[30] L. I. Kuncheva and C. J. Whitaker, "Measures of diversity in classifier ensembles and their relationship with the ensemble accuracy," *Mach. Learn.*, vol. 51, no. 2, pp. 181–207, May 2003.

[31] [Online]. Available: http://www.gavrila.net/Computer_Vision/Research/Pedestrian_Detection/pedestrian_detection.html

A Sampling Theorem Approach to Traffic Sensor Optimization

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***Abstract*—With the objective of minimizing the total cost, which includes both sensor and congestion costs, the authors adopted a novel sampling theorem approach to address the problem of sensor spacing optimization. This paper presents the analysis and modeling of the power spectral density of traffic information as a 2-D stochastic signal using highly detailed field data. The field data were captured by the Next-Generation SIMulation (NGSIM) program in 2005. To the best knowledge of the authors, field data with such a level of detail were previously unavailable. The resulting model enables the derivation of a characterization curve that relates sensor error to sensor spacing. The characterization curve, concurring in general with observations of a previous work, provides much more detail to facilitate sensor deployment. Based on the characterization curve and a formulation relating sensor error to congestion cost, the optimal sensor spacing that minimizes the total cost can be determined.**

***Index Terms*—Sampling theorem, sensor optimization, spectral domain analysis, traffic congestion, traffic sensing.**

I. Introduction

The United States has 47 000 mi (75 640 km) of Interstate Highways [1]. If half of the roads were monitored by traffic sensors with one every one third of a mile, as is typically adopted in practice, approximately 70 000 sensors would be required. Assuming each sensor has a lifetime cost of $30 000 [2], the total cost will amount to about $2 billion. If, somehow, one determines that 20% of the sensors are unnecessary in the sense that their existence does not provide additional information, a savings of roughly $400 million is expected.

Little research has been conducted on the subject of optimal traffic sensor deployment or on the potential savings from such optimization. Current practice is mainly based on rudimentary studies or none at all. For example, Georgia Navigator [3], which is Georgia's Intelligent Transportation System (ITS), chooses to install sensors every one third of a mile along its major highways. The rationale is that this is the distance that a vehicle traverses assuming an average traffic speed of 60 mi/h (96.6 km/h) over a data aggregation interval of 20 s.

Therefore, the strategic problem that this paper attempts to address is traffic sensing optimization, including optimizing sensor deployment strategies and minimizing the uncertainty of the system of interest. To achieve this goal, we sought an interdisciplinary collaboration and developed an analytical approach based on the sampling theorem.

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In this approach, traffic information such as flow, speed, and density is obtained from actual detailed vehicle trajectories collected by Cambridge Systematics, Inc., under the auspices of the Next-Generation SIMulation (NGSIM) program [4].

The NGSIM program has collected data sets of vehicle trajectories from actual live traffic video footages. Advances in technology have enabled the NGSIM program to capture vehicle trajectories to a level of detail that was previously impossible. Six sets of data, each containing detailed vehicle trajectories of actual automobiles traveling on two different freeways under real-life actual driving conditions during the morning and evening peak periods, were used in this paper. The morning and evening peak periods correspond to the times of day where traffic information is of greatest importance. This paper focuses on the most important times to traffic management instead of worst-case scenarios or extreme cases.

Signal processing techniques and the multidimensional sampling theorem will be used to glean an understanding of traffic information, which is treated as a 2-D stochastic signal in the space–time domain. A model of the power spectral density (PSD) of traffic information as a 2-D stochastic signal will then be derived. Using the derived model, we attempt to determine a characterization curve that relates sensor spacing to the error from the sensor. With this relationship, we determine the optimal sensor spacing.

Our contributions include the following:

1) a novel sampling theorem approach to address the optimal sensor deployment problem;
2) an analytical model of the PSD of traffic information as a 2-D stochastic signal;
3) the normalized mean-square error (NMSE) and sensor spacing characterization curve;
4) procedure and formulation to determine the optimal sensor spacing that minimizes the total cost.

The next section provides a review of related work. Section III details the spectral characteristics and the modeling of traffic information and presents the NMSE and sensor spacing characterization curve. Section IV discusses the optimization of sensor spacing. This is followed by a conclusion.

II. Review of Related Work

In 1979, the Federal Highway Administration published a guideline for locating freeway sensors [5] based on empirical studies. The report concluded the following: 1) that any sensor spacing below 1000 ft (304.8 m) generally produces relatively little or no increase in effectiveness; 2) that sensor spacing over 2500 ft (762 m) produces unsatisfactory performance according to the criteria defined in the report; and 3) that there exists a cost-effectiveness tradeoff for sensor spacing between 1000 ft (304.8 m) and 2500 ft (762 m). This report is similar to our research in that both addressed the sensor optimization problem and led to consistent findings. The differences between the two are as follows: 1) The report is based on empirical studies, whereas our approach is analytical; 2) the report employed a different set of criteria than ours to evaluate sensor deployment strategies; and 3) the report gives categorical recommendations, whereas our approach yields more details over the whole spectrum of sensor spacing, which might be of greater interest to practitioners.

A few studies [6]–[9] addressed the sensor location problem in a traffic system, i.e., the minimum number of sensors and their associated locations to facilitate the estimation of origin–destination (O–D) matrices. These studies differ from our research in different ways. First, the objective of these studies was to estimate O–D matrices, whereas that of ours is to optimize sensor spacing and minimize the uncertainty of the subject system. Second, these studies employed strong assumptions about sensor locations (e.g., a link contains at most one sensor or sensors appear at nodes/intersections only), whereas in our approach, a sensor may appear at any location as appropriate, and a link with more than one sensor is possible. Although nodes/intersections are typical sources of traffic congestion, removing the restriction with regard to sensor location is important because capacity constraints may apply at midblock locations due to curves, grades, and accidents. Eisenman *et al.* [10] provided a conceptual framework of the sensor location problem and analyzed the sensitivity of the estimation and prediction quality to the number and locations of sensors. The major differences between the work of Eisenman *et al.* and our work are as follows: First, rather than solving for optimal sensor locations as we do, Eisenman *et al.* assume the given sensor deployment scenarios and evaluate their effects. Second, from the perspective of O–D estimation, the work of Eisenman *et al.* emphasizes high volume links where sensors are desirable, whereas our work deploys sensors to minimize a more general cost function, including sensor costs and costs due to the loss of information. It might be interesting to find out how our work compares to these efforts. Unfortunately, a meaningful comparison has been very difficult because their stark differences in objective and approach provide no common basis for such a comparison.

Fujito *et al.* [11] investigated the impact of sensor spacing along freeways on the computation of performance measures. Sisiopiku *et al.* [12], Thomas [13], and Oh *et al.* [14] tried to find optimal loop detector locations to improve traveler information such as travel times. Woods [15] identified the need to optimize the spacing of detectors and monitor stations in high-occupancy-vehicle lane operations. MacHutchon and Ryan [16] called to optimize sensor locations for fog detection. Other applications of the sensor location problem include dilemma zone [17] and actuated signal control [18].

Applications of the sensor location problem are also found in many engineering areas other than transportation. Goulias [19] studied optimal placement of pavement temperature sensors. Stubbs and Park [20] applied Shannon's sampling theorem to reconstruct exact mode shapes of a structural system from a limited number of sampling points. Shi *et al.* [21] developed a method to prioritize sensor locations according to their ability to localize structural damage. Papadimitriou *et al.* [22] presented a statistical methodology to optimally locate sensors in a structure for structural model updating. Instead of monitoring structures, Berry *et al.* [23] studied sensor placement in municipal water networks, and the model to optimize the placement of sensors is based on a mixed-integer program. Also, based on a mixed-integer linear program, Propato *et al.* [24] proposed a model that identifies optimal sensor locations for water quality monitoring. Ucinski [25] studied the optimal sensor placement in a distributed system to maximize the accuracy of parameter identification in a 2-D spatial domain.

Discussions on the Shannon sampling theorem and related signal processing concepts can be found in [26]–[29]. Models for 2-D stochastic processes are described in [30], whereas [31] introduced a family of spectral density–covariance function pairs for 2-D stochastic processes. We obtained several spectral density functions of demonstration models that serve as good candidates for the modeling process from [31]. Traffic flow fundamentals are covered in [32].

In summary, to our knowledge, there is no existing study that employed the same approach and accomplished the same goal as ours. More specifically, our research distinguishes itself from existing literature by the following: 1) It is an analytical approach based on the sampling theorem; 2) it not only yields the optimal sensor spacing but also characterizes the relative merits of the entire spectrum of sensor spacing; and 3) instead of restricting sensors to be at nodes or one per link, as done in existing studies, our research allows the sensor to be anywhere, and the consideration of deploying a sensor is how it helps reduce the cost function.

III. Spectral Characteristics and Modeling of Traffic Information

A. Theory and Spectral Characteristics of Traffic Information

Theoretical derivations based on [32]–[35] indicate that there exists a strong correlation between density values of neighboring points in the space–time domain. This gives a theoretical basis for the sampling theorem approach. Analysis of field data using definitions and techniques in [36] further verifies the validity of this approach.

Central to this discussion is the Shannon sampling theorem [26], which states that for a function $f(t)$ that contains no frequencies higher than W cycles per second, it is completely determined by giving its ordinates at a series of points spaced $1/(2W)$ seconds apart. The theorem can be extended to multidimensional signals.

In the context of the 2-D traffic information signal, sampling is carried out in both the time and space domains. In the time domain, the sampling rate translates to the intervals over which traffic information is aggregated. In the space domain, the sampling rate relates to the spacing between two adjacent sensors, i.e., how closely the sensors are placed apart decides how close each sample of the signal is taken to the next.

In our analysis using the NGSIM data sets, we arbitrarily set the sampling rate for both time and space at $f_{s,0} = 1/20$. This corresponds to a data aggregation interval of 20 s and a sensor spacing of 20 ft, which is equal to 6.096 m. It is a common practice to use a data aggregation interval of 20 s. Too short a data aggregation interval may give rise to noisy peaky readings, whereas long data aggregation intervals may lead to inaccurate readings. A sensor spacing of 20 ft (6.096 m), which corresponds to the average length of a car, is inconceivable in practice but is chosen as such to sample the signal at a high frequency to capture the high-frequency components.

To obtain the PSD of traffic information, we first determine its 2-D autocorrelation. The signal can be considered stationary since the data were collected over relatively short periods of time during the peak period, and the freeway condition did not change, i.e., there were no facilities like on/off ramps. The Fourier transform of the 2-D autocorrelation function gives the PSD of the signal.

Magnitude plots of the PSD reveals that the 2-D traffic information signal has a high concentration of power in the low-frequency contents. This implies that most of the spectral content can be captured using a relatively lower sampling rate, which corresponds to a larger sensor spacing.

From the PSD, it is possible to derive the NMSE associated with a particular sampling rate. The NMSE, which takes a value between zero and one, is given by

$$\mathrm{NMSE} = \frac{\sum_{i=1}^{N_t}\sum_{j=1}^{N_x}(X_{i,j}-\hat{X}_{i,j})^2}{\sum_{i=1}^{N_t}\sum_{j=1}^{N_x}(X_{i,j})^2} \tag{1}$$

where

N_t, N_x maximum index for time and space indexes, respectively, in the frequency domain;

$X_{i,j}$ PSD magnitude at time index i and space index j;

$\hat{X}_{i,j}$ magnitude of the truncated PSD due to a lower sampling rate at i and j.

In this case, the NMSE is a measure of the power loss due to the spectral content not captured by the sampling, i.e., the spectral content that is greater than half the sampling rate.

The information carried by a signal is commensurate with power, and any loss of power can be related to information loss. By converting the sampling rate into sensor spacing, we have the means to relate the NMSE to sensor spacing. A suitable model for the PSD remains to be developed to have a continuous characterization curve that relates the NMSE to sensor spacing.

B. Modeling the PSD

We desire a model with a closed-form defining equation to rigorously and completely characterize the relationship between the NMSE and sensor spacing. Several demonstration models are found in [31], of which the simplest model has a spectral density function defined by

$$S(n_1, n_2) = \frac{1}{(n_1^2 + n_2^2 + 1)} \tag{2}$$

where n_1 and n_2 are frequency-domain variables. Plotting the spectral density function given by (2) gives a plot that has a shape similar to that given by the magnitude plot of the PSD obtained from the field data.

For the purpose of modeling, we introduce a parameter c and rewrite (2) as

$$S(\chi, \tau, c) = \frac{c}{(\chi^2 + \tau^2 + c)} \tag{3}$$

where χ and τ are frequency-domain variables that correspond to the space and time variables in the space–time domain, respectively, and c is a real constant.

The presence of c in the numerator serves to normalize the model to facilitate the modeling process. By varying c, we attempt to find a model that best fits the PSD points obtained from the NGSIM data sets. The value of c that gives the minimum root-mean-square error (RMSE) was determined for each of the six NGSIM freeway data sets. We found that c ranged between 1.944 and 2.01 with a mean value of $\bar{c} = 1.981$.

The model defined by $S(\chi, \tau, \bar{c})$ is then used to characterize the variation of the NMSE with sensor spacing. The complete characterization is depicted in Fig. 1. Data points from the data sets, as well as the upper and lower limits of the characterization curve, are also presented. The upper and lower limits that correspond to the models are given by $S(\chi, \tau, c)$, where $c = 1.944$ sets the upper limit, and $c = 2.01$ sets the lower limit.

Fig. 1 shows a good fit between the model and NGSIM data. In particular, the narrow band bounded by the upper and lower limits closely follows the curve. While it is possible that other models may fit the data equally well or even better, we opt to stay with the current model and leave the search for other, possibly better, models as a future research topic.

IV. Sensor Optimization

A. Discussion

With reference to Fig. 1, we observe that for any sensor spacing equal to or less than 1000 ft (304.8 m), the NMSE is less than 0.05. The stretch between 1000 ft (304.8 m) and 2500 ft (762 m) is almost linear and corresponds to the reported cost-effectiveness tradeoff region. With a sensor spacing greater than 2500 ft (762 m), the characterization curve reveals an NMSE of approximately one third. Thus, the sampling theorem helps us examine and explain the performance of sensors as a function of sensor spacing.

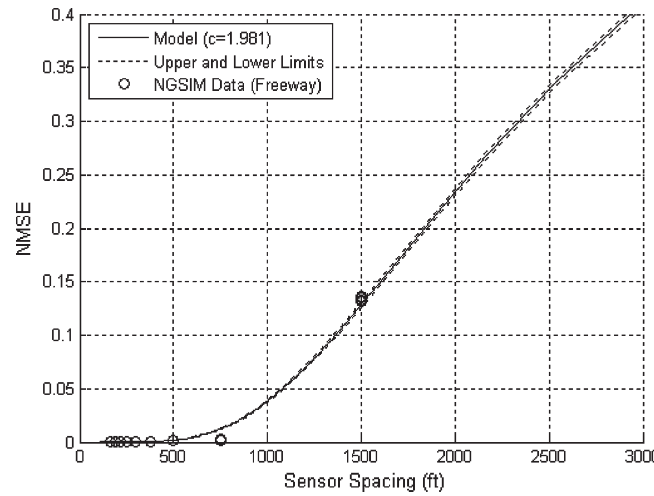

Fig. 1. NMSE as a function of sensor spacing based on the PSD model.

Although [5] adopted an empirical approach and used a different set of criteria, it allows a cross-comparison between findings of that study and ours. Noticeably, our research provides many more details over the whole spacing spectrum and is more practically appealing to practitioners.

B. Optimization

The report [5] also included a discussion on cost-effectiveness analysis to help practitioners decide on the most cost-effective sensor spacing. To this end, we offer a scientific and analytical approach based on the sampling theorem to determine the optimal sensor spacing.

The objective of our approach is to minimize the total cost $t(d)$, which includes the cost to install, operate, and maintain sensors and the cost incurred by motorists who are caught in congestion. Hence, the objective function is given by

$$\text{Minimize } t(d) = s(d) + c(d) \tag{4}$$

where

- d sensor spacing;
- $c(d)$ annual cost of congestion due to sensor deficiency, as will be explained later;
- $s(d)$ annual cost of sensors, which includes both capital and operating costs ($s(d) = ns_C$);
- s_C annual cost per sensor;
- n number of sensors involved ($n = (L/d) + 1$, where L is the length of the road section monitored by sensors, and "1" accounts for the extra sensor needed to close the road section).

Therefore, $s(d)$ can be expressed as

$$s(d) = s_C \left(\frac{L}{d} + 1 \right). \tag{5}$$

The authors are not aware of any formulation that relates the NMSE to the cost incurred by motorists due to congestion. As such, a formulation based on the traffic flow theory was developed to define the function $c(d)$. The formulation assumes that the freeway is managed

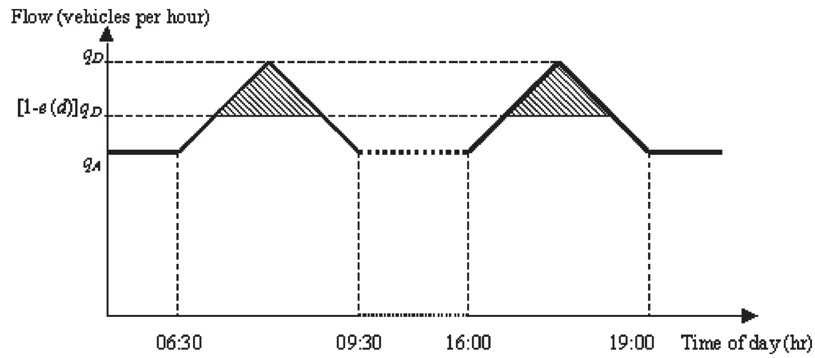

Fig. 2. Flow during peak periods with shaded area representing vehicles caught in congestion due to the sensor error.

by a traffic management center (TMC) that will not allow the level of service (LOS) [38] provided by the freeway lane to degrade beyond LOS D. To achieve this objective, the TMC monitors the real-time condition of the freeway using the sensors placed d distance apart and diverts traffic to alternative routes once the freeway operates at LOS D. In this formulation, congestion arises when the error from the sensors causes the TMC to activate the diversion measures too late. It is also assumed that congestion occurs only during peak periods and traffic operates at LOS A at other times. During a peak period, the traffic may build up toward the conditions that define LOS D. To formulate the congestion cost and for the sake of simplicity, it is assumed that traffic linearly builds up toward the peak, as is depicted in Fig. 2, which presents the variation of flow q with the time of day, where the service volume at LOS X is denoted by q_X. The linear model, while simplifying the formulation, captures the dominant characteristic of traffic flow during peak periods, i.e., a buildup toward a peak value. Other models that reasonably represent peak traffic behavior will also be valid. The value of the service volume or traffic flow at each LOS is obtained from [38].

The formulation of $c(d)$ accounts for the cost due to congestion arising from the error from the system of sensors. The error can cause the system of sensors to either overreport or underreport the actual flow of the traffic. Overreporting refers to the situation where the system of sensors gives a reading higher than the actual value. This does not cause any congestion. On the other hand, underreporting occurs when the system gives a lower reading than the actual value, i.e., $q_{\text{reading}} = (1 - e(d))q_{\text{actual}}$. Note that the error $e(d)$ takes a value between 0 and 1. This results in traffic congestion. Due to underreporting, congestion occurs at $(1 - e(d))q_D$. It is assumed that the sensor system is equally likely to over- or underreport. Ignoring other effects for simplicity, the shaded triangles in Fig. 2 represent the vehicles that are caught in congestion due to the underreporting of the sensors. The area of the shaded triangles gives the number of vehicles caught in congestion.

With this formulation, the annual cost of congestion due to the sensor error is given by

$$c(d) = N_Y N_P p c_V L D_P \left(\frac{q_D^2}{2(q_D - q_A)} \right) (e(d))^2 \tag{6}$$

where

- N_Y number of days in a year;
- N_P number of peak periods in a day;
- D_P duration of peak periods (in hours);
- p probability of underreporting;
- c_V cost of congestion per vehicle mile;
- q_X service volume at LOS X, where $X = $ A or D.

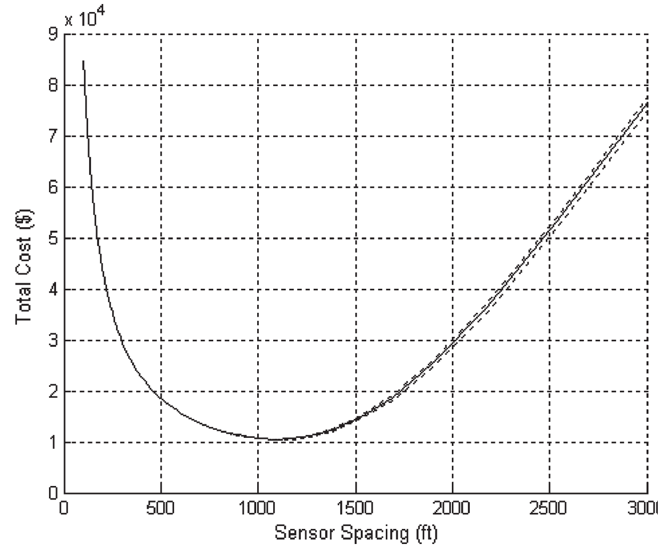

Fig. 3. Total cost $t(d)$ as a function of sensor spacing d defined by (7).

For any deployment of sensors, denoting the random variables of the actual flow value and the sensor reading using Q and Q', respectively, we can write

$$E\left[|Q - Q'|\right] \leq \left(E\left[|Q - Q'|\right]\right)^{0.5} = \sqrt{\text{NMSE}} \left(E[Q^2]\right)^{0.5}.$$

Based on the congestion cost formulation, in the case of underreporting when $Q = q_D$ (a degenerate random variable)

$$E\left[|q_D - Q'|\right] \leq \sqrt{\text{NMSE}} \left(E\left[q_D^2\right]\right)^{0.5}$$
$$\implies E[Q'] \geq (1 - \sqrt{\text{NMSE}})q_D.$$

This gives a lower bound on the value of sensor reading when the actual flow value is q_D. In other words, $e(d) \leq \sqrt{\text{NMSE}}$. For brevity, we write NMSE instead of NMSE(d) while bearing in mind that NMSE is also a function of d. The practitioner can choose to use $e(d) = \sqrt{\text{NMSE}}$ for the most conservative result. Alternatively, investigating a suitable function that relates $e(d)$ and $\sqrt{\text{NMSE}}$ is a good future research topic.

For illustrative purposes, we assume that $e(d) = \text{NMSE} \leq \sqrt{\text{NMSE}}$. The inequality $\text{NMSE} \leq \sqrt{\text{NMSE}}$ is true since

$0 \leq \text{NMSE} \leq 1$. Hence, (4) can be rewritten as

$$\text{Minimize}: \; t(d) = s_C\left(\frac{L}{d}+1\right) + N_Y N_P p c_V L D_P \times \left(\frac{q_D^2}{2(q_D-q_A)}\right)(\text{NMSE})^2. \quad (7)$$

An illustrative example where actual values are substituted into (7) can be the following. The annual cost of a traffic sensor s_C is estimated as \$1600 based on [37]. The cost of congestion per vehicle c_V is estimated as \$0.26 based on [39]. The probability of underreporting is assumed to be 0.5. The number of days can be taken as 365, and there are two peaks in a day. Each peak lasts for 3 h. The service volumes at LOS A, i.e., q_A, and LOS D, i.e., q_D, are 700 and 1850, respectively, according to [38]. A road length of 1 mi is assumed.

Fig. 3 presents the variation in the total cost $t(d)$ as a function of sensor spacing d based on (7) using the aforementioned values. The optimal sensor spacing is found to be 1086 ft (331 m). The dotted curves in Fig. 3 are the upper and lower limits of the congestion cost, which are derived in the same way as those limits in Fig. 1.

In [5], a cost-effectiveness tradeoff for sensor spacing is reported. In the proposed approach, the optimal sensor spacing for a given cost function $s(d)$ is provided instead. This difference in output hinders any numerical comparison between the two methods. In addition, close to three decades of technological advancement and economic changes that separate the current work and [5] further makes any comparison between the two less purposeful.

V. Conclusion

Based on the sampling theorem, we analyzed the spectral characteristics of traffic information and modeled its PSD. Using the obtained model, a characterization curve that relates the NMSE to sensor spacing was obtained. Using a formulation that relates sensor spacing d to the cost incurred by motorists due to congestion, the optimal sensor spacing that minimizes the total cost was determined. Although the analysis presented in this paper is based on traffic data collected by existing sensors, the findings provide valuable insights into new deployment of traffic sensors.

The superiority of the proposed approach over an empirical method, such as that employed in [5], partly lies in its extensibility. Using tunable parameters, different models for different circumstances, e.g., lower speed limit, road curvature, etc., can be developed without the need for fresh new data for each circumstance. In addition, the current approach can be used with any type of sensor technology as long as a cost function can be determined. The possibility or ease with which an empirical method can be extended to account for the changing circumstances or new sensor technology is not obvious.

References

[1] "National Transportation Statistics 2006," *Bureau of transportation statistics*, Jun. 6, 2006. [Online]. Available: http://www.bts.gov/publications/national_transportation_statistics/2006/

[2] California Center for Innovative Transportation, *Traffic surveillance-road-based technologies*, Jun. 5, 2005. [Online]. Available: http://www.calccit.org/itsdecision/serv_and_tech/Traffic_Surveillance/road-based/roadside/video_sum.html

[3] Georgia Navigator, Sep. 1, 2006. [Online]. Available: http://www.georgia-navigator.com/

[4] NGSIM, *Next generation simulation*, Jun. 2005, Oakland, CA: Cambridge Systematics. [Online]. Available: http://ngsim.camsys.com/

[5] H. M. Koble, G. M. Anderson, and R. G. Goldblatt, "Formulation of guidelines for locating freeway sensors," Federal Highway Admin., Washington, DC, FHWA-RD-78-137, 1979.

[6] L. Bianco, G. Confessore, and P. Reverberi, "A network based model for traffic sensor location with implications on O/D matrix estimates," *Transp. Sci.*, vol. 35, no. 1, pp. 50–60, Feb. 2001.

[7] Y. Liu, X. Lai, and G.-L. Chang, "Optimal detector locations for OD matrix estimation," in *Proc. 8th Int. Conf. Appl. Adv. Technol. Transp. Eng.*, Beijing, China, 2004.

[8] A. P. Tomas, "A note on sensor location for traffic counting at roundabouts: Solutions for a particular cost function," Universidade do Porto, Porto, Portugal, DCC-01-03, 2001. DCC-FC & LIACC.

[9] H. Yang and J. Zhou, "Optimal traffic counting locations for origin-destination matrix estimation," *Transp. Res., Part B: Methodol.*, vol. 32, no. 2, pp. 109–126, Feb. 1998.

[10] S. M. F. Eisenman, Z. Xiang, M. Xuesong, and S. Hani, "Number and location of sensors for real-time network traffic estimation and prediction: Sensitivity analysis," in *Proc. Transp. Res. Rec.*, 2006, vol. 1964, pp. 253–259.

[11] I. Fujito, R. Margiotta, W. Huang, and W. Perez, "Effect of sensor spacing on performance measure calculations," in *Proc. Transp. Res. Rec.*, 2006, vol. 1945, pp. 1–11.

[12] V. P. Sisiopiku, N. M. Rouphail, and A. Santiago, "Analysis of correlation between arterial travel time and detector data from simulation and field studies," in *Proc. Transp. Res. Rec.*, 1994, vol. 1457, pp. 166–173.

[13] G. B. Thomas, "Optimal detector location on arterial streets for advanced traveler information system," Ph.D. dissertation, Arizona State Univ., Tempe, AZ, 1999.

[14] S. Oh, B. Ran, and K. Choi, "Optimal detector location for estimating link travel speed in urban arterial roads," in *Proc. 82nd Transp. Res. Board Annu. Meeting*, Washington, DC, 2003. CD-ROM.

[15] D. Woods, "Optimal detector locations for HOV lane operations," Texas Transp. Inst., College Station, TX, FHWA/TX-95/1392-7, 1994.

[16] K. R. MacHutchon and A. Ryan, "Fog detection and warning: A novel approach to sensor location," in *Proc. 5th African Electr. Technol. Conf.*, Cape Town, South Africa, 1999.

[17] D. Woods and L. Koniki, "Optimizing detector placement for high speed isolated signalized intersections using vehicular delay as the criterion. Interim report," Texas Transp. Inst., College Station, TX, FHWA/TX-94/1392-3, 1994.

[18] T.-P. Hsu, *Optimization of Detector Location for Traffic Actuated Signal Control*. Karlsruhe, Germany: Inst. Verkehrswesen, Univ. Karlsruhe, 1991.

[19] D. Goulias, *Methodology for Optimal Placement of Pavement Temperature Sensors*. New York: Univ. Transp. Res. Center, Region II, 1995. City College.

[20] N. Stubbs and S. Park, "Optimal sensor placement for mode shapes via Shannon's sampling theorem," *Microcomput. Civ. Eng.*, vol. 11, no. 6, pp. 411–419, 1996.

[21] Z. Shi, S. Law, and L. Zhang, "Optimum sensor placement for structural damage detection," *J. Eng. Mech.*, vol. 126, no. 11, pp. 1173–1179, Nov. 2000.

[22] C. Papadimitriou, J. L. Beck, and S. K. Au, "Entropy-based optimal sensor location for structural model updating," *J. Vib. Control*, vol. 6, no. 5, pp. 781–800, 2000.

[23] J. W. Berry, L. Fleischer, W. E. Hart, C. A. Phillips, and J.-P. Watson, "Sensor placement in municipal water networks," *J. Water Resour. Plan. Manage.*, vol. 131, no. 3, pp. 237–243, 2005.

[24] M. Propato, O. Piller, and J. G. Uber, "A sensor location model to detect contaminations in water distribution networks," in *Proc. World Water Environ. Resour. Congr.*, Anchorage, AK, 2005.

[25] D. Uciñski, "Optimal sensor placement in parameter estimation of distributed systems," in *Proc. 14th Int. Symp. MTNS*, Perpignan, France, 2000.

[26] C. E. Shannon, "Communications in the presence of noise," *Proc. Inst. Radio Eng.*, vol. 37, no. 1, pp. 10–21, Jan. 1949.

[27] A. J. Jerri, "The Shannon sampling theorem—Its various extensions and applications: A tutorial review," *Proc. IEEE*, vol. 65, no. 11, pp. 1565–1596, Nov. 1977.

[28] A. J. Jerri, "Correction to 'The Shannon sampling theorem—Its various extensions and applications: A tutorial review,"' *Proc. IEEE*, vol. 67, no. 4, p. 695, Apr. 1979.

[29] D. E. Dudgeon and R. M. Mersereau, *Multidimensional Digital Signal Processing*. Englewood Cliffs, NJ: Prentice-Hall, 1984.

[30] V. Heine, "Models for two-dimensional stationary stochastic processes," *Biometrika*, vol. 42, no. 1/2, pp. 170–178, Jun. 1955.

[31] A. V. Vecchia, "A general class of models for stationary two-dimensional random processes," *Biometrika*, vol. 72, no. 2, pp. 281–291, Aug. 1985.

[32] A. D. May, *Traffic Flow Fundamentals*. Englewood Cliffs, NJ: Prentice-Hall, 1989.

[33] M. Lighthill and G. Whitham, "On kinematic waves. II. A theory of traffic flow on long crowded roads," *Proc. R. Soc. Lond., A Math. Phys. Sci.*, vol. 229, no. 1178, pp. 317–345, May 1955.

[34] P. I. Richards, "Shock waves on the highway," *Oper. Res.*, vol. 4, no. 1, pp. 42–51, Feb. 1956.

[35] B. D. Greenshields, "A study of traffic capacity," in *Proc. Highw. Res. Board*, 1934, vol. 14, pp. 468–477.

[36] L. Edie, "Discussion on traffic stream measurements and definitions," in *Proc. 2nd Int. Symp. Theory Traffic Flow*, Paris, France, 1963.

[37] Intelligent Transportation System Joint Program Office, *Equipment costs for roadside detection*, Jan. 5, 2007. [Online]. Available: http://www.itscosts.its.dot.gov/

[38] *Highway Capacity Manual*, Transp. Res. Board, Washington, DC, 2000.

[39] Victoria Transport Policy Institute, *Transportation Cost and Benefit Analysis—Congestion Costs*, Sep. 5, 2006. [Online]. Available: http://www.vtpi.org/tca/tca0505.pdf