision probability of ϵ is not achievable at the trailing vehicle. In this case we set $r^* = x$. The new achievable success probability, $1 - \epsilon'$, can then be obtained from (7)where ζ is calculated from (5) using again the optimal data rate from Table 1 but this time a transmission range of $r^* = x$.

To see the numerical implications of our design, we assume a highway of N lanes. With independent Poisson traffic on each lane and all lanes having the same density, the resulting process of the interferers is itself Poisson with intensity Nk. We let $\tau_{PR} = 2$ seconds and $l = 500$ bytes.

Figure 1(a) shows the achievable success probability $1 - \epsilon'$ for a target collision probability of $\epsilon = 0.05$, as the inter-vehicle spacing varies. We study highways with different number of lanes, corresponding to different values of vehicle densities. As can be seen from the figure, for a 1-lane highway, the achievable success probability rises sharply over a short interval to reach its target value $1 - \epsilon$. This rise is more gradual as the number of lanes increases. Interestingly, according to Figure 1(b), the optimal access probability is somewhat constant over the range of inter-vehicle spacings where $\epsilon' > \epsilon$ and then starts decaying. Moreover the figure suggests that when the target success probability is achievable i.e. $\epsilon' = \epsilon$, the optimal access probability is the same for all vehicular densities, however, according to Figure 1(c), the corresponding reaching range is lower at higher densities.

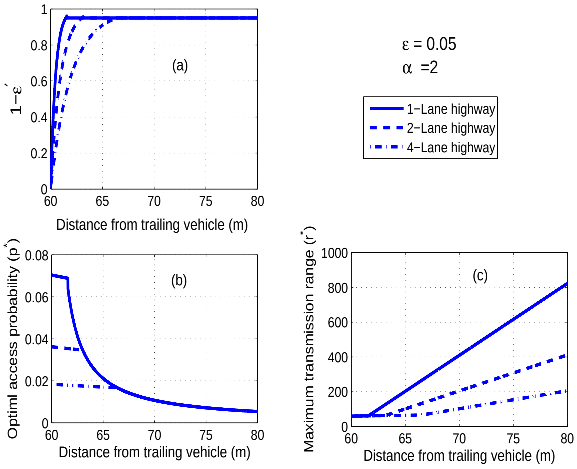

Figure 1: Achievable delay-bounded packet success probability, optimal access probability, and maximum transmission range, all as a function of inter-vehicle spacing. $v_i = v_{i+1} = 30\frac{m}{s}$, $k = 20$ *veh/km/lane* .

3. REFERENCES

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