The Development of a Moving Staggered Mesh Method for Incompressible Flows

Prof. Blair Perot & Xing Zhang

19th Biennial Conference on Numerical Analysis Dundee, June 26-29, 2001
Motivation

• Develop a numerical method suitable for complex unsteady flows with:
  – Moving walls or surfaces
  – Time dependent solutions

• Efficient, Accurate, Parallel
Staggered Mesh

- Primary variable: velocity normal to cell faces

Cartesian  Unstructured
Staggered Mesh Properties

• No pressure modes. (No locking, no damping required)
• Efficient. (75% fewer unknowns in 2D, 60% fewer in 3D)
• Conserves: mass, momentum, kinetic energy, vorticity
• Discrete integration by parts, operator symmetry, operator orthogonality…
• Allows exact projection. (elimination of pressure)
Exact Projection

- Define a discrete streamfunction
  - single component along edges in 3D (nodes in 2D)
  - Reduced unknowns
  - No temporal error
  - Exactly incompressible
  - Reduced iteration compared to a pressure solve

\[
\begin{bmatrix}
A & G \\
D & 0
\end{bmatrix}
\begin{bmatrix}
u \\
p
\end{bmatrix}
= 
\begin{bmatrix}
r \\
0
\end{bmatrix}
\Rightarrow
C^T AC\psi = C^T r
\]
Moving Mesh Implementation

• Based on Reynolds Transport Theorem for a moving control volume.
• Mesh moves as the solution updates.
• *Not* an interpolation process.
  – Conservative (including volume).
  – Not diffusive.
  – Preserves temporal solution accuracy.
  – Simple (change in convective flux term)
Mesh Relaxation

- Damped Spring Relaxation

\[ \nabla \cdot k \nabla x^{n+1} - \lambda^{-1} (x^{n+1} - x^n) = 0 \]

- Parallel
- Much faster than remeshing.
- Naturally leads to good node distribution.
Semi-Lagrangian Motion

Fr = 0.7

Comparison of numerical and analytical results

Acceleration due to gravity

Time period of standing waves (s)

Analytical
Numerical

Video Clip
Mesh Flipping

Before Flipping

After Flipping

• Flipping = *local* remeshing.

• Maintains high mesh quality at low cost.

• Currently based on Delaunay mesh criteria.

3D Flipping:

2 tetrahedra ↔ 3 tetrahedra
Anisotropic Mesh Relaxation

- Variable spring constant $\Rightarrow$ mesh adaptation.
- Fixed computational cost.
Surface Tracking

- Exact mass and volume conservation.
- High resolution of free-surface motion and physics (surface tension).
- Can simulate very large distortions.
Free Surface Physics

Damped Oscillating Cylinder

Period of oscillation within 1% of analytical result
3D Implementation

3D Oscillating Drop

3D Sloshing
Parallel Implementation

Uses MPI (message passing)

Implemented on a PC Cluster
(60 CPUs, 100 Mbps Ethernet)
Summary

Demonstration of Moving Staggered Mesh:

- Free-Surface Problems
- Solution Adaptation
- 3D Flows
- Parallel Implementation

perot@ecs.umass.edu
Mesh Smoothing

Before

After