Direct Numerical Simulations of Turbulent Flows over Superhydrophobic Surfaces

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Direct numerical simulations are used to investigate the drag reducing performance of superhydrophobic surfaces in turbulent channel flow. Slip velocities, wall shear stresses, and Reynolds stresses are considered for a variety of superhydrophobic surface micro-feature geometry configurations at a friction Reynolds number of $Re_\tau = 180$. For the largest micro-feature spacing of 90 $\mu$m an average slip velocity over 75% of the bulk velocity is obtained, and the wall shear stress reduction is nearly found to be nearly 40%. The simulation results suggest that the mean velocity profile near the superhydrophobic wall continues to scale with the wall shear stress, but is offset by a slip velocity that increases with increasing microfeature spacing.

1. Introduction

Significant effort has been placed on the development of surfaces which reduce the amount of drag experienced by a fluid as it passes over the surface. Drag reduction in turbulent flows can be achieved via a number of very different mechanisms including the addition of polymers (Lumley (1969)), the addition of bubbles (Sanders et al. (2006)), riblets (Bechert et al. (1997)), compliant walls (Hahn et al. (2002)), and active blowing and suction (Kim (1999)). Laminar drag reduction is much harder to achieve. Macro-scale laminar drag reduction is possible with liquids using surface or fluid electric charges (Maynes & Webb (2003)), and via surface chemistry (surface hydrophobicity) (Tretheway & Meinhart (2002)). Recent work (see Ou et al. (2004); Ou & Rothstein (2005); P. Joseph et al. (2006)) has shown that liquid laminar drag reduction is achievable in larger channels using superhydrophobic surfaces. Superhydrophobic surfaces combine hydrophobic chemistry with micron scale topological features to produce slip lengths on the order of tens of microns.

In this work it will be demonstrated that superhydrophobic surfaces can also produce significant drag reduction for liquids operating in the turbulent regime. Experiments by S. Gogte et al. (2005) and early results from Daniello & Rothstein (2008) were at high enough Reynolds numbers to be turbulent and showed some drag reduction could be achieved. A theoretical analysis by Fukagata et al. (2006) (that assumes classical scaling arguments still hold when a superhydrophobic surface is present) suggests how a small alteration of the laminar sublayer can affect the entire turbulent boundary layer and alter the subsequent drag. More recently, Min & Kim (2004, 2005) performed a direct numerical simulation (DNS) in a channel at a shear stress Reynolds number, $Re_\tau = 180$ in which the turbulence was fully resolved but the superhydrophobic surface itself was modeled. The surface boundary condition was modeled as $u_{wall} = b \left( \frac{du}{dy} \right)_{wall}$, where $b$ was an assumed
slip length. In the current work, both the turbulence and the surface boundary conditions are fully resolved. At the Reynolds number simulated in this work, the surface micro-features have a size that ranges from $17^+\text{wall units}$ wide to $34^+\text{wall units}$ wide, which is below the streak width of $100^+\text{wall units}$. Superhydrophobic surfaces can be made using a variety of micro-features. In this work, regular arrays of micro-ridges and micro-posts are used, so that the micro-topology is very well characterized. However, superhydrophobic surfaces are also effective with random post arrangements (plasma etched plastics and the Lotus leaf being two common examples). The physical arrangements of micro-ridge and micro-post surfaces considered in this work are illustrated in Figure 1. The SHS is only placed on the bottom wall of the channel.

The spacing and size of the micro-features allows a liquid-gas interface to be supported between the individual posts or ridges. The gaps between post/ridges allow a nonzero average velocity to be present on the surface (called the slip velocity) and can result in drag reduction. The role of hydrophobicity in superhydrophobic surfaces is simply to keep the liquid out of the gaps. Normal surface chemistry would tend to pull the fluid into the gaps via a strong capillary action. While the chemical hydrophobicity of the surface (or its magnitude as measured by the static contact angle) has absolutely no affect on the drag reduction properties of the superhydrophobic surface, it does dictate the maximum pressure that the free-surface can support before the free-surface fails and the liquid is forced (by the static pressure) to move into the gap (Ou et al. (2004)).

In this investigation, only the liquid is simulated. The top surface of each micro-feature is taken to be a no-slip boundary on the liquid, and the suspended liquid-gas interface between the micro-features is simulated as a flat and shear-free boundary on the liquid. While the drag is locally zero on the free-surface (between the posts/ridges), it is much higher than normal over the post/ridges because the fluid rapidly decelerates there. The net drag is the sum of these two effects, and depends on which effect dominates. While it is often stated that the drag reduction is a result of reduced wall contact with the fluid, this is an incomplete explanation of the phenomena, as the spacing between the microfeatures is also critically important.

The boundary assumptions on the liquid mean that the free-surface in these simulations is assumed to experience no out-of-plane deflection. The deflection is quantified by the maximum free-surface displacement, $s$. When the deflection ratio $s/w$ (where $w$ is the gap width) is greater than roughly $20\%$ (the exact value depends on the hydrophobic contact angle) the surface will fail. For small deflections ($< 20\%$) the deflection is well

![Figure 1. Representations of the superhydrophobic surfaces. Subfigure (a) shows ridges, arranged on a plane with periodic boundary conditions, with width $d$, spacing $w$, and channel height $H$. Subfigure (b) shows posts, arranged similar to the ridges shown in (a). Posts are square and spaced evenly in the streamwise ($x$) and spanwise ($z$) directions. Note these diagrams are not to scale.](image-url)
approximated by \( s/w \approx (w \Delta p)/(8 \sigma) \), where \( \sigma \) is the surface tension. For our maximum gap size of 90\( \mu \text{m} \) and water with \( \sigma = 7 \times 10^{-2} \), this means that pressures up to 700 Pa produce small deflections (< 10%). The work of Ybert et al. (2007) confirms that the affect of small deflections (< 20%) on laminar drag reduction is negligible. The assumption of shear-free flow on the free-surface is reasonable if the micro-posts and micro-ridges are tall enough (i.e. same order of magnitude as the spacing). If the posts/ridges are significantly shorter than the gap width, then the circulating air in the gap between the micro-features could cause some drag on the free-surface and cause such a surface to produce less drag reduction (more total drag) than these DNS calculations predict.

2. Direct Numerical Simulations

A CFD code was developed to perform direct numerical simulation of turbulent channel flow over surfaces with varying boundary conditions. The governing Navier-Stokes equations are numerically solved using a second-order accurate Cartesian staggered mesh method with classical projection for the pressure solution. A second-order accurate, three step, low storage Runge Kutta scheme is used to advance the solution in time. The mesh has non-uniform spacing in the vertical direction (normal to the surfaces), in order to allow greater resolution near the channel’s top and bottom walls. A Cartesian mesh is well suited to the channel and micro-feature geometry being investigated. A staggered scheme is employed because it conserves mass and momentum, as well as vorticity and kinetic energy. There is no numerical viscosity or artificial dissipation in this scheme (Perot (2000)), which is vital to accurately predicting the turbulent energy cascade (see Mittal & Moin (1998)). The code is fully parallel, using MPI libraries, and optimized for execution on supercomputers.

The computational domain is a box of ratio 6:2:3 (length:height:width). \( x \) is the streamwise direction of length \( L_x \), \( y \) the vertical direction of height \( L_y \), and \( z \) the spanwise direction of width \( L_z \). For the data presented in this paper, all resolutions are 128 grid points in each direction, except the 15\( \mu \text{m}-15\mu \text{m} \) case, which has 256 grid points in each direction. The boundary conditions consist of alternating regions of no-slip and no-shear, which correspond to the top of the micro-feature (a post or ridge) and the interface supported between micro-features, respectively. Several ridge and post configurations were examined, where the feature width \( d \) and feature spacing \( w \) were varied (see Figure 1). Eight ridges (in the spanwise direction) were present for all ridge simulations, except for the 15\( \mu \text{m}-15\mu \text{m} \) case which had 16 across the channel. Similarly, a minimum of eight posts in the spanwise direction, and sixteen in the streamwise direction, were present for all post simulations. Note that in the case of ridges, the configurations are referred to by the ratio of their spanwise width (in \( z \)) to spanwise spacing (in \( z \)). For example, a ridge which is 30\( \mu \text{m} \) wide and spaced at 30\( \mu \text{m} \) is a “30\( \mu \text{m}-30\mu \text{m} \) ridge”. All micro-ridge cases investigated involve uniformly spaced ridges of equal width. Similarly, all micro-post cases investigated involve square posts (whose widths \( d \) in both the streamwise and spanwise direction are equal) that are uniformly spaced (spacing \( w \) in both the streamwise and spanwise direction is equal). For clarity, the symbols used for each geometry remain consistent throughout the figures and are: regular channel flow (benchmark) – –; 15\( \mu \text{m}-15\mu \text{m} \) ridges □; 30\( \mu \text{m}-30\mu \text{m} \) ridges △; 30\( \mu \text{m}-50\mu \text{m} \) ridges ◇; 30\( \mu \text{m}-90\mu \text{m} \) ridges ▽; 30\( \mu \text{m}-30\mu \text{m} \) posts ▲; 30\( \mu \text{m}-50\mu \text{m} \) posts ◆; 30\( \mu \text{m}-90\mu \text{m} \) posts ▼. In every case, the sum of each feature width and spacing (\( w+d \)) is kept constant at 16 grid points and 67.5+ wall units.

The instantaneous variables calculated in the CFD code are averaged. As such, each subdomain around a single feature produces a set of temporally averaged velocity and
pressure data, denoted $\bar{u}'$ and $\bar{p}'$. These temporal averages are then ensemble averaged (denoted by angle brackets) over all the post/ridges in the simulation to produce $\langle u'^2 \rangle$ and $\langle \bar{p}' \rangle$. The Reynolds stresses are calculated as $R(x, y, z) = \langle (u_i - \langle u_i \rangle)(u_j - \langle u_j \rangle) \rangle$. For graphical presentation, we calculate velocity averages over $xz$ planes to remove the local micro-feature variations, $U(y) = \langle \bar{u}' \rangle^2 = \frac{1}{\tau_w L_z} \int_0^{L_x} \int_0^{L_z} (\bar{u}')^2 \, dxdz$. Similarly, the planar averaged Reynolds stresses are $R_{ij}(y) = \langle \bar{u}' \bar{v}' \rangle = \frac{1}{\tau_w L_z} \int_0^{L_x} \int_0^{L_z} R \, dxdz$. Finally, the bulk velocity, $U_{bulk} = \frac{1}{\tau_w} \int_{-\frac{L_z}{2}}^{\frac{L_z}{2}} U(y) \, dy$, is used as a velocity scale, noting the center of the channel is located at $y = 0$. The slip velocity, $U_{slip} = U(-L_y/2)$ is the $xz$ planar average of the streamwise velocity at the lower wall. This slip velocity is a macroscopic value, as the actual velocities at the superhydrophobic surface are either zero (above the no-slip feature) or some non-zero value (above the interface). The average wall shear stress $\tau_w = (\tau_w^T + \tau_w^B)/2$ is calculated by averaging the top wall shear stress $\tau_w^T = \mu \frac{dU}{dy} \bigg|_{y=L_y/2}$ and the bottom wall shear stress $\tau_w^B = \mu \frac{dU}{dy} \bigg|_{y=-L_y/2}$ where $\mu$ is the viscosity. The top and bottom wall shear stresses are also macroscopic quantities. At statistical steady-state, the average wall shear stress $\tau_w$ is directly related to the average channel pressure gradient $\frac{dp}{dz}$ by $\tau_w = (2/L_y) \frac{dp}{dz}$. Note the pressure gradient and not the mass flux is prescribed in these calculations.

3. Validation

Previous DNS of turbulent channel flow performed by Moser et al. (1998) was employed as a means of benchmarking the CFD code. In Figure 2 the planar averaged streamwise velocity profile and the four non-zero components of the planar averaged Reynolds stress tensor are compared to the results of Moser et al. (1998) with quantitative agreement at a friction Reynolds number $Re_T = (u_T L_y)/\nu = 180$. The friction velocity is given by $u_T = (\tau_w/\rho)^{1/2}$. The results are plotted against the non-dimensional channel height $y^+ = u_T y/\nu$, where $y$ is the vertical position in the channel. The left end of the axis in Figure 2 cooresponds to the bottom wall of the channel (at $y = -L_y/2$), and the right end corresponds to the center of the channel (at $y = 0$). For the standard channel flow, the velocity and Reynolds stress profiles are symmetric about the channel’s vertical center plane. This assumption of symmetry does not apply to the subsequent simulations of channels with an superhydrophobic lower surface and standard upper surface.

4. Results

Velocity profiles and Reynolds stresses were computed for the four different ridge cases. Figure 3(a) shows the mean velocity profiles normalized by the average friction velocity $u_T$. The channel height $(y)$ is normalized by the channel half-height $\delta = L_y/2$. The standard channel flow data is provided (with a dashed line) as a reference. Results from the ridge cases show slip velocities and peak mean velocities that increase with increased width to spacing ratio $d/w$. In addition, the location of the peak velocity moves closer to the superhydrophobic surface. In Figure 3(b), the velocity profiles are shown close to the superhydrophobic surface and plotted against $y^+$. In this figure both axis are scaled by the bottom wall shear stress $\tau_w^B$. Note that the 15µm-15µm ridge case was performed with 256 grid points in the vertical $(y)$ direction, which is why it has a data point closer to the superhydrophobic surface. This resolution increase was necessary in order to properly resolve the smaller feature sizes and spacings.
Figure 2. Velocity profiles and Reynolds stresses calculated with the present code (−−) for turbulent channel flow at $Re_{τ} = 180$ compared with results of Moser et al. (1998) (symbols). (a) Near wall velocity profiles, scaled by $u_τ$. (b) Planar averaged Reynolds stresses scaled by $u_τ^2$. $R_{11}(\circ)$, $R_{22}(\oplus)$, $R_{33}(\otimes)$, and $R_{12}(\bullet)$.

Figure 3. Ridge cases: Planar averaged velocity profiles. $15\mu m - 15\mu m (□), 30\mu m - 30\mu m (△), 30\mu m - 50\mu m (♦)$, and $30\mu m - 90\mu m (▽)$, with standard channel flow (−). (a) Velocity profiles scaled by the average friction velocity. (b) Velocity profiles near the superhydrophobic surface (left), scaled by the bottom wall (superhydrophobic surface) friction velocity. Symbols are employed to differentiate lines, and do not reflect actual data points.

The Reynolds stresses $R_{ij}(y)$ are shown in Figure 4 and are scaled by the average friction velocity squared, $u_τ^2$. Since the mean pressure gradient remains constant in these simulations, a net reduction in drag or shear on the lower (superhydrophobic) boundary leads to a corresponding increase in the net shear on the top (regular) wall. With reduced shear, turbulence levels decrease near the superhydrophobic boundary. The turbulence levels increase on the regular boundary because of the higher shear now present near the upper wall. In the middle of the channel, the slope of the shear stress ($R_{12}$) must balance the fixed pressure gradient, and this is why these curves are all parallel in this region. Note that, due to the slip velocities present in the $x$ and $z$ directions, $R_{11}$ and $R_{33}$ become non-zero at the superhydrophobic surface.

Figures 5 and 6 show $U(y)$ and $R_{ij}(y)$ for posts. Similar to the results found with ridges, the slip velocities and peak mean velocities for the post cases increase with increased $d/w$. For posts, the slip velocities tend to be larger than those from equivalently sized
Figure 4. Ridge cases: Reynolds stresses scaled by $u'^{2}$, $15 \mu m - 15 \mu m (\square)$, $30 \mu m - 30 \mu m (\triangle)$, $30 \mu m - 50 \mu m (\diamond)$, and $30 \mu m - 90 \mu m (\bigtriangleup)$, with standard channel flow (−). Symbols are employed to differentiate lines, and do not reflect actual data points.

Figure 5. Post cases: Planar averaged velocity profiles. $30 \mu m - 30 \mu m (\blacktriangle)$, $30 \mu m - 50 \mu m (\blacklozenge)$, and $30 \mu m - 90 \mu m (\blacktriangledown)$, with standard channel flow (−). (a) Velocity profiles scaled by the average friction velocity. (b) Velocity profiles near the superhydrophobic surface (left), scaled by the bottom wall (superhydrophobic surface) friction velocity. Symbols are employed to differentiate lines, and do not reflect actual data points.
or spaced ridges. Similar to results found with ridges, $R_{11}$ and $R_{33}$ become non-zero at the superhydrophobic surface, and peak values for $R_{11}$, $R_{22}$, and $R_{33}$ decrease near the superhydrophobic surface. The asymmetry present in $R_{11}$, $R_{22}$, and $R_{33}$ is more pronounced than that found in the ridged case. In addition, the peak in $R_{11}$ for both the 30$\mu$m – 50$\mu$m and 30$\mu$m – 90$\mu$m post cases moves so that the maximum occurs on the superhydrophobic surface itself.

5. Discussion

The presence of superhydrophobic surface micro-features has a significant affect on the behavior of turbulent channel flow. Marked changes in the velocity profiles, Reynolds stresses, and wall shear stress are observed for a variety of micro-ridge and micro-post geometries. These results are consistent with the recent experimental work of Daniello & Rothstein (2008), who have investigated turbulent flow over superhydrophobic surfaces with similar micro-feature geometries.

Figures 3(b) and 5(b) suggest that the essential scaling properties of turbulent boundary layers remain intact even when an superhydrophobic surface is present and significant drag reduction is occurring for that boundary layer. In those figures, $u^+ = C + y^+$ to
Figure 7. $R_{12}$, scaled by the square of the bottom wall friction velocity, $(u_\tau)^2$, at the superhydrophobic surface, for streamwise ridges $15\mu m - 15\mu m(\square)$, $30\mu m - 30\mu m(\triangle)$, $30\mu m - 50\mu m(\diamond)$, and $30\mu m - 90\mu m(\bigtriangledown)$ and square uniformly spaced posts $30\mu m - 30\mu m(\blacksquare)$, $30\mu m - 50\mu m(\blacklozenge)$, and $30\mu m - 90\mu m(\blacktriangle)$, with standard channel flow $R_{12}(\cdots)$. Symbols are employed to differentiate lines, and do not reflect actual data points.

within 10% for all the cases considered (where $C$ is the normalized slip velocity). For this result it was important to scale with the local (bottom) boundary shear stress. Figure 7 investigates if this scaling also holds for the turbulence as well as the mean flow. This figure shows $R_{12}$, profiles for all seven cases investigated, scaled by the bottom wall shear stress. The profiles all collapse onto the standard channel flow profile ($\cdots$) near the bottom wall (superhydrophobic surface), suggesting that the structure of the near-wall turbulence has not fundamentally changed. Less surprisingly (and not shown), the same behavior is also exhibited at the top wall if the $R_{12}$ profiles are scaled by the top wall shear stress.

The slip and drag reduction properties of the superhydrophobic surface are summarized in Figure 8. Drag reduction performance increases with increased feature spacing $w$. This trend is consistent for both ridges and posts, as the slip velocity attains nearly 65% of the bulk velocity for ridges, and over 75% of the bulk velocity for posts, with a width to spacing ratio just above 0.3. For equal ridge width to spacing ratios $d/w$, more slip is achieved with larger micro-features. This is clearly seen in Figure 8(a), where the slip velocity at $15\mu m - 15\mu m$ ridges is nearly 40% lower than the slip velocity for the $30\mu m - 30\mu m$ ridges. A similar trend for the drag reduction is found in Figure 8(b), where the bottom wall shear stress reduction for the $15\mu m - 15\mu m$ ridges is nearly 30% lower that that for the $30\mu m - 30\mu m$ ridges. For a width to spacing ratio $d/w$ of unity, the $30\mu m - 30\mu m$ ridges consistently outperform the $15\mu m - 15\mu m$ ridges. This indicates that the actual size of the features, and not simply the ratio of width to spacing (or the percentage surface area contacted), plays an important role in the surface’s drag-reduction performance. Smaller features lead to diminished drag-reduction performance. When comparing posts and ridges, it is clear that for a given ratio of micro-feature size to spacing, $d/w$, posts yield higher slip velocities and larger shear stress reductions when compared to ridges. The performance advantage of posts over ridges appears to increase with increased feature spacing, $w$.

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Figure 8. Drag reduction performance of the superhydrophobic surface with ridges and posts. See Figure 1 for definitions of $d$ and $w$. (a) Slip velocity at the superhydrophobic surface as a percentage of the bulk velocity for varying feature width and spacing. (b) Wall shear stress reduction at the superhydrophobic surface, normalized by the average wall shear stress, for varying feature width and spacing. Symbols are connected by lines to illustrate trends.

REFERENCES


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