The Effect of VII Market Penetration on Safety and Efficiency of Transportation Networks

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***Abstract*—The introduction of the Vehicle Infrastructure Integration (VII) initiative by the US Department of Transportation (USDOT) has recently attracted the attention of researchers and given rise to a plethora of studies, each one seeking to enhance the safety and efficiency of transportation networks in one way or another. Moreover, vehicles communicating with each other and with the infrastructure, render their drivers with more information on their surroundings, extending the horizon of drivers and making them more vigilant to face the unexpected. In this study, we provide a mathematical framework which foretells the effect of deploying VII-enabled vehicles, such as VII-assisted and VII-automated, on the overall safety and efficiency. We show that inter-vehicular communications improves the throughput of vehicles by reducing the Perception-Reaction times of drivers. Furthermore, we exhibit that this is obtained without needing to compromise over safety requirements. Our framework can be utilized by decision makers to anticipate the effect of VII on transportation networks prior to their real-life deployment.**

***Index Terms*—Vehicular Ad-Hoc Networks (VANETs), Vehicle-Infrastructure Integration (VII), Dedicated Short Range Communications (DSRC), Perception-Reaction (P-R) time.**

I. INTRODUCTION

With the ever increasing production of vehicles and their inevitable role in everyday life, transportation networks are drawing the attention of industry and academia more than any other time. Despite the undeniable beneficial impacts of transportation networks on our lives, there are a numerous factors by which they impair our everyday life. Without no doubt, many of us have experienced being trapped in heavy traffic, wasting our time and energy resources. Traffic congestion wastes 40 percent of travel time on average, unnecessarily consumes about 2.3 billion gallons of fuel per year, and adversely impacts the environment. More importantly traffic accidents are held responsible for a good portion of death causes. Annually more than 40,000 people are killed and much more injured in highway traffic accidents in the united states alone [1]. Vehicle Infrastructure Integration (VII) [2], a major initiative at the United States Department of Transportation (USDOT), proposes to use Dedicated Short Range Communications (DSRC) to establish vehicle-vehicle and vehicle-roadside communications to deliver timely information to save lives, reduce congestion, and improve quality of life.

The network of communicating vehicles forms a Vehicular Ad-Hoc Network (VANET) on roads. VANET is an emerging area, and due to the potentially dramatic improvements it renders in terms of safety, highway efficiency, and driver convenience, has attracted attention from both academia and industry in the US, EU, and Japan. The most important feature of VANETs is their ability to extend the horizon of drivers and on-board devices and thus to improve road traffic safety, efficiency and comfort [3, 4]. VANET will enable a wide range of novel applications such as accident avoidance messaging, congestion sensing, traffic metering, and general information services [5–7]. The allocation of 75 MHz in the 5.9 GHz band for DSRC may also enable future delivery of rich media content to vehicles at short to medium ranges via both inter-vehicle and roadside-vehicle communication [8].

The VII Initiative envisions that each future vehicle will be equipped with an On-Board Equipment (OBE) which includes a DSRC transceiver (also called On-Board Unit, OBU), a Global Positioning System (GPS) receiver, and a computer. Also equipped with similar devices, roadside equipment (RSE) will be deployed at selected roadside locations. Therefore, vehicles will be able to communicate with each other and with the roadside by means of DSRC. As a result, drivers will be able to respond to their driving environment earlier, i.e. shorter Perception-Reaction (P-R) times than drivers without VII assistance (P-R time is the duration of time from the moment a phenomena happens to when the driver reacts with a preventive action). In addition, partially or fully automated driving systems will be devised to further reduce the P-R time and hence the variation in responses. Thus it will be possible to organize compact platoons (where vehicles follow each other closely at high speeds without sacrificing safety) on highways to dramatically increase throughput. Moreover, traffic management centers will be able to optimize system-wide operations with real-time feedback from OBEs and RSEs. Our goal in this paper is to study the effect of VII deployment on the safety and efficiency of transportation systems.

In the near future, a traffic stream may consist of mixed vehicles operated under different driving modes: A vehicle may be operated without VII assistance (non-VII thereafter), by a human driver with VII assistance (VII-assisted, where the driver is alarmed to take the appropriate action), or by a VII-enabled automated system (VII-automated, where the OBU itself is in charge of driving). Here we assume VII-automated vehicles to be equipped with technologies such as Adaptive Cruise Control (ACC) enabling it to consistently (and automatically) adjust its distance with its leader.

Here we propose a mathematical framework which can be utilized as a tool to foresee the effect of the above VII-enabled vehicles on safety and efficiency, as they penetrate through the

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market. First, In section II, we propose a mobility paradigm according to which vehicles move in platoons (clusters) on highways and derive the average number of clusters through probabilistic methods. Next, in section III, the effect of VII on safety and efficiency is considered. Here the decreased P-R time of drivers gained through VII plays a major role in our analysis. As intuitive as it seems, in a VII-enabled zone, where vehicles exchange safety information on their current status and possible hazards, drivers reside in a higher state of alertness, which decreases their effective P-R time. Hence, maintaining their speed, drivers can reduce their distance with the leading car. One can see that this phenomena results in higher throughput without sacrificing safety.

II. CLUSTERING PARADIGM

In this section we provide a simple model to depict vehicle platoons on highways. We assume that n vehicles are randomly placed on a single lane highway of length L. Initially each driver chooses a speed uniformly at random from $[v_{min} \quad v_{max}]$, as its desired speed. It follows that if a vehicle's speed is higher than at least one of the cars preceding it, it will join the cluster ahead; otherwise, it will trail back and form a new cluster. By joining the cluster ahead it would leave a safety distance with its leader, proactively avoiding collisions. Hence, if the vehicles are numbered from 1 to n from the beginning to the end of the road, the ith vehicle would form a new cluster with probability $\frac{1}{i}$ and join the cluster ahead with probability $\frac{i}{i+1}$. We will see in the next section that this inter-vehicle gap is a function of VII market penetration. Note that we neglect the possibility of vehicles overtaking each other in a single lane highway.

Figure 1 shows the clustering probabilities for up to 5 vehicles. The numbers on the branches separated by commas, show the length of the clusters and the number in the circle to which the branch ends, is the probability of having that specific clustering configuration. For example the branch identified with $(2, 2, 1)$ represents the configuration of having 2,2 and 1 vehicles in the 1st, 2nd and 3rd clusters respectively. As can be seen from the tree, this happens with the probability of $\frac{1}{40}$. Note how the probabilities are derived through the following example: Assume that we have a $(2, 2)$ configuration. The 5th vehicles either joins the cluster ahead with probability $\frac{4}{5}$, hence arriving at a $(2, 3)$ arrangement with probability $\frac{3}{24} \times \frac{4}{5} = \frac{1}{10}$; or trails back with probability $\frac{1}{5}$, resulting in the $(2, 2, 1)$ configuration with probability $\frac{3}{24} \times \frac{1}{5} = \frac{1}{40}$ (See dotted section of Figure 1). By extending this tree for an arbitrary number of vehicles, one can derive all the clustering probabilities.

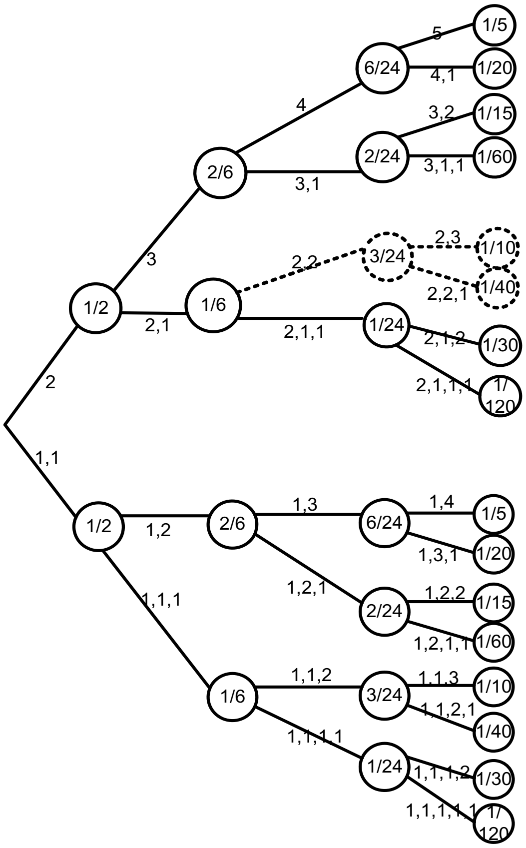

Fig. 1. Decision tree demonstrating clustering probabilities.

In what follows we will compute the average number of clusters. Lets define x_i as:

$$x_i = \begin{cases} 1 & \textit{if the ith vehicle is a clusterhead} \\ 0 & otherwise \end{cases} \tag{1}$$

With this definition it can be seen that if C is the mean number of clusters,

$$\begin{aligned} C =& E[\sum_{i=1}^{n} X_i] = \sum_{i=1}^{n} E[X_i] = \sum_{i=1}^{n} p_i = \sum_{i=1}^{n} \frac{1}{i} \\ &= 1 + \ln(n)(1 + o(1)) \end{aligned} \tag{2}$$

Where, $p_i = Prob.(X_i = 1)$ and $o(1) \to 0$ as $n \to \infty$, where n is the number of vehicles on the road.

III. SAFETY AND EFFICIENCY ANALYSIS

In this section we study the enhancements VII renders in terms of safety and efficiency. Moreover we show that VII deployment can increase the throughput (of vehicles) on roads without sacrificing safety. Note that the terms flow and throughput will be used interchangeably hereafter to address the number of vehicles passing an arbitrary point in unit time.

In equilibrium conditions, the following relation exists between traffic flow characteristics, *flow*(q), *density*(k) and *speed* (v) [9]:

$$q = kv \tag{3}$$

Density can be expressed in terms of the total number of vehicles ,N, on a section of road of length L as $k = \frac{N}{L}$. As stated earlier we assume a constant speed for all vehicles, Hence we have:

$$E[q] = \frac{v}{L} E[N] \tag{4}$$

Thus, in what follows we derive the expectation of N, the number of vehicles on the road. Note that we have:

$$\sum_{i=1}^{N-1} S_i + Nl = L \tag{5}$$

Where, S_i is the gap between the i *th* and the $i-1$ *th* vehicle and l is the average length of a vehicle. Taking the expectation of both sides, using Wald's equation [10] and sorting the terms, we have:

$$E[N] = \frac{L + E[S]}{l + E[S]} \tag{6}$$

Note that, as we shall later see, the S_i's are i.i.d random variables (as they should be in order to be able to utilize Wald's equation), hence we have dropped the notation for their i-dependency in (6). The application of Wald's equation also requires $E[S] < \infty$ which is true, since (as we will see) we assume bounded inter-vehicle and inter-cluster gaps. The next step would be to determine the stochastic properties of S, the gap between the vehicles. Here we introduce a car following model in which vehicles allow for a safety distance between themselves and their leading vehicle. The safety distance should be such that in the event when the leader (vehicle $i-1$) applies a sudden brake and slows down with maximum deceleration B_{i-1}, the follower (vehicle i) should be able to safely stop behind it after going through a P-R time and a deceleration process at a comfortable rate b_i ($|b_i| < |B_i|$). In Figure 2 we denote by $x_i, \dot{x}_i$ and $\ddot{x}_i$ the position, speed and acceleration of the ith vehicle respectively. Note that if we denote by x_{i-1}^* and x_i^* the stopping position of the leader and follower, respectively, we will have:

$$x_{i-1}^* = x_{i-1}(t) - \frac{\dot{x}_{i-1}^2(t)}{2B_{i-1}} \tag{7}$$

$$x_i^* = x_i(t) + \dot{x}_i\tau_i - \frac{\dot{x}_i^2(t)}{2b_i} \tag{8}$$

Where τ is the Perception-Reaction (P-R) time. To ensure safety, we must have $x_{i-1}^* - l_{i-1} \geq x_i^*$. Hence we have for the inter-vehicle gap, S:

$$S = \tau\dot{x} + G\dot{x}^2 \tag{9}$$

where, $G = \frac{1}{2B} - \frac{1}{2b}$ and the subscript i has been dropped for for simplicity. Note that this inter-vehicle gap is different for VII-assisted, automated and non-VII cars due to the disparate P-R time associated with each. Note that for our analysis we assume that α percent of the vehicles are VII-assisted, β percent are VII-automated and the rest are non-VII-enabled. In Figure 3, we establish probability distributions for the P-R time in different regimes. As can be seen from the figure, in vehicles with no VII utilization, the P-R time is substantially larger than cases with VII assistance. With VII assistance, thanks to the information and warnings the driver obtains through its OBU, the P-R time reduces as a result of drivers' augmented state of alertness. Note how the non-VII regime suffers a higher variance in driver P-R time than the VII-assisted regime. For VII-automated cars the P-R time is deterministically specified by hardware which is in the order of micro seconds.

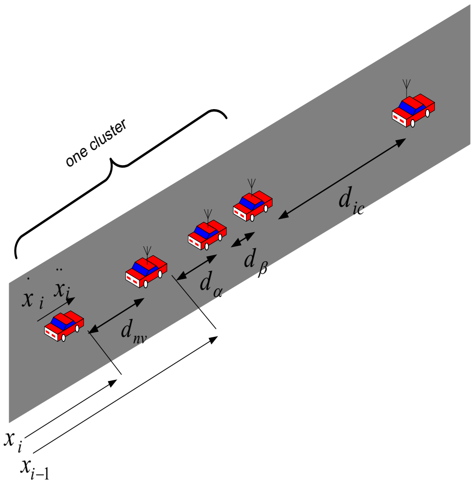

Fig. 2. Clustering configuration and inter-vehicular gaps.

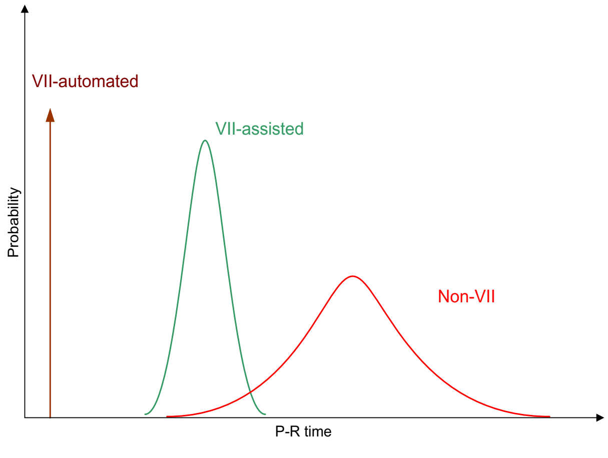

Fig. 3. Perception-Reaction times for different classes of vehicles.

According to (9) and discussions following it, we can identify four different kinds of inter-vehicle gaps, S. First is the inter-cluster gap, d_{ic}, which is the distance between the last vehicle of the preceding cluster and the first vehicle of a following cluster. The distance between a non-VII-enabled car and its leader (within a cluster) is referred to as d_{nv}. This gap is shown with d_α and d_β for VII-assisted and VII-automated cars, respectively. Note that generally we have $d_{ic} > d_{nv} > d_\alpha > d_\beta$ due to the difference in P-R time of various vehicles (See Figure 2).

Note that a VII-automated vehicle always follows its leading car and is never a cluster head (except probably the first car on the road). One important issue to account for is that a VII-assisted vehicle decreases its distance with its leader to d_α, only when it has the information of all its (r) surrounding vehicles; that is all those neighboring vehicles should either be VII-assisted or automated in order to be able to communicate with that specific VII-assisted vehicle.

Due to the above explanations, the gap between the vehicles has the following stochastic characteristics (*w.p.* denotes ”with probability”).

$$S = \begin{cases} d_\alpha & w.\ p. \quad p_\alpha = \alpha \frac{\binom{(\alpha+\beta)n}{r}}{\binom{n}{r}} \frac{n-C}{n} \\ d_\beta & w.\ p. \quad p_\beta = \beta \frac{\binom{(\alpha+\beta)n}{r}}{\binom{n}{r}} \\ d_{ic} & w.\ p. \quad p_{ic} = \frac{C}{n} \\ d_{nv} & w.\ p. \quad p_{nv} = 1 - p_\alpha - p_\beta - p_{ic} \end{cases} \tag{10}$$

In the above, note that C represents the average number of clusters in the system. For a VII-assisted vehicle (which is not a cluster head with probability $\frac{n-C}{n}$) to decrease its distance with its leader to d_α, it should be able to communicate with its immediate r preceding vehicles, which happens with probability $\frac{\binom{(\alpha+\beta)n}{r}}{\binom{n}{r}}$. All other probabilities are derived similarly. Also, knowing that $C = O(\ln n)$ from the previous section and for the case when $r = o((\alpha+\beta)n)$, (10) is simplified to:

$$S = \begin{cases} d_\alpha & w.\ p. \quad p_\alpha = \alpha(\alpha+\beta)^r(1-o(1)) \\ d_\beta & w.\ p. \quad p_\beta = \beta(\alpha+\beta)^r(1-o(1)) \\ d_{ic} & w.\ p. \quad p_{ic} = o(1) \\ d_{nv} & w.\ p. \quad p_{nv} = 1-(\alpha+\beta)^{r+1}(1-o(1)) \end{cases} \tag{11}$$

Having the inter-vehicle gap probabilities, we can now compute $E[S]$ as:

$$E[S] = p_\alpha E[d_\alpha] + p_\beta E[d_\beta] + p_{ic} E[d_{ic}] + p_{nv} E[d_{nv}] \tag{12}$$

Replacing terms from (11), we would have:

$$\begin{aligned} E[S] =& (\alpha(\alpha+\beta)^r E[d_\alpha] + \beta(\alpha+\beta)^r E[d_\beta] + \\ & (1-(\alpha+\beta)^{r+1} E[d_{nv}]))(1+o(1)) \end{aligned} \tag{13}$$

Substituting $E[S]$ from (13) into (6) to derive $E[N]$ and further replacing $E[N]$ in (4), we would have the expected value of vehicular flow as a function of α, β and the mean Perception-Reaction times. To asymptotically derive this value we have:

$$E[q] = \frac{v}{L}\frac{L+E[S]}{l+E[S]} \overset{L\to\infty}{=} \frac{v}{l+E(S)} \tag{14}$$

Where $L \to \infty$ is true when the number of vehicles, $n \to \infty$. Also, due to (13), we asymptotically have: $E[S] = (\alpha+\beta)^r(\alpha E[d_\alpha] + \beta E[d_\beta]) + (1-(\alpha+\beta)^{r+1})E[d_{nv}]$ as $n \to \infty$.

In Figure 4 We’ve depicted the average flow as a function of α and β. Here, α and β are shown as a percentage on the x and y axes respectively. Also flow has (normalized) units of vehicles/second. As one can see, VII deployment can increase traffic flow by up to 2 times in comparison with no VII utilization. More inspiring is the fact that this efficiency is attained without sacrificing safety. By looking at Figure 4, one can see that for a fixed $\beta(\alpha)$, throughout increases by increasing $\alpha(\beta)$; although the augmentation in throughput obtained by increasing β is greater than the one achieved by increasing α. Moreover for a fixed amount of $\alpha+\beta$, the higher the value of β, the higher the throughput.

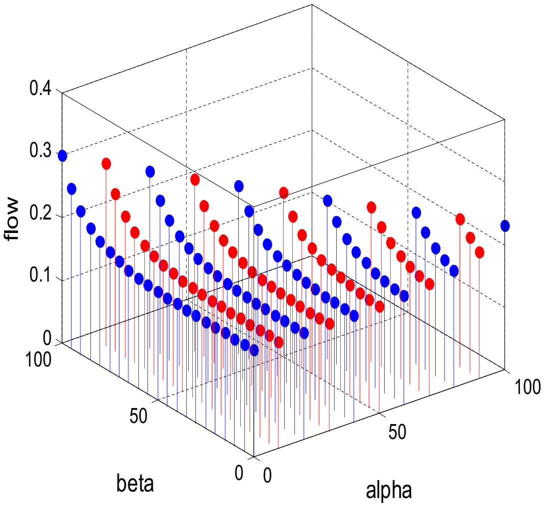

Fig. 4. Vehicle flow as a function of α and β. $v = 25\frac{m}{s}$, $l = 5m$, $r = 4$

IV. CONCLUSION

In this paper we investigated the effect of VII deployment on safety and efficiency in transportation networks. We studied how the gradual introduction of VII-enabled vehicles into transportation networks enhances vehicular throughput while at the same time maintaining safety. We learned that the enhancement in throughput is the result of lower Perception-Reaction time for the drivers of VII-enabled vehicles. The lower P-R time is due to the inter-vehicle communications, making the drivers more attentive. Furthermore we introduced kinds of VII-enabled vehicles such as VII-assisted and VII-automated and studied the effect of each on safety and efficiency. We concluded that VII-automated and VII-assisted vehicles both play their roles in increasing the overall throughput with the former having a greater effect than the latter. Our framework can be utilized by decision makers to foresee the effect of introducing different classes of VII-enabled vehicles into the transportation network prior to their actual deployment.

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