Results on Finite Wireless Networks on a Line

A. Eslami, M. Nekoui, and H. Pishro-Nik

***Abstract*— Today, due to the vast amount of literature on large-scale wireless networks, we have a fair understanding of the asymptotic behavior of such networks. However, in real world we have to face finite networks for which the asymptotic results cease to be valid. We refer to networks as being finite when the number of nodes is less than a few hundred. Here we study a model of wireless networks, represented by random geometric graphs. In order to address a wide class of the network's properties, we study the threshold phenomena. Being extensively studied in the asymptotic case, the threshold phenomena occurs when a graph theoretic property (such as connectivity) of the network experiences rapid changes over a specific interval of the underlying parameter. Here, we find an upper bound for the threshold width of finite line networks represented by random geometric graphs. These bounds hold for all monotone properties of such networks. We then turn our attention to an important non-monotone characteristic of line networks which is the medium access (MAC) layer capacity, i.e. the maximum number of possible concurrent transmissions. Towards this goal, we provide an algorithm which finds a maximal set of concurrent non-interfering transmissions and further derive lower and upper bounds for the cardinality of the set. Using simulations, we show that these bounds serve as reasonable estimates for the actual value of the MAC-layer capacity.**

***Index Terms*— Finite Wireless Networks, Threshold phenomena, MAC-layer capacity, Random geometric graphs, Unreliable sensor grids.**

I. Introduction

There currently exists a vast amount of literature on the asymptotic analysis of different properties for large-scale random networks [1]–[10]. However, in real world we have to face small or moderate-size networks which consist of a limited number of nodes. As previously shown by the authors, asymptotic results often cease to be valid for such networks [11], [12]. Here, we study a model which is extensively used in analyzing these networks: the random geometric graph. In a random geometric graph, vertices are distributed randomly according to a uniform distribution and there exists an edge between any two vertices not more than a specific distance apart.

We first study the threshold phenomena for monotone properties in finite wireless networks modeled by random geometric graphs. A monotone graph property is a graph property such that if a graph H satisfies it, every graph G on the same vertex set obtained by adding edges to H also satisfies the property. Note that many of the graph properties such as connectivity, bearing a complete subgraph of a specific size, or having a specific minimum degree are monotone properties. What makes the monotone properties so interesting is that the probability of having a monotone property in a large random graph jumps from a value near 0 to a value close to 1 in a relatively short interval of the communication radius. The length of this interval- known as the threshold width- has been under close scrutiny in percolation theory, statistical physics, cluster analysis and some related issues in computer science, economics and political sciences. The asymptotic behavior of the threshold phenomena for random geometric graphs is well-studied in [1]–[5] where some upper bounds have been derived for the threshold width of the monotone properties. Here, we aim to analyze the threshold phenomena when the graph consists of a finite number of nodes. In this paper, we will first find an upper bound for the threshold width of the monotone properties in finite one-dimensional random geometric graphs, called *random interval graphs*. Moving on, we extend our approach to other models of random networks such as networks with random Poisson node deployment and unreliable sensor grids. Note that previous studies on finite networks are limited to specific properties such as coverage and connectivity (see for example [11]–[16]). However, our method is a comprehensive one which leads to a bound, true for all monotone properties.

We then move on to study a non-monotone characteristic of finite wireless networks which is the MAC-layer capacity. The problem of capacity has been investigated extensively for different models of wireless networks (see for example [7], [10]). However, almost all previous analytic results are asymptotic since they consider large-scale networks. In the second part of this paper, we study the MAC-layer capacity in random line networks. Asymptotic MAC-layer capacity of ad hoc wireless networks is studied in [17]. The MAC-layer capacity is defined in [17] as the maximum possible number of concurrent transmissions at the medium access layer. However, the asymptotic result obtained there is not as precise when we consider finite networks [11]. In this paper, we analyze the average MAC-layer capacity for finite line networks. Here we obtain lower and upper bounds for the MAC-layer capacity. We also provide an algorithm which finds the exact value for the MAC-layer capacity along with a set of active links which achieves it. Our simulations show that our bounds are good estimates for real values.

The rest of the paper is organized as follows. In section II, we derive upper bounds for the threshold width of one-dimensional finite networks. We follow on by analyzing the MAC-layer capacity of random line networks in section III. The paper is concluded in section IV.

II. Threshold Phenomena in Finite Line Networks

In this section we provide an upper bound on the threshold width of finite wireless networks on a line. Consider n points distributed uniformly and independently in the d-dimensional unit cube $[0,1]^d$. Given a fixed distance $r > 0$, connect two points if their Euclidean distance is at most r. Such graphs are called random geometric graphs, and are denoted by $G(n, r)$, as in [18]. Random geometric graphs are better suited than

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This work is supported by National Science Foundation under grants CCF-0728970 and ECCS-0636569.

more combinatorial classes (such as Bernoulli random graphs) to model problems where the existence of an edge between two different nodes depends on their spatial distance. As a result, random geometric graphs have received increased attention in recent years in the context of distributed wireless networks such as sensor networks (see for example, [1], [6], [7]). In these graphs, the probability of a monotone property is an increasing function of r and when $r = \sqrt{d}$, the graph is a complete graph which satisfies every monotone property. In [2], the authors show that all monotone graph properties have a sharp threshold for large random geometric graphs. For a definition of the sharp threshold see [19]. However, the goal of most of the previous studies is to address the asymptotic behavior of the threshold phenomena. In this paper, we only consider one-dimensional random geometric graphs, called random interval graphs. In fact, random geometric graphs of higher dimensions are usually much more difficult to analyze. We believe that studying the threshold phenomena for finite one-dimensional networks could serve as the cornerstone of analyzing higher dimensions.

We now explain some notations and some definitions we need to state our results. The key idea in our analysis is to relate the behavior of monotone properties to the weight of the "bottleneck" matching (to be defined later) of the bipartite graph whose vertex sets are obtained by distributing n points uniformly and independently on $[0, 1]$. Such a relation has been exploited in [2] to find an upper bound on the threshold width in the asymptotic case. Here, we describe the concept of bottleneck matching and its relation with monotone properties. Recall that in a bipartite graph with vertex sets V_1 and V_2, a perfect matching is a bijection (a one-to-one and onto mapping) $\phi : V_1 \to V_2$, such that each $v \in V_1$ is adjacent to $\phi(v) \in V_2$. Thus a perfect matching is a disjoint collection of edges that covers every vertex. If the graph is weighted, then we define the weight of the matching as the maximum weight of any edge in the matching. A *bottleneck matching* is a perfect matching with the minimum weight. Let S_1 and S_2 denote two sets of n points each, where the points are i.i.d., chosen uniformly at random from the set $[0, 1]$. Form the complete bipartite graph on (S_1, S_2) and let the weight of an edge be the Euclidean distance between its endpoints. Let M_n denote the bottleneck matching weight of this graph. If A is a monotone property, then for $0 < \epsilon < 1$, let $r(n, \epsilon) = \inf\{r > 0 : Pr\{G(n, r) \text{ has property } A\} \geq \epsilon\}$. We define the threshold width of A as $\tau(A, \epsilon) = r(n, 1 - \epsilon) - r(n, \epsilon)$ when $0 < \epsilon < 1/2$. In [2], the authors linked the weight of the bottleneck matching with the threshold width of the monotone properties in a theorem which we repeat here.

Theorem 1: If $Pr\{M_n > \gamma(n)\} \leq p$ for some function $\gamma(n)$ and some constant p, then $\tau(A, \sqrt{p})$ of any monotone property A is at most $2\gamma(n)$.

According to this theorem, if we can find an upper bound on the probability $Pr\{M_n > \gamma(n)\}$ then we can use it to find an upper bound on the threshold width. We first state a lemma about the weight of the bottleneck matching for two sets of points on the unit interval. We omit the proof of this lemma due to the limited space.

Lemma 1: Let S_1 and S_2 be two sets of points each, where the points are chosen uniformly and randomly from the set $[0, 1]$. Let $\hat{S}_1 = X_1, X_2, ..., X_n$ and $\hat{S}_2 = Y_1, Y_2, ..., Y_n$ be the points ordered according to their positions on $[0, 1]$, i.e. $X_1 < X_2 < ... < X_n$ and $Y_1 < Y_2 < ... < Y_n$. Then the bottleneck matching is the perfect matching $\phi : S_1 \to S_2$ such that $\phi(X_i) = Y_i$ for $i = 1, 2, ..., n$. Accordingly, the weight of the bottleneck matching is $M_n = \max_{i=1,...,n} |Y_i - X_i|$.

Now we need to find an upper bound for $Pr\{\max_{i=1,...n} |Y_i - X_i| > \gamma\}$ for every γ.

Theorem 2: For the two sets of random points defined in Lemma 1 and for every $\gamma > 0$, we have

$$Pr\{M_n > \gamma\} \leq \sum_{i=1,...n} 2 \int_0^\infty f_i(u + \gamma) F_i(u) du, \quad (1)$$

where $f_i(u) = i\binom{n}{i} u^{i-1}(1 - u)^{n-i}$, and

$$F_i(u) = I_u(i, n + 1 - i) = \sum_{j=i}^{n} \binom{n}{j} u^j (1 - u)^{n-j},$$

where $I_u(i, n + 1 - i)$ is the regularized incomplete beta function with parameters i and $n + 1 - i$.

Proof: Using Union bound, we have

$$\begin{aligned} Pr\{M_n > \gamma\} &= Pr\{\max_{i=1,...,n} |Y_i - X_i| > \gamma\} \\ &\leq \sum_{i=1,...,n} Pr\{|Y_i - X_i| > \gamma\}. \quad (2) \end{aligned}$$

X_i and Y_i are the ith order statistics of the uniform distribution and have Beta distribution with parameters i and $n-i+1$ (see [20] chapter 7). Note that if we denote the PDF and CDF of X_i (or Y_i) by $f_i(u)$ and $F_i(u)$, respectively, then

$$\begin{aligned} f_i(u) =& \frac{n!}{(i-1)!(n-i)!} u^{i-1}(1 - u)^{n-i} = \\ & i\binom{n}{i} u^{i-1}(1 - u)^{n-i}, \\ F_i(u) =& I_u(i, n + 1 - i) = \sum_{j=i}^{n} \binom{n}{j} u^j (1 - u)^{n-j}. \end{aligned}$$

On the other hand, since X_i and Y_i are independent random variables, we know that

$$Pr\{|Y_i - X_i| > \gamma\} = 2 \int_0^\infty f_i(u + \gamma) F_i(u) du,$$

which along with (2) gives (1). ■

We can evaluate (1) for different values of γ and n, and hence find the upper bound for the threshold width. Note that Theorem 2 gives an upper bound for every monotone graph property and it is not limited to a specific property. The bound for $n = 50$ is depicted in Figure 1. Comparing our bound against the actual value of the threshold width for some famous graph properties, we observed that the bound does not provide a tight approximation for them. It remains as an open problem to see whether our bound is tight for any monotone property. It is noteworthy that for large n's, the bound leads to better estimates as it will converge to the bound in [2] for the asymptotic case.

An interesting point about the upper bound of Theorem 2 is that Theorem 1 and Lemma 1 hold for any two independent sets of random points that have the same size and the same distribution. Therefore, given the PDF and CDF of the order statistics of an arbitrary random variable, we can use them as $f_i(u)$ and $F_i(u)$ in (1) to find a version of Theorem 2 for that

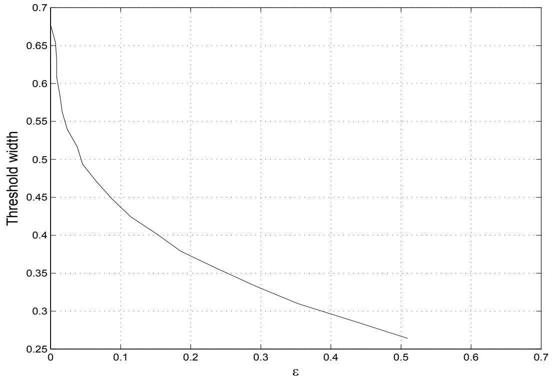

Fig. 1. Upper bound on the threshold width of the monotone properties for $G(50, r)$.

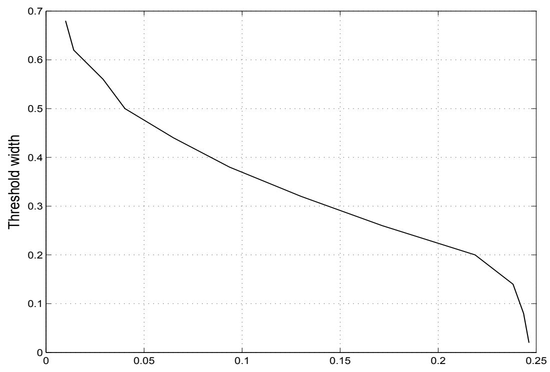

Fig. 2. Upper bound on the threshold width of the monotone properties for a one-dimensional unreliable sensor grid with parameters $n = 100$, $m = 35$.

random variable. As an example, suppose that the nodes in our line network are placed according to a Poisson point process with parameter λ. Then, for $i = 1, ..., n$ we can derive (see [20] chapter 7)

$$f_i(u) = \frac{\lambda^i u^{i-1} e^{-\lambda u}}{(i-1)!}, \quad \text{and} \quad F_i(u) = 1 - \sum_{m=0}^{i-1} \frac{(\lambda u)^m e^{-\lambda u}}{(m-1)!}.$$

Using $f_i(u)$ and $F_i(u)$ as above, we can find the upper bound of the threshold width for the geometric graphs generated by a Poisson point process.

Now, we consider an unreliable sensor grid on the unit interval which consists of n equidistant sensor nodes such that m of them are active. Note that all subsets of size m of the n nodes are equally probable to be active. For a given r, if the distance between two active nodes is less than r, there is a link between them. We can study the threshold phenomena for this unreliable grid when r ranges between 0 and 1. Note that as in [12], the probability curve would be piecewise constant for every graph property. Assuming that the grid nodes are located at the points $\frac{k}{n}$, $k = 1, ..., n$, $f_i(u)$ can be easily derived as

$$f_i(u) = \sum_{k=i}^{n-(m-i)} \frac{\binom{k-1}{i-1}\binom{n-k}{m-i}}{\binom{n}{m}} \delta(u - \frac{k}{n}). \quad (3)$$

Substituting (3) in (1), we have found the upper bound shown in Figure 2 on $\tau(A, \epsilon)$ for $n = 100$ and $m = 35$.

III. MAC-Layer Capacity

In this section we study the MAC-layer capacity of finite networks in line deployments. Given a graph $G(V, E)$, the goal is to choose a subset of the edges on which transmissions can occur without conflicting with one another. That is, if transmission along (s, t) and (s', t') are occurring simultaneously, then none of the edges (s, s'), (s, t'), (s', t), (t, t') should be present in the graph. The set of edges that can be so chosen is called a *D2-Matching* (Distance-2 Matching). The problem of finding a D2-matching of maximum cardinality is called D2EMIS [17]. There, it is shown that for a wide class of MAC protocols including IEEE 802.11, the MAC-layer capacity can be modeled as a maximum D2-matching (D2EMIS) problem in the underlying wireless network. In this paper, we define $MAC(n, r)$ as the average, over all configurations of nodes, of the cardinality of the D2EMIS for a random interval graph[1] $G(n, r)$. Note that $MAC(n, r)$ is the average value of the maximum size of the D2-matching on $G(n, r)$. We first provide analytical lower and upper bounds on $MAC(n, r)$. Then, we propose an algorithm to find the exact value of the size of the D2EMIS for any arbitrary node configuration. Using this algorithm, we compare our bounds to the actual value of the capacity.

A. Lower Bound on the MAC-Layer Capacity

In this section we introduce a lower bound on the MAC-layer capacity which is a combination of two different bounds. First, recall that a connected component of size k of a graph G is a maximal connected subgraph of G with k vertices. For a random interval graph $G(n, r)$, let us denote the number of connected components of size k by C_n^k and the total number of the connected components by C_n.

Theorem 3: For a line network modeled by a random interval graph $G(n, r)$ we have

$$MAC(n, r) \geq 1 + (n-3)(1-r)^n - (n-2)(1-2r)^n. \quad (4)$$

Proof: The proof is based on the number of the connected components in the network's graph. Since transmissions in different components do not conflict, every connected component of size greater than one can contribute at least one transmission to $MAC(n, r)$. Therefore, the average number of the concurrent transmissions is always larger than the average number of the connected components of size greater than one. The average number of the total connected components and the average number of the isolated vertices (connected components of size one) for a random interval graph are calculated in [21], Theorems 1 and 4. Using this, we have

$$\begin{aligned} MAC(n, r) \geq & E[C_n] - E[C_n^1] = 1 + (n-1)(1-r)^n - \\ & (n-2)(1-2r)^n - 2(1-r)^n = \\ & 1 + (n-3)(1-r)^n - (n-2)(1-2r)^n. \end{aligned}$$

■

Now, we prove the following lemma which leads us to a different lower bound on $MAC(n, r)$.

Lemma 2: Given a line network modeled by $G(n, r)$ and an interval I of length l on the line, let $P(l)$ be the probability

[1] one-dimensional random geometric graphs are referred to as random interval graphs [18].

of having at least one link in I. Then

$$P(l)=\begin{cases} 1-(1-l)^n-nl(1-l)^{n-1} & \text{if } l\le r,\\ 1-\sum_{k=0}^{\min(\lceil \frac{l}{r}\rceil,n)}\binom{n}{k}[l-(k-1)r]^k\times (1-l)^{n-k} & \text{if } l>r \end{cases} \tag{5}$$

Proof: If $l\le r$ then $P(l)$ is equal to the probability of having at least 2 nodes in I which is $1-(1-l)^n-nl(1-l)^{n-1}$ for n nodes distributed uniformly and independently on $[0,1]$. In the case of $l>r$, we find the probability of having no link in an interval of length l which we denote by $P_{nl}(l)$. Then we will have $P(l)=1-P_{nl}(l)$. For $P_{nl}(l)$, we have

$$P_{nl}(l)=\sum_{k=0}^{\min(\lceil \frac{l}{r}\rceil,n)} Pr\{\text{no link in } I|\ k \text{ nodes in } I\}\times Pr\{k \text{ nodes in } I\}= \sum_{k=0}^{\min(\lceil \frac{l}{r}\rceil,n)} Pr\{\text{no link in } I|\ k \text{ nodes in } I\}\times \binom{n}{k}\ l^k\ (1-l)^{n-k}. \tag{6}$$

Therefore, it suffices to find the probability of having no link in I given that there are k nodes in it. For $k=0$ and 1, this probability is trivially 1. It is easy to verify that given that $k\ge 2$ nodes are in the arbitrary interval $I=[x_i,x_i+l]$, they are distributed independently and uniformly on I. We need to find the probability of the event that these k nodes have spacings larger than r. To achieve this, we define two sets whose ratio of their volumes is the sought probability. The first set is the set of all configurations of k points in I whose volume is l^k. The other one is the set of all the configurations of k points in I, $k\le\lceil\frac{l}{r}\rceil$, for which all the $k-1$ spacings between the points are larger than r. This is, in fact, equivalent to the set of k points drawn uniformly and independently from a subinterval of length $l-(k-1)r$ of I. The volume of this set is $(l-(k-1)r)^k$. Hence, substituting $\frac{(l-(k-1)r)^k}{l^k}$ as $Pr\{$no link in $I\mid k$ nodes in $I\}$ in (6), we will find $P_{nl}(l)$ which leads us to $P(l)$ in (5). ∎

Theorem 4: For a line network modeled by $G(n,r)$, define $P(l)$ as above and $m(l)=\lfloor\frac{1}{l+r}\rfloor$ for $0<l<1$. Then

$$MAC(n,r)\ge\max_{l\in[0,1]}\left\{m(l)P(l)+P\big(1-m(l)(l+r)\big)\right\}. \tag{7}$$

Proof: The proof is based on a constructive algorithm which finds a number of possible concurrent transmissions on the unit-length line. Consider the intervals of length l in Figure 3 which are a distance r apart. We have $m(l)=\lfloor\frac{1}{l+r}\rfloor$ of these intervals which are denoted by $I_1,I_2,...,I_{m(l)}$. Also, there may be an interval of length $1-m(l)(l+r)$ at the end of the line which we denote by $I_{m(l)+1}$. Note that all these intervals do not necessarily contain an edge. However, the edges contained in $I_1,I_2,...,I_{m(l)+1}$ are at least a distance r apart and can be in the D2-matching. Therefore, the average number of the concurrent transmissions obtained in this way is equal to the average number of the intervals containing at least one edge. Let X be the number of such intervals. To find $E[X]$, we assign an indicator random variable X_i to each interval I_i which is one if there exist at least one edge in that interval and is zero otherwise. Then, we have $X=\sum_{i=1}^{m(l)+1}X_i$ and $E[X]=\sum_{i=1}^{m(l)+1}E[X_i]$. But according to Lemma 2, $E[X_i]=Pr\{X_i=1\}=P(l)$ for $i=1,2,...,m(l)$, and $E[X_{m+1}]=Pr\{X_{m+1}=1\}=P(1-m(l)(l+r))$. We can maximize $E[X]$ over l which gives us (7). ∎

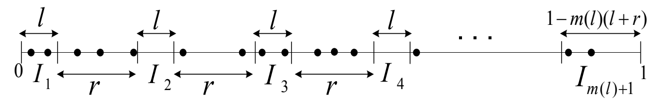

Fig. 3. Intervals corresponding to the constructive lower bound on MAC-layer capacity. Note that in the figure above we have $X_1=1$, $X_2=0$, $X_3=1$, $X_4=0$, and $X_{m(l)+1}=1$.

A lower bound on $MAC(n,r)$ can be obtained from maximum of the lower bounds given by Theorems 3 and 4.

B. Upper Bound on the MAC-Layer Capacity

In this section the upper bound on the MAC-layer capacity is addressed via a theorem which results from a combination of two bounds.

Theorem 5: For a line network modeled by $G(n,r)$ we have

$$MAC(n,r)\le\min(\sum_{k=1}^{n}E[C_n^k]\times\lceil\frac{k-1}{3}\rceil,\lceil\frac{1}{r}\rceil),$$

where

$$E[C_n^k]=\sum_{j=0}^{1}\binom{n-k-1}{1-j}\binom{2}{j}\times \sum_{i=0}^{k-1}\binom{k-1}{i}(-1)^i\ \times\big(1-(2-j+i)r\big)_+^n \tag{8}$$

with $a_+=a$ for positive a and $a_+=0$ otherwise.

Proof: Consider a connected component of size 2. This component contributes one transmission to $MAC(n,r)$. Now, consider components of size 3 and 4. According to the definition of the set of edges in a D2-matching, these components also contribute at most 1 edge to the D2EMIS. In fact, any edge chosen for D2-matching precludes at least two other edges from participating in the matching. Therefore, a component of size k can support at most $\lceil\frac{k-1}{3}\rceil$ concurrent transmissions. Thus, the average number of the concurrent transmissions is smaller than the sum of the average number of the connected components of size k times $\lceil\frac{k-1}{3}\rceil$. Average number of the connected components of size k in a random interval graph is given in [21] as (8).

On the other hand, every transmission covers at least an interval r of the line. That is, if we pick an edge as a member of the D2EMIS we can not pick any other edge for D2EMIS in a distance less than r from the first one. Therefore, there can not be more than $\lceil\frac{1}{r}\rceil$ concurrent transmissions. This completes the proof. ∎

C. Algorithm for Exact Value of the MAC-Layer Capacity

As we mentioned earlier, transmissions in different connected components do not conflict. Therefore, to find the D2EMIS, it suffices to give an algorithm for finding the maximum possible number of concurrent transmissions in every component of size greater than one. We now propose an algorithm to find the maximum number of concurrent

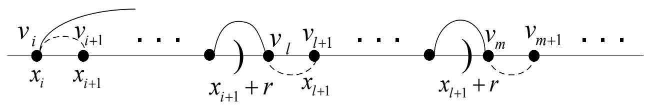

Fig. 4. Algorithm to find the exact value of the MAC-layer capacity. Dashed lines are the edges chosen by the algorithm as members of the D2EMIS.

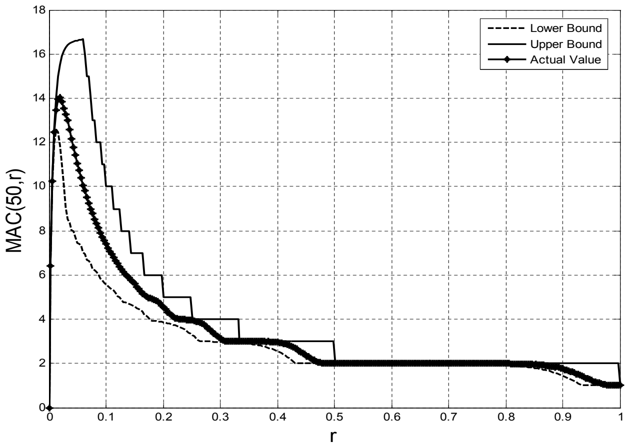

Fig. 5. Actual value of $MAC(50, r)$ along with the lower and upper bounds of Theorems 3, 4 and 5.

transmissions in a connected component. Assume that the first vertex of the component, v_i, is at location x_i and the second vertex, v_{i+1}, is located at x_{i+1}, as it is shown in Figure 4. We choose the first edge of the component, connecting v_i to v_{i+1}, to participate in the D2EMIS. So, none of the vertices in the interval $(x_{i+1}, x_{i+1} + r]$ can participate in the D2EMIS. We call the interval $[x_i, x_{i+1} + r]$ an *interference interval*. Then, we consider the first vertex located after $x_{i+1} + r$ which is v_l, and choose the edge (v_l, v_{l+1}) as another member of the D2EMIS. Again, none of the vertices within range r of v_{l+1} can participate in D2EMIS. We repeat this until we reach the end of the component. It is easy to see that this greedy choice is optimal. Consider an optimal algorithm which does not choose the first edge of the component, hence leads to an edge in D2EMIS in an interference interval larger than $[x_i, x_{i+1}+r]$. Assume that the last vertex of the component is v_p and is located at x_p. So, this algorithm has to choose the second edge of the D2EMIS from an interval shorter than $(x_{i+1} + r, x_p]$. Note that our algorithm chooses the first edge in $(x_{i+1}+r, x_p]$ as the second edge of the D2EMIS. In fact, moving toward the end of the component, our algorithm always selects its next edge for the D2EMIS from an interval at least as large as the one for the optimal algorithm. Therefore, the optimal algorithm cannot find a larger D2-matching than our algorithm.

To find $MAC(n, r)$, we need to find the average size of the D2-matching obtained by the above algorithm. However, analyzing this algorithm to find the exact value of $MAC(n, r)$ might be difficult. Figure 5 shows the exact value of $MAC(50, r)$ resulted from simulations and using the proposed algorithm, compared against the lower and upper bounds given by Theorems 3, 4 and 5. It can be easily checked that both the lower and upper bounds are asymptotically tight, regarding [17], as they are maximized at $r = \Theta(\frac{1}{n})$ resulting in a maximum of $\Theta(n)$.

IV. Conclusion

In this paper, we studied the threshold phenomena and MAC-layer capacity in finite wireless networks on a line. We considered random geometric graphs as a model for wireless networks which is used extensively in the literature. We derived an upper bound for the threshold width of such finite networks which holds for every monotonic graph property. We also studied the problem of MAC-layer capacity for finite line networks. MAC-layer capacity is an example of non-monotonic characteristics of networks. We provided an algorithm for finding its exact value and also derived lower and upper bounds. Through simulations, we verified that our bounds can give quite a good estimate of the actual value of the MAC-layer capacity.

References

[1] M. Franceschetti and R. Meester, *Random Networks for Communication: From Statistical Physics to Information Systems*. Cambridge University Press, 2008.

[2] A. Goel, S. Rai, and B. Krishnamachari, "Monotone properties of random geometric graphs have sharp thresholds," *Annals of Applied Probability*, vol. 15, pp. 2535–2552, 2005.

[3] B. Krishnamachari, S. B. Wicker, R. Bjar, and M. Pearlman, "Critical density thresholds in distributed wireless networks," in *Communications, Information and Network Security*, Kluwer Publishers, 2002.

[4] G. L. Mccolm, "Threshold functions for random graphs on a line segment," *Combinatorics, Probability and Computing*, vol. 13, pp. 373–387, 2004.

[5] S. Muthukrishnan and G. Pandurangan, "The bin-covering technique for thresholding random geometric graph properties," in *Proceedings of the sixteenth annual ACM-SIAM symposium on Discrete algorithms*, pp. 989–998, 2005.

[6] P. Gupta and P. Kumar, "Critical power for asymptotic connectivity in wireless networks," *Stochastic Analysis, Control, Optimization and Applications: A Volume in Honor of W.H. Fleming, W.M. McEneaney, G. Yin and Q. Zhang (Eds.)*, 1998.

[7] P. Gupta and P. R. Kumar, "The capacity of wireless networks," *IEEE Trans. Inform. Theory*, vol. 46, no. 2, pp. 388–404, 2000.

[8] S. Shakkottai, R. Srikant, and N. Shroff, "Unreliable sensor grids: Coverage, connectivity and diameter," *In the proceedings of IEEE INFOCOM'03, San Francisco, CA, April 2003.*

[9] S. Kumar, T. H. Lai, and J. Balogh, "On k-coverage in mostly sleeping sensor network," in *MobiCom*, September 2004.

[10] P. Gupta and P. R. Kumar, "Towards an information theory of large networks: an achievable rate region," *IEEE Trans. Inform. Theory*, vol. 49, pp. 1877–1894, 2003.

[11] H. Pishro-Nik and F. Fekri, "Analysis of wireless ad-hoc and sensor networks in finite regime," in *5th Annual IEEE Communications Society Conference on Sensor, Mesh and Ad Hoc Communications and Networks (SECON)*, pp. 73–81, 2008.

[12] H. Pishro-Nik, "Analysis of finite unreliable sensor grids," in *WiOpt'06*, pp. 243–254, April 2006.

[13] P. Balister, B. Bollobas, A. Sarkar, and S. Kumar, "Reliable density estimates for coverage and connectivity in thin strips of finite length," in *MobiCom '07: Proceedings of the 13th annual ACM international conference on Mobile computing and networking*, pp. 75–86, 2007.

[14] M. Desai and D. Manjunath, "On the connectivity in finite ad hoc networks," *Communications Letters, IEEE*, vol. 6, pp. 437–439, October 2002.

[15] N. Karmachandani, D. Manjunath, D. Yogeshwaran, and S. K. Iyer, "Evolving random geometric graph models for mobile wireless networks," in *Proceedings of the Fourth International Symposium on Modeling and Optimization in Mobile, Ad Hoc, and Wireless Networks, (WIOPT 2006)*, 2006.

[16] A. Ghasemi and S. Nader-Esfahani, "Exact probability of connectivity in one-dimensional ad hoc wireless networks," *IEEE Communications Letters*, vol. 10, pp. 251–253, 2006.

[17] H. Balakrishan, C. L. Barrett, V. S. A. Kumar, M. V. Marathe, and S. Thite, "The distance-2 matching problem and its relationship to MAC-layer capacity of ad hoc wireless networks," *IEEE J. Select. Areas Commun.*, vol. 22, pp. 1069 –1079, 2004.

[18] M. Penrose, *Random Geometric Graphs*. Oxford University Press, 2003.

[19] J. Kahn, G. Kalai, and N. Linial, "The influence of variables on boolean functions," in *Proceedings of 29th Annual Symposium on Foundations of Computer Science*, pp. 68–80, 1988.

[20] A. Papoulis and S. U. Pillai, *Probability, Random Variables and Stochastic Processes*. McGraw-Hill, 4th ed., 2001.

[21] E. Godehardt and J. Jaworski, "On the connectivity of a random interval graph," *Random Struct. Algorithms*, vol. 9, no. 1-2, pp. 137–161, 1996.