Fundamental Tradeoffs in Vehicular Ad Hoc Networks

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ABSTRACT

Due to their advantages in terms of safety, efficiency and comfort, Vehicular Ad Hoc Networks (VANETs) have recently drawn the attention of researchers from among a wide spectrum of engineering fields. Although transportation and communications engineers have independently delved into issues related with vehicular networks, analyzing them from their own perspectives; the void of a more comprehensive study which blends the theory of the two and seeks to address their mutual dependencies, is evident in the current literature. In this paper we initiate this surge by studying the interactions of traffic flow, safety and communications capacity within a simple transportation system. To that end we first render mathematical realizations for such criteria and study how the new technology can affect them and their mutual interactions. More specifically, the tradeoffs inherent in the capacity-flow and flow-safety relations have been analyzed. Our study helps foresee the effect of the gradual introduction of communications-enabled vehicles on the safety and efficiency of transportation networks before their actual deployment. [1]

Categories and Subject Descriptors

C.2.1 [**Computer-Communication Networks**]: Network Architecture and Design-Wireless Communication.

General Terms

Theory, Human Factors.

Keywords

Communications capacity, IntelliDrive^{SM}, Traffic Flow Theory, VANET.

1. INTRODUCTION

Recently, Vehicular Ad Hoc Networks (VANETs) have been under the spotlight of researchers and authorities concerned with enhancing daily driver experiences in terms of safety, efficiency and comfort. IntelliDrive^{SM}-formerly known as Vehicle Infrastructure Integration (VII)-a major initiative at the United States Department of Transportation (USDOT), proposes to use Dedicated Short Range Communications (DSRC) to establish vehicle-vehicle and vehicle-roadside communications to deliver timely information to save lives, reduce congestion, and improve quality of life [1]. IntelliDrive^{SM} provides the capability for vehicles to identify threats and hazards on the roadway and communicate this information over Vehicular Ad Hoc Networks (VANETs) to give drivers alerts and warnings. This not only enables the drivers to take preventive actions against upcoming threats, but also shortens their Perception-Reaction[2] (P-R) times with respect to that of the drivers without IntelliDrive^{SM} assistance. As we shall see, the reduction in driver P-R time results in the formation of compact, high speed platoons (and hence higher flow of vehicles) at a certain safety requirement, or alternatively, increased safety at a fixed amount of flow.

Not surprisingly, VANETs have undergone the scrutiny of mostly transportation and communication engineers in recent years. However, the relation between on-road communications and traffic efficiency has not yet been quantified nor mathematically analyzed. Within the transportation society, there are numerous papers which address how such driver-assisting technologies as cruise control can impact vehicular traffic flow [2]. However, they do not account for the available capacity for inter-vehicle communications in their frameworks. In yet another world of wireless communications, specific characteristics of VANETs such as its highly dynamic topology, delay-sensitive applications and constrained deployment region have lead to the outgrowth of an abundant number of VANET-specific physical, MAC and routing layer schemes [3, 4]. MAC and network layer issues for urban deployments of VANETs have also been addressed by the authors in [5]. The goal in most of these schemes is to establish reliable point-to-point communications between vehicles. To the best of the authors knowledge, there has little been done, if any, to study the effect of communications on traffic flow capacity and stability.

With the above introduction, the lack of a unifying theory which integrates the fundamental concepts of the two afore-

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[2]P-R time is the duration of time from the moment a phenomena happens to when the driver reacts with a preventive action.

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mentioned engineering disciplines and seeks to address measures of both is evident in the current literature. In this paper we take some initial steps in this prospect. Towards this goal, we first provide mathematical representations of safety, flow and communications capacity. Here, we define safety as the probability of a collision free driving experience between two successive vehicles. We further provide a microscopic interpretation of traffic flow. This shall later prove useful when we address flow-safety tradeoffs in VANETs. Also, due to the stringent delay requirements imposed on VANET-specific applications, inter-vehicular communications within a cluster of vehicles is considered to be single hop broadcast. With the above tools and concepts in hand, we shall study the ties and tradeoffs inherent between traffic flow, safety, and communications capacity and see how IntelliDrive^{SM} can affect them. The effect of IntelliDrive^{SM} on the flow-safety relation is studied. Further, we shall see that although there exists a tradeoff between the communications capacity and traffic flow in the initial stages of IntelliDrive^{SM} deployment, they would have a proportionate relationship after a certain percentage of the vehicles are equipped.

What is the importance of studying such tradeoffs? First, communications capacity relates to safety and comfort of driving while flow relates to efficiency, thus our results render a tradeoff between safety, comfort and efficiency. Second, in the design of automated highways, we have the option of choosing parameters such as vehicle density and such tradeoffs help us find an optimum operating point in terms of both transportation and communications capacity.

The rest of the paper is organized as follows; In section 2 we elaborate the requisite preliminaries for our analysis. Section 3 includes the main course of the paper where the effect of IntelliDrive^{SM} on flow, safety and capacity and their mutual dependencies are considered. The paper is finally concluded in section 4.

2. SYSTEM MODEL AND PRELIMINARIES

2.1 Car-following model

Here we introduce a car following model that ensures safe driving. Car following models have long been studied by transportation engineers. Models such as Pipes, General Motors (GM) and Gipps account for different car following behaviors [6]. Here, the gap between two adjacent vehicles should be such that in case of a sudden brake by the leader, the follower should be able to safely stop behind it with a comfortable deceleration rate. According to Gipps, this gap should be such that having applied a sudden brake, the final location of the leader minus its length, be greater than the final location of the follower which has gone through a comfortable deceleration process. However, we have realized that although this is a necessary condition, it is not sufficient and that to avoid collisions, the safety condition should be consistently checked during the whole of the deceleration process rather than just at the stopping point. Here we elaborate the results but skip the derivation details due to lack of space. Assume that vehicle i is following its leader vehicle $i-1$. We shall denote by $x_i(t)$, $v_i(t)$, $b_i(t)$, $B_i(t)$ and $l_i(t)$ the location, speed, comfortable deceleration, maximum deceleration and length of the i^{th} vehicle. The time dependence of the latter notions shall be dropped hereafter for notational simplicity. Also assume that the PDF of the P-R time of the follower driver follows a truncated normal distribution between τ_i^{max} and τ_i^{min} with mean μ_i and variance σ_i.

PROPOSITION 1. *Consider two vehicles following each other with speeds v_i and v_{i-1} when the leader abruptly decelerates with rate B_{i-1}. The follower, after going through an initial Perception-Reaction time of τ_i, decelerates with a comfortable rate of b_i. With this setting and in order to avoid collisions, the initial spacing between the two, $s_i = x_{i-1} - x_i$, should satisfy (1).*

2.2 The Effect of IntelliDrive^{SM} on Driver Behavior

We conjecture that the effect of IntelliDrive^{SM} on driver behavior is due to two main factors. Intuitively, the more aware a driver is made of its surrounding environment beyond its eyesight, the lower should be its P-R time and also the chances of applying a sudden brake. Moreover, we propose that the higher the number of immediate equipped predecessors of a vehicle, before the first non-equipped one, the lower is its driver P-R time and the chances of it applying a sudden brake[3]. However, having the information of other vehicles beyond a certain distance is not considered to affect the driver P-R time. This distance is dependent on the specific time-varying traffic conditions. For example, in high-flow traffic, you would need to know about further distances upstream than when you are in a less mobile environment. In our analysis, $I_i(t)$ shall denote the number of predecessor vehicles beyond which having the information of others does not affect the driver P-R time of vehicle i.

The relevant information of a vehicle such as its location, speed and acceleration are transmitted via its DSRC radio to its surrounding vehicles inside the cluster. The useful lifetime (or acceptable delay to deliver a packet) is assumed to be the time interval between the generation of two subsequent data packets (which is 100 ms for a 10 Hz GPS device). A high speed vehicle moving with a speed of $90mph$ typically moves less than 2.5 meters during the useful lifetime. Hence, a scheme which can deliver data at the above rate (a packet every 100 ms) fulfils the safety requirements as the inter-vehicle spacings are much more than 2.5 meters at high speeds (due to the Greenshields model). Now, all vehicles in a specific interference range contend for channel access to transmit their packet within its useful lifetime. Note that usually the number of cars in the interference range of a specific vehicle is less than a hundred. Also the time needed to transmit a single packet is determined by the packet length and chosen data rate. If we assume a 6 Mbps data rate for vehicles operating in the 10 MHz control channel (as specified in the DSRC standard), and 250 Bytes per packet [4], transmission of each packet would approximately take 340 microseconds. This example is to show that even in the worst cases in terms of contention for media access (low data rate and large number of interfering cars), a TDMA strategy is able to successfully deliver all the required packets within their useful lifetimes (100 interferers $\times 340\mu s << 100ms$) and hence satisfy the safety requirements even at high vehicular flow.

[3]Note that the first unequipped (e.g not mounted with IntelliDrive^{SM}-enabled On-Board Equipment) leader vehicle, impedes the further shortening of a specific driver's P-R time. This is because one always accounts for the worst case i.e. the unequipped vehicle abruptly braking, and him not being informed.

$$s_i \geq \begin{cases} \tau_i^2 \frac{B_{i-1}}{2} + \tau_i(\frac{b_i+B_{i-1}}{b_i} v_i - v_{i-1}) + \frac{B_{i-1}+b_i}{2b_i^2} v_i^2 - \frac{v_i v_{i-1}}{b_i} & v_i \geq \frac{2b_i}{b_i+B_{i-1}} v_{i-1} - \frac{2b_i B_{i-1}}{b_i+B_{i-1}} \tau_i \\ \tau_i^2 \frac{B_{i-1}}{2} + \tau_i(v_i - v_{i-1}) & otherwise \end{cases} \quad (1)$$

A vehicle that gains a sufficient amount of information regarding its surroundings, is typically informed about an irregularity in traffic flow much sooner. Hence, it takes it less time to perceive its leader vehicle's sudden braking when that happens. Also, as the attention of assisted drivers has already been drawn to an accident before the driver is needed to react, the variance in their perception time (and hence the variance of P-R time) is much lower than drivers of non-equipped vehicles. The above observation is supported by [7] which notes that the most important variable that affects driver P-R time is driver expectation which can affect the P-R time by a factor of 2. There, the author concludes that an unexpected event can increase both the perception and reaction time of the driver. It is further emphasized that driver attention is a graded function. Based on the above, we assume that the mean and variance of driver P-R time decreases linearly with the number of equipped vehicles proceeding it[4], and reaches its minimum for the case where the driver is informed about all $I_i(t)$ leading vehicles. Figure 1 shows the variation of driver P-R time with the number of IntelliDrive^{SM}-enabled vehicles proceeding it. In the figure, X denotes the number of leading equipped vehicles before the first non-equipped vehicle. The maximum declaration rate of a driver is also considered to decrease linearly with X. Note that the linear relation is considered in order to expedite the mathematical analysis and that other decreasing functions would yield similar results of this paper.

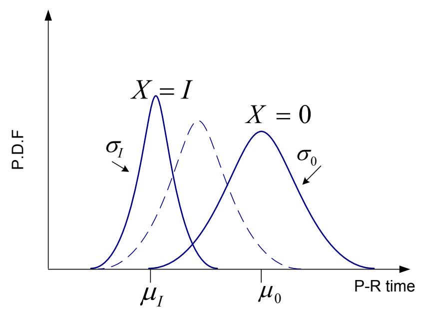

Figure 1: Probability distribution function of driver Perception-Reaction in the presence IntelliDrive^{SM}. $\mu_0 = 2s$, $\mu_I = 1s$, $\sigma_0 = 0.1s$, $\sigma_I = 0.05s$.

2.3 Model for successful transmission

In our capacity analysis, we adopt the generalized physical model for successful transmissions. Here, the transmission rate between two nodes is inversely proportional to their distance.

DEFINITION 1. *[8](Generalized Physical model) The transmission rate , W_{ij}, between transmitter i and receiver j is computed according to Shannon's channel capacity formula as:*

$$W_{ij} = B \log(1 + \frac{P d_{ij}^{-\alpha}}{BN_0 + \sum_{k \neq i, k \in \tau} P|X_k - X_j|^{-\alpha}}) \quad (2)$$

where B is the channel bandwidth, d_{ij} is the distance between nodes i and j, P is the transmission power, α is the path loss exponent, $\frac{N_0}{2}$ is the noise power spectral density and τ is the set of simultaneously transmitting nodes.

The specific parameter values used in simulations of this paper are: $N_0 = 10^{-9}$ Watts/Hz, $P = 10$ Watts, $\alpha = 2$, and $B = 10$ Mbps.

3. FUNDAMENTAL TRADEOFFS

In this section we mathematically define traffic flow, safety and communication capacity and later on see how IntelliDrive^{SM} affects them and their mutual interactions. As we shall see, the pivotal part of the analysis is based on the reduction of driver P-R time due to the presence of inter-vehicular communications. According to Proposition 1, this would help vehicles shorten their spacing with their leaders while maintaining their speed, and hence increase the traffic flow. Also, for a fixed amount of flow, the safety between two vehicles could be enhanced. Further, as we consider single-hop broadcast within a cluster, two main factors would influence the communications capacity where one is the shortening of the cluster length and the other is the increased number of channel access inquiries. Both being due to IntelliDrive^{SM} deployment, the former has a constructive, as opposed to the later's destructive, effect on capacity. In the following subsections we study these issues in detail.

3.1 Traffic flow

Let us start with the definition of flow:

DEFINITION 2. *Assume n vehicles on a highway. Then the traffic flow is defined as $f = \frac{n}{\sum_{i=2}^{n} \frac{s_i}{v_i}}$, where $s_i = x_{i-1} - x_i$ is the spacing between two consecutive vehicles when passing the point of observation.*

Here we investigate the most and the least effect of IntelliDrive^{SM} on traffic flow. Consider an IntelliDrive^{SM} market penetration rate of γ percent. This means that vehicles are equipped with probability γ and not equipped otherwise. Hence, the number of equipped vehicles in a stream of n follows a binomial distribution with parameter γ. Following from the discussion in section 2.2, it can be shown that IntelliDrive^{SM} most enhances traffic flow when all the equipped vehicles follow one another successively. Here, a group of successive equipped vehicles is referred to as an *equipped cluster*. The least effect, on the other hand, is for a configuration with the greatest number of disjoint equipped clusters. For example for the range of equipped vehicles from 0 all the way to $\frac{n}{2}$, the least effect of IntelliDrive^{SM} is virtually zero which happens when every other vehicle is equipped. In this case, each vehicle is

[4]Based on the discussion in the previous paragraph, we assume that the equipped vehicles can successfully deliver at the required rate to the neighbors.

either non-equipped or its leader is non-equipped despite itself being equipped; both cases impeding it from enjoying the benefits of IntelliDrive[SM]. Here we provide a lower bound for the expected value of traffic flow.

$$\begin{aligned} E[f] &= \sum_{j=0}^{n} E[f|n_\gamma = j] \binom{n}{j} \gamma^j (1-\gamma)^{n-j} \\ &\overset{①}{\geq} \sum_{j=0}^{n} \frac{n}{\sum_{i=2}^{n} \frac{1}{v_i} E[s_i|n_\gamma = j]} \binom{n}{j} \gamma^j (1-\gamma)^{n-j} \end{aligned} \quad (3)$$

Where n_γ is the number of equipped vehicles and ① follows from Definition 2, the linearity of expectation and further from Jensen's[5] inequality. Note that (3) is in general true for any configuration of vehicle locations and speeds on the highway. Here we illustrate the results for vehicles within a cluster. Let $S_{i|j}$ denote the inter-vehicle spacing between vehicles i and $i-1$ when there are j equipped vehicles in the cluster. We shall have according to Proposition 1 (as for vehicles in a cluster we have $\Delta v_i = 0$):

$$S_{i|j} = r_{i|j}\tau_{i|j}^2 + q_{i|j}\tau_{i|j} + p_{i|j} \quad (4)$$

Where $r_{i|j} = \frac{B_{i-1|j}}{2}$, $q_{i|j} = \frac{B_{i-1|j}}{b_i}v$, and $p_{i|j} = \frac{B_{i-1|j}-b_i}{2b_i^2}v^2$ are constants. Also, $\tau_{i|j}$ is a random variable denoting driver i's P-R time when there are j equipped vehicles within the cluster. Hence we shall have:

$$E[s_i|n_\gamma = j] = E[S_{i|j}] = r_{i|j}(\mu_{i|j}^2 + \sigma_{i|j}^2) + q_{i|j}\mu_{i|j} + p_{i|j} \quad (5)$$

Where $\mu_{i|j}$ and $\sigma_{i|j}^2$ are the mean and the variance of $\tau_{i|j}$. Note that different orderings of the j equipped vehicles would lead to different distributions for $\tau_{i|j}$ (and hence different values for $\mu_{i|j}$ and $\sigma_{i|j}^2$) and also different values for the constants $p_{i|j}$, $q_{i|j}$ and $r_{i|j}$. Here, utilizing (5) to evaluate (3), the least and the most effect of IntelliDrive[SM] on the flow of the cluster is shown in Figure 2. We also observed that when $n \gg I$, flow is independent of the total number of vehicles that reside inside a cluster. Hence, the departure of vehicles from, or their addition to a cluster due to exists, lane changes and merging, and overtaking does not affect the flow of the cluster (as long as the average penetration rate of IntelliDrive[SM] remains constant).

3.2 Safety

Here we first describe the general characteristics that a valid safety function essentially needs to fulfil and then introduce a probabilistic safety function consistent with our car following model. We shall then see how IntelliDrive[SM] affects safety and also study its variation with traffic flow.

DEFINITION 3. *Let $h_i(s_i, v_i, \Delta v_i)$ denote the safety function between the i^{th} and the $(i-1)^{\text{th}}$ vehicle, where s_i, v_i and $\Delta v_i = v_i - v_{i-1}$ denote the spacing, speed of the follower, and the difference in speed of the two vehicles. A valid safety function h_i should satisfy the following characteristics:*
1. $\frac{\partial h_i}{\partial s_i} > 0$ *2.* $\frac{\partial h_i}{\partial v_i}|_{\Delta v_i = \text{cte}} < 0$ *3.* $\frac{\partial h_i}{\partial \Delta v_i} < 0$.

This states that a valid safety function should always be an increasing function of the inter-vehicle spacing. However, the safety between two vehicles decreases when their relative speed or the speed of the follower (for a fixed relative speed) increases. In our study we formalize the desired safety function as the probability of not having an accident between two following vehicles. Hence h_i always takes values between zero and one. According to our car following model a safety of one corresponds to an inter-vehicle spacing well beyond the danger zone; whereas a safety of zero asserts an imminent accident in case of the leader's abrupt deceleration. More formally, the safety function between two vehicles at any given time, is the probability that their spacing is greater than the threshold value in Proposition 1. The safety function can be derived for three different regimes based on the relative speed of the leader and follower vehicles. For the case of vehicles within a cluster where $\Delta v_i = 0$, we have:

$$h_i = P(\tau_i \leq \sqrt{\frac{2s_i}{B_{i-1}} + \frac{v_i^2}{B_{i-1}b_i}} - \frac{v_i}{b}) \quad (6)$$

Notice that the safety function is expressed in terms of the (truncated normal) distribution of the P-R time. As conjectured earlier through Definition 3, h_i turns out to be a function of s_i, v_i, and Δv_i. Note that this notion of safety is rather conservative as it accounts for the worst case scenario which is the leader vehicle's sudden braking.

We now study how deploying IntelliDrive[SM] can affect inter-vehicular safety. Let E_i^γ be the event that vehicle i is equipped. Also let $E_i^{\gamma^j}, j \in \{0, \cdots, I-1\}$ be the event that *exactly* j of the immediate predecessors of vehicle i are also equipped. Finally, let $E_i^{\gamma^I}$ be the event that *at least* I of the immediate predecessors of vehicle i are equipped. Notice that the events $E_i^{\gamma^j}, j \in \{0, \cdots, I\}$ are mutually disjoint, and partition the space. Hence they have a valid probability mass function of:

$$P(E_i^{\gamma^j}) = \gamma^j(1-\gamma) \qquad \forall j \in \{0, \cdots, I-1\} \quad (7)$$

$$P(E_i^{\gamma^I}) = \gamma^I \quad (8)$$

This way we can use the law of total probability to write:

$$h_i = (1 - P(E_i^\gamma))h_{i|E_i^{\gamma_0}} + P(E_i^\gamma)\sum_{j=1}^{I} h_{i|E_i^{\gamma_j}} P(E_i^{\gamma_j}) \quad (9)$$

Here $h_{i|E_i^{\gamma_j}}$ is the probability of not having a collision, given that exactly j (at least j in case $j = I$) of vehicle

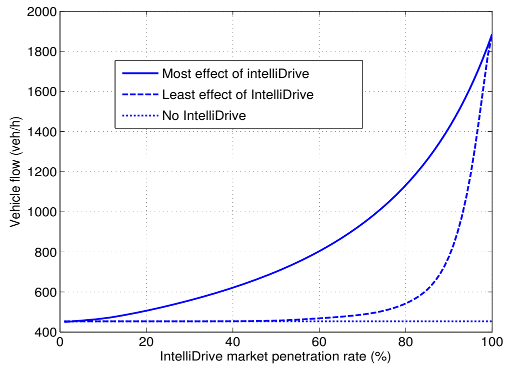

Figure 2: Traffic flow as a function of IntelliDrive[SM] market penetration rate. $n = 16$, $v = 45mph$, $I = 8$.

[5]Jensen's inequality states that $E[f(x)] \geq f(E[x])$ when f is a convex function.

i's predecessors are equipped. The value of $h_{i|E_i^{\gamma_j}}$ can be obtained from (6) by adopting the appropriate distribution for τ_i for each value of j. Plotting (9), Figure 3 (left) depicts the improvement IntelliDrive^{SM} can render in terms of safety. Notice how as the improvement in safety might not be as eye-catching up until penetration rates of about 50%, it suddenly picks up power for larger penetration rates. Figure 3 (right) shows the tradeoff inherent between traffic flow and safety.

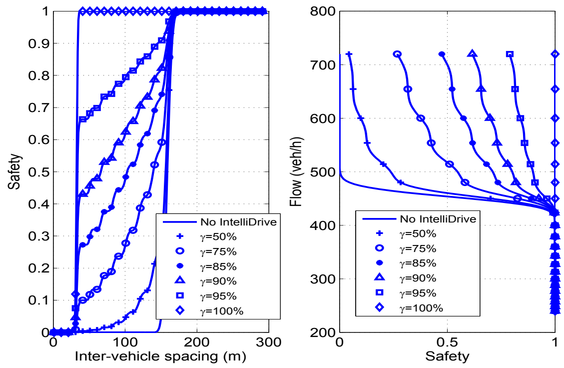

Figure 3: Safety as a function of inter-vehicle spacing for various IntelliDrive^{SM} market penetration rates (left), Flow-safety tradeoff (right). $v = 45mph$, $I = 8$.

3.3 Communications capacity

Communications within a cluster is broadcast in the sense that the data of any vehicle should be received by all others within that cluster[6]. In order to fulfil the stringent delay requirements of VANET applications, we assume vehicles are scheduled to transmit in a TDMA[7] manner and reach the furthest within the cluster in a single transmission, during the course of which the data is also delivered to all other vehicles in the cluster. We also assume that for each transmitter, capacity-achieving Gaussian channel codes are assumed to support the achievable rate of the furthest receiver. This way, if node i can broadcast at a rate λ_i, we shall say that a per-node rate of Λ is achievable within the cluster if:

$$\Lambda = \inf_{i \in C} \lambda_i \tag{10}$$

Where C is the set of all vehicles in the cluster. Due to the definition of the generalized physical model, Λ is the achievable transmission rate of the first or the last vehicle within a cluster (since in this case the transmission range is maximum and equals the cluster length). Note that Λ is a random variable as a result of inter-vehicle spacings being random. Here we study how its expectation, $E[\Lambda]$, varies with IntelliDrive^{SM} market penetration rate.

$$E[\Lambda] = \sum_{j=1}^{n} E[\Lambda | n_\gamma = j] P(n_\gamma = j)$$
$$= \sum_{j=1}^{n} E[\frac{W \log(1 + \frac{PD_j^{-\alpha}}{WN_0})}{j}] \binom{n}{j} \gamma^j (1-\gamma)^{n-j} \tag{11}$$

Where the last equation is a result of vehicles deploying a simple TDMA strategy among themselves to access the channel and $D_j = \sum_{i=1}^{n} S_{i|j}$ is the length of the cluster when there are a total of j equipped vehicles in it (correspondingly $S_{i|j}$ is the inter-vehicle spacing between vehicles i and $i-1$ when there are j equipped vehicles in the cluster). It also reflects the fact that $E[\Lambda | n_\gamma = 0] = 0$ which is why the summation on j starts from 1.

In order to compute the expectation in (11) we need to have the distribution of D_j. For that we first need to derive the PDF's of the $S_{i|j}$'s. Note that when all vehicles within a cluster have the same speed, the $S_{i|j}$'s are distributed according to (4). Note that for a specific ordering of the j equipped vehicles, the $\tau_{i|j}$'s in (4) are independent but *not* identically distributed normal random variables with distinct means and variances. Consequently the $S_{i|j}$'s themselves are also independent random variables each with the CDF:

$$F_{S_{i|j}}(s_{i|j}) = P(p_{i|j} + q_{i|j}\tau_{i|j} + r_{i|j}\tau_{i|j}^2 \leq s_{i|j})$$
$$= P(|\tau_{i|j} + \frac{q_{i|j}}{2r_{i|j}}| \leq \sqrt{\frac{s_{i|j} - p_{i|j}}{r_{i|j}} + \frac{q_{i|j}^2}{4r_{i|j}^2}}) \tag{12}$$

Noting that all $\tau_{i|j}$'s are normally distributed, the PDF of each $S_{i|j}$ can be obtained using Leibnitz's rule to differentiate (12) with respect to $s_{i|j}$. Now, as they are all independent, we can perform a convolution on the PDF's of the $S_{i|j}$'s to derive the distribution of D_j. However, here we utilize the Central Limit Theorem under Lyapunov's condition [9] which states that the sum of a number of independent (but not necessarily identically distributed) random variables converges to a normal random variable in case they all have finite means and finite standard deviations. Due to the specific (bell-shaped) distribution of the $S_{i|j}$'s we observed that the CLT renders a perfect approximation for the PDF of D_j even when we have only two vehicles in a cluster, i.e $n = 2$. Hence we use the approximation $D_j = \mathcal{N}(\mu_{D_j}, \sigma_{D_j}^2)$ where μ_{D_j} and $\sigma_{D_j}^2$ are as in (13) and (14) respectively, where $\mu_{i|j}$ and $\sigma_{i|j}$ are the mean and variance of driver i's P-R time when there are a total of j equipped vehicles in the cluster.

$$\mu_{D_j} = \sum_{i=1}^{n} E[S_{i|j}] = \sum_{i=1}^{n} p_{i|j} + q_{i|j}\mu_{i|j} + r_{i|j}(\mu_{i|j}^2 + \sigma_{i|j}^2) \tag{13}$$

$$\sigma_{D_j}^2 = \sum_{i=1}^{n} E[S_{i|j}^2] - E^2[S_{i|j}]$$
$$= \sum_{i=1}^{n} 4r_{i|j}\mu_{i|j}\sigma_{i|j}^2(r_{i|j}\mu_{i|j} + q_{i|j}) + \sigma_{i|j}^2(2r_{i|j}^2\sigma_{i|j}^2 + q_{i|j}^2) \tag{14}$$

We can now use this distribution of D_j to compute $E[\Lambda]$ in (11). Note that as mentioned before, for any j, the specific ordering of the j equipped vehicles is what influences the

[6]Here still, only the information of at most the first immediate I neighbors affects the driver P-R time.

[7]Note that by establishing coordination between the nodes, a TDMA scheme attains maximum communications capacity, and that other distributed media access schemes such as ALOHA and CSMA would result in lower capacities. Hence the capacity results in this paper serve as upper bounds for the practical scenarios.

values of $\mu_{i|j}$ and $\sigma_{i|j}$. For example, a succession of the j equipped vehicles, as opposed to their intermittent placement, leads to the least possible μ_{D_j} (due to lower values of $\mu_{i|j}$ and $\sigma_{i|j}$) and consequently the highest possible value for $E[\Lambda]$. Figure 4 (top) demonstrates the most and the least effect of IntelliDrive^{SM} on the broadcast capacity using the above normal approximation for the cluster length.

Moving a step further and in order to avoid finding the expectation of the logarithmic term, we can alternatively use the following lower bound on $E[\Lambda]$ which shall prove to be rather close to the actual value. Note that by the double differentiation of the logarithmic term in (11), it can easily be verified that for positive values of α, the logarithmic term is a convex function of D_j. Hence, utilizing Jensen's inequality we shall have:

$$E[\Lambda] \geq \sum_{j=1}^{n} \frac{W}{j} \log(1 + \frac{P\mu_{D_j}^{-\alpha}}{WN_0}) \binom{n}{j} \gamma^j (1-\gamma)^{n-j} \qquad (15)$$

We observed that this lower bound can actually serve as a fine approximation for the real value of $E[\Lambda]$. For example, for the specific case considered in Figure 4, the lower bound always resided within the %0.9 range of the real value.

Two different trends are evident in the capacity-IntelliDrive^{SM} curve in Figure 4(top). There is always a phase where capacity decreases with the increase in market penetration rate. This is primarily due to the TDMA channel sharing strategy adopted by the vehicles in the cluster. In this phase, the reduction in cluster length as a result of deploying IntelliDrive^{SM} is not enough to compensate for the reduction in capacity due to a greater number of vehicles wanting to access the channel. However, there is always a specific penetration rate beyond which the constructive effect of the former on capacity, proves more compelling than the destructive effect of the latter, and as a result, the capacity actually increases with the increase in IntelliDrive^{SM} market penetration rate. This threshold market penetration rate occurred at $\gamma = 55\%$ for the case considered in Figure 4. For an ordering of the equipped vehicles leading to the least effect of IntelliDrive^{SM} on capacity and flow, the same story applies though here the threshold IntelliDrive^{SM} market penetration rate is higher and set at 65%.

According to the above discussion and prior ones regarding traffic flow, one can see that flow as a function of capacity behaves considerably different over different ranges of IntelliDrive^{SM} market penetration rates. Moreover, there is a proportionate relationship between communications capacity and flow for IntelliDrive^{SM} market penetration rates of above $\gamma = 55\%$. It is seen that for lower penetration rates, a trade off exists between the two. That is, flow comes at the expense of capacity and vice versa. See Figure 4 (bottom).

4. CONCLUSION

In this paper we aimed at unraveling the relationships between safety, communications capacity and traffic flow in Vehicular Ad Hoc Networks. In brief, it was shown that IntelliDrive^{SM} always has a positive effect on traffic flow as it decreases driver Perception-Reaction time and hence allows for high speed compact clusters. Further, it was shown that for a fixed amount of flow, IntelliDrive^{SM} can help considerably increase the safety between two adjacent vehicles. As for the per-node communications throughput, it experienced

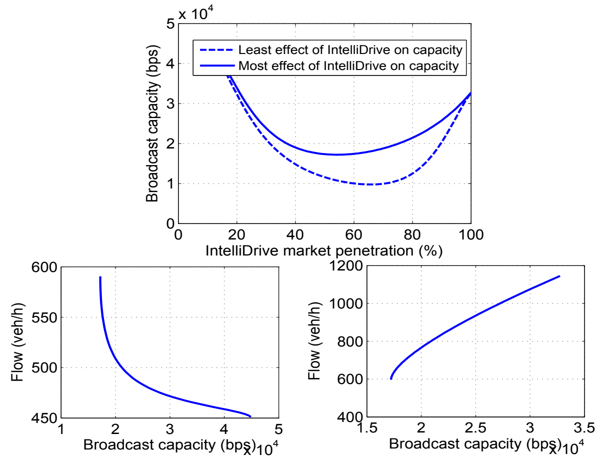

Figure 4: Broadcast capacity as a function of IntelliDrive^{SM} market penetration rate (top). Flow-capacity tradeoff (bottom). $n = 16$**,** $v = 45$***mph*** **and** $I = 8$**.**

a decline in the early stages of IntelliDrive^{SM} deployment, but started to rise up after a threshold market penetration rate. The same conditions determined the relation between communications capacity and vehicular flow, where the latter increased with the former beyond the threshold market penetration rate, whereas a tradeoff existed between the two in the early stages.

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