Optimal Policies with Convertible Lead Times

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June 2003

Keywords: Inventory, Dynamic Programming, Expediting, Emergency Orders, Dual Supply Modes.

Abstract

A retailer facing Poisson demands places orders at a linear cost from a supplier with fixed lead time l. The retailer has the option of converting (expediting) each order, at a cost, over a certain time interval after the order is originally placed. A converted order arrives $l_e < l$ units of time after it is converted. We show that a threshold base–stock policy is optimal in an order-for-order basis. Under such a policy the retailer places an order to bring his inventory position up to its base–stock whenever a demand arrives. An order is converted the first time, if any, where the residual lead time exceeds a time threshold related to the number of demands seen since the order was placed.

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1 Introduction

Consider a retailer facing Poisson demand at rate λ. The retailer can choose between two linear ordering cost modes: regular and emergency. Under regular ordering, the unit ordering cost is c. A unit purchased at cost c arrives after l units of time, where l is a positive constant. On the other hand an emergency order has unit cost $c_e > c$ and lead time $l_e < l$. We will assume that a regular order can be converted into an emergency order over the interval $[0, l - l_e]$. A regular order converted into an emergency order arrives l_e units of time after it is converted. Thus, a conversion can occur only if the converted order arrives earlier than originally scheduled. We will assume that the cost of converting an order is a constant $K_e \geq c_e - c$. The case $K_e = c_e - c$ corresponds to the case where there is no penalty associated with canceling a regular order. In addition to purchasing and converting costs, the retailer incurs linear holding and stock-out penalty costs at rates h and p, respectively. Unsatisfied demand is backlogged. Since all demands are eventually satisfied, the problem is to find the policy that minimizes the long run average converting, holding, and stock-out penalty cost.

We can think of the action of converting a regular order into an emergency order as "expediting" the order. In a purchasing setting, we can think of two purchasing sources where we have the option of canceling a regular order within an interval of time after it is placed. Intuitively, converting an order may be attractive after an unexpected burst in sales renders a regular order hopelessly late.

The inventory control problem posed by the availability of two supply modes with different costs and lead times has been extensively studied in the literature, dating back to 1961 (Barankin (1961)). Most of the models consider a periodic review setting (Barankin (1961), Daniel (1962), Fukuda (1964), Bulinskaya (1964), Veinott (1966), Whittmore and Saunders (1977), Teunter and Vlachos (2001), Tagaras and Vlachos (2001), Vlachos and Tagaras (2001)) and optimal policies are derived only under restrictive assumptions.

The problem under continuous review has been addressed by Moinzadeh and Nahmias (1988) and Moinzadeh and Schmidt (1991). The former considers fixed ordering costs for both regular and expedited orders. The authors develop a heuristic that is an extension of the standard (Q, R) policy. The latter assumes that demand and fixed ordering costs are small compared to the holding cost and therefore a one for one ordering policy is reasonable, as in the current paper. They propose an $(S-1, S)$ inventory model where emergency orders are placed whenever the net inventory is below a certain level $\hat{S}$ and the remaining lead time of the $\hat{S}+1$th unit in the pipeline is greater than the emergency lead time. In this manner, not only current inventory but also information on the outstanding orders is considered before placing an expensive emergency order. Given the described policy structure, they calculate the optimal parameters S and $\hat{S}$. Their results show considerable savings associated with the dual supply strategy. Bradley (2003) studies a related problem of dual sourcing in capacitated settings using a Brownian model.

In these papers the decision is whether to order through the regular and/or emergency modes. In our model, however, we also allow for orders to be expedited within a certain time after being placed. Expediting in a make-to-order production environment, from the perspective of the supplier rather than the buyer, has been addressed in Arslan et al. (2001). They characterize the structure of optimal policies and show that expediting occurs only when the number of orders on hand reaches a certain threshold value.

In our model, the cost, per unit, of an optimal base stock policy that uses only regular orders is

$$C = c + \min_n G(n, l)$$

where the minimization is over the non-negative integers,

$$G(n, a) = hE(T_n - a)^+ + pE(a - T_n)^+, \tag{1}$$

E denotes expectation, T_n is the sum of n independent exponential random variables each with parameter λ, and a is any constant. Notice that $G(0, a) = -ha$ if $a < 0$.

On the other hand, the cost, per unit, of an optimal base stock policy that uses only emergency orders is

$$C_e = c_e + \min_n G(n, l_e),$$

where the minimization is again over the non-negative integers. We want to develop an optimal policy, that uses both types of orders. Necessarily, the average cost of an optimal policy is at most $\min\{C_e, C\}$.

Suppose that the inventory manager uses base stock level b, an integer, to place regular orders which may or may not be converted at a later time. When is it optimal to convert a regular order into an emergency order? For fixed b, this is a stopping time problem, but we also need to optimize over b.

To solve this problem we will use, for *each* outstanding order, the pair (n, t) as the state, where n is the target demand to be satisfied by the order and t is the remaining time until the regular order arrives. In our analysis, we assume that each outstanding order is permanently linked to its target demand. We thus study the problem in an order-for-order basis. When a regular order is placed, the state for that order is (b, l). After the first arrival, say at time $T_1 = t_1$, the state changes to $(b-1, l-t_1)$, etcetera. When an order with state (n, t) is expedited, its state becomes (n, l_e) even though it may arrive earlier than some other orders targeted for earlier demands. We derive the optimal policy under this assumption. Moreover, we would like to point out that there are interesting cases in which it provides an overall optimal policy. This is the case when the order is placed in anticipation of a demand that corresponds to the kth customer arrival, an idea first suggested by Katircioglu (1996). For instance, consider a two-echelon serial supply chain in which the retailer faces Poisson demands and economies of scale in ordering, while the supplier replenishes its stock from an outside source with ample stock, linear cost and a lead time of l. The supplier's objective is to minimize total inventory and penalty costs. In such a system, Gallego et al. (2000) show that if the retailer uses a (Q, r) inventory policy and fully shares demand information with the supplier, the optimal policy for the supplier calls for ordering Q units to raise its inventory to mQ whenever the retailer's inventory position drops to $r+n$, where $n = b^* - (m-1)Q$, m is an integer such that $(m-1)Q < b^* \leq mQ$, b^* is the smallest integer satisfying $P(N_l \leq b) > \frac{p}{p+h}$ and N_l is the number of customer arrivals during the lead time, l. Observe that b^* is simply the optimal stock level for a supplier faced by unit retailer orders, i.e. for $Q = 1$. In this setting, the order placed by the supplier at a time t

is targeted to satisfy a retailer order that will be generated exactly when the b^*th customer arrives. Thus, when $m = 1$, the order is targeted for this particular demand, exactly as in our assumption. Now, if the supplier has the option to expedite orders, the policy derived in this paper is optimal.

In the remainder of the paper, we first introduce a simple myopic policy, the analysis of which sheds light into the structure of optimal policies. In section 3, we show the optimality of a threshold base stock policy. We also discuss efficient policy computation and implementation issues. In section 4 we present a computational study to assess the benefits of the mixed (regular-emergency) ordering strategy and the gains of optimal over myopic policies.

2 A Myopic Policy

Although the optimal decision of whether or not to convert is made by comparing the cost of converting now to the expected cost of the optimal continuation policy, a reasonable myopic heuristic is to compare the expected cost of converting now, $K_e + G(n, l_e)$, to the expected cost of keeping the regular order until it arrives $G(n, t)$. The analysis of this heuristic policy will be useful in establishing the form of the optimal policy in the next section. Let $H(n, u) = G(n, l_e + u) - G(n, l_e)$; that is, the savings associated with expediting an order that is destined to satisfy the nth customer demand, from a remaining lead time of $l_e + u$ to l_e.

Proposition 1 *There exists an increasing sequence of positive numbers* $\{u_o, u_1, \ldots\}$ *such that* $H(n, u_n) = K_e$, $H(n, u) < K_e$ *over* $[0, u_n)$, *and* $H(n, u) > K_e$ *for* $u > u_n$.

<u>**Proof:**</u> Simple calculus yields

$$G(n, l) = p(l - \frac{n}{\lambda}) + (h + p) \int_l^\infty P(T_n > x)dx, \tag{2}$$

and

$$H(n, u) = pu - (h + p) \int_{l_e}^{l_e+u} P(T_n > x)dx, \tag{3}$$

which is easily seen to be decreasing in n and strictly convex in u. Since $H(n,0) = 0$ and $\lim_{u\to\infty} H(n,u) = \infty$, it follows that there exist a unique $u_n > 0$ satisfying $H(n,u_n) = K_n$ with $H(n,u) < K_n$ for $u \in [0,u_n)$ and $H(n,u) > K_n$ for $u > u_n$. Finally, since $H(n,u)$ is decreasing in n, it follows that $H(n,u) < H(n-1,u) < K_n$ for all $u \in [0,u_{n-1})$, so $u_n > u_{n-1}$.

□

The proposition tells us that it is better to convert than to keep the order until it arrives whenever $t \geq l_e + u_n$ at state (n,t). Otherwise we will keep the regular order, but not necessarily until it arrives. Instead, we will keep the order until the next demand arrives and reconsider the decision at that time. We can express this policy more formally as follows: Starting from state (b,l), let $0 \equiv T_o \leq T_1 \leq T_2 \leq \ldots \leq T_b$ be the demand arrival epochs. The myopic policy calls for converting the order at demand arrival epoch T_n, if the time to go exceeds the threshold for the target demand, i.e., if $l - T_n \geq l_e + u_{b-n}$, or equivalently, if $T_n \leq l - l_e - u_{b-n}$. Thus, the demand arrival epoch at which we convert under the myopic policy is random and is given by

$$\tau_m^b = \inf\{T_n : T_n \leq l - l_e - u_{b-n}, n \in \{0,\ldots,b\}\ \}, \tag{4}$$

where $\tau_m^b = \infty$ (corresponding to not converting) whenever the set is empty.

What is the expected cost $V_m(b,l)$, excluding the unit variable cost c, of using the myopic heuristic? Clearly

$$V_m(0,t) = \begin{cases} G(0,t) & \text{if } t < l_e + u_o, \\ K_e + G(0,l_e) & \text{otherwise.} \end{cases}$$

Now, suppose we have available the function $V_m(n-1,\cdot)$ for some $n \geq 1$. The cost $V_m(n,\cdot)$ is given by

$$V_m(n,t) = \begin{cases} EV_m(n-1,t-T) & \text{if } t < l_e + u_n, \\ K_e + G(n,l_e) & \text{otherwise.} \end{cases}$$

where T has an exponential distribution with parameter λ. Under the heuristic, it makes sense to select the base stock level, say b_m to be the smallest integer minimizing $V_m(\cdot,l)$. The expected cost of the heuristic is thus $V_m(b_m,l)$.

3 Optimal Policy

We now turn to the analysis of the optimal policy. Let $V(n,t)$ be the optimal cost per unit, excluding the unit variable cost c, starting from state (n,t). At $(0,t)$, $V(0,t) = G(0,t)$ for $t < l_e$, while

$$V(0,t) = \min\{K_e + G(0,l_e), G(0,t)\},$$

for $t \geq l_e$. Putting this together, we have

$$V(0,t) = \begin{cases} G(0,t) & \text{if } t < l_e, \\ \min\{K_e + G(0,l_e), G(0,t)\} & \text{otherwise.} \end{cases}$$

We can now write a recursive equation to compute $V(n,\cdot)$ given $V(n-1,\cdot)$ for $n \geq 1$. This is given by

$$V(n,t) = \begin{cases} G(n,t) & \text{if } t < l_e, \\ \min\{K_e + G(n,l_e), EV(n-1,t-T)\} & \text{otherwise,} \end{cases}$$

where T is an exponential random variable with parameter λ. The recursion indicates that it is too late to convert if $t \leq l_e$. On the other hand, if $t > l_e$ then we must select the best course of action between converting now and delaying the decision until the next arrival. The exponential assumption ensures that there is nothing to be gained by allowing a conversion option between demand arrivals. An optimal policy can be obtained by finding an integer, say b, that minimizes the function $V(\cdot, l)$.

In our analysis of this problem we will repeatedly use the following lemma.

Lemma 1 *Let T be an exponential random variable with parameter λ. Then $EG(n,a-T) = G(n+1,a)$.*

<u>Proof:</u>

$$\begin{aligned} EG(n,a-T) &= E[hE(T_n + T - a)^+ + pE(a - T - T_n)^+] \\ &= hE(T_{n+1} - a)^+ + pE(a - T_{n+1})^+ \\ &= G(n+1,a), \end{aligned}$$

where $T_{n+1} = T_n + T$ is the sum of $n+1$ independent exponential random variables, each with parameter λ.

$\square$

Let $F(0,v) = G(0, l_e + v) - G(0, l_e)$ and for $n \geq 1$ let $F(n,v) = EV(n-1, l_e + v - T) - G(n, l_e)$. Let n_e be the smallest minimizer of $G(n, l_e)$.

Lemma 2 *There exists an increasing sequence of finite positive numbers* $\{v_o, v_1, \ldots, v_{n_e}\}$ *such that* $F(n, v_n) = K_e$, $F(n,v) < K_e$ *for* $v \in [0, v_n)$ *and* $F(n,v) > K_e$ *for* $v > v_n$. *Moreover,* $v_n \geq u_n$ *for all* $n = 0, 1, \ldots, n_e$.

Proof: Clearly $v_o = u_o$, $F(0,v) < K_e$ on $[0, v_o)$ and $F(0,v) > K_e$ on account of $F(0,v) = H(0,v)$, so the result holds for $n = 0$. Suppose that the result holds up to some $n \in \{0, \ldots, n_e - 1\}$. We will show that the result holds up to $n+1$. From the inductive hypothesis there exists a $v_n > 0$ such that

$$V(n, l_e + v) = \begin{cases} G(n, l_e + v) & \text{if } v < 0, \\ EV(n-1, l_e + v - T) & \text{if } 0 \leq v < v_n, \\ K_e + G(n, l_e) & \text{if } v \geq v_n. \end{cases}$$

Lemma 1 and the fact that $V(n-1, l_e + v) = G(n-1, l_e + v)$ for all $v < v_o$ implies that $EV(n-1, l_e+v-T) = EG(n-1, l_e+v-T) = G(n, l_e+v)$ holds for all $v < v_o$. Consequently,

$$V(n, l_e + v) = \begin{cases} EV(n-1, l_e + v - T) & \text{if } v < v_n, \\ K_e + G(n, l_e) & \text{if } v \geq v_n. \end{cases}$$

Let us now consider the function $F(n+1, \cdot)$. Clearly

$$\begin{aligned} F(n+1, v) &= EV(n, l_e + v - T) - G(n+1, l_e) \\ &\leq EG(n, l_e + v - T) - G(n+1, l_e) \\ &= G(n+1, l_e + v) - G(n+1, l_e) \\ &= H(n+1, v) \end{aligned}$$

implies that $F(n+1, v) \leq H(n+1, v) < K_e$ for all $v \in [0, u_{n+1})$. On the other hand, since $V(n,t) = K_e + G(n, l_e)$ for all $t > l_e + v_n$, it follows that

$$\lim_{v \to \infty} EV(n, l_e + v - T) = K_e + G(n, l_e).$$

Consequently,

$$\begin{aligned}\lim_{v\to\infty} F(n+1,v) &= K_e + G(n,l_e) - G(n+1,l_e)\\ &> K_e\end{aligned}$$

on account of $G(n+1,l_e) < G(n,l_e)$ for $n < n_e$. The above shows that $F(n+1,v) = K_e$ has no roots in the interval $[0,u_{n+1})$ and at least one root in the interval $[u_{n+1},\infty)$. To show that there is a unique root in the interval $[u_{n+1},\infty)$ we will use the variation diminishing property of the exponential distribution, see Karlin (1968). This property states that if $f(\cdot)$ is function with a unique sign change from $-$ to $+$ then $g(t) = Ef(t-T)$ has at most one sign change from $-$ to $+$. To show how this result applies notice that

$$\begin{aligned}F(n+1,v) - K_e &= EV(n,l_e+v-T) - G(n+1,l_e) - K_e\\ &= EV(n,l_e+v-T) - EG(n,l_e-T) - K_e\\ &= E[V(n,l_e+v-T) - G(n,l_e-T) - K_e],\end{aligned}$$

the result follows since $V(n,l_e-t+v) - G(n,l_e-t) - K_e$ has a unique sign change from $-$ to $+$ over $v \geq 0$, namely at $v = t+v_n$. This shows that $F(n+1,v) - K_e$ has a unique root, say v_{n+1}, of $F(n+1,v) = K_e$ such that $F(n+1,v) < K_e$ on $v \in [0,v_{n+1})$ and $F(n+1,v) > K_e$ on $v > v_{n+1}$. Since $F(n+1,u) < K_e$ for all $v \in [0,u_{n+1})$ it follows that $u_{n+1} \leq v_{n+1}$. It remains to be shown that $v_n < v_{n+1}$. To see this notice that $V(n,l_e+v-t) - G(n,l_e-t) - K_e < 0$ on $v < v_n$ for all $t \geq 0$, consequently,

$$\begin{aligned}F(n+1,v) - K_e &= E[V(n,l_e+v-T) - G(n,l_e-T) - K_e]\\ &< 0\end{aligned}$$

on $v \leq v_n$ implying that $v_n < v_{n+1}$.

□

To establish the form of the optimal policy we now investigate the behavior of the function $F(n,\cdot)$ when $n > n_e$.

Lemma 3 *$F(n,v) \leq K_e$ for all $v \geq 0$ and all $n > n_e$.*

Proof: Clearly,

$$\lim_{v \to \infty} F(n_e + 1, v) = K_e + G(n_e, l_e) - G(n_e + 1, l_e) \leq K_e.$$

Since $F(n_e + 1, v) - K_e$ can have at most one sign change, and this must be from $-$ to $+$, $F(n_e+1, 0) - K_e < 0$ and $F(n_e+1, v) - K_e \leq 0$ as $v \to \infty$, it must be that $F(n_e+1, v) - K_e \leq 0$ for all $v \geq 0$. Now assume that the result is true for some $n > n_e$, so $F(n, v) \leq K_e$ for all $v \geq 0$. From the variation diminishing property, it follows that $F(n + 1, v) \leq K_e$ for all $v \geq 0$, completing the proof.

□

The value function for the optimal policy is given by

Theorem 1 *Let $v_n = \infty$ for all $n > n_e$, then*

$$V(n, l_e + v) = \begin{cases} EV(n - 1, l_e + v - T) & \text{if } v < v_n, \\ K_e + G(n, l_e) & \text{if } v \geq v_n. \end{cases}$$

Proof: The functional form for $V(n, l_e + v)$ for $n \in \{0, \ldots, n_e\}$ follows directly from Lemma 2. The functional form for $n > n_e$ follows from Lemma 3.

□

Notice that for $n > n_e$ it is optimal to wait until n drops to n_e before considering whether or not to convert an order. Consequently, for $n > n_e$

$$V(n, l_e + v) = EV(n_e, l_e + v - T_{n-n_e})$$

where T_k is the sum of k independent exponential random variables, each with parameter λ.

An optimal base stock level, say b^*, is obtained by finding the smallest minimizer of $V(\cdot, l)$. The average cost of the resulting optimal policy is $V(b^*, l)$.

Suppose we start at state (b^*, l), and let $0 \equiv T_o \leq T_1 \leq T_2 \leq \ldots \leq T_{b^*}$ be the demand arrival epochs after placing the regular order. Let τ^{b^*} be the optimal conversion time. An optimal policy calls for converting the order at T_n, the arrival epoch of the nth demand, if $l - T_n \geq l_e + v_{b^*-n}$, or equivalently, if $T_n \leq l' - v_{b-n}$, where $l' = l - l_e$. Thus, we can write the policy as

Corollary 1

$$\tau^{b^*} = \inf\{T_n : T_n \leq l' - v_{b-n}, n \in \{0, \dots, b^*\}\ \}. \tag{5}$$

Notice that the optimal policy calls for converting immediately, i.e., at state (b^*, l) if $0 = T_o \leq l' - v_{b^*}$ or equivalently if $l' \geq v_{b^*}$.

3.1 Efficiently Computing the Thresholds and the Value Functions

It remains to find an effective way to recursively compute the $V(n,t)$ functions. It is easy to see that $v_o = K_e/p$. The issue is how to compute the threshold values v_n for all $n \leq n_e$. Suppose we have available the function $V(n,\cdot)$. Recall that $V(n+1,t) = EV(n,t-T)$ for $t \leq l_e + v_{n+1}$, where v_{n+1} is the root of the equation

$$EV(n, t-T) = K_e + G(n+1, l_e).$$

Since $v_{n+1} > v_n$, it is enough to consider $t = l_e + v_n + v \geq l_e + v_n$. Over that interval we can write

$$EV(n, l_e + v_n + v - T) = V(n, l_e + v_n)P(T \leq v) + V(n+1, l_e + v_n)P(T > v).$$

Observe that when $T \leq v$ then $l_e + v_n + v - T \geq l_e + v_n$ and therefore the order will be expedited with cost $K_e + G(n, l_e) = V(n, l_e + v_n)$. Given $T > v$, however, we observe that $EV(n, l_e + v_n + v - T | T > v) = EV(n, l_e + v_n - T) = V(n+1, l_e + v_n)$, where the first equality is simply due to the memoryless property of the exponential.

Consequently the problem of finding v_{n+1} reduces to that of finding the root of

$$V(n, l_e + v_n)P(T \leq v) + V(n+1, l_e + v_n)P(T > v) = K_e + G(n+1, l_e).$$

This exercise results in

$$v_{n+1} = v_n - \frac{1}{\lambda} \ln \left[\frac{G(n, l_e) - G(n+1, l_e)}{V(n, l_e + v_n) - V(n+1, l_e + v_n)} \right],$$

where we have used the fact that $V(n, l_e + v_n) = K_e + G(n, l_e)$.

Notice that the only new quantity that is required is $V(n+1, l_e+v_n)$, but since $v_n < v_{n+1}$ it turns out that $V(n+1, l_e + v_n) = EV(n, l_e + v_n - T)$, which is computable since $V(n, \cdot)$ is available.

3.2 Policy Implementation

To implement the policy we need to decide, at each arrival epoch, whether or not to expedite outstanding orders. Let $(t_o, t_1, \ldots, t_{n_e}, \ldots, t_b)$ be the vector of residual lead times corresponding to all orders. According to equation (5) it is optimal to convert order $n \in \{0, \ldots, n_e\}$ if and only if $t_n > l_e + v_n$. Let E be the subset of orders that are expedited, and let T be the time until the next demand. At that time the state is updated as follows:

$$t_n = \begin{cases} (l_e - T)^+ & \text{if } n+1 \in E, \\ (t_{n+1} - T)^+ & \text{if } n+1 \in \{1, \ldots, b\} \cap \overline{E}, \\ l & \text{if } n = b. \end{cases}$$

Notice that the state space of the policy is of dimension $b+1$. Deciding which orders to convert requires at most n_e comparisons, and the state update is fairly simple.

4 Computational Study

In this section we carry out an extensive computational study to evaluate the benefits of expediting and the performance of the policies proposed above. For that purpose, we compare the base-stock levels and costs associated with the optimal and myopic policies derived in Sections 2 and 3, respectively, with those associated with never or always expediting. These policies will be referred to as the *Never Expedite (NE)* and *Expedite Immediately (EI)* policies, respectively. The first one uses only regular orders, with a unit cost of $c + G(\bar{n}, l)$ where $\bar{n} = \arg\min G(n, l)$, and the second uses only emergency orders, with a unit cost of $c_e + G(n_e, l_e)$, as explained in Section 1. The base-stock levels and costs associated with

Myopic and *Optimal* policies are determined using the dynamic programming approach explained in the corresponding sections. All the costs reported exclude the variable purchasing cost c, since it is constant for all policies.

The parameters used for our analysis are as follows:

$l = 40, h = 1$	$le = 10, 20, 30$	$Ke = 10, 50, 100$	$p = 9, 19, 39, 99$	$\lambda = 0.1, 1, 3$

We consider $K_e = c_e - c$. That is, there is no penalty cost associated with cancelling a regular order, but the retailer pays a significantly higher price for the emergency order.

The average percent cost reduction relative to either the NE or EI policies, whichever leads to lower cost in each case, is reported in Table 1. We also report the resulting reductions in base-stock levels relative to the NE policy, where only regular orders are placed.

Table 1: Performance comparison of the different policies.

	Cost (%)	*Base-Stock (%)*
Optimal Policy	5.53825693	0.834144307
Myopic Policy	1.29498288	1.584874183

Tables 2-10 in the Appendix report the cost per unit and optimal base-stock levels associated with the different policies for each of the parameter settings.

Observe that the myopic policy yields only 25% of the optimal savings on average. The difference between the optimal and myopic policies increases as l_e decreases. This is intuitive: the smaller the expedited lead time, the more the chances the supplier gets to expedite at different points in the future. Observe that the Myopic policy will always expedite a higher or equal number of orders than the Optimal policy. Any order that is expedited by the Optimal policy must also be expedited by the Myopic policy. Either it was expedited at an earlier point in time or it is profitable to expedite it at the same time as in the Optimal policy. Consequently, the base-stock levels are generally lower for the Myopic policy. However, there are a few cases (8 out of the 108 tested) for which the base-stock level is higher in the Myopic policy. These are cases with low expedited lead time ($l_e = 10$) and high stock-out penalty cost relative to the cost of expediting. Again, this shows how the optimal solution takes advantage of the option to expedite in the future when l_e is low. Finally, we observe that as the penalty cost, p, increases, the benefits obtained by expediting grow substantially.

5 Conclusions

In this paper we have shown that significant benefits can be obtained by exercising the option to expedite orders. The policy that minimizes expediting, holding and stock-out costs in an order-for-order basis is determined by a number ($n_e + 1$) of threshold values and is thus easily computable and implementable. Our computational results show that the decision on whether or not to expedite must take into account the possibility of doing so at a future time, since a myopic policy yields only 25% of the actual savings.

The evaluation of the benefits obtained through expediting orders can prove useful in crafting contracts between retailer and supplier. The supplier could offer the emergency ordering option at an initial contract fee in addition to the higher cost per unit. Based on the expected savings, the retailer will decide whether or not to purchase the emergency option. The remaining issue though is to assess the value that this type of contract creates for the supplier. This problem has been studied in Wang, Cohen and Zheng (2002).

Acknowledgement: We are grateful to Dr. Kaan Katircioglu (IBM) for his helpful discussions on the topic.

6 References

Arslan, H., Ayhan, H., T. V. Olsen, 2001. Analytic models for when and how to expedite in make-to-order systems, IIE Transactions, 33, 1019–1029.

Barankin, E.W., 1961. A Delivery-Lag Inventory Model With an Emergency Provision, Naval Res. Logis. Quart, 8, 285-311.

Bradley, J., 2003. A Brownian Approximation of a Production-Inventory System with a Manufacturer that Subcontracts, working paper, Jonhnson Graduate School of Business, Cornell University.

Bulinskaya, E., 1964. Some Results Concerning Optimal Inventory Policies, Theory Probability Applications, 9, 502–507.

Daniel, K.H., 1962. Multistage Inventory Models and Techniques. In: Scarf, Gilford, Shelly (eds.) , Stanford University Press, Stanford, CA, Chapter 2.

Fukuda, Y., 1964. Optimal Policy for the Inventory Problem With Negotiable Leadtime, Management Science, 10, 690-708.

Gallego, G., Ying J., Katircioglu K., Leung Y. T., 2000. When to Share Demand Information in a Simple Supply Chain. Working paper, Columbia University.

Karlin, S., 1968. Total Positivity, Vol 1, Stanford University Press, Stanford, CA, 1968.

Katircioglu, K., 1996. Essays in Inventory Control, Thesis, University of British Columbia, Faculty of Commerce and Business, pp. 70–105.

Moinzadeh, K., Nahmias, S, 1988. A continuous review model for an inventory system with two supply modes, Management Science, 34, 761–773.

Moinzadeh, K. and Schmidt, C., 1991. An (S-1,S)Inventory System With Emergency Orders, Operations Research, 39, 308–321.

Tagaras, G., Vlachos, D., 2001. A periodic review inventory system with emergency replenishments, Management Science, 47, 415–429.

Teunter, R., Vlachos, D., 2001. An Inventory System With Periodic Regular Review and Flexible Emergency Review, IIE Transactions, 33, 625–635.

Veinott, A.F.Jr., 1966. The Status of Mathematical Inventory Theory, Management Science, 12, 745–777.

Vlachos, D, Tagaras, G., 2001. An inventory system with two supply modes and capacity constraints, International Journal of Production Economics, 72, 41–58.

Wan, Y., Cohen, M. A., Zheng, Y.S., 2002. Differentiating Customer Service on the Basis of Delivery Lead-Times, IIE Transactions, 34, 979–989.

Whittmore, A.S., Saunders, S., 1977. Optimal Inventory Under Stochastic Demand With Two Supply Options, SIAM J. Appl. Math., 32, 293-305.

A Appendix

Table 2: $l_e = 10, \lambda = 0.1$

Ke	p	NE		EI		Optimal Policy		Myopic Policy	
		$G(\bar{n}, l)$	$\bar{n}$	$K_e + G(n_e, l_e)$	n_e	$V(b^*, l)$	b^*	$V_m(b_m, l)$	b_m
10	**9**	51.03	5	30.36	2	28.46	5	30.36	2
	19	92.06	5	34.67	3	33.49	5	34.67	3
	39	174.12	5	39.33	3	37.93	5	39.33	3
	99	420.30	5	44.35	4	43.93	5	44.35	4
50	**9**	51.03	5	70.36	2	41.55	5	51.03	5
	19	92.06	5	74.67	3	53.34	5	74.67	3
	39	174.12	5	79.33	3	64.10	5	79.33	3
	99	420.30	5	84.35	4	77.60	5	84.35	4
100	**9**	51.03	5	120.36	2	48.47	5	51.03	5
	19	92.06	5	124.67	3	69.03	5	92.06	3
	39	174.12	5	129.33	3	89.35	5	129.33	3
	99	420.30	5	134.35	4	113.16	5	134.35	4

Table 3: $l_e = 20, \lambda = 0.1$

Ke	p	NE		EI		Optimal Policy		Myopic Policy	
		$G(\bar{n}, l)$	$\bar{n}$	$K_e + G(n_e, l_e)$	n_e	$V(b^*, l)$	b^*	$V_m(b_m, l)$	b_m
10	**9**	51.03	5	37.51	4	35.47	5	37.51	4
	19	92.06	5	44.50	5	43.24	5	44.50	5
	39	174.12	5	49.00	5	49.00	5	49.00	5
	99	420.30	5	62.49	5	62.49	5	62.49	5
50	**9**	51.03	5	77.51	4	47.98	5	51.03	4
	19	92.06	5	84.50	5	68.82	5	84.50	5
	39	174.12	5	89.00	5	88.10	5	89.00	5
	99	420.30	5	102.49	5	102.49	5	102.49	5
100	**9**	51.03	5	127.51	4	50.87	5	51.03	5
	19	92.06	5	134.50	5	83.59	5	135.03	4
	39	174.12	5	139.00	5	119.58	5	139.00	5
	99	420.30	5	152.49	5	152.49	5	152.49	5

Table 4: $l_e = 30, \lambda = 0.1$

Ke	**p**	**NE**		**EI**		**Optimal Policy**		**Myopic Policy**	
		$G(\bar{n}, l)$	$\bar{n}$	$K_e + G(n_e, l_e)$	n_e	$V(b^*, l)$	b^*	$V_m(b_m, l)$	b_m
10	**9**	51.03	5	43.46	5	43.46	5	43.46	5
	19	92.06	5	56.92	5	56.92	5	56.92	5
	39	174.12	5	83.85	5	83.85	5	83.85	5
	99	420.30	5	164.62	5	164.62	5	164.62	5
50	**9**	51.03	5	83.46	5	50.99	5	51.03	5
	19	92.06	5	96.92	5	87.06	5	123.87	5
	39	174.12	5	123.85	5	123.85	5	123.85	5
	99	420.30	5	204.62	5	204.62	5	204.62	5
100	**9**	51.03	5	133.46	5	51.03	5	51.03	5
	19	92.06	5	146.92	5	91.90	5	92.06	5
	39	174.12	5	173.85	5	161.76	5	173.85	5
	99	420.30	5	254.62	5	254.62	5	254.62	5

Table 5: $l_e = 10, \lambda = 1$

Ke	**p**	**NE**		**EI**		**Optimal Policy**		**Myopic Policy**	
		$G(\bar{n}, l)$	$\bar{n}$	$K_e + G(n_e, l_e)$	n_e	$V(b^*, l)$	b^*	$V_m(b_m, l)$	b_m
10	**9**	11.45	48	15.87	14	10.25	46	11.64	47
	19	13.59	51	17.07	15	11.68	47	13.64	50
	39	15.50	53	18.11	17	12.93	49	15.63	52
	99	17.86	55	19.34	18	14.38	50	17.86	55
50	**9**	11.45	48	55.87	14	11.45	48	11.45	48
	19	13.59	51	57.07	15	13.45	50	13.64	50
	39	15.50	53	58.11	17	14.96	52	15.63	52
	99	17.86	55	59.34	18	16.83	54	17.86	55
100	**9**	11.45	48	105.87	14	11.45	48	11.45	48
	19	13.59	51	107.07	15	13.59	51	13.64	50
	39	15.50	53	108.11	17	15.47	53	15.63	52
	99	17.86	55	109.34	18	17.46	55	17.86	55

Table 6: $l_e = 20, \lambda = 1$

Ke	**p**	**NE**		**EI**		**Optimal Policy**		**Myopic Policy**	
		$G(\bar{n}, l)$	$\bar{n}$	$K_e + G(n_e, l_e)$	n_e	$V(b^*, l)$	b^*	$V_m(b_m, l)$	b_m
10	**9**	11.45	48	18.19	26	11.16	47	13.07	45
	19	13.59	51	19.77	28	13.00	49	14.90	48
	39	15.50	53	21.16	29	14.62	51	17.28	50
	99	17.86	55	22.86	31	16.55	53	19.26	53
50	**9**	11.45	48	58.19	26	11.45	48	11.45	48
	19	13.59	51	59.77	28	13.59	51	13.59	51
	39	15.50	53	61.16	29	15.50	53	17.28	50
	99	17.86	55	62.86	31	17.68	55	19.26	53
100	**9**	11.45	48	108.19	26	11.45	48	11.45	48
	19	13.59	51	109.77	28	13.59	51	13.59	51
	39	15.50	53	111.16	29	15.50	53	15.50	53
	99	17.86	55	112.86	31	17.86	55	19.26	53

Table 7: $l_e = 30, \lambda = 1$

Ke	**p**	**NE**		**EI**		**Optimal Policy**		**Myopic Policy**	
		$G(\bar{n}, l)$	$\bar{n}$	$K_e + G(n_e, l_e)$	n_e	$V(b^*, l)$	b^*	$V_m(b_m, l)$	b_m
10	**9**	11.45	48	19.95	37	11.45	48	11.45	48
	19	13.59	51	21.83	39	13.58	51	13.59	51
	39	15.50	53	23.52	41	15.45	53	15.50	53
	99	17.86	55	25.60	43	17.67	55	17.86	55
50	**9**	11.45	48	59.95	37	11.45	48	11.45	48
	19	13.59	51	61.83	39	13.59	51	13.59	51
	39	15.50	53	63.52	41	15.50	53	15.50	53
	99	17.86	55	65.60	43	17.86	55	17.86	55
100	**9**	11.45	48	109.95	37	11.45	48	11.45	48
	19	13.59	51	111.83	39	13.59	51	13.59	51
	39	15.50	53	113.52	41	15.50	53	15.50	53
	99	17.86	55	115.60	43	17.86	55	17.86	55

Table 8: $l_e = 10, \lambda = 3$

Ke	**p**	**NE**		**EI**		**Optimal Policy**		**Myopic Policy**	
		$G(\bar{n}, l)$	$\bar{n}$	$K_e + G(n_e, l_e)$	n_e	$V(b^*, l)$	b^*	$V_m(b_m, l)$	b_m
10	**9**	6.53	134	13.32	37	6.43	134	6.53	133
	19	7.71	138	13.94	39	7.30	137	7.71	138
	39	8.78	142	14.51	41	8.04	139	8.78	142
	99	10.06	146	15.20	43	8.89	142	10.06	146
50	**9**	6.53	134	53.32	37	6.53	134	6.53	133
	19	7.71	138	53.94	39	7.71	138	7.71	138
	39	8.78	142	54.51	41	8.78	142	8.78	142
	99	10.06	146	55.20	43	10.06	146	10.06	146
100	**9**	6.53	134	103.32	37	6.53	134	6.53	134
	19	7.71	138	103.94	39	7.71	138	7.71	138
	39	8.78	142	104.51	41	8.78	142	8.78	142
	99	10.06	146	105.20	43	10.06	146	10.06	146

Table 9: $l_e = 20, \lambda = 3$

Ke	**p**	**NE**		**EI**		**Optimal Policy**		**Myopic Policy**	
		$G(\bar{n}, l)$	$\bar{n}$	$K_e + G(n_e, l_e)$	n_e	$V(b^*, l)$	b^*	$V_m(b_m, l)$	b_m
10	**9**	6.53	134	14.65	70	6.53	134	8.22	127
	19	7.71	138	15.50	73	7.71	138	9.31	132
	39	8.78	142	16.28	76	8.73	142	10.44	136
	99	10.06	146	17.21	79	9.84	145	11.53	141
50	**9**	6.53	134	54.65	70	6.53	134	6.53	134
	19	7.71	138	55.50	73	7.71	138	9.31	132
	39	8.78	142	56.28	76	8.78	142	10.44	136
	99	10.06	146	57.21	79	10.06	146	11.53	141
100	**9**	6.53	134	104.65	70	6.53	134	6.53	134
	19	7.71	138	105.50	73	7.71	138	7.71	138
	39	8.78	142	106.28	76	8.78	142	10.44	136
	99	10.06	146	107.21	79	10.06	146	11.53	141

Table 10: $l_e = 30, \lambda = 3$

Ke	**p**	**NE**		**EI**		**Optimal Policy**		**Myopic Policy**	
		$G(\bar{n}, l)$	$\bar{n}$	$K_e + G(n_e, l_e)$	n_e	$V(b^*, l)$	b^*	$V_m(b_m, l)$	b_m
10	**9**	6.53	134	15.67	102	6.53	134	6.53	134
	19	7.71	138	16.70	106	7.71	138	7.71	138
	39	8.78	142	17.64	109	8.78	142	8.78	142
	99	10.06	146	18.76	113	10.06	146	10.06	146
50	**9**	6.53	134	55.67	102	6.53	134	6.53	134
	19	7.71	138	56.70	106	7.71	138	7.71	138
	39	8.78	142	57.64	109	8.78	142	8.78	142
	99	10.06	146	58.76	113	10.06	146	10.06	146
100	**9**	6.53	134	105.67	102	6.53	134	6.53	134
	19	7.71	138	106.70	106	7.71	138	7.71	138
	39	8.78	142	107.64	109	8.78	142	8.78	142
	99	10.06	146	108.76	113	10.06	146	10.06	146