Managing Flexible Capacity in a Make-to-Order Environment*

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Abstract

Recent changes in the marketplace are forcing manufacturers, whose production schedules have traditionally been driven by demand forecasts, to transform their build-to-stock production systems into make-to-order ones. In such systems, new strategies are needed to accommodate demand variability in the absence of finished goods inventory. Manufacturing flexibility is one lever that can be pulled to hedge against both long-term forecast uncertainty and short-term (period-to-period) variability of orders received. Hence, flexibility can play a key role in improving the manufacturer's market responsiveness. However, it is not clear how a flexible multi-plant multi-product manufacturing system should be managed. While there are multiple ways of allocating customer orders to available capacity at the different plants to optimize sales, the overall performance of the supply chain varies significantly with the capacity allocation policy choice. Our main objective is to determine the impact of flexibility and allocation policies on supply chain performance in order to consider it when *designing* and *managing* a make-to-order production system. We focus on a two-plant two-product manufacturing setting. We show that while flexibility increases sales, it also leads to an increase in production variability, resulting in larger variability upstream in the supply chain and significantly higher component inventory levels. The magnitude of this increase varies substantially depending on the allocation policy used. We also quantify the impact of flexibility on the manufacturer's outbound transportation costs and the trade-off between inventory levels and outbound transportation costs in a flexible system. Based on this analysis, we provide managerial insights on the value of flexible capacity to hedge against long-term and short-term demand uncertainty in a make-to-order setting and discuss strategies that can be used to effectively manage it.

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1 Introduction and Motivation

Recent developments in supply chain management and e-commerce, as well as changes in consumer expectations, are having a fundamental impact on the way businesses are run and managed. As the pressure mounts to give consumers exactly what they want, where they want it, when they want it, at a competitive price, manufacturers are looking for ways to move from predominantly make-to-stock to predominantly make-to-order production systems. In such systems, the final product is manufactured to consumers' orders; however, some major components of these final products may have long procurement lead-times and need to be kept in stock to achieve fast "order-to-delivery" times for the final products.

Manufacturers moving towards a make-to-order environment necessitate new manufacturing strategies to manage the variability in customer orders. One strategy is to invest in flexible manufacturing capacity. The type of flexibility we consider results from being able to produce different products in the same plant at the same time. Specifically, we consider a manufacturing system with multiple plants and products, where investments in flexible capacity allow some plants to produce more than one product at a time, and in turn, the same product to be manufactured at several sites. While the decision of where and how much flexibility to install (i.e., selecting a particular *flexibility configuration*) must be made long before production starts, how this flexibility is later managed will have a large impact on the overall performance of the supply chain. Our objective is therefore to determine the magnitude of this impact, and to consider it when *designing* and *managing* the manufacturing system. While previous research has focused on the benefits of flexibility to hedge against forecast uncertainty in the long term, we concentrate on understanding the benefits and costs of managing the flexible capacity to absorb demand variability in the short term. Due to the complexity of the system, we study various rational capacity allocation policies that are easy to implement in practice. The performance measures we consider are sales, induced variability at suppliers, manufacturer's component inventory, and outbound distribution cost. We do not explicitly consider inbound distribution costs because it would require specific knowledge of the suppliers' locations. As product variety and the use of flexible manufacturing equipment at each plant increases, unit production costs and cycle times will be generally higher. The study of this effect is beyond the scope of this paper. Recent works by Bradley and Blossom (2001), Thonemann and Bradley (2001), Dupalga, Hahn and Hur (1996), Yano and Rachamadugu (1991) and others address the management and impact of product variety and product mix within a manufacturing plant in various settings.

The benefits of flexibility to hedge against forecast uncertainty are well-known: increased sales and higher capacity utilization (Jordan and Graves (1995)). However, the impact of manufacturing flexibility on supply chain performance, i.e., on variability in upstream production and manufacturers' operational costs, in a make-to-order environment has not been explored. Adding flexibility results in increased variability in the production level of each product at each plant, because a surge in the demand of any product may require shifting the production of other products among different plants. Consequently, component inventory levels and associated inventory costs in a flexible production environment are generally higher. Moreover, component suppliers will observe higher variability and spread it up the supply chain, increasing system costs throughout. In addition, manufacturing the same product in different plants leads to lower volumes of the required components shipped to each plant, for single-sourced components. This results in higher shipping costs and potentially longer lead-times for the components. A simple EOQ analysis shows that, in the case of constant demand, splitting volume equally between two plants increases total transportation set-up and inventory costs by a factor of $\sqrt{2}$. On the other hand, outbound transportation cost will be lower and order-to-delivery times will be faster, due to the increased ability to produce closer to the customer. Consequently, a model that balances the benefits of flexibility not only against the initial investment cost, but also against changes in operational costs, will offer new insights in a cost-benefit analysis of manufacturing flexibility.

The remainder of this paper is organized as follows. In the next section, we briefly review the related literature. In Section 3, we introduce a three-stage framework that captures the steps of the capacity and flexibility planning process in a make-to-order environment. Then, we introduce a stylized two-plant two-product model to analytically study the impact of flexible capacity on the performance metrics discussed above under various capacity management strategies. Analytical results are presented in Section 5.1 and numerical results in Section 5.2. In Section 5.3, we extend our two-plant two-product model to include outbound distribution and quantify the trade-off between inventory and transportation costs in the flexible setting. Finally, we conclude with a summary of our research contributions, managerial take-aways, and suggestions for future research.

2 Literature Review

Manufacturing flexibility has received much attention in recent years, especially after the advent of flexible and computer-controlled manufacturing systems; see Beach *et al.* (2000), De Toni and Tonchia (1998), Kouvelis (1992), and Sethi and Sethi (1990) for extensive reviews. In this section, we discuss the literature that considers investment decisions in flexible capacity and addresses the value of flexibility.

Much research has focused on determining optimal investment decisions on flexible and dedicated capacity at the strategic level. Eppen, Martin, and Schrage (1989) consider a scenario-based approach to update capacity and flexibility investments over a planning horizon; Fine and Freund (1990) characterize the optimal investments in flexible and dedicated capacity assuming concave revenue functions and using a non-linear stochastic programming formulation; Gupta, Gerchack and Buzacott (1992) study the dependence of the optimal investment decisions on available initial capacities; Caulkins and Fine (1989) consider a multi-period setting, in which the firm can hold inter-period inventories, and analyze it under the assumption that all uncertainty is resolved after the capacity investment decisions, but before production and inventory decisions are made. Jordan and Graves (1995) analyze the benefits of manufacturing process flexibility and conclude that the increase in expected sales and capacity utilization resulting from full flexibility (each plant is able to produce all products) can be achieved almost fully with limited or partial flexibility (each plant produces only a few of the products) by assigning products to plants such that the product-plant assignment graph is connected (*chaining*). More recently, Graves and Tomlin (2000) study the extent to which these findings apply to multi-stage supply chains, and develop insights to guide the choice of process flexibility in such settings. Van Mieghem (1998) characterizes the optimal investment decisions as a function of product margins, investment costs, and multivariate demand uncertainty, and shows that it can be advantageous to invest in flexible resources even when the two products are perfectly positively correlated, but have different profit margins.

A related stream of work focuses on substitutable inventory systems, where demand for a specific product can be satisfied by other products; see Khouja (1999) for a review. For instance, Bassok, Anupindi, and Akella (1999) study a single-period multi-product inventory problem, where demand for a specific product can be substituted by any product of higher value. In the service sector, Netessine, Dobson, and Shumsky (2000) consider a similar substitution structure, where services may be upgraded by one class. They show that an increase in correlation leads to an alternating pattern in adjustments to optimal capacity decisions: if the optimal capacity of one resource increases, then the optimal capacity of the adjacent resource decreases.

In the aforementioned work, the benefits of capacity flexibility are studied considering a two-stage sequential model: In the first stage, dedicated and flexible capacity investment decisions are made under uncertain demand; in the second stage, demand is realized and available capacity is allocated to products. Even the approach of Caulkins and Fine (1989), who consider inter-period inventories, can be reduced to a two-stage model, since all uncertainty is resolved in the first period, before production and inventory decisions are made. Thus, the above models do not address how to manage flexible capacity to accommodate period-to-period demand variability in the system and the resulting performance of the supply chain. To study these issues, we propose a new capacity investment decision framework that explicitly considers flexible capacity management strategies and their impact on sales, production and upstream variability, and outbound distribution cost.

3 A Three-Stage Capacity Planning Framework

The capacity planning process in a make-to-order environment should evolve as follows (see Figure 1). First, capacity and flexibility investment decisions are made long before production starts (3 to 5 years in the automotive industry). Second, overall allocation policies and associated component inventory policies are set at a time when aggregate demand is fairly well known, but there is a need to hedge against period-to-period demand variability. Third, production capacity is allocated to different products at each period after orders have been received.

At Stage 1, the decisions of how much capacity to install and the level of flexibility to invest in are made based on long-term demand forecasts, the price of flexible and dedicated equipment, the associated production costs and speed, and, as we will argue in this paper, the resulting projected performance of the supply chain. Of course, competitive issues need to be considered at this strategic stage as well.

An overall allocation policy, determined in Stage 2, will specify how plant capacities are to be

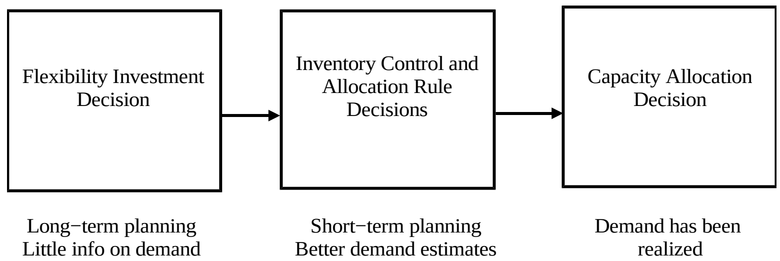

Figure 1: The three-stage approach

allotted to satisfy demand of each product when orders are received. Although the final product is made to order, the need for component inventories and other production resources requires that appropriate allocation and inventory parameters be determined before demand uncertainty is resolved. Modifying these allocation and associated inventory policies requires significant operational changes, which will most likely disrupt production and affect current contracts with suppliers and third-party logistics providers. Consequently, it is not practical to vary them often. Hence, the policies are determined in the medium term, say every quarter, to fit the characteristics of the production system and improve system performance. For example, consider a product that can be built in multiple plants in a flexible setting. We may assign this product mainly to a single plant, and each period any excess demand is routed to other plants with spare capacity (if any). This *prioritized* policy may result in more effective manufacturing and lower inbound transportation cost due to the higher production volumes and close proximity of dedicated component suppliers. On the other hand, we may choose to split production of this particular product at each period among several plants that can produce it. Such a *distributed* policy will have its benefits as well: changes in production levels can be easily accommodated since several plants are trained in the manufacturing of the product, and outbound transportation cost will generally be reduced due to the disperse location of the plants. Yet another attractive policy would be to allocate plant capacities to product demands based on their *profit margins.* These different allocation policies may lead to the same total shortage in the system since they make full use of the flexible capacity at every period; but as we will discuss below, they result in significantly different production variability, component inventory levels, and outbound transportation costs. If constantly changing the mix and quantities of products within the system in the short-term proves to be impractical, flexibility can simply be used to hedge against the long-term forecast uncertainty in the system by dedicating an appropriate proportion of system capacity to each product at stage 2 and keeping the capacity allocation constant in the medium term.

Finally, at Stage 3, customer orders are received every period and assigned to plants using the allocation policy previously determined in Stage 2. These final allocation decisions are thus constrained by capacity investments made in the first stage, and by the overall allocation and inventory policy decisions made in the second stage.

The introduction of an intermediate stage allows us to include tactical decisions in the capacity planning framework and to distinguish between two types of demand uncertainty:

1. *Long-term uncertainty*, which represents the high forecast error encountered at the early investment stage, Stage 1.

2. *Short-term uncertainty*, which represents the variability in the number of orders of each product received in each period, encountered at Stage 2.

In this paper, we study the effectiveness of manufacturing flexibility to hedge against both long-term and short-term demand uncertainty. For that purpose, we determine the impact of a given flexibility configuration on the performance of the supply chain when using various rational allocation policies. The performance metrics we consider are sales, induced variability at suppliers, manufacturer's inventory and outbound distribution cost. Such an analysis is useful at both tactical and strategic levels. At the tactical level, it allows us to compare the different allocation policies and identify under what conditions each of them should be implemented. At the strategic level, supply chain performance is an important factor in the design of the production system and should thus be incorporated into a cost-benefit analysis of manufacturing flexibility.

4 Model and Assumptions

Allocation policies can become quite complex in general flexible settings with multiple plants and products; consequently, they have so far only been dealt with through simulation (see Muriel and Somasundaram (2001)). To gain insight through analytical models, we study a stylized two-plant two-product model under practical (rather than optimal) allocation policies. Two general types of allocation policies will be used: fully flexible policies that make full use of the flexible capacity every period, and partially flexible policies that only change the capacity allocated to each product in the medium term. The former react to both long-term and short-term uncertainty. The latter allow us to single out the effectiveness of flexibility to hedge against long-term uncertainty. Comparing the two, we can understand the costs and benefits of using flexibility to accommodate short-term or period-to-period variability.

We consider a two-stage supply chain (see Figure 2), composed of two plants that make two final products to order, and two suppliers, each replenishing the plants with a major component of each final product. Manufacturing plants are capacitated, while suppliers are not. We study the performance of two capacity flexibility configurations: *no flexibility*, where each plant is dedicated to a single product, and *full flexibility*, where both plants can make both products. In what follows, we use the terms "non-flexible" and "dedicated" interchangeably.

Given the flexibility configuration and the allocation policy implemented, the sequence of events is as follows: (1) Customer orders are received at the beginning of each period and allocated to the plant(s) that can manufacture them, up to plant capacity; (2) orders for components are placed and will be received L periods later. Backlogging is not allowed, and excess demand at any period is thus lost. Since we consider a pure make-to-order environment, there is no final product

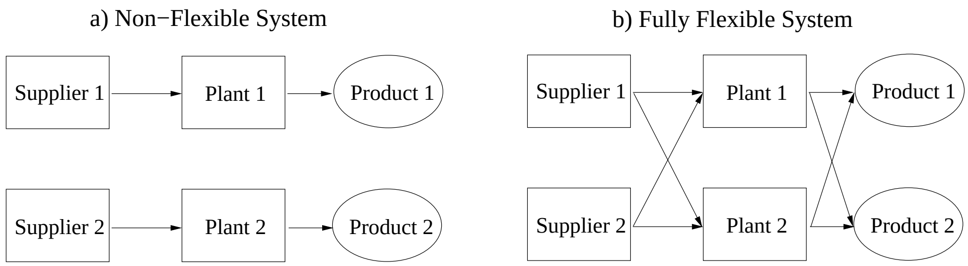

Figure 2: Two-plant two-product models: Dedicated versus flexible settings.

inventory. Hence, sales at each period are equal to the amount produced in that period. We assume that a flexible plant can dedicate anywhere from 0 to 100% of its capacity to manufacture any of the products that it can produce. In addition, we consider that each product requires a single, product-specific component that is single-sourced. Hence, there is no component inventory pooling opportunity, and the variability observed by each component supplier is determined by the total production variability of the corresponding product in the manufacturing system. Simulation experiments considering various components with different lead-times suggest that our findings can be generalized to the case of multiple product-specific components (see Muriel and Somasundaram (2001)). The restriction to product-specific components is of particular interest to the automotive industry, since they comprise 70% of all components and have typically higher costs and longer lead-times. Extending this analysis to multiple common components and to a variety of options that customers can select from within each basic product is a direction for future research, as discussed in Section 6.

In this paper, our major focus is on *production variability*, rather than on inventory in the system. Component inventory levels will only be used as a system-wide measure of the impact of variability. Unlike production variability, inventories across plants can be added up to a single performance indicator useful in comparing different policies and configurations. Consequently, in our analysis of different configurations and allocation policies, we ignore component inventory restrictions so as to us to fully observe the induced production variability and devise appropriate inventory policies. We explicitly consider the effect of inventory constraints in Section 5.1.4 through a simulation study to validate our results.

In what follows, we let product i be the *main product* produced in plant i, $i = 1, 2$, and, therefore, the only one in the absence of flexibility. We let C denote the capacity of each plant. We assume that customer orders are independent (across products and over time), normally distributed and stationary. D_i denotes the normal period demand distribution for product i, $i = 1, 2$, with mean μ_{D_i} and standard deviation σ_{D_i}, for $i = 1, 2$. Observe that the normal demand distribution considered is a continuous approximation of the discrete product demand. In addition, the probability of negative values in this normal distribution is assumed to be negligible. In our analysis, this is justified for demand coefficients of variation (c.v.), $\frac{\sigma_{D_i}}{\mu_{D_i}}$, of at most 0.25, which are typical for the period-to-period variability observed in the automotive industry.

Clearly, when there is no manufacturing flexibility in the system, each plant will only produce its *main product*. Thus, X_i^{NF}, the production quantity of product i at plant i in period t, $i = 1, 2$, under *no flexibility* is given by

$$X_i^{NF} = \min\{D_i, C\}.$$

We omit the dependence on time since the process is stationary. For the case of full flexibility, where both plants can manufacture both products, we investigate the performance of various simple capacity allocation policies. For each $i, j \in \{1, 2\}$, let X_{ij}^A denote the quantity of product i allocated to plant j in the flexible system under allocation policy A, and let $i^c \equiv \{1, 2\} \setminus \{i\}$. We first consider policies that *optimize total sales* by hedging against both forecast and short-term demand uncertainties.

Fully Flexible Allocation Policies

We refer to allocation policies that make full use of the flexible capacity at each period as *fully flexible policies*. The total system production associated with any of these policies at any period is thus the minimum of total system demand and total system capacity: $\sum_{i=1}^{2}\sum_{j=1}^{2} X_{ij}^A = \min\{2C, \sum_{i=1}^{2} D_i\}$. Thus, they maximize sales each period.

Although infinitely many such allocation policies are possible, we focus on the following set of policies that are simple to implement and attractive in practice. Observe that these policies represent extreme cases that can help develop insights.

Symmetric Allocation Policies

These policies give the same preference to each product in the capacity allocation decision. We analyze two types of symmetric policies:

1. *Symmetric prioritized allocation policy (SymP):* Production of each product is concentrated in its main plant, while the unsatisfied demand is allocated to the spare capacity of the other plant. Thus, as much demand for product i as capacity permits is allocated to plant i, $i = 1, 2$; that is,

$$X_{ii}^{SymP} = \min\{\ D_i,\ C\ \}, \qquad X_{ii^c}^{SymP} = \min\{\ D_i - X_{ii}^{SymP},\ C - X_{i^c i^c}^{SymP}\ \}$$

2. *Symmetric distributed allocation policy (SymD):* Production of each product is distributed among the two plants; that is, a specified portion of the demand for product i is allocated to plant i, $i = 1, 2$, and the remaining demand to the spare capacity of the other plant. In what follows, we focus on the following type of a distributed policy:

$$X_{ii}^{SymD} = \min\{\ \frac{1}{2}D_i,\ C\ \}, \qquad X_{ii^c}^{SymD} = \min\{\ D_i - X_{ii}^{SymD},\ C - X_{i^c i^c}^{SymD}\ \}$$

Of course it is possible to allocate a different portion (other than half) of each product demand to its main plant. However, in this case the closed-form expression of the allocation policy needs to be modified so as to achieve the minimum possible shortage for every demand realization[1] and the analysis becomes quite cumbersome. Since our objective is to gain insight through simple models, we focus on the given type of distributed policies.

We note, however, that there are other natural ways of evenly distributing demand across plants. A proportional allocation policy, in which the capacity of each plant is allocated to each product in proportion of their demands at each period, is considered in the extensive simulation study by Muriel and Somasundaram (2001). Their results show robust, outstanding performance of the proportional allocation for scenarios with multiple plants and products. In addition, a more complex distributed policy that takes advantage of customer locations will be introduced in Section 5.3.

Profit-Based Allocation Policies

These policies give preference to the most profitable product, and are thus attractive when the difference in product profit margins is high. We assume that sales of product 1 are preferred over those of product 2. Again, prioritized- and distributed-type of allocation policies are possible, as given below.

1. *Profit-based prioritized allocation policy (ProfitP)*: This policy is similar to the symmetric prioritized allocation policy, except that now product 1 also gets priority on the second plant. That is,

$$X_{11}^{ProfitP} = \min\{D_1, C\}, \qquad X_{12}^{ProfitP} = \min\{D_1 - X_{11}^{ProfitP}, C\}$$
$$X_{22}^{ProfitP} = \min\{D_2, C - X_{12}^{ProfitP}\}, \quad X_{21}^{ProfitP} = \min\{D_2 - X_{22}^{ProfitP},\ C - X_{11}^{ProfitP}\}$$

2. *Profit-based distributed allocation policy (ProfitD):* Under this policy, demand of product 1 is split between the two plants, as follows:

$$X_{11}^{ProfitD} = \min\{\frac{D_1}{2},\ C\}, \quad X_{12}^{ProfitD} = \min\{D_1 - X_{11}^{ProfitD},\ C\}$$
$$X_{21}^{ProfitD} = \min\{C - X_{11}^{ProfitD},\ \frac{D_2}{2}\}, \quad X_{22}^{ProfitD} = \min\{C - X_{12}^{ProfitD},\ D_2 - X_{21}^{ProfitD}\}$$

As discussed in Section 3, each of these policies is attractive in practice for different reasons. Prioritized policies concentrate production of a product mostly in a single plant, leading to lower manufacturing and inbound distribution costs due to economies of scale; distributed policies may allow producing closer to the customer's location, and thus, result in shorter delivery times to customers, see Section 5.3; profit based policies are economically attractive when the products have significantly different profit margins.

Fully flexible policies may involve shifting the capacity allocated to each product every period. To reduce the managerial complexity in the flexible system, partially flexible allocation policies

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[1] Consider the case where $C = 100$, $D_1 = 150$, and $D_2 = 30$ units. If we first allocate $(2/3)^{rd}$ of each product demand to its main plant and the unsatisfied demand to the spare capacity of the other plant, only 170 units will be produced (instead of the optimal 180).

that only allow allocation changes in the medium term might be preferable. These policies are detailed in the following.

Partially Flexible Allocation Policies

Fixed-Capacity allocation policies (Fixed)

This type of policies assigns the overall flexible system capacity (of $2C$) to each product in proportion of their mean demands. The capacity allocated to each product will not change from period to period. However, different quantities of each product will be produced each period in response to product demands, up to the fixed capacity levels. That is, the fixed capacity, FC_i, assigned to product i is given by

$$FC_i = 2C \times \frac{\mu_{D_i}}{\mu_{D_1} + \mu_{D_2}}, \text{ for } i = 1, 2,$$

and the production of product i in the system under this allocation policy at a particular period is,

$$X_i^{Fixed} = min\{D_i, FC_i\}, \text{ for } i = 1, 2.$$

Observe that how we allocate the capacity reserved for each product to the different plants will not affect the production variability in the system. In fact, in terms of sales, inventory, and production variability, this setting is equivalent to a dedicated setting with plant capacities of FC_1 and FC_2, respectively. The *fixed-capacity* policy will not lead to maximum sales, since it only hedges against the long-term forecast uncertainty. Comparing its performance with that of fully flexible policies allows us to extract the actual benefits and costs of using flexibility to accommodate the period-to-period variability in orders received. In this manner, we can study the trade-off between further revenues, higher operational costs, and managerial complexity.

5 Analysis and Results

In this section, we quantify the increase (or reduction) in total sales, production variability, variability observed upstream, component inventory levels, and outbound transportation costs in the flexible versus the non-flexible systems, using the allocation policies described above.

We start our analysis in Section 5.1 assuming that demand for both products follows independent and identically distributed normal distributions with mean equal to plant capacity. This is referred to as the *balanced system*. Although the assumption of mean demand of each product equal to plant capacity may seem restrictive, this case is important because flexibility is most valuable when "capacity is neither too tight nor too loose" (Jordan and Graves (1995)). Furthermore, this case allows us to determine the pure value of flexibility to hedge against the short-term demand variability in the absence of forecast error.

Then we extend our analysis to *unbalanced systems*, where system capacity may not match overall demand (*system unbalance*) and mean demands ***may not match the capacity allocated to them in the dedicated system*** (*product unbalance*). Through numerical integration, we determine the effectiveness of flexibility to hedge against short- and long-term uncertainties as system and product unbalance grows. Finally, in Section 5.3, we incorporate customer locations into our two-plant two-product model and use the resulting *location-based* model to study the impact of flexibility on the manufacturer's outbound distribution in balanced and unbalanced systems. Appendixes A - F include the proofs of all results and detailed computational results.

5.1 The Balanced System

In this section, we study a *balanced system* in which there is no forecast (long-term) error; i.e., the demands for both products are independent and identically distributed normal distributions with mean equal to plant capacity. We analytically determine the expected sales and production variability in the non-flexible and flexible systems under the different capacity allocation policies described above, assuming component inventories are non-constraining. This provides an upper bound on sales and parameters necessary to derive the amount of component inventory to be held at each plant. The effect of inventory constraints is analyzed via simulation at the end of the section.

Recall that when there is no manufacturing flexibility in the system, the production quantity of product i at plant i, $i = 1, 2$, is given by $X_i^{NF} = \min\{D_i, C\}$. The mean and variance of this random variable can be derived as follows (see Appendix B and Winkler, Roodmand and Britney (1972)).

Theorem 5.1 *Considering normal product demands with mean μ_{D_i} and standard deviation σ_{D_i}, the mean and variance of the production level of product i, $i = 1, 2$, in the non-flexible environment are given by:*

$$E(X_i^{NF}) = C + F_i(C)[\ \mu_{D_i} - C\ ] - \frac{e_{D_i}(C)}{\sqrt{2\pi}}\ \sigma_{D_i},$$

$$Var(X_i^{NF}) = [\ F_i(C) - \Big(\frac{e_{D_i}(C)}{\sqrt{2\pi}}\Big)^2\ ]\ \sigma_{D_i}^2 + F_i(C)\ [1 - F_i(C)][\ \mu_{D_i}^2 + C^2\ ] -$$
$$[1 - 2F_i(C]\ \frac{e_{D_i}(C)}{\sqrt{2\pi}}\ (\mu_{D_i} - C)\sigma_{D_i} - 2\ F_i(C)\ [1 - F_i(C)]\ C\mu_{D_i},$$

where $F_i(.)$ is the cumulative distribution function of demand D_i and $e_{D_i}(C) = exp(\frac{-(C-\mu_{D_i})^2}{2\sigma_{D_i}^2})$.

In the *balanced* system, we let μ_D and σ_D denote the common mean and standard deviation of the demands, that is, $\mu_{D_i} = \mu_D = C$ and $\sigma_{D_i} = \sigma_D$, for $i = 1, 2$. Then, Theorem 5.1 reduces to:

Corollary 5.2 *Considering identically distributed normal product demands with mean $\mu_D = C$ and standard deviation σ_D, the mean and variance of the production level of product i, $i = 1, 2$, in the non-flexible environment are given by:*

$$E(X_i^{NF}) = C - \frac{1}{\sqrt{2\pi}}\ \sigma_D \approx C - 0.40\sigma_D, \qquad Var(X_i^{NF}) = [\ \frac{1}{2} - \frac{1}{2\pi}]\ \sigma_D^2 \approx 0.34\ \sigma_D^2$$

Observe that the capacity constraint truncates the demand at plant capacity, C. This results in production levels having lower expected value and variability, as shown in the theorem. As demand variability grows, production variability grows proportionally, but the expected production quantity decreases. This is intuitive because the higher demand values get truncated while the lower demand values drive the mean production down.

Corollary 5.2 highlights the inefficiency of a tightly capacitated make-to-order system to cope with uncertain demand, since $0.40\sigma_D$ potential sales are lost. To reduce lost sales one can either add more capacity or more flexibility to the system. The effect of adding more capacity can be easily analyzed using Theorem 5.1 for different capacity levels. In what follows, we focus on the effect of adding flexibility.

In the dedicated setting, the variability in demand of the two products often leads to one plant having *idle capacity*, while the other plant is not able to satisfy all demand for its corresponding product. However, if both plants are flexible, and any of the fully flexible allocation policies previously described is used, the total system capacity of $2C$ will be fully shared by the two products, and capacity shortages will occur only when both plants are producing at capacity, i.e., when $D_1 + D_2 > 2C$. Our objective is to determine the magnitude of the resulting increase in sales, as well as the changes in upstream variability, inventory and transportation costs brought about by the different allocation policies in the flexible system. For that purpose, we first determine the mean and variability of production levels in the flexible system for each of the fully flexible allocation policies. In what follows, we make use of the approximation $P(0 < D_i < 2C) \approx 1$, $i = 1, 2$. This is justified for $\mu_D = C$ and demand coefficients of variation in the second stage no larger than 0.25, which are the cases of interest for our industrial partner.

The production quantities under the symmetric prioritized allocation policy can be written as:

$$X_{ii}^{SymP} = \min \{ \ D_i, \ C \ \} = X_i^{NF}, \qquad X_{ii^c}^{SymP} = \min \{ \ (D_i - C)^+, \ (C - D_{i^c})^+ \ \}$$

We can then determine the expected values and variabilities (all derivations are provided in Appendix B).

Theorem 5.3 *Considering independent and identically distributed normal product demands with mean $\mu_D = C$ and standard deviation σ_D, the mean and production variability of product i, $i = 1, 2$, in plants i and i^c under the symmetric prioritized allocation policy in the flexible environment are given by:*

$$E(X_{ii}^{SymP}) = E(X_i^{NF}) = C - \frac{1}{\sqrt{2\pi}} \sigma_D \approx C - 0.40 \ \sigma_D;$$
$$Var(X_{ii}^{SymP}) = Var(X_i^{NF}) = [\ \frac{1}{2} - \frac{1}{2\pi}] \ \sigma_D^2 \approx 0.34 \ \sigma_D^2;$$
$$E(X_{ii^c}^{SymP}) = [\ \frac{1}{\sqrt{2\pi}} - \frac{1}{2\sqrt{\pi}} \ ]\sigma_D \approx 0.12 \ \sigma_D;$$
$$Var(X_{ii^c}^{SymP}) = [\frac{1}{4} - \frac{(5\sqrt{2} - 4)}{4\sqrt{2\pi}}] \ \sigma_D^2 \approx 0.08 \ \sigma_D^2.$$

Under the above policy, each plant capacity is first allocated to its main product, and thus the associated production quantity is identical to that in the non-flexible system. On the other hand, each plant i, $i = 1, 2$, will allocate some spare capacity to its secondary product, i^c, only when $D_i < C$ and $D_{i^c} > C$. Therefore, the production quantity of product i^c at plant i will be zero 75% of the time and hence will have a very small variance, as shown in Theorem 5.3.

The production quantities under the symmetric distributed allocation policy are given by:

$$X_{ii}^{SymD} = \min \{ \frac{1}{2} D_i, \ C \}, \qquad X_{ii^c}^{SymD} = \min \{ \frac{1}{2} D_i, \ (C - \frac{1}{2} D_{i^c})^+ \}$$

Theorem 5.4 *Considering independent and identically distributed normal product demands with mean $\mu_D = C$ and standard deviation σ_D, the mean and production variability of product i, $i = 1, 2$, in plants i and i^c under the distributed allocation policy in the flexible environment are given by:*

$$E(X_{ii}^{SymD}) = \frac{1}{2} \ C; \qquad Var(X_{ii}^{SymD}) = \frac{1}{4} \ \sigma_D^2;$$
$$E(X_{ii^c}^{SymD}) = \frac{1}{2} \ C - \frac{1}{2\sqrt{\pi}} \ \sigma_D \approx \frac{1}{2} \ C - 0.28 \ \sigma_D;$$
$$Var(X_{ii^c}^{SymD}) = \frac{0.6817}{4} \ \sigma_D^2 \approx 0.17 \ \sigma_D^2.$$

Since under our assumptions $P(0 < D_i < 2C) \approx 1$, the production of product i at plant i is simply $\frac{1}{2} D_i$, with a variance of $\frac{1}{4} \ \sigma_D^2$. The production quantity of the secondary product in a plant is lower and less variable, though, because it is constrained by the demand for the primary product.

As discussed above, the profit-based prioritized allocation policy works as follows:

$$X_{11}^{ProfitP} = \min\{D_1, C\}, \qquad X_{12}^{ProfitP} = \min\{(D_1 - C)^+, C\}$$
$$X_{21}^{ProfitP} = \min\{(C - D_1)^+, (D_2 - C)^+\}, \qquad X_{22}^{ProfitP} = \min\{D_2, C, (2C - D_1)^+\}$$

By definition, $X_{11}^{ProfitP} = X_{11}^{SymP}$, and $X_{21}^{ProfitP} = X_{21}^{SymP}$. Thus, it only remains to determine the expected value and variance of $X_{12}^{ProfitP}$ and $X_{22}^{ProfitP}$.

Theorem 5.5 *Considering independent and identically distributed normal product demands with mean $\mu_D = C$ and standard deviation σ_D, the mean and production variability of each product at each plant under the profit-based prioritized allocation policy in the flexible environment are given by:*

$$E(X_{11}^{ProfitP}) = C - \frac{1}{\sqrt{2\pi}} \ \sigma_D \approx C - 0.40 \ \sigma_D;$$
$$Var(X_{11}^{ProfitP}) = [\ \frac{1}{2} - \frac{1}{2\pi}] \ \sigma_D^2 \approx 0.34 \ \sigma_D^2;$$
$$E(X_{12}^{ProfitP}) = \frac{1}{\sqrt{2\pi}} \ \sigma_D \approx 0.40 \ \sigma_D;$$
$$Var(X_{12}^{ProfitP}) = [\frac{1}{2} - \frac{1}{2\pi}] \ \sigma_D^2 \approx 0.34 \ \sigma_D^2;$$

$$E(X_{21}^{ProfitP}) = [\ \frac{1}{\sqrt{2\pi}} - \frac{1}{2\sqrt{\pi}}\ ]\sigma_D \approx 0.12\ \sigma_D;$$

$$Var(X_{21}^{ProfitP}) = [\frac{1}{4} - \frac{(5\sqrt{2}-4)}{4\sqrt{2\pi}}]\ \sigma_D^2 \approx 0.08\ \sigma_D^2;$$

$$E(X_{22}^{ProfitP}) = C - \frac{(\sqrt{2}+1)}{2\sqrt{\pi}}\ \sigma_D \approx C - 0.68\ \sigma_D;$$

$$Var(X_{22}^{ProfitP}) = [\frac{3}{4} - \frac{(2\sqrt{2}+1)}{4\pi}]\ \sigma_D^2 \approx 0.45\ \sigma_D^2.$$

Observe that the demand for product 1 will be fully satisfied under this allocation policy (since our assumptions imply that $P(D_1 < 2C) \approx 1$). In addition, note that the variance of production of product 1 is identical in both plants. This is intuitive: Since $X_{11}^{ProfitP}$ is a normal truncated to the value of C, it takes up the variability associated with the left side of the bell curve, while $X_{12}^{ProfitP}$ takes up the variability on the right side of the symmetric curve. Regarding the product with the lower profit margin (product 2), its total production is reduced, on average, by about 0.28 σ_D from its production level under symmetric policies, to give priority to the more profitable product 1. This is coupled with a significant increase in the variability of production of the second product at its main plant ($\approx 0.11\ \sigma_D^2$) due to the added uncertainty resulting from its dependency on the demand of the first product routed to the second plant.

We note that the symmetric and profit-based distributed allocations lead to identical production levels and variability, but for different plant-product combinations. Thus, the results in Theorem 5.4 can be easily translated to the profit-based distributed policy.

Corollary 5.6 *Considering independent and identically distributed normal product demands with mean $\mu_D = C$ and standard deviation σ_D, the mean and production variability of each product at each plant under the profit-based distributed allocation policy in the flexible environment are given by:*

$$E(X_{11}^{ProfitD}) = E(X_{12}^{ProfitD}) = \frac{1}{2}\ C; \qquad Var(X_{11}^{ProfitD}) = Var(X_{12}^{ProfitD}) = \frac{1}{4}\ \sigma_D^2;$$

$$E(X_{21}^{ProfitD}) = E(X_{22}^{ProfitD}) = \frac{1}{2}\ C - \frac{1}{2\sqrt{\pi}}\ \sigma_D \approx \frac{1}{2}\ C - 0.28\ \sigma_D;$$

$$Var(X_{21}^{ProfitD}) = Var(X_{22}^{ProfitD}) = \frac{0.6817}{4}\ \sigma_D^2 \approx 0.17\ \sigma_D^2.$$

The following sections use the results derived in Theorems 5.3–5.5 and Corollaries 5.2 and 5.6 to study the impact of flexibility on sales, component suppliers, and on the manufacturer's inventory levels.

5.1.1 Sales

In the make-to-order environment under consideration, sales are equivalent to total system production. Hence, the increase in total sales in the flexible system (versus the non-flexible base case) can be easily obtained from the previous results.

Corollary 5.7 *Considering independent and identically distributed normal product demands with mean $\mu_D = C$ and standard deviation σ_D, adding flexibility results in an increase in expected sales (or, equivalently, expected production) per period of*

$$2 \left[\frac{1}{\sqrt{2\pi}} - \frac{1}{2\sqrt{\pi}} \right] \sigma_D \approx 0.23 \; \sigma_D$$

for all fully flexible allocation policies.

Observe that this expression is an upper bound on the increase in expected sales, since we are not considering inventory constraints. It holds for any *fully flexible allocation*, because they all satisfy demand up to total system capacity and thus lead to identical sales.

The corollary shows how the expected increase in sales grows as the independent demands for the two products become more variable around their means. This is intuitive: as demand uncertainty (σ_D) increases, the magnitude of idle capacity and shortages that can be prevented in the flexible environment is higher, resulting in the larger expected increase in sales.

5.1.2 Variability Observed Upstream

For each product, we consider a single vendor that supplies the major component specific to that product system-wide. In this section, we study the variability observed by each supplier in the flexible system, and compare it to that in the non-flexible setting. We refer to the variability observed by supplier i, who provides the component specific to product i, $i = 1, 2$, as the *system variability* associated with product i, and denote it by $Var(System_i^A)$, where A represents the allocation policy under consideration; this is the variability associated with the orders for the corresponding component from both manufacturing plants. Clearly, system variability will depend on the manufacturer's inventory policy. Here, we will simply consider that suppliers experience the total variability in demand for their particular component in the manufacturing system, i.e., the variability in the system-wide production level of the final product from period to period. This is consistent with the orders generated by a manufacturer following a "modified order-up-to inventory policy" that we introduce in Section 5.1.3. Again we consider the case of normal independent and identically distributed product demands with mean equal to capacity.

In the non-flexible setting, the *system variability* associated with the production of product i, $i = 1, 2$, is simply given by:

$$Var(System_i^{NF}) \equiv Var(X_i^{NF}) = \left[\frac{1}{2} - \frac{1}{2\pi} \right] \sigma_D^2 \approx 0.34 \; \sigma_D^2, \quad \text{by Theorem 5.2,}$$

since the unique component of product i will only be required by plant i. In the fully flexible setting, however, the *system variability* associated with product i and a particular capacity allocation policy A is given by $Var(System_i^A) \equiv Var(X_{ii}^A + X_{ii^c}^A)$, where $A = SymP,\ SymD,\ ProfitP$ *or* $ProfitD$, and $i = 1, 2$. Thus,

$$Var(System_i^A) = Var(X_{ii}^A) + Var(X_{ii^c}^A) + 2Cov(X_{ii}^A, X_{ii^c}^A)$$

Recall that these production variability terms were characterized in Theorems 5.3- 5.5 and Corollary 5.6. Thus, it only remains to find the covariance terms (see Appendix C), which leads to the following theorem.

Theorem 5.8 *Considering independent and identically distributed normal product demands with mean $\mu_D = C$ and standard deviation σ_D, the system variability associated with product i, $i = 1, 2$, in the flexible system is given by:*

1. $Var(System_i^{SymP}) = [\ \frac{3}{4} - \frac{3}{4\pi}\ ]\ \sigma_D^2 \approx 0.51\ \sigma_D^2$, *under the symmetric prioritized allocation policy.*

2. $Var(System_i^{SymD}) = \frac{2.6817}{4}\ \sigma_D^2 \approx 0.67\ \sigma_D^2$, *under the symmetric distributed allocation policy.*

3. $Var(System_i^{ProfitP}) = Var(System_i^{ProfitD}) = \begin{cases} \sigma_D^2, & for\ i = 1, \\ [1 - \frac{1}{\pi}]\sigma_D^2 \approx 0.68\ \sigma_D^2, & for\ i = 2, \end{cases}$
under both prioritized and distributed profit-based policies.

Since the profit-based policies are not symmetric, the resulting system variability is product-dependent: The total production variability of the first product is simply equal to demand variability since, under this allocation policy and our assumptions, all demand for product 1 will always be satisfied. The high system variability of product 2 is due to the effect of product 1 variability on the total production of product 2. Under the symmetric prioritized and distributed policies, system capacity limits the production of both products, leading to lower production variability. The distributed policy leads to the second highest system variability for both suppliers: this is because under this allocation, total production of product i, $i = 1, 2$, varies between $1/2D_i$ and D_i depending on the magnitude of the demand for the other product (observe that the total production of product i is given by $min\{D_i, \frac{1}{2}D_i + C - \frac{1}{2}D_{i^c}\}$). Under the symmetric prioritized policy, however, the effect on the total production of product i of the variability of demand for product i^c is not as pronounced since each product is guaranteed at least C units of capacity.

Figure 3 depicts how the system standard deviation of component demand changes in the dedicated system and in the flexible system under the symmetric and profit-based, prioritized and distributed allocation policies as the demand coefficient of variation grows.

In the figure, the graphs profit-based (1) and profit-based (2) correspond to product 1 and 2, respectively, under the profit-based policies. Observe that the system standard deviation corresponding to product 2 under the distributed policy and the profit-based policies are very close, as shown in Theorem 5.8.

These results show that suppliers observe significantly higher variability in the flexible system as compared to the non-flexible system, and the magnitude of this increase varies wildly with the

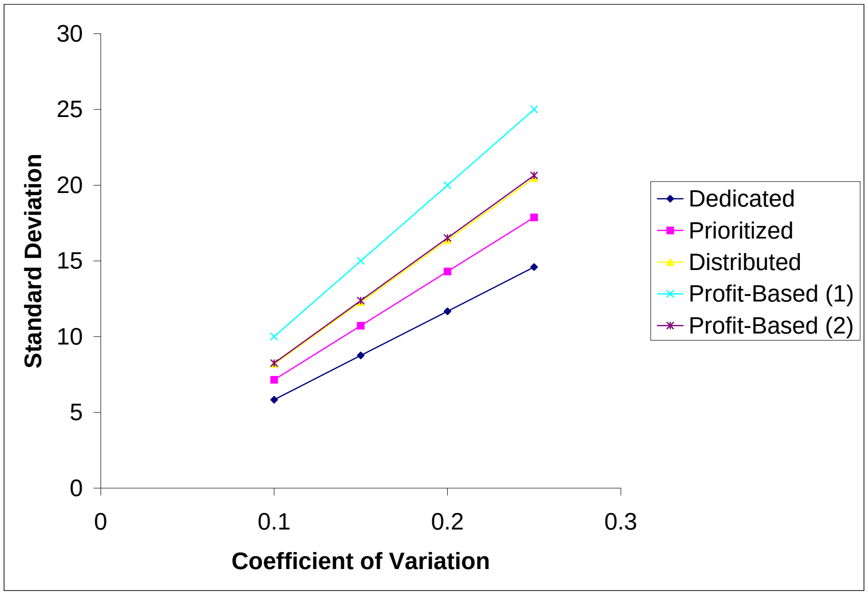

Figure 3: Standard deviation of component demands observed by suppliers in the flexible and non-flexible settings

allocation policy implemented. The increase in the standard deviation of component demand is 22% under the symmetric prioritized policy, 40% under the symmetric distributed policy, and 71% and 41% for products 1 and 2, respectively, under the profit-based policies.

The next section discusses the impact of the increased production variability on the manufacturer's inventory levels.

5.1.3 Manufacturer's Component Inventory Levels

In this section, we consider, for each product, a major component that needs to be kept in inventory due to its long lead-time of L periods, while the final product is made-to-order. We evaluate the effect of the higher production variability on the manufacturer's component inventory levels, using parameters typical in the automotive industry. For this purpose, we determine the increase in component inventory levels in the flexible system under the different allocation policies with respect to our base case, the non-flexible system. As in the previous sections, we report exact analytical results for the balanced system. Numerical results for unbalanced systems will be provided in Section 5.2.

In our analysis, we consider order-up-to (or base-stock) inventory policies, because they are simple, easily implementable, and are commonly used in practice. It is well-known (see Kapuscinski and Tayur (1998) and Zipkin (2000)) that such policies are optimal for multi-stage uncapacitated and single-stage capacitated systems, when excess demands are backordered and setup costs are negligible. Kapuscinski and Tayur (1998) present a simulation-based method using infinitesimal perturbation analysis to find the best order-up-to policies for multi-stage capacitated systems. More recently, Parker and Kapuscinski (2001) show that a modified echelon base-stock policy, in which the order-up-to levels are truncated by the production capacity and by the available inventory upstream, is optimal for capacitated two-stage systems, where the final stage is the bottleneck. In the case of lost sales, however, base-stock policies have been shown to be optimal only in the uncapacitated single-stage setting with a lead-time of one period. For longer lead-times, obtaining an optimal inventory policy requires solving a dynamic program with high-dimensional states and order-up-to policies are often used as an approximation. Metters (1997) stresses the difficulty of solving the capacitated (lost sales) case and proposes heuristics for finding the order-up-to quantities in a single-stage problem. Since our focus is not on finding optimal inventory policies but on understanding the effect of flexibility on inventories, we propose a simple *modified order-up-to policy* based on normal demands and a given service level. We believe that the conclusions obtained using this policy are fairly general, since it is really the production variability, which has been calculated exactly in the previous section, that drives the level of inventory in the system.

In our system, demand for each component is triggered by production of the final product and hence truncated by the plant capacity constraint. Thus, the total inventory (on-hand plus on-order) should never exceed the production capacity over the lead-time (i.e., $C \times L$). Consequently, we consider a *modified order-up-to inventory policy*, in which the safety stock of components of product i at plant j is limited to the average unused plant capacity during the lead-time (i.e., $L(C - \mu_{ij})$), as defined below.

Definition 5.1 *A modified order-up-to policy consists on ordering components of product i at plant j every period to bring the system inventory up to*

$$L\mu_{ij} + \min\{z\sqrt{L}\sigma_{ij}, L(C - \mu_{ij})\},$$

where μ_{ij} and σ_{ij} are the mean and standard deviation of the production level of product i at plant j, respectively, and z is a safety factor corresponding to a specified stock-out probability (for normally distributed demands). Thus, the average level of component inventory is given by:

$$\frac{\mu_{ij}}{2} + \min\{z\sqrt{L}\sigma_{ij}, L(C - \mu_{ij})\} \tag{1}$$

The following observations justify the use of this inventory policy: (1) In our simulation studies, we observe that the production quantities under distributed allocation policies are well approximated by normal distributions. (2) In the dedicated and symmetric prioritized allocations, for the main products at each plant, the total system inventory of LC ensures that component inventory is always available over the lead time; that is, the capacity constraint is always encountered before the inventory one. (3) The production quantities of secondary products in prioritized-type policies are highly skewed to the right. In this case, the normal safety stock calculation underestimates the system requirements (see Section 5.1.4 for more details and a better approximation). Overall, we

Coefficient of Variation ($\frac{\sigma_D}{\mu_D}$)			0.10	0.15	0.20	0.25
Increase in Sales (%)			1.22	1.86	2.54	3.24
Increase in Inventory (%) ($z = 1.64$)	Symmetric Prioritized Policy (SymP)	L=2weeks	12.55	17.88	22.68	27.05
		L=3weeks	14.13	19.57	24.24	28.28
		L=4weeks	15.16	20.50	24.89	28.55
	Distributed Policies (SymD and ProfitD)	L=2weeks	24.61	35.04	44.46	53.02
		L=3weeks	24.25	33.59	41.59	48.53
		L=4weeks	22.77	30.80	37.39	42.89
	Profit-Based Prioritized Policy (ProfitP)	L=2weeks	23.93	34.07	43.24	51.56
		L=3weeks	27.21	37.68	46.65	54.43
		L=4weeks	27.64	37.38	45.38	52.06

Table 1: Percent increase in expected sales and component inventory levels under full flexibility relative to no flexibility for typical parameter values in the automotive industry.

have an upper bound on sales and a lower bound on inventory, which means that we are in fact underestimating the trade-off between the increase in sales and inventory under flexibility.

In what follows, we study the performance of the flexible system, under the fully flexible allocation policies discussed above, using the modified order-up-to inventory policy. Considering, again, a balanced system and using the results in Theorems 5.2–5.5, Corollaries 5.6 and 5.7, and Equation (1), we quantify the effect of flexibility on the manufacturer's sales and component inventory levels for these allocation policies as the demand variability and procurement lead-times grow. The resuls are reported in Table 1, considering a 95% service level, which corresponds to a safety factor, z, of 1.64 in the case of normal demand for components.

This analysis leads to the following insights for balanced systems:

1. The higher the variability of product demand, the higher the benefits of flexibility, since inventory increases at a lower rate than sales as variability grows. Thus, the benefits of adding flexibility increase as product demand becomes more variable, but due to the higher inventory levels, not as much as one would predict.

2. Component inventory levels are highly dependent on the allocation policy implemented: the symmetric prioritized allocation policy results in significantly lower component inventory levels than the symmetric distributed and both profit-based policies.

3. The percent increase in inventory levels under the distributed policies relative to the dedicated case decreases as the procurement lead-time increases. This occurs because the amount of inventory in the dedicated case is constrained by plant capacity over the lead-time and will thus grow proportionally to L, while it grows proportional to $\sqrt{L}$ in the flexible system under the distributed policies.

These results demonstrate the significant increase in component inventory levels – roughly 10% for each percentage point increase in sales in the balanced system – when flexibility is added to the system. In addition, they suggest that as long as profit margins are not a major factor, the symmetric prioritized allocation policy should be used in a flexible environment since it leads to significantly lower inventory levels and supplier variability. If, however, considerably different profit margins make a profit-based policy more attractive, its implementation will require higher inventory levels. The difference in the inventory levels resulting from the profit-based prioritized and distributed policies is quite small; the distributed policy tends to perform better as component lead times increase. A natural question at this point is whether these properties continue to hold for unbalanced systems. Through numerical integration, we evaluate, in Section 5.2, the case where mean demand is not equal to plant capacity (see also details in Appendix F and G).

Our next step is to evaluate the performance of the modified order-up-to policy and the effect of component inventory constraints on the system.

5.1.4 Simulation Study: Impact of Inventory Constraints

The simplifying assumptions made in the analytical treatment raise the need for validation of our results. First, the analysis of sales and variability was carried out under the assumption of infinite component inventory availability. How much will sales be negatively affected and production variability curtailed by the additional inventory constraints? Second, inventory in the system was calculated using an approximate policy, the modified order-up-to policy, and led to quite different amounts of required safety stock under the different allocation policies studied. Will the various policies lead to equal sales once inventory constraints are included?

To address these issues, we simulated the two-plant two-product setting with independent normal demands of mean equal to capacity and coefficients of variation of 0.15 and 0.25. We generated 5000 periods and used the theoretical mean and variance of the production levels characterized in Theorems 5.1-5.5 to calculate the initial inventory levels for each allocation policy. The simulation results are presented in Appendix D.

The average values of the different performance metrics in the simulation are very close to the theoretical ones derived in the previous sections. The difference is well below 1% for all policies and performance measures. Safety stocks are slightly higher in the simulation since initial inventory levels are calculated using the (higher) theoretical sales and variability values, in the absence of inventory constraints, and thus overestimated. The variability observed by suppliers is lower in the simulation since component inventory constrains the extreme cases; except in the dedicated case, in which inventory is never constraining. As conjectured, the average sales vary with the allocation policy used, but the simulation results show that the difference is very small. We observe that the frequency of stock-outs is between 4 and 5% for the distributed-type policies, in accordance with the selected safety factor $z = 1.64$ or 95% service level. However, the modified order-up-to calculation leads to frequent stock-outs (8% for secondary product components) in prioritized allocation policies due to the skewedness of the distribution of production of the secondary products. To overcome this problem, we tested the prioritized policy using a variation of the modified order-up-to policy that takes into account the shape of the distribution of production of secondary products; see Appendix

D for details. We found that it leads to higher sales and lower inventories than any of the other policies, demonstrating the superiority of the prioritized allocations in the balanced case.

Comparing the results for coefficients of variation of 0.15 versus 0.25, we observe that higher demand variability results in a very similar frequency of stockouts, but a larger impact of the inventory constraints on sales because the stockouts are of higher magnitude.

5.2 The Unbalanced System

In this section, we evaluate unbalanced systems that result from forecast errors at the investment stage. Two types of unbalance may be encountered:

1. *System unbalance:* total system capacity and demand are out of sync due to errors in the forecast of overall system demand.

2. *Product unbalance:* the relative magnitudes of the demands of the different products do not match the capacities allocated to them.

Intuitively, there is little flexibility can do against the first type of unbalance. However, it can significantly improve the performance of a system with product unbalance. As we shall see, flexibility is the most advantageous when the system is balanced but product unbalance is significant. In a more precise manner, if C is the capacity available for each product in the dedicated base case, we could measure system unbalance by $\sum_i \mu_{D_i} - 2C$ and product unbalance by $(\mu_{D_1} - \mu_{D_2})^+$. We will refer to growing system unbalance, in general, when $(\sum_i \mu_{D_i} - 2C)^+$ increases, to positive system unbalance when $\sum_i \mu_{D_i} > 2C$, and to negative system unbalance when $\sum_i \mu_{D_i} < 2C$. Although we are assuming here equal capacities at both plants, the definitions of unbalance can be easily extended for general capacities. In particular, for systems with general plant capacities of C_1 and C_2, product unbalance could be defined as $(\frac{\mu_{D_1}}{C_1} - \frac{\mu_{D_2}}{C_2})^+$.

Our objective is to understand how the effectiveness of flexibility depends on forecast and short-term uncertainties. When the mean demands for the two products are not equal, the benefits of flexibility are two-fold. First, flexibility allows a better tactical assignment of available capacity to actual demands, thus hedging against the long-term forecast error. Second, it provides a buffer against short-term or period-to-period variability by making it possible to redistribute the capacity according to the specific period demand realizations. The first effect has been well-captured in the extant literature, considering a single production period (e.g. Jordan and Graves (1995)). To analyze the second effect, we need to be able to discern the costs and benefits attributable to each type of uncertainty. For that purpose, we now include in our analysis the fixed-capacity policy, which partitions the available capacity in proportion to the mean product demands in the medium term and works as a non-flexible system with those capacity levels for each product in the short term. Clearly, this policy is solely using flexibility to hedge against the long-term forecast uncertainty, and thus, the associated benefits and costs are fully attributable to the first effect. Observe that it is important to consider the fixed-capacity policy as a benchmark, because it allows for a more stable production environment, with a fixed proportion of the total capacity assigned to each product, and thus will be much easier to implement in practice.

In what follows, we consider independent product demands with different means varying in the set $\{60, 75, 90, 100, 110, 125, 140\}$, capacity of both plants fixed at 100, demand coefficients of variation of 0.15 and 0.25, component procurement lead-times of 2 weeks, and a z value of 1.64. All these parameters are typical for our industry partner in the automotive industry (after scaling the demands and capacities).

Using numerical integration, we characterize the mean and variance of production levels for the various values of the mean product demands (see Appendix B). Based on these, we then evaluate the performance measures–sales, upstream variability and manufacturer's component inventory levels–for the different allocation policies. In the following sections, we discuss the effect of system and product unbalances on each performance measure. While we include some highlights of the numerical results for coefficients of variation of 0.15 within the text, the complete set of numerical results is presented in Appendix F and Appendix G. To easily interpret the tables, note that: (1) The cases on the *main diagonal* (upper left to lower right corner) represent zero product unbalance, but different levels of system unbalance. (2)The cases on each diagonal parallel to the main diagonal have similar levels of product unbalance, and as one moves away from the main diagonal towards the upper-right or lower-left parallels, product unbalance increases. (3) The *perpendicular diagonal* (lower left to upper right corner) represents cases with zero system unbalance, but different levels of product unbalance. (4) Each of its parallels represents cases with similar level of system unbalance and as one moves away from the main perpendicular diagonal towards the upper-left or lower-right parallels, system unbalance increases. Some diagonals in the tables are bolded to facilitate the observation of such effects.

5.2.1 Sales

We first evaluate the benefits of using flexibility to hedge against *forecast error* using the fixed-capacity allocation policy, and then observe the incremental benefit obtained by using flexibility to accommodate the *short-term order variability* with fully flexible policies.

Table 2 presents the expected percent increase in sales obtained when using the fixed-capacity policy over the non-flexible case. We observe that the fixed-capacity policy is very effective to improve sales when the system exhibits product unbalance. As product unbalance increases and system unbalance decreases (i.e., as we move away from the main diagonal and closer to the perpendicular diagonal) the benefits gained through the partially flexible strategy grow. The highest increase in sales thus occurs for mean demands of 60 and 140 and amounts to 17% for a demand coefficient of variation of 0.15 (14% for coefficient of variation of 0.25, see Appendix F). The benefits decrease as period-to-period variability increases, since the fixed-capacity policy only buffers against the long-term forecast uncertainty.

The following table, Table 3, reports the additional increase in sales (over the non-flexible base case) when using a fully flexible policy instead of the fixed-capacity policy. Hence, it shows how

$C = 100$, $\frac{\sigma_{D_i}}{\mu_{D_i}} = 0.15$, $i = 1, 2$ Increase in Sales (%)	product 1: Mean Demand (μ_{D_1})							
		60	**75**	**90**	**100**	**110**	**125**	**140**
product 2: Mean Demand (μ_{D_2})	**60**	0.0	**0.0**	1.1	**3.6**	7.2	**13.0**	17.7
	75	0.0	0.0	**0.6**	2.2	**4.4**	8.0	**10.6**
	90	1.1	0.6	0.0	**0.4**	1.4	**3.1**	4.5
	100	3.6	2.2	0.4	0.0	**0.3**	1.2	**2.0**
	110	7.2	4.4	1.4	0.3	0.0	**0.3**	0.7
	125	13.0	8.0	3.1	1.2	0.3	0.0	**0.1**
	140	17.7	10.6	4.5	2.0	0.7	0.1	0.0

Table 2: Percent increase in sales when using flexibility to hedge against long-term forecast uncertainty (i.e., fixed-capacity versus dedicated)

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the benefits of flexibility to hedge against the short-term variability change with varying product and system unbalances.

$C = 100$, $\frac{\sigma_{D_i}}{\mu_{D_i}} = 0.15$, $i = 1, 2$ Increase in Sales (%)	product 1: Mean Demand (μ_{D_1})							
		60	**75**	**90**	**100**	**110**	**125**	**140**
product 2: Mean Demand (μ_{D_2})	**60**	0.0	0.0	0.1	0.2	0.6	**1.4**	**1.7**
	75	0.0	0.1	0.4	0.9	**1.5**	**1.8**	**1.6**
	90	0.1	0.4	1.2	**1.7**	**1.9**	**1.7**	1.2
	100	0.2	0.9	**1.7**	**1.9**	**1.8**	1.4	0.9
	110	0.6	**1.5**	**1.9**	**1.8**	1.5	1.0	0.6
	125	**1.4**	**1.8**	**1.7**	1.4	1.0	0.6	0.4
	140	**1.7**	**1.6**	1.2	0.9	0.6	0.4	0.2

Table 3: Additional percent increase in sales when using flexibility to accommodate the short-term variability (i.e., percent increase in sales when using a fully flexible policy minus that resulting from using a fixed-capacity allocation policy).

Full flexibility is the most advantageous in cases where system demand and system capacity are well balanced, as those highlighted in bold in the table. Still, the gain is not even 2% over the more stable fixed-capacity allocation. While the benefits decrease sharply with system unbalance, the effect of product unbalance is very mild. Finally, using flexibility to accommodate period-to-period variability is always much more advantageous in cases of high demand variability, as one would expect (with maximum increases of 1.86% and 3.24% for coefficients of variation of 0.15 and 0.25, respectively, for balanced systems, see Appendix F).

In conclusion, we find that the partially flexible policy is very effective to improve sales in cases of high product unbalance, low system unbalance, and low period-to-period variability in customer orders. Additional flexibility, i.e. fully flexible policies, will be valuable when the system is well balanced and period-to-period demand variability high.

5.2.2 Variability Observed Upstream

The full numerical tables included in Appendix G report the variability observed by the suppliers of each product under the different allocation policies. In general, we observe that when mean product demands are unbalanced, the variability in component orders for the product with higher mean demand significantly increases. The product with lower demand, however, experiences a moderate increase or even a decrease in variability, specially when using the fixed-capacity policy, since its production becomes more constrained by the overall system capacity restriction. The increase in variability of component orders associated with a particular product grows as demand for that product surges. This is in part due to demand variability being proportional to its mean (15 % or 25% in the cases studied). At the same time, the variability of the demand for the components associated with the other product decreases. That is, for a fixed level of demand of product 1, as the demand for product 2 increases, the variability in orders of product 1 components decreases while the variability in orders of product 2 components increases.

The increase in induced component demand variability is quite high and varies wildly with the allocation policy used. In the majority of the cases, the flexible policies can be listed in order of ascending system variability: fixed-capacity, symmetric prioritized, symmetric distributed, and both profit-based policies. Fully flexible allocation policies lead to much higher upstream variability than the partially flexible fixed-capacity policy.

5.2.3 Manufacturer's Component Inventory Levels

For easy comparison between cases with different product demands, the tables below report the relative percent increase in inventory, defined as the percent increase in inventory per percentage point increase in sales (both over the dedicated case), for the different allocation policies.

As in Section 5.2.1, we start by studying the performance of the fixed-capacity policy to understand the effect that using flexibility to hedge against the long-term forecast uncertainty has on inventory. The results are summarized in Table 4 for a demand coefficient of variation of 0.15 for each product. Again higher coefficients of variation are reported in Appendix F and are included in our discussion of the results.

The fixed-capacity policy leads to a moderate reduction in inventory when the total system capacity is tight; see the lower right corner of Table 4. This is because the production of both products becomes tightly constrained after the available capacity is reorganized according to the mean product demands, leading to lower production variability. When the total mean demand is below capacity, however, a considerable increase in inventory can occur. This occurs because allocating more capacity to the product with highest demand leads to an increase it its production variability.

Next, we study the impact of accommodating the short-term variability on component inventory through the use of fully-flexible allocation policies.

For all the fully flexible allocation policies considered, the increase in inventory per percentage point increase in sales decreases with product unbalance (i.e., as we move away from the main

$C = 100$, $\frac{\sigma_{D_i}}{\mu_{D_i}} = 0.15$, $L = 2$ weeks		product 1: Mean Demand (μ_{D_1})						
		60	**75**	**90**	**100**	**110**	**125**	**140**
product 2: Mean Demand (μ_{D_2})	**60**	N/A	7.43	5.86	5.92	4.31	2.13	0.93
	75	7.43	0.00	5.95	6.16	2.46	0.38	-0.29
	90	5.86	5.95	0.00	-2.16	-2.28	-2.36	-2.39
	100	5.92	6.16	-2.16	0.00	-2.54	-2.63	-2.66
	110	4.31	2.46	-2.28	-2.54	0.00	-2.80	-2.83
	125	2.13	0.38	-2.36	-2.63	-2.80	0.00	-2.94
	140	0.93	-0.29	-2.39	-2.66	-2.83	-2.94	0.00

Table 4: Percent increase in inventory per percentage point increase in sales: Fixed-Capacity over dedicated. N/A stands for non-applicable.

SYMP: $C = 100$, $\frac{\sigma_{D_i}}{\mu_{D_i}} = 0.15$, $L = 2$ weeks		product 1: Mean Demand (μ_{D_1})						
		60	**75**	**90**	**100**	**110**	**125**	**140**
product 2: Mean Demand (μ_{D_2})	**60**	N/A	**33.2**	7.9	**5.4**	4.1	**3.0**	2.3
	75	**33.2**	33.2	**8.5**	5.5	**4.1**	3.1	**2.6**
	90	7.9	**8.5**	9.5	**7.7**	6.1	**4.9**	4.5
	100	**5.4**	5.5	**7.7**	9.6	**9.4**	8.2	**7.6**
	110	4.1	**4.1**	6.1	**9.4**	12.3	**13.2**	12.5
	125	**3.0**	3.1	**4.9**	8.2	**13.2**	21.0	**24.2**
	140	2.3	**2.6**	4.5	**7.6**	12.5	**24.2**	37.7

Table 5: Percent increase in inventory per percentage point increase in sales: symmetric prioritized over dedicated.

diagonal through the parallel diagonals) and increases as system unbalance increases (i.e., as we move away from the perpendicular diagonal).

The inventory and sales tables presented in this section show that hedging against short-term uncertainty through the use of a fully flexible policy is most beneficial when system unbalance is small; i.e., for the cases that fall on the main perpendicular diagonal and the two diagonals parallel to it shown in bold in Table 3. Even then, the gain in sales was quite low–below 1.9% and 3.3% for coefficients of variation of 0.15 and 0.25, respectively. Consequently, we focus on the inventory requirements of the different fully flexible policies *for cases with low system unbalance.* We observe the following:

1. The symmetric prioritized policy outperforms the symmetric distributed policy by yielding lower production variability and, thus, requiring lower inventory.

2. On the other hand, inventory requirements under the profit-based prioritized and distributed policies are very close, with the prioritized policy leading to slightly lower inventory.

3. Profit-based policies will yield higher revenues by favoring the more profitable products.

SYMD: $C = 100$, $\frac{\sigma_{D_i}}{\mu_{D_i}} = 0.15$, $L = 2$ weeks		product 1: Mean Demand (μ_{D_1})						
		60	**75**	**90**	**100**	**110**	**125**	**140**
product 2: Mean Demand (μ_{D_2})	**60**	N/A	**11.82**	6.23	**6.24**	4.73	**3.50**	3.11
	75	**11.82**	11.23	**6.73**	6.61	**5.13**	4.17	**4.04**
	90	6.23	**6.73**	9.06	**11.02**	9.78	**8.85**	8.91
	100	**6.24**	6.61	**11.02**	19.41	**22.69**	23.03	**23.33**
	110	4.73	**5.13**	9.78	**22.69**	38.49	**52.77**	57.95
	125	**3.50**	4.17	**8.85**	23.03	**52.77**	126.83	**192.35**
	140	3.11	**4.04**	8.91	**23.33**	57.95	**192.35**	449.87

Table 6: Percent increase in inventory per percentage point increase in sales: symmetric distributed over dedicated.

However, they result in higher variability in general. Thus, the gain in revenues must be traded-off with the additional costs brought about by the induced variability in the system.

Nevertheless, distributed policies will have the advantage of producing closer to the customer, thus reducing outbound distribution cost and order-to-delivery times. Quantifying the associated reduction in outbound transportation cost for the manufacturer in the flexible environment is the focus of the next section.

5.3 Manufacturer's Outbound Distribution

A fully flexible environment will also enable the manufacturer to satisfy customers from their closest plants. To achieve significant savings in distribution costs and shipping times to customers, however, a distributed production system, which we found to require higher inventories in general, must be in place. In this section, we determine the associated reduction in outbound transportation costs and study the trade-off between increased inventory levels and lower transportation costs in the flexible system.

For that purpose, we incorporate customer locations into our two-plant two-product system and analytically determine lower bounds on the reduction in outbound transportation costs in the flexible system. We then evaluate the integrated effect of flexibility on component inventory levels and outbound distribution through a simulation study.

5.3.1 Location-Based Model

To evaluate the impact on outbound distribution, we extend our two-plant two-product model to a "location-based" model, where customers are independently, uniformly distributed in a region $\mathcal{R}$ of area A, and assume that each customer demands a single product. As before, we assume that the overall demands for the two products are independent normal distributions. The total shipping cost is considered to be proportional to the total distance traveled to satisfy each customer directly from her corresponding plant[2] and distances are calculated using a rectilinear (right-angle) distance metric[3].

To make use of the customer location information, each of the allocation policies described in our original model (see Section 4) needs to be slightly modified. The *symmetric prioritized* allocation policy will still assign to plant i as many customers of product i as possible, but give priority to those in its "production area" (i.e., customers who are closer to it than to the other plant). Any excess demand is routed to the other plant. On the other hand, under the *symmetric distributed* policy, plant i will give priority to customers in its production area, and within those, preference to customers of product i. When capacity for one plant is exceeded by the demand in its production area, surplus orders are assigned to the spare capacity, if any, of the other plant. The profit-based prioritized and distributed policies can be extended in a similar fashion. In what follows, we refer to these modified policies as location-based policies and to plant $i's$ production area as region i, $i = 1, 2$. Note that since the proportion of product i, $i = 1, 2$, allocated to its main plant in the modified distributed policies is dependent on the customer locations (instead of the fixed proportion of $\frac{1}{2}$), the resulting production variability will be higher than that of the original distributed policies, SymD and ProfitD. Thus, we distinguish between these policies by referring to the location-based distributed policies as SymDL and ProfitDL. The location-based prioritized policies, however, do not modify the quantities of each product allocated to each plant by the original prioritized policies, and hence we will keep the same notation of SymP and ProfitP.

In what follows, we start by evaluating the effect of the *location-based symmetric distributed allocation policy, SymDL,* on outbound transportation cost. Other allocation policies are considered in Section 5.3.3.

5.3.2 Analytical Study

In this section, we develop lower bounds on the reduction in unit outbound shipping costs gained in the flexible environment by using the *location-based symmetric distributed allocation policy, SymDL.*

Let u^{NF} be the expected distance traveled per unit sold in the dedicated environment, and u^{SymDL} be the unit distance traveled in the fully flexible environment under the location-based distributed allocation policy. In the following, we estimate and compare u^{NF} and u^{SymDL}.

Let (X_{0j}, Y_{0j}) represent the coordinates of plant j in the region, for $j = 1, 2$. Consider an arbitrary customer, k, with coordinates (X_k, Y_k) uniformly distributed in the service area, and let $Z_{kj} = |X_k - X_{oj}| + |Y_k - Y_{oj}|$ denote the rectilinear distance between customer k and plant j, $j = 1, 2$. Larson and Odoni (1981) show that, when plant and customer locations are independently,

[2]This approximation has been commonly used in the literature, especially in the context of "location-allocation" problems; see, for instance, Erlebacher and Meller (2000).

[3]Such a distance metric is often used due to the approximate rectilinear patterns of city streets and the U.S. interstate highway system (Larson and Odoni (1981), p. 95). If rectilinear distances are not appropriate, a linear adjustment factor can be used to approximate the correct distance function from the rectilinear.

uniformly distributed in the region,

$$E(Z_{kj}) = c_o\sqrt{A}, \tag{2}$$

where c_o is $\frac{2}{3}$ if region $\Re$ is a square (of area A). This expression also gives a good approximation for *fairly compact and fairly convex areas* (Larson and Odoni (1981), p. 135). Observe that in the dedicated case, each customer is allocated to the only plant that manufactures her choice of product, and thus the expected distance traveled per unit, u^{NF}, can be approximated by $\frac{2}{3}\sqrt{A}$.

Next, consider the case where both plants are fully flexible and the location-based symmetric distributed allocation policy is used.

For each customer k, $k \geq 1$, let $Z_{k(1)} \equiv \min_j\{Z_{kj}\}$ denote the distance between this customer and her closest plant, and similarly, let $Z_{k(2)} \equiv \max_j\{Z_{kj}\}$ denote the distance between customer k and the further plant. When plant and customer locations are independently, uniformly distributed in the region, we derive (see Appendix D):

$$E[Z_{k(1)}] = c_1\sqrt{A}, \qquad E[Z_{k(2)}] = c_2\sqrt{A}, \tag{3}$$

where c_1 is 0.478 and c_2 is 0.8558 when $\Re$ is a square. As before, these expressions also provide good approximations for fairly compact and fairly convex areas.

Similar expressions can be obtained when plant locations are given. For example, if we consider the area (unit square) divided into two identical triangles, that is, $(X_{01}, Y_{01}) = (0.25, 0.25)$ and $(X_{02}, Y_{02}) = (0.75, 0.75)$, then $c_o = 0.625, c_1 = 0.3958, c_2 = 0.8542$.

We are now ready to derive the expected total distance traveled in the flexible setting. For this purpose, we first need to determine the expected number of customers served within and outside each plant's production area. The expected distance traveled per unit, u^{SymDL}, will then be approximated by the ratio of expected total distance to expected total number of customers served.

Let $X^{SymDL}_{j,\text{``in''}}$ and $X^{SymDL}_{j,\text{``out''}}$ denote the total number of customers, served by plant j, who are within and outside this plant's production area, respectively, under the location-based symmetric distributed allocation policy. We first observe that since the demand for each product, D_i, $i = 1, 2$, is independently and normally distributed with mean μ_{D_i} and standard deviation σ_{D_i}, the total demand, $D_T \equiv \sum_{i=1}^{2} D_i$, is also normally distributed with mean $\sum_{i=1}^{2} \mu_{D_i}$ and standard deviation $\sqrt{\sum_{i=1}^{2} \sigma^2_{D_i}}$. Let N_j, $j = 1, 2$, denote the total number of customers in plant j's production area. To determine the distribution of N_j, we associate a bernoulli random variable, B_k, with each customer $k = 1, 2, \cdots, D_T$, as follows:

$$B_k = \begin{cases} 1, & \text{if } Z_{k1} < Z_{k2} \\ 0, & \text{otherwise.} \end{cases}$$

Thus, $N_1 = \sum_{k=1}^{D_T} B_k$ and $N_2 = \sum_{k=1}^{D_T}(1 - B_k)$. Letting $p \equiv P(Z_{k1} < Z_{k2})$, for $k \geq 1$, we can write the expected value and variance of N_1 as:

$$\mu_{N_1} \equiv E(N_1) = E(D_T)\ E(B_k) = \sum_{i=1}^{2} \mu_{D_i}\ p$$

$$\sigma_{N_1}^2 \equiv Var(N_1) = E(D_T)Var(B_k) + [E(B_k)]^2 Var(D_T) = \sum_{i=1}^{2} \mu_{D_i}\ p(1-p) + p^2 \sum_{i=1}^{2} \sigma_{D_i}^2$$

The expectation and variance of random variable N_2 can be derived similarly. Observe that variable N_1 given $D_T = n$ (variable N_2 given $D_T = n$) is binomial with parameters n and p (n and $1-p$). Simulation studies suggest that, although variables N_j, $j = 1,2$, take only integer values, it is reasonable to approximate each by a normal distribution with mean μ_{N_j} and standard deviation σ_{N_j}. We denote its associated normal cumulative distribution function by $F_{N_j}(.)$ and let $e_{N_j}(C) = exp(\frac{-(C-\mu_{N_j})^2}{2\sigma_{N_j}^2})$, with *exp* denoting the exponential function.

Recall that under the location-based distributed allocation policy, each plant j first allocates its capacity to customers within its production area, thus serving $X_{j,\text{“in”}}^{SymDL} = \min\{C, N_j\}$ of them. Using the normal approximation of N_j, we can write (see Appendix E.2):

$$E[X_{j,\text{“in”}}^{SymDL}] = C + F_{N_j}(C)(\mu_{N_j} - C) - \frac{e_{N_j}(C)}{\sqrt{2\pi}}\ \sigma_{N_j} \tag{4}$$

In addition, if needed, plant j will allocate its remaining capacity to those customers who are in the other plant's production area, but cannot be served by it. Thus, plant j will serve an additional $X_{j,\text{“out”}}^{SymDL} = \min\{(C - N_j)^+,\ (N_{j^c} - C)^+\}$ customers who are, in fact, closest to plant j^c, for $j, j^c \in \{1,2\}$ and $j^c \neq j$. We obtain the following *upper bound* on $E[X_{j,\text{“out”}}^{SymDL}]$ (see Appendix E.2 for the derivation):

$$\begin{aligned} E[X_{j,\text{“out”}}^{SymDL}] \leq \min\{\ &[\ C - \mu_{N_j} + \frac{e_{N_j}(C)}{F_{N_j}(C)\ \sqrt{2\pi}}\ \sigma_{N_j}\ ]\ F_{N_j}(C), \\ &[\ -C + \mu_{N_{j^c}} + \frac{e_{N_{j^c}}(C)}{(1 - F_{N_{j^c}}(C))\ \sqrt{2\pi}}\ \sigma_{N_{j^c}}\ ]\ [1 - F_{N_{j^c}}(C)]\ \} \end{aligned} \tag{5}$$

Then, the expected total distance traveled, in the flexible environment under the location-based distributed policy, to serve all customers assigned to plant j, which we denote by $E[Distance_j^{SymDL}]$, can be calculated as follows.

$$\begin{aligned} E[Distance_j^{SymDL}] &= E[\sum_{k=1}^{X_{j,\text{“in”}}^{SymDL}} Z_{k(1)} + \sum_{k=1}^{X_{j,\text{“out”}}^{SymDL}} Z_{k(2)}] \\ &= E[X_{j,\text{“in”}}^{SymDL}]\ E[Z_{k(1)}] + E[X_{j,\text{“out”}}^{SymDL}]\ E[Z_{k(2)}], \quad \text{by Wald's Equation} \\ &= E[X_{j,\text{“in”}}^{SymDL}]\ c_1\sqrt{A} + E[X_{j,\text{“out”}}^{SymDL}]\ c_2\sqrt{A} \end{aligned} \tag{6}$$

Finally, we approximate the expected shipping cost per unit in the flexible environment by,

$$\begin{aligned} u^{SymDL} &= \frac{\sum_{j=1}^{2} E[Distance_j^{SymDL}]}{\sum_{j=1}^{2}(E[X_{j,\text{``}in\text{''}}^{SymDL}] + E[X_{j,\text{``}out\text{''}}^{SymDL}])} \\ &\leq c_1\sqrt{A} + \frac{(c_2 - c_1)\sqrt{A}\ \sum_{j=1}^{2} E[X_{j,\text{``}out\text{''}}^{SymDL}]}{\sum_{j=1}^{2} E[X_{j,\text{``}in\text{''}}^{SymDL}]} \end{aligned}$$

Substituting Equations (4) and (5) and the appropriate values of c_1 and c_2 (based on the plant locations) in the above expression, we obtain a *lower bound* on $100 \times \frac{u^{NF} - u^{SymDL}}{u^{NF}}$, the percent reduction in the expected unit shipping cost in the flexible environment under the location-based distributed allocation policy, relative to the dedicated case.

Figure 4 below and Table 14 in Appendix E.3 report the lower bounds on the percent reduction in expected shipping cost per unit for normal demand distributions with different coefficients of variation and means. We consider cases where plants have a capacity of 100 units each and are located at coordinates $(X_{01}, Y_{01}) = (0.25, 0.25)$ and $(X_{02}, Y_{02}) = (0.75, 0.75)$ in the unit square. Figure 4 illustrates the impact of demand variability and system unbalance on outbound distribution cost, considering identical demand distributions for the two products; Table 14 focuses on the effect of product unbalance. This analysis yields the following insights:

1. For all scenarios studied, full flexibility leads to a significant reduction (at least 31.73%) in expected unit shipping cost.
2. This reduction is more pronounced when system unbalance is high. This is intuitive; the flexible system yields lower unit shipment cost in these cases because capacity of each plant is almost entirely allocated to orders within its production area in the flexible environment.
3. The lower period-to-period variability of product demand, the higher the reduction in the expected shipping cost per unit. This is because as demand variability increases, the demand that each plant faces from customers closest to it becomes more variable. As a result, the average number of customers satisfied from the further plant in the flexible system increases, leading to higher shipping costs.

Furthermore, these trends continue to hold when demands of the two products are unbalanced (see Table 14 in Appendix E.3). The question, however, is how much more inventory will be required in the flexible system to take full advantage of customer locations. This is addressed through simulation in the next section.

5.3.3 Simulation Study: Inventory/Transportation Trade-Off

In this section, we evaluate the effect of the various location-based allocation policies, introduced in Section 5.3.1, on component inventory and outbound distribution. The simulation study is also useful to assess the tightness of the lower bound derived in the previous section.

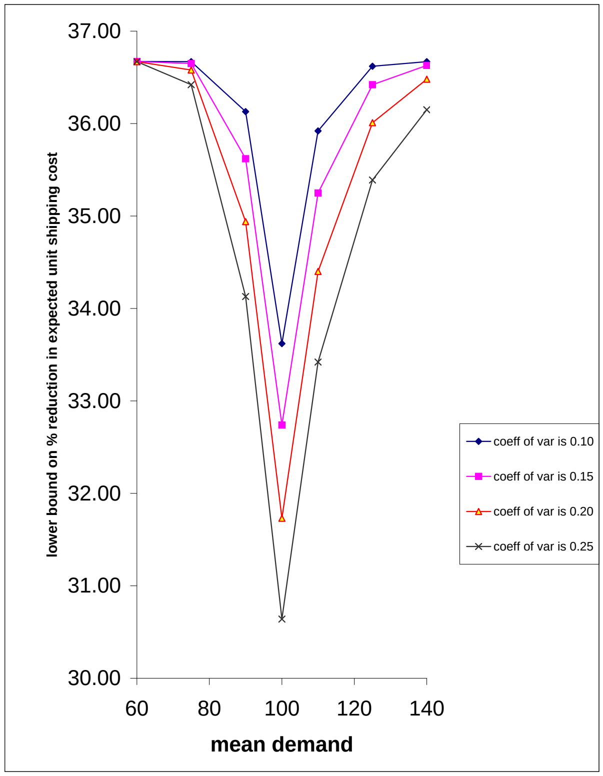

Figure 4: Lower bounds on the percent reduction in the expected shipping cost per unit in the flexible system when product demands are identically distributed

We again assume that product demands are independent, plant locations are fixed at coordinates $(X_{01}, Y_{01}) = (0.25, 0.25)$ and $(X_{02}, Y_{02}) = (0.75, 0.75)$ in the unit square, and customer locations are independently, uniformly distributed in the unit square. To allow for comparison between the simulation and previous analytical results, we ignore the inventory constraints and determine inventory requirements based on the expected sales and the production variability in the simulation. Each scenario, characterized by product demands (μ_{D_1}, μ_{D_2}), is replicated 2000 times with different, independently generated sets of customer locations. Over these replications, we determine the average sales, production variability and the average outbound distribution cost per unit sold for the dedicated system and for the various allocation policies in the flexible system. The full simulation results are reported in Appendix G, where product and system unbalance, as well as different coefficients of variation (0.15 and 0.25) are tested considering a component procurement lead-time of 2 weeks, a z value of 1.64, and plant capacities of 100 each. Within this section we will only include Table 7, which summarizes the simulation results for all allocation policies, for the case of $\mu_{D_1} = 100$ and varying μ_{D_2}. Observe that since all these allocation policies optimize sales in our simple two-plant two-product model, they all result in the same average sales.

$\mu_{D_1} = 100$: $C = 100$, $\frac{\sigma_{D_i}}{\mu_{D_i}} = 0.15$, $L = 2$ weeks, $z = 1.64$									
	% incr.	% reduction in unit outbound cost				% increase in inventory			
μ_{D_2}	in sales	SymP	SymDL	ProfitDL	ProfitP	SymP	SymDL	ProfitDL	ProfitP
60	4.49	1.60	39.59	39.59	1.87	23.75	38.33	38.33	23.75
75	3.91	1.17	39.52	39.52	1.87	17.55	36.12	36.26	18.27
90	2.37	0.81	38.52	38.52	2.04	16.89	39.62	39.65	24.58
100	2.25	0.83	37.31	37.31	2.97	20.18	54.78	55.26	34.04
110	1.77	0.58	37.16	37.19	3.26	18.25	60.01	60.82	41.67
125	2.77	0.72	37.06	37.11	3.28	26.69	63.04	65.29	48.87
140	3.21	0.81	36.76	36.79	3.42	20.54	69.54	69.41	49.41

Table 7: Percent change in sales, component inventory levels, and unit outbound distribution cost for different allocation policies in the flexible setting relative to the non-flexible one ($\mu_{D_1} = 100$).

The magnitude of savings in transportation gained by the distributed allocations is very high, especially in the symmetric settings. However, these savings come at the expense of much higher inventory levels. As expected, the amount of inventory in the location-based distributed policies is higher than in the original distributed policies due to the additional variability in the production quantities at each plant induced by the variability on customer locations. On the other hand, the inventory levels under the prioritized policies correspond to those obtained through numerical integration in Section 5.2.3, since the associated production quantities do not depend on customer locations. We conclude that taking advantage of the flexible environment to service customers from their closest plants requires a significant amount of additional component inventory.

Simulation results also suggest that the lower bound derived in Section 5.3.2 (and tabulated in Table 14 in Appendix E.3 for these cases) is quite strong; the actual percent reduction in the expected unit shipping cost in the flexible environment exceeds the lower bound by only 3% on average.

The reduction in expected outbound cost per unit is quite similar for symmetric and profit-based distributed policies, SymDL and ProfitDL. This is because, for the demand ranges tested in the simulation study, each plant satisfies customers mostly within its own production area under the profit-based distributed policy, in a similar way as under the symmetric distributed policy (however, the mix of products satisfied are different under both policies). When $\mu_{D_1} < C$, the reductions in expected unit outbound distribution cost for symmetric and profit-based prioritized policies are fairly close as well, since both allocations are identical when the demand of the preferred product is below capacity. However, as μ_{D_1} increases above plant capacity, the profit-based prioritized policy will give priority to the excess product 1 demand at plant 2, which will mainly consist of region 2 customers. Thus, in such cases, the reduction in unit outbound distribution cost will be higher for the profit-based prioritized policy than that for the symmetric prioritized policy.

In summary, the major take-aways from our simulation study are:

- There is a significant trade-off between component inventory levels and outbound distribution costs in the allocation decision; policies that require lower component inventory (because they concentrate production of each product to a primary plant) lead to higher unit outbound distribution cost, and the opposite holds for policies that focus on satisfying customers from their closest plant.

- As observed in Section 5.2, the increased revenue resulting from dedicating more resources to a more profitable product when its demand exceeds plant capacity requires a large increase in component inventory levels.

6 Conclusions and Future Research Directions

We introduce a three-stage capacity planning framework that considers strategic capacity investment decisions at the first stage, tactical capacity allocation and inventory control decisions at the intermediate stage, and the assignment of customer orders to available capacity at the final stage. In a make-to-order environment, we study the impact of using flexible capacity to hedge against both forecast uncertainty at the time capacity investments are made and short-term variability of customer orders. How flexibility is managed to allocate customer orders to plant capacity will ultimately drive the performance of the supply chain. For a simple two-plant two-product model, we quantify the effects of adding flexibility on sales, demand variability observed upstream, component inventory levels, and outbound distribution cost, under various rational capacity allocation policies.

Our analysis uncovers several important factors:

1. Partially flexible policies: The fixed-capacity policy, which shifts the capacity allocation only in the medium term, provides an excellent strategy to hedge against long-term uncertainty. It is most advantageous when product demands are unbalanced, capacity is tight, and short-term variability is moderately low. Furthermore, it leads to a reduction in component inventory and induced variability at suppliers over the dedicated system in such cases.

2. Fully flexible policies: The additional increase in sales obtained through the use of the fully flexible policies, which may change the capacity allocation every period to accommodate short-term order variability, is quite low. The highest benefits occur when the system is well-balanced, i.e. when total system capacity is very close to expected system demand. Even in these cases, the gains in sales come at the expense of a steep increase in component inventory and upstream demand variability. However, the benefits of a fully flexible policy increase as the short-term variability in product demand increases.

3. Prioritized vs. Distributed Policies: Distributed policies, which split production of each product between the two plants, lead to lower outbound transportation costs because production can be scheduled closer to the customer. However, they require higher component inventory than prioritized policies, which concentrate production of each product primarily to a single plant, because the amount of each product to be produced at each plant is more variable. We

find that this trade-off between inventory and outbound distribution is significant and needs to be considered in determining which allocation policy to use.

4. Profit-based vs. Symmetric Policies: Profit-based policies lead to an increase in revenues by favoring the most profitable product, but at the expense of higher inventory levels than symmetric prioritized policies. When the mean demand for the most profitable product is higher than plant capacity, the additional variability induced by a profit-based policy should be traded off with the additional revenue obtained by selling more of the most profitable product.

In fact, which allocation policy to use depends on the magnitude of system and product unbalance, the period-to-period variability, the profit margins, and the costs associated with inventory holding and outbound distribution. The practical implication of this insight is quite significant: *As demand patterns and cost parameters change over time, medium-term planning decisions, such as overall capacity allocation and inventory policies, should be continuously revisited and changed if appropriate.*

Other issues need to be considered when determining a capacity allocation policy. For example, in practice it may not always be desirable to use a prioritized allocation policy because the resulting low volumes of production of the second product will lead to higher manufacturing costs due to the higher number of setups, slow learning, and other lost benefits of scale. Finally, the managerial complexity associated with fully flexible policies, which may require shifting the production levels every period, may not be worthwhile when considering the modest gains in sales and steep increase in inventory over the stable fixed capacity policy.

In summary, our results demonstrate: (1) the significant impact that flexible capacity has on the operational performance of the supply chain, and (2) how the magnitude of this impact is highly dependent on the way the flexible capacity is *managed* in the system (i.e., by the capacity allocation policy implemented). While manufacturing flexibility allows the company to satisfy more demand and produce closer to the customer's location, it requires significantly higher levels of component inventory and results in higher upstream variability. Consequently, these changes in operational performance need to be considered when designing and managing the flexible capacity in a make-to-order environment. This highlights the importance of managing flexible systems appropriately, and hence the need for more rigorous models that (1) capture the interaction between different periods over a planning horizon, (2) determine overall optimal allocation and component inventory policies, and (3) find the optimal flexibility configuration.

Numerous extensions of the models studied deserve further attention. While evaluating the performance of different capacity allocation policies, we assume that product demands are independent and that each product requires a single component, which is unique to that particular product. However, in real life, product demands are correlated over time and among products, and multiple components - some unique, some shared among different products- are required to assemble the final product. Moreover, some of the demand not satisfied in a particular period could be backlogged and added to the following production period if customers are willing to wait.

In addition, other capacity allocation policies, such as policies that consider both inbound and outbound distribution costs when allocating capacity to customers in different locations, might be attractive in practice. Finally, the effect on supply chain performance of different flexibility configurations in production systems with multiple plants and products needs to be studied. An extensive simulation study presented in the companion paper, Muriel and Somasundaram (2001), addresses most of these issues.

The transformation from make-to-stock to make-to-order producers is progressing rapidly. While some manufacturers, such as Dell Computers, have perfected the make-to-order model for modular assembly, much work needs to be done in non-modular industries to improve capacity investment decision making, selection of capacity allocation and inventory policies, and pricing.

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References

Andreou, S. A. (1990), A Capital Budgeting Model for Product-Mix Flexibility. *Journal of Manufacturing Operations Management* **3**, pp. 5–23.

Bassok, Y. , R. Anupindi and R. Akella (1999), Single-period Multiproduct Inventory Models with Substitution. *Operations Research* **47**, No. 4, pp. 632–642.

Beach, R. , A. P. Muhlemann, D. H. R. Price, A. Paterson and J. A. Sharp (2000), A Review of Manufacturing Flexibility. *European Journal of Operational Research* **122**, pp. 41–57.

Bradley, J. R. and A. P. Blossom (2001), Using product-Mix Flexibility to Implement a Make-to-Order Assembly Line. Working Paper, Cornell University.

Caulkins, J. P. and C. H. Fine (1989), Seasonal Inventories and the Use of Product-Flexible Manufacturing Technology. MIT Working Paper #3011-89-MS.

Eppen, G. D., R. K. Martin and L. Schrage (1989), A Scenario Approach to Capacity Planning. *OR Practice* **37**, No. 4, pp. 517–527.

Erlebacher, S. J. and R. D. Meller (2000), The Interaction of Location and Inventory in Designing Distribution Systems. *IIE Transactions* **32**, No. 2, pp. 155–166.

Fine, C. H. and R. M. Freund (1990), Optimal Investment in Product-Flexible Manufacturing Capacity. *Management Science* **36**, No. 4, pp. 449–466.

Graves, S. C. and B. T. Tomlin (2000), Process Flexibility in Supply Chains. Under review for *Management Science.*

Gupta, D., Y. Gerchak and J. A. Buzacott (1992), The Optimal Mix of Flexible and Dedicated Manufacturing Capacities: Hedging Against Demand Uncertainty. *International Journal of*

Production Economics **28**, pp. 309–319.

Jones, R. A. and J. M. Ostroy (1984), Flexibility and Uncertainty. *Rev. Economic Studies* **51**, pp. 13–32.

Jordan, W. C. and S. C. Graves (1995), Principles and Benefits of Manufacturing Process Flexibility. *Management Science* **41**, No. 4, pp. 577–594.

INCLUDE KAPUSCISNKI AND TAYUR'S BOOK CHAPTER

in Tayur, S., R. Ganesham and M. Magazine (2000), Quantitative Models in Supply Chain Management. Kluwer Academic Publishers.

Khouja, M. (1999), The Single-period (News-vendor) Problem: Literature Review and Suggestions for Further Research. *Omega* **27**, pp. 537–553.

Kouvelis, P. (1992), Design and Planning Problems in Flexible Manufacturing Systems: A Critical Review. *Journal of Intelligent Manufacturing* **3**, pp. 75–99.

Larson, R. C. and A. R. Odoni (1981), *Urban Operations Research.* Prentice-Hall, Inc., New Jersey.

Muriel, A. and A. Somasundaram (2001), Simulation Study of the Impact of Manufacturing Flexibility on Sales, Inventory and Transportation Costs. Working paper, University of Massachusetts, Amherst.

Netessine, S., G. Dobson and R. A. Shumsky (2000), Flexible Service Capacity: Optimal Investment and the Impact of Demand Correlation, *Proceedings of the MSOM 2000 Conference*, Ann Arbor.

Parker, R. and R. Kapuściński (2001), Optimal Inventory Policies for a Capacitated Two-Echelon System. Working paper, University of Michigan.

Ross, S. M. (2000), *Introduction to Probability Models.* Seventh Edition, Harcourt Academic Press, CA.

Sarhan, A. E. and B. G. Greenberg (1962), *Contribution to Order Statistics.* Wiley Publications in Statistics.

Sethi, A. K. and S. P. Sethi (1990), Flexibility in Manufacturing: A Survey. *The International Journal of Flexible Manufacturing Systems* **2**, pp. 289–328.

Stalk, G. (1988). Time-The Next Source of Competitive Advantage. Harvard Business Review, July-August 1998.

Stalk, G., S. Stephenson and T. King (1996). Searching for Fulfillment: Breakthroughs in Order-to-delivery Process in the Auto Industry. The Boston Consulting Group.

Triantis, A. J. and J. E. Hodder (1990), Valuing Flexibility as a Complex Option. *The Journal of Finance* **45**, No. 2, pp. 549–565.

Van Mieghem, J. A. (1998), Investment Strategies for Flexible Resources. *Management Science* **44**, No. 8, pp. 1071–1078.

Winkler, R. L., G. M. Roodmand and R. R. Britney (1972), The Determination of Partial Moments. *Management Science* **19**, No. 3, pp. 290–296.

Zipkin, P. H. (2000), Foundations of Inventory Management. McGraw-Hill.

A Fundamentals

In this section of the Appendix we derive some basic expressions that will be used in determining the performance of the two-plant two-product model introduced in Section 4, in non-flexible and flexible settings.

We consider a normal random variable D with mean μ_D, standard deviation σ_D, probability density function (pdf) $f(.)$ and cumulative distribution function (CDF) $F(.)$. We let $\phi(.)$ and $\Phi(.)$ denote the pdf and CDF, respectively, of the standard normal distribution. As defined in the body of the paper, $e_D(y) = exp(\frac{-(y-\mu_D)^2}{2\sigma_D^2})$ and $(x)^+$ equal to x if $x \geq 0$, and 0, otherwise. Finally, we use y, y_1 and y_2 to denote given values and let

$$X = \min\{\beta D,\ y\},$$

where $0 < \beta \leq 1$. In what follows, we derive the first and second moments of X, and then those of other variables that will be needed later in the analysis.

A.1 First Order Moments

In what follows, we denote the first order partial moments (expectations) of random variable D as

$$E_{-\infty}^{y}(D) = \int_{-\infty}^{y} Df(D)dD, \quad \text{and} \qquad E_{y}^{+\infty}(D) = \int_{y}^{+\infty} Df(D)dD$$

¿From Winkler et al. (1972), we have:

$$E_{-\infty}^{y}(D) = \mu_D\ F(y) - \sigma_D\ \frac{e_D(y)}{\sqrt{2\pi}} \tag{7}$$

$$E_{y}^{+\infty}(D) = \mu_D\ [1 - F(y)] + \sigma_D\ \frac{e_D(y)}{\sqrt{2\pi}} \tag{8}$$

Thus,

$$\begin{aligned} E(X) &= \int_{-\infty}^{\frac{y}{\beta}} \beta Df(D)dD + y\ [\ 1 - F(\frac{y}{\beta})\ ] \\ &= \beta\ E_{-\infty}^{\frac{y}{\beta}}(D) + y\ [\ 1 - F(\frac{y}{\beta})\ ] \\ &= y + F(\frac{y}{\beta})[\ \beta\mu_D - y\ ] - \beta\ \frac{e_D(\frac{y}{\beta})}{\sqrt{2\pi}}\ \sigma_D, \quad \text{by Equation (7)} \end{aligned} \tag{9}$$

Also, we observe that,

$$E(D|D \leq y) = \frac{1}{F(y)}\ E_{-\infty}^{y}(D) = \mu_D - \sigma_D\ \frac{e_D(y)}{F(y)\ \sqrt{2\pi}} \tag{10}$$

$$E(D|D \geq y) = \frac{1}{[1 - F(y)]}\ E_{y}^{+\infty}(D) = \mu_D + \sigma_D\ \frac{e_D(y)}{[1 - F(y)]\ \sqrt{2\pi}} \tag{11}$$

$$E(D|y_1 < D < y_2) = \frac{1}{[F(y_2) - F(y_1)]} \; [\; E_{-\infty}^{y_2}(D) - E_{-\infty}^{y_1}(D) \;]$$
$$= \mu_D + \frac{1}{[F(y_2) - F(y_1)]} \frac{\sigma_D}{\sqrt{2\pi}} (e_D(y_1) - e_D(y_2)) \quad (12)$$

A.2 Second Order Moments

We define the second order partial moments of random variable D as

$$E_{-\infty}^{y}(D^2) = \int_{-\infty}^{y} D^2 f(D) dD, \quad \text{and} \quad E_{y}^{+\infty}(D^2) = \int_{y}^{+\infty} D^2 f(D) dD$$

¿From Winkler et al. (1972), we have:

$$E_{-\infty}^{y}(D^2) = (\mu_D^2 + \sigma_D^2)\; F(y) - \sigma_D\; (y + \mu_D)\; \frac{e_D(y)}{\sqrt{2\pi}} \quad (13)$$

$$E_{y}^{+\infty}(D^2) = (\mu_D^2 + \sigma_D^2)\; [1 - F(y)] + \sigma_D\; (y + \mu_D)\; \frac{e_D(y)}{\sqrt{2\pi}} \quad (14)$$

Thus,

$$E(X^2) = \int_{-\infty}^{y} \beta^2 D^2 f(D) dD + y^2 \; [\; 1 - F(y) \;]$$
$$= \beta^2 \; E_{-\infty}^{y}(D^2) + y^2 \; [\; 1 - F(y) \;]$$
$$= \beta^2 \; [(\mu_D^2 + \sigma_D^2)\; F(y) - \sigma_D\; (y + \mu_D)\; \frac{e_D(y)}{\sqrt{2\pi}}] + y^2 \; [\; 1 - F(y) \;] \quad (15)$$

Hence, we obtain:

$$Var(X) = E(X^2) - [E(X)]^2$$
$$= \beta^2 [\; F(\frac{y}{\beta}) - (\frac{e_D(\frac{y}{\beta})}{\sqrt{2\pi}})^2 \;]\; \sigma_D^2 + F(\frac{y}{\beta})\; [1 - F(\frac{y}{\beta})][\; \beta^2 \mu_D^2 + y^2 \;]$$
$$- [1 - 2F(\frac{y}{\beta})]\; \frac{e_D(\frac{y}{\beta})}{\sqrt{2\pi}}\; (\beta^2 \mu_D - \beta y)\sigma_D - 2\beta\; F(\frac{y}{\beta})\; [1 - F(\frac{y}{\beta})]\; y\mu_D \quad (16)$$

Again, we also observe that,

$$E(D^2|D \leq y) = \frac{1}{F(y)}\; E_{-\infty}^{y}(D^2)$$
$$= \mu_D^2 + \sigma_D^2 - \sigma_D\; (y + \mu_D)\; \frac{e_D(y)}{F(y)\; \sqrt{2\pi}} \quad (17)$$

$$E(D^2|D \geq y) = \frac{1}{1 - F(y)}\; E_{y}^{+\infty}(D^2)$$
$$= \mu_D^2 + \sigma_D^2 + \sigma_D\; (y + \mu_D)\; \frac{e_D(y)}{[1 - F(y)]\sqrt{2\pi}} \quad (18)$$

$$E(D^2|y_1 < D < y_2) = \frac{1}{[F(y_2) - F(y_1)]}\ [\ E^{y_2}_{-\infty}(D^2) - E^{y_1}_{-\infty}(D^2)\ ] \tag{19}$$
$$= \mu_D^2 + \sigma_D^2 + \frac{1}{[F(y_2) - F(y_1)]}\ \frac{\sigma_D}{\sqrt{2\pi}}[(y_1 + \mu_D)e_D(y_1) - (y_2 + \mu_D)e_D(y_2)]$$

A.3 First and Second Order Moments of Other Useful Variables

Assume that D_i, $i = 1, 2$, are independent and identically distributed (i.i.d.) normal random variables with mean $\mu_D = y$, and standard deviation σ_D. Consider the associated random variables $Z_1 = (\frac{D_1 - y}{\sigma_D}|D_1 > y)$ and $Z_2 = (\frac{y - D_2}{\sigma_D}|D_2 < y)$. Since D_1 and D_2 are i.i.d. and symmetric around y, Z_1 and Z_2 are also i.i.d., with pdf

$$g(z) = \begin{cases} 2\ \phi(z), & \text{for } z \geq 0, \\ 0, & \text{otherwise.} \end{cases}$$

and CDF $G(z) = 2\Phi(z) - 1$, for $z \geq 0$.

Now, let $Z_{(1)} = \min\{Z_1, Z_2\}$ and $Z_{(2)} = \max\{Z_1, Z_2\}$. We can write the pdf, $h_{(1)}(.)$, and CDF, $H_{(1)}(.)$, of $Z_{(1)}$, as:

$$h_{(1)}(z) = 2[1 - G(z)]\ g(z), \qquad H_{(1)}(z) = 1 - [1 - G(z)]^2 \tag{20}$$

Similarly, the pdf, $h_{(2)}(.)$, and CDF, $H_{(2)}(.)$, of $Z_{(2)}$, are:

$$h_{(2)}(z) = 2G(z)\ g(z), \qquad H_{(2)}(z) = G(z)^2 \tag{21}$$

Thus, the expected value of $Z_{(1)}$ is:

$$\begin{aligned} E[Z_{(1)}] &= \int_{-\infty}^{+\infty} z h_{(1)}(z)\ dz \\ &= \int_{-\infty}^{+\infty} 2z\ [1 - G(z)]\ g(z)\ dz \\ &= 8\int_{0}^{+\infty} z[1 - \Phi(z)]\ \phi(z)\ dz \\ &= 8\ [\ \int_{0}^{+\infty} z\phi(z)dz - \int_{0}^{+\infty} z\Phi(z)\phi(z)dz\ ] \end{aligned} \tag{22}$$

Using again (8), the partial moment in (22) can be written as,

$$\int_{0}^{+\infty} z\phi(z)dz = \frac{1}{\sqrt{2\pi}} \tag{23}$$

In order to evaluate $\int_0^{+\infty} z\Phi(z)\phi(z)dz$ in (22), observe that $\frac{\delta\phi(z)}{\delta z} = \phi'(z) = -z\phi(z)$ for the standard normal distribution. Substituting this and using integration by parts leads to:

$$\begin{aligned}\int_0^{+\infty} z\Phi(z)\phi(z)dz &= -\int_0^{+\infty} \Phi(z)\phi'(z)dz \\ &= -[\ (\Phi(z)\phi(z))\ |_0^{+\infty} - \int_0^{+\infty} \phi^2(z)dz], \\ &= \frac{1}{2\sqrt{2\pi}} + \frac{1}{4\sqrt{\pi}} \end{aligned} \tag{24}$$

Substituting (47) and (53) in (22), we get:

$$\begin{aligned} E[Z_{(1)}] &= 8[\ \frac{1}{\sqrt{2\pi}} - \frac{1}{2\sqrt{2\pi}} - \frac{1}{4\sqrt{\pi}}\ ] \\ &= \frac{4}{\sqrt{2\pi}} - \frac{2}{\sqrt{\pi}} \end{aligned} \tag{25}$$

In a similar fashion, we derive:

$$E[Z_{(1)}^2] = 1 - \frac{2}{\pi} \tag{26}$$

and

$$E[Z_{(2)}] = \frac{2}{\sqrt{\pi}} \tag{27}$$

$$E[Z_{(2)}^2] = 1 + \frac{2}{\pi} \tag{28}$$

B Analysis of Production Levels: Expectation and Variance

In this section, we determine the expected value and variance of the production level of each product at each plant in the two-plant two-product model introduced in Section 4, under various capacity allocation policies in the flexible and non-flexible systems.

We assume that D_i, demand for product i, $i = 1, 2$, are independent random variables, each normally distributed with mean μ_{D_i}, standard deviation σ_{D_i}, pdf $f_i(.)$ and CDF $F_i(.)$. When we consider the case with identical demands, we let μ_D and σ_D denote the common mean and standard deviation of the demands, that is, $\mu_{D_i} = \mu_D$ and $\sigma_{D_i} = \sigma_D$, for $i = 1, 2$. We let C denote the capacity of each plant. Finally, we let X_i^{NF} be the production level of product i at plant i, $i = 1, 2$, in the non-flexible system, and X_{ij}^A be the production level of product i at plant j, $i, j = 1, 2$, in the flexible system under capacity allocation policy A, $A = SymP,\ SymD,\ ProfitP\ or\ ProfitD$.

B.1 Non-Flexible System

When plants are dedicated to producing a single product, we have $X_i^{NF} = \min\{D_i, C\}$, for $i = 1, 2$. Substituting $\beta = 1$ and $y = C$ in Equations (9) and (16), we obtain

$$E(X_i^{NF}) = C + F_i(C)[\mu_{D_i} - C] - \frac{e_{D_i}(C)}{\sqrt{2\pi}}\ \sigma_{D_i} \tag{29}$$

$$\begin{aligned} Var(X_i^{NF}) &= [\ F_i(C) - (\frac{e_{D_i}(C)}{\sqrt{2\pi}}\ )^2\ ]\ \sigma_{D_i}^2 + F_i(C)[1 - F_i(C)]\ (\mu_{D_i}^2 + C^2) \\ &\quad - [1 - 2F_i(C)]\ \frac{e_{D_i}(C)}{\sqrt{2\pi}}\ [\mu_{D_i} - C]\sigma_{D_i} - 2F_i(C)[1 - F_i(C)]\ C\mu_{D_i} \end{aligned} \tag{30}$$

When $\mu_{D_i} = C$, Equations (29) and (30) reduce to:

$$E(X_i^{NF}) = C - \frac{1}{\sqrt{2\pi}}\ \sigma_{D_i} \tag{31}$$

$$Var(X_i^{NF}) = [\ \frac{1}{2} - \frac{1}{2\pi}]\ \sigma_{D_i}^2 \tag{32}$$

B.2 Flexible System

The production level of each product at each plant will depend on the capacity allocation policy used. In what follows, we find the expected value and variability of production of each product at each plant under each of the allocation policies introduced in Section 4.

B.2.1 Symmetric Prioritized Allocation Policy

By definition, we have

$$E(X_{ii}^{SymP}) = E(X_i^{NF}), \quad \text{and} \qquad Var(X_{ii}^{SymP}) = Var(X_i^{NF}), \quad \text{for} \quad i = 1, 2$$

Observe that,

$$X_{ij}^{SymP} = \min\ \{\ (D_i - C)^+,\ (C - D_j)^+\ \} = \begin{cases} C - D_j, & \text{if } D_j < C \text{ and } D_i - C > C - D_j \\ D_i - C, & \text{if } D_i > C \text{ and } D_i - C < C - D_j \\ 0, & \text{otherwise.} \end{cases}$$

Thus, we can write,

$$\begin{aligned} E(X_{ij}^{SymP}) &= \int_C^\infty \int_{2C-D_i}^C (C - D_j) f_j(D_j) f_i(D_i)\ dD_j\ dD_i \\ &\quad + \int_0^C \int_C^{2C-D_j} (D_i - C) f_i(D_i) f_j(D_j)\ dD_i\ dD_j \\ E[(X_{ij}^{SymP})^2] &= \int_C^\infty \int_{2C-D_i}^C (C - D_j)^2 f_j(D_j) f_i(D_i)\ dD_j\ dD_i \end{aligned}$$

$$+\int_0^C \int_C^{2C-D_j} (D_i - C)^2 f_i(D_i) f_j(D_j)\ dD_i\ dD_j \tag{33}$$

The integrals in (33) are solved numerically to calculate the expected value and variance of production levels and, in turn, the expected sales and component inventory levels reported in the tables of Appendix F and Appendix G. However, when demands are identically distributed with mean demand equal to plant capacity, we find the exact solution analytically. For that purpose, let $\mu_{D_i} = \mu_D = C$, $\sigma_{D_i} = \sigma_D$, and $F_i(.) = F(.)$, for $i = 1, 2$. The moments of X_{ij}^{SymP} can be written as:

$$\begin{aligned}
E(X_{ij}^{SymP}) &= E\ [\ \min\ \{(D_i - C),\ (C - D_j)\ |\ D_i > C \text{ and } D_j < C\ \}\ ]\ \ P(D_i > C \text{ and } D_j < C) \\
&= E\ [\ \min\ \{(\frac{D_i - C}{\sigma_D}\ |\ D_i > C),\ (\frac{C - D_j}{\sigma_D}\ |\ D_j < C)\ \}\ ]\ \sigma_D\ F(C)\ [1 - F(C)] \\
&= [\ \frac{4}{\sqrt{2\pi}} - \frac{2}{\sqrt{\pi}}\ ]\ \sigma_D\ F(C)\ [1 - F(C)], \qquad \text{by Equation (25)} \\
&= [\ \frac{1}{\sqrt{2\pi}} - \frac{1}{2\sqrt{\pi}}\ ]\sigma_D, \qquad \text{since } F(C) = 1 - F(C) = \tfrac{1}{2} \text{ when } \mu_D = C
\end{aligned} \tag{34}$$

Observe that the associated increment in sales, which we denote by ΔS^{SymP}, is

$$\begin{aligned}
\Delta S^{SymP} &= E(X_{12}^{SymP}) + E(X_{21}^{SymP}) \\
&= 2\ [\ \frac{1}{\sqrt{2\pi}} - \frac{1}{2\sqrt{\pi}}\ ]\sigma_D.
\end{aligned}$$

This proves the first part of Theorem 5.7.

Similarly, using Equation (26), we obtain

$$\begin{aligned}
E\ [(X_{ij}^{SymP})^2] &= E\ [\ (\min\ \{(D_i - C),\ (C - D_j)\ |\ D_i > C \text{ and } D_j < C\ \})^2\ ]\ \ P(D_i > C \text{ and } D_j < C) \\
&= [\ \frac{1}{4} - \frac{1}{2\pi}\ ]\ \sigma_D^2
\end{aligned}$$

This leads to the following:

$$\begin{aligned}
Var(X_{ij}^{SymP}) &= E\ [(X_{ij}^{SymP})^2] - [E(X_{ij}^{SymP})]^2 \\
&= [\frac{1}{4} - \frac{(5\sqrt{2} - 4)}{4\sqrt{2\pi}}]\ \sigma_D^2
\end{aligned} \tag{35}$$

B.2.2 Symmetric Distributed Allocation Policy

Letting $\beta = \frac{1}{2}$ and $y = C$ in Equations (9) and (16), we get:

$$\begin{aligned}
E(X_{ii}^{SymD}) &= C + F_i(2C)[\ \frac{1}{2}\mu_{D_i} - C\ ] - \frac{e_{D_i}(2C)}{2\sqrt{2\pi}}\ \sigma_{D_i} \\
Var(X_{ii}^{SymD}) &= \frac{1}{4}\ [\ F_i(2C) - (\frac{e_{D_i}(2C)}{\sqrt{2\pi}})^2\ ]\ \sigma_{D_i}^2 + F_i(2C)[1 - F_i(2C)][\ \frac{1}{4}\mu_{D_i}^2 + C^2\ ]
\end{aligned}$$

$$- [1 - 2F_i(2C)] \frac{e_{D_i}(2C)}{\sqrt{2\pi}} \ [\ \frac{1}{4}\mu_{D_i} - \frac{1}{2}C\ ]\ \sigma_{D_i} - F_i(2C)(1 - F_i(2C))\ C\mu_{D_i}$$

When $\mu_{D_i} = \mu_D = C$, $\sigma_{D_i} = \sigma_D$, for $i = 1, 2$, and for demand coefficients of variation such that $F_i(2C) \approx 1, e_{D_i}(2C) \approx 0$, the above expressions reduce to:

$$E(X_{ii}^{SymD}) = \frac{1}{2}\ C, \quad \text{for} \quad i = 1, 2 \tag{36}$$

$$Var(X_{ii}^{SymD}) = \frac{1}{4}\ \sigma_D^2, \quad \text{for} \quad i = 1, 2 \tag{37}$$

When $i \neq j$, for the general case we have:

$$X_{ij}^{SymD} = \begin{cases} C - \frac{1}{2}D_j, & \text{if } \frac{1}{2}D_j < C \text{ and } \frac{1}{2}D_i > C - \frac{1}{2}D_j \\ \frac{1}{2}D_i, & \text{if } \frac{1}{2}D_j < C \text{ and } \frac{1}{2}D_i < C - \frac{1}{2}D_j \\ 0, & \text{otherwise.} \end{cases}$$

Thus, we can write,

$$\begin{aligned} E(X_{ij}^{SymD}) &= \int_0^{2C} \int_{2C-D_j}^{\infty} (C - \frac{1}{2}D_j) f_j(D_j) f_i(D_i)\ dD_i\ dD_j \\ &+ \int_0^{2C} \int_0^{2C-D_j} \frac{1}{2}D_i f_j(D_j) f_i(D_i)\ dD_i\ dD_j \\ E[(X_{ij}^{SymD})^2] &= \int_0^{2C} \int_{2C-D_j}^{\infty} (C - \frac{1}{2}D_j)^2 f_j(D_j) f_i(D_i)\ dD_i\ dD_j \\ &+ \int_0^{2C} \int_0^{2C-D_j} (\frac{1}{2}D_i)^2 f_j(D_j) f_i(D_i)\ dD_i\ dD_j \end{aligned} \tag{38}$$

The integrals in (38) are solved numerically to calculate the expected value and variance of production levels and, in turn, the expected sales and component inventory levels, reported in the tables of Appendix F and G. However, when demands are identically distributed with mean demand equal to plant capacity, the exact solution can be determined analytically in a similar way as for the prioritized policy:

$$E(X_{ij}^{SymD}) = \frac{1}{2}\ C - \frac{1}{2\sqrt{\pi}}\ \sigma_D, \quad \text{for} \quad i, j = 1, 2,\ i \neq j \tag{39}$$

$$Var(X_{ij}^{SymD}) = \frac{0.681690114}{4}\ \sigma_D^2, \quad \text{for} \quad i, j = 1, 2,\ i \neq j \tag{40}$$

where the term 0.681690114 is the variance of the minimum of two iid standard normal distributions, and is tabulated in Sarhan and Greenberg (1962).

It is easy to see that the associated increase in sales is $\Delta S^{SymD} = \Delta S^{SymP}$, as stated in Corollary 5.7, since the total sales under both policies are given by $min\{D_1 + D_2, 2C\}$.

B.2.3 Profit-Based Allocation Policies

We will only consider the profit-based prioritized policy, since the expected value and variance of production levels for the profit-based distributed policy can be easily derived from those for the symmetric distributed policy.

We let $X_{ij}^{ProfitP}$ be the production quantity of product i at plant j under the profit-based allocation policy. In what follows, we assume, without loss of generality, that product 1 has a higher priority than product 2. Thus, the profit-based prioritized allocation policy is as follows:

$$X_{11}^{ProfitP} = \min\{D_1, C\}, \qquad X_{12}^{ProfitP} = \min\{(D_1 - C)^+, C\}$$
$$X_{21}^{ProfitP} = \min\{(C - D_1)^+, (D_2 - C)^+\}, \qquad X_{22}^{ProfitP} = \min\{D_2, C, (2C - D_1)^+\}$$

By definition, $X_{11}^{ProfitP} = X_{11}^{SymP}$, and $X_{21}^{ProfitP} = X_{21}^{SymP}$. Thus, it only remains to determine the expected value and variance of $X_{12}^{ProfitP}$ and $X_{22}^{ProfitP}$. In what follows, we let $p = 1 - F_1(C)$ and $q = 1 - F_1(2C)$.

Using Equations (12) and (19) in the following derivation, we obtain:

$$\begin{aligned}
E(X_{12}^{ProfitP}) &= Cq + E(D_1 - C|C < D_1 < 2C)(p-q) \\
&= C(2q-p) + E(D_1|C < D_1 < 2C)(p-q) \\
&= C(2q-p) + \mu_{D_1}(p-q) + \frac{\sigma_{D_1}}{\sqrt{2\pi}}(e_{D_1}(C) - e_{D_1}(2C)) \\
E[(X_{12}^{ProfitP})^2] &= C^2 q + E[(D_1 - C)^2|C < D_1 < 2C](p-q) \\
&= C^2 q + E(D_1^2|C < D_1 < 2C)(p-q) \\
&\quad + C^2(p-q) - 2CE(D_1|C < D_1 < 2C)(p-q) \\
&= C^2 p + \mu_{D_1}^2(p-q) + \sigma_{D_1}^2(p-q) - 2C\mu_{D_1}(p-q) \\
&\quad + \frac{\sigma_{D_1}}{\sqrt{2\pi}}[(\mu_{D_1} - C)e_{D_1}(C) - \mu_{D_1} e_{D_1}(2C)] \\
Var(X_{12}^{ProfitP}) &= E[(X_{12}^{ProfitP})^2] - [E(X_{12}^{ProfitP})]^2 \\
&= C^2(p - (2q-p)^2) + \mu_{D_1}^2[(p-q) - (p-q)^2] + \\
&\quad \sigma_{D_1}^2[\ (p-q) - \frac{1}{2\pi}(e_{D_1}(C) - e_{D_1}(2C))^2\ ] - 2C\mu_{D_1}(p-q)(1+2q-p) + \\
&\quad \frac{\sigma_{D_1}}{\sqrt{2\pi}}[\ (\mu_{D_1} - C - 2C(2q-p) - 2\mu_{D_1}(p-q))\ e_{D_1}(C) \\
&\quad -(\mu_{D_1} - 2C(2q-p) - 2\mu_{D_1}(p-q))\ e_{D_1}(2C)\ ]
\end{aligned}$$

Observe that this is a closed for expression for the general case. Furthermore, when $\mu_{D_1} = C$, we have:

$$E(X_{12}^{ProfitP}) = C\ [1 - F_1(2C)] + \frac{[1 - e_{D_1}(2C)]}{\sqrt{2\pi}}\ \sigma_{D_1} \tag{41}$$

$$E[(X_{12}^{ProfitP})^2] = C^2[1 - F_1(2C)] + [F_1(2C) - F_1(C)]\ \sigma_{D_1}^2 - \frac{e_{D_1}(2C)}{\sqrt{2\pi}}\ \sigma_{D_1} C \tag{42}$$

The parameters of the distribution of production quantities of product 2 at plant 2, however, are calculated using numerical integration in the case of general mean demands:

$$X_{22}^{ProfitP} = \begin{cases} D_2 & \text{if } D_1 < C \text{ and } D_2 < C \\ C & \text{if } D_1 < C \text{ and } D_2 > C \\ D_2 & \text{if } C < D_1 < 2C \text{ and } D_2 < 2C - D_1 \\ 2C - D_1 & \text{if } C < D_1 < 2C \text{ and } D_2 > 2C - D_1 \\ 0 & \text{if } D_1 > 2C \end{cases}$$

$$\begin{aligned} E[X_{22}^{ProfitP}] = & \int_0^C \int_0^C D_2 f_2(D_2) f_1(D_1) dD_2 dD_1 + CP[D_1 < C]P[D_2 > C] \\ & + \int_C^{2C} \int_0^{2C-D_1} D_2 f_2(D_2) f_1(D_1) dD_2 dD_1 \\ & + \int_C^{2C} \int_{2C-D_1}^{\infty} (2C - D_1) f_2(D_2) f_1(D_1) dD_2 dD_1 \end{aligned}$$

$$\begin{aligned} E[(X_{22}^{ProfitP})^2] = & \int_0^C \int_0^C D_2^2 f_2(D_2) f_1(D_1) dD_2 dD_1 + C^2 P[D_1 < C]P[D_2 > C] \\ & + \int_C^{2C} \int_0^{2C-D_1} D_2^2 f_2(D_2) f_1(D_1) dD_2 dD_1 \\ & + \int_C^{2C} \int_{2C-D_1}^{\infty} (2C - D_1)^2 f_2(D_2) f_1(D_1) dD_2 dD_1 \end{aligned}$$

When $\mu_{D_i} = \mu_D = C$ and $\sigma_{D_i} = \sigma_D$, $i = 1, 2$, we assume that $P(D_1 > 2C) = 0$, which is reasonable when the mean demand is close to plant capacity, and for demand coefficients of variation encountered in stage 2. Then, we obtain a closed form expression of the parameters of the distribution of production quantities as follows.

$$\begin{aligned} E(X_{22}^{ProfitP}) &= E[\min\{D_2, C, (2C - D_1)^+\}] \\ &= E[\min\{D_2, C\}|D_1 < C]P[D_1 < C] \\ &\quad + E[2C - D_1|2C > D_1 > C, D_2 > C] \; P[2C > D_1 > C]P[D_2 > C] \\ &\quad + E[\min\{D_2|D_2 < C, 2C - D_1|2C > D_1 > C\}] \; P[2C > D_1 > C]P[D_2 < C] \\ &= E[\min\{D_2, C\}] \; P[D_1 < C] + E[2C - D_1|D_1 > C]P[D_1 > C] \; P[D_2 > C] \\ &\quad + CP[D_1 > C]P[D_2 < C] \\ &\quad - \sigma_D E[\; \max\{\frac{C - D_2}{\sigma_D}|D_2 < C, \; \frac{D_1 - C}{\sigma_D}|D_1 > C\} \;] \; P[D_1 > C]P[D_2 < C] \\ &= (C - \frac{1}{\sqrt{2\pi}}\sigma_D) \; \frac{1}{2} + 2C \; \frac{1}{4} - (C + \frac{2}{\sqrt{2\pi}}\sigma_D) \; \frac{1}{4} + C\frac{1}{4} - \frac{2}{\sqrt{\pi}} \; \sigma_D \frac{1}{4} \\ &= C - \frac{(\sqrt{2} + 1)}{2\sqrt{\pi}} \; \sigma_D \end{aligned} \tag{43}$$

Similarly,

$$
\begin{aligned}
E[(X_{22}^{ProfitP})^2] &= E[(\min\{D_2, C\})^2]\ P[D_1 < C] \\
&\quad +[\ 4C^2 - 4CE(D_1|2C > D_1 > C) \\
&\quad +E(D_1^2|2C > D_1 > C)\ ]\ P[2C > D_1 > C]P[D_2 > C] \\
&\quad +E[\ (\ \min\{\ D_2|D_2 < C, 2C - D_1|2C > D_1 > C\ \}\ )^2\ ] \\
&\quad \times P[2C > D_1 > C]P[D_2 < C] \\
&= E[(\min\{D_2, C\})^2]\ P[D_1 < C] \\
&\quad +[\ 4C^2 - 4CE(D_1|D_1 > C) + E(D_1^2|D_1 > C)\ ]\ P[D_1 > C]P[D_2 > C] \\
&\quad +C^2\ P[D_1 > C]P[D_2 < C] \\
&\quad -2C\sigma_D\ E[\ \max\{\frac{C - D_2}{\sigma_D}|D_2 < C,\ \frac{D_1 - C}{\sigma_D}|D_1 > C\}\ ]\ P[D_1 > C]P[D_2 < C] \\
&\quad +\sigma_D^2\ E[\ (\max\{\frac{C - D_2}{\sigma_D}|D_2 < C,\ \frac{D_1 - C}{\sigma_D}|D_1 > C\})^2\ ]\ P[D_1 > C]P[D_2 < C] \\
&= \frac{1}{2}[\ (C^2 + \sigma_D^2)\frac{1}{2} - \sigma_D C\frac{2}{\sqrt{2\pi}} + \frac{C^2}{2}\ ] + C^2 - C(C + \frac{2}{\sqrt{2\pi}}\ \sigma_D) \\
&\quad +\frac{1}{4}(C^2 + \sigma_D^2 + \sigma_D C\frac{4}{\sqrt{2\pi}}) + \frac{C^2}{4} - \frac{C\sigma_D}{2}\frac{2}{\sqrt{\pi}} + \frac{\sigma_D^2}{4}(1 + \frac{2}{\pi}) \\
&= C^2 +\ (\frac{3}{4}\ + \frac{1}{2\pi})\ \sigma_D^2 - \frac{(\sqrt{2} + 1)}{\sqrt{\pi}}\ \sigma_D C \qquad (44)
\end{aligned}
$$

Again, it can be shown that $\Delta S^{ProfitP} = \Delta S^{SymD} = \Delta S^{SymP}$, as stated in Corollary 5.7.

C Analysis of System Production Variability Observed by the Suppliers

We consider the same two-plant two-product model, in which product demands, D_i, $i = 1, 2$, are independent and identically distributed normals with mean equal to plant capacity, i.e., $\mu_D = C$, and standard deviation σ_D. We refer to *system production variability of a product* as the total variability of production of that product in the manufacturing system. This is an important measure, since this is the variability that the suppliers of components associated with that product face. Clearly, the system variability in a non-flexible system, which we denote by $Var(System_i^{NF})$, is simply equal to the variability of production of the product in its associated plant; that is, $Var(System_i^{NF}) = Var(X_i^{NF})$. On the other hand, the system variability in a flexible system will depend on the allocation policy used. For a given allocation policy A, we denote system variability of product i, $i = 1, 2$, by

$$Var(System_i^A) \equiv Var(\sum_{j=1}^{2} X_{ij}^A) = Var(X_{ii}^A) + Var(X_{ij}^A) + 2Cov(X_{ii}^A, X_{ij}^A).$$

Throughout this section, we use the following approximations: $P(D_i < 2C) \approx 1$, $P(D_i > 0) \approx 1$, and $e_{D_i}(2C) = exp(\frac{-(2C-\mu_D)^2}{2\sigma_D^2}) \approx 0$, for $i = 1, 2$. These approximations are justified for the coefficients of variation of demand observed by our industry partner, and given that mean demand is equal to plant capacity, C.

In what follows, we study the system variability resulting from the four allocation policies considered previously, namely, symmetric, prioritized and distributed, and profit-based, prioritized and distributed, policies. Observe that to quantify the system variability of these allocation policies, it only remains to determine the covariance term.

C.1 Symmetric Prioritized Allocation Policy

We first determine the system variability under the prioritized allocation policy in the flexible system. For this purpose, we need to derive the covariance term.

$$\begin{aligned} Cov(X_{ii}^{SymP}, X_{ij}^{SymP}) &= E[X_{ii}^{SymP} \times X_{ij}^{SymP}] - E[X_{ii}^{SymP}]\ E[X_{ij}^{SymP}], \quad \text{by definition} \\ &= E[\ C \min\{(D_i - C), (C - D_j) \mid D_i > C \text{ and } D_j < C\ \}\ ] \\ &\quad \times P(D_i > C \text{ and } D_j < C) - E[X_{ii}^{SymP}]\ E[X_{ij}^{SymP}] \\ &= C\ E[X_{ij}^{SymP}] - E[X_{ii}^{SymP}]\ E[X_{ij}^{SymP}], \quad \text{by definition} \\ &= E[X_{ij}^{SymP}]\ (\ C - E[X_{ii}^{SymP}]\ ) \\ &= \frac{1}{\sqrt{2\pi}}\ [\ \frac{1}{\sqrt{2\pi}} - \frac{1}{2\sqrt{\pi}}\ ]\ \sigma_D^2, \quad \text{by Equations (31) and (34)} \\ &= [\ \frac{1}{2\pi} - \frac{1}{2\sqrt{2\pi}}\ ]\ \sigma_D^2 \end{aligned} \tag{45}$$

Thus, using Equations (32), (35) and (45)

$$\begin{aligned} Var(System_i^{SymP}) &= Var(X_{ii}^{SymP}) + Var(X_{ij}^{SymP}) + 2Cov(X_{ii}^{SymP}, X_{ij}^{SymP}) \\ &= [\ \frac{1}{2} - \frac{1}{2\pi}]\ \sigma_D^2 + [\frac{1}{4} - \frac{(5\sqrt{2}-4)}{4\sqrt{2\pi}}]\ \sigma_D^2 + 2\ [\ \frac{1}{2\pi} - \frac{1}{2\sqrt{2\pi}}\ ]\ \sigma_D^2, \\ &= [\ \frac{3}{4} - \frac{3}{4\pi}\ ]\ \sigma_D^2 \end{aligned}$$

C.2 Symmetric Distributed Allocation Policy

Next we analyze the system variability under the distributed policy in the flexible system. Again, we derive the covariance term

$$Cov(X_{ii}^{SymD}, X_{ij}^{SymD}) = E[X_{ii}^{SymD} \times X_{ij}^{SymD}] - E[X_{ii}^{SymD}]\ E[X_{ij}^{SymD}].$$

In what follows, we repeatedly use the approximations of $P(0 < D_i < 2C) = P(0 < D_j < 2C) \approx 1$, which are reasonable for the cases we are interested in. Hence, we can write

$$X_{ii}^{SymD} \times X_{ij}^{SymD} = \begin{cases} \frac{D_i}{2}\,(C - \frac{1}{2}D_j), & \text{if } \frac{1}{2}D_i > C - \frac{1}{2}D_j \\ \frac{1}{4}D_i^2, & \text{if } \frac{1}{2}D_i < C - \frac{1}{2}D_j \\ 0, & \text{otherwise.} \end{cases}$$

Thus,

$$\begin{aligned} E(X_{ii}^{SymD} \times X_{ij}^{SymD}) = &\frac{1}{2}\int_0^{2C}\int_{2C-D_j}^{\infty} D_i(C - \frac{1}{2}D_j) f(D_j) f(D_i)\, dD_i\, dD_j \\ &+ \frac{1}{4}\int_0^{2C}\int_0^{2C-D_j} D_i^2 f(D_j) f(D_i)\, dD_i\, dD_j \end{aligned} \tag{46}$$

We first solve for the term $\frac{1}{4}\int_0^{2C}\int_0^{2C-D_j} D_i^2 f(D_j) f(D_i)\, dD_i\, dD_j$ in Equation (46), which can be written as:

$$\begin{aligned} \frac{1}{4}\int_0^{2C} f(D_j)\, E_{-\infty}^{(2C-D_j)}(D_i^2) = &\frac{1}{4}(C^2 + \sigma_D^2)\int_0^{2C} F(2C - D_j)\, f(D_j)\, dD_j \\ &- \frac{3C\sigma_D}{4\sqrt{2\pi}}\int_0^{2C} e_{D_i}(2C - D_j)\, f(D_j)\, dD_j \\ &+ \frac{\sigma_D}{4\sqrt{2\pi}}\int_0^{2C} D_j\, e_{D_i}(2C - D_j)\, f(D_j)\, dD_j, \quad \text{by Equation (13)} \end{aligned} \tag{47}$$

Next we observe that if we let $\tilde{f}$ denote the pdf of a normal random variable $\tilde{D}$ with mean C and standard deviation $\sigma_D/\sqrt{2}$, then we can write:

$$e_{D_i}(2C - D_j)\, f(D_j) = \frac{1}{\sqrt{2}}\,\tilde{f}(D_j).$$

Thus,

$$\int_0^{2C} e_{D_i}(2C - D_j)\, f(D_j)\, dD_j = \frac{1}{\sqrt{2}}\int_0^{2C} \tilde{f}(D_j)\, dD_j \approx \frac{1}{\sqrt{2}} \tag{48}$$

In addition, since $\mu_D = C$, the symmetry of the normal distribution implies that,

$$f(D_j) = f(2C - D_j), \quad \text{and} \quad 1 - F(D_j) = F(2C - D_j) \tag{49}$$

Thus,

$$\int_0^{2C} F(2C - D_j)\, f(D_j)\, dD_j = -\,\frac{1}{2}\,F^2(2C - D_j)\,\Big|_0^{2C} = \frac{1}{2} \tag{50}$$

$$\int_0^{2C} D_j\, e_{D_i}(2C - D_j)\, f(D_j)\, dD_j = \frac{1}{\sqrt{2}}\int_0^{2C} D_j\, \tilde{f}(D_j)\, dD_j$$

$$= \frac{1}{\sqrt{2}}\, E_0^{2C}(\tilde{D}_j). \tag{51}$$

Here the partial expectation, $E_0^{2C}(\tilde{D}_j)$, can be approximated by the full expectation, C, since $P(0 < \tilde{D}_j < 2C) = 1$.

Substituting Equations (7), (48), (50) and (51) in Equation (47), we obtain:

$$\frac{1}{4}\int_0^{2C}\int_0^{2C-D_j} D_i^2 f(D_j) f(D_i)\ dD_i\ dD_j = \frac{C^2}{8} + \frac{\sigma_D^2}{8} - \frac{C\sigma_D}{4\sqrt{\pi}} \tag{52}$$

Next, we solve for the term $\frac{1}{2}\int_0^{2C}\int_{2C-D_j}^{\infty} D_i\ (C - \frac{1}{2}D_j)\ f(D_j)\ f(D_i)\ dD_i\ dD_j$ in Equation (46), which can be written as:

$$\frac{C}{2}\int_0^{2C} E_{2C-D_j}^{\infty}(D_i)\ f(D_j)\ dD_j - \frac{1}{4}\int_0^{2C} D_j\ E_{2C-D_j}^{\infty}(D_i)\ f(D_j)\ dD_j \tag{53}$$

Rewriting $E_{2C-D_j}^{\infty}(D_i)$, the partial expectation of D_i, as in Equation (8), and solving for the resulting terms in a way similar to that of Equation (47), we obtain:

$$E(X_{ii}^{SymD} \times X_{ij}^{SymD}) = \frac{C^2}{8} + \frac{\sigma_D^2}{8} - \frac{C\sigma_D}{8\sqrt{\pi}} + \frac{C}{4}\int_0^{2C} D_j\ F(2C - D_j)\ f(2C - D_j)\ dD_j \tag{54}$$

Finally, we solve for the term $\int_0^{2C} D_j\ F(2C - D_j)\ f(2C - D_j)\ dD_j$ in the above expression. This is done as follows.

$$\begin{aligned}\int_0^{2C} D_j\ F(2C - D_j)\ f(2C - D_j)\ dD_j &= \int_0^{2C} D_j\ [1 - F(D_j)]\ f(D_j)\ dD_j, \quad \text{by Equation (49)}\\ &= \int_0^{2C} D_j\ f(D_j)\ dD_j - \int_0^{2C} D_j\ F(D_j)\ f(D_j)\ dD_j \end{aligned} \tag{55}$$

Again, the first term $\int_0^{2C} D_j\ f(D_j)\ dD_j$ is the partial expectation, $E_0^{2C}(D_j)$, which, in this case, can be approximated by the full expectation, $E_0^{2C}(D_j) \approx C$, since $P(0 < D_j < 2C) = 1$. For the second term, we observe that $\frac{\delta f(D_j)}{\delta D_j} = f'(D_j) = (\frac{C - D_j}{\sigma_D^2})\ f(D_j)$. Thus, we can write:

$$\begin{aligned}-\int_0^{2C} D_j\ F(D_j)\ f(D_j)\ dD_j &= \sigma_D^2\int_0^{2C}(\frac{-D_j + C}{\sigma_D^2})\ F(D_j)\ f(D_j)\ dD_j - C\int_0^{2C} F(D_j)\ f(D_j)\ dD_j\\ &= \sigma_D^2\int_0^{2C} F(D_j)\ f'(D_j)\ dD_j - C\ \frac{1}{2}, \quad \text{by Equation (50)}\\ &= -\frac{\sigma_D}{2\sqrt{\pi}} - \frac{C}{2}, \quad \text{using integration by parts.}\end{aligned} \tag{56}$$

Substituting Equation (56) in (55), we obtain:

$$\int_0^{2C} D_j\ F(2C - D_j)\ f(2C - D_j)\ dD_j = \frac{C}{2} - \frac{\sigma_D}{2\sqrt{\pi}} \tag{57}$$

Finally, substituting (57) in (54), and using (36) and (39), we obtain the covariance term as:

$$\begin{aligned} Cov(X_{ii}^{SymD}, X_{ij}^{SymD}) &= E[X_{ii}^{SymD} \times X_{ij}^{SymD}] - E[X_{ii}^{SymD}]\ E[X_{ij}^{SymD}] \\ &= \frac{\sigma_D^2}{8} \end{aligned} \tag{58}$$

Thus, substituting (37), (40), and (58) in the following, we obtain:

$$\begin{aligned} Var(System_i^{SymD}) &= Var(X_{ii}^{SymD}) + Var(X_{ij}^{SymD}) + 2Cov(X_{ii}^{SymD}, X_{ij}^{SymD}) \\ &= \left(\frac{0.6817}{4} + \frac{1}{2} \right) \sigma_D^2 \end{aligned}$$

C.3 Profit-Based Allocation Policies

We now derive the covariance term under profit-based prioritized and distributed policies in the flexible system. Observe that the total system-wide production of each product is identical for both policies. We can thus restrict our analysis to the profit-based prioritized policies. Since these policies are not symmetric, the associated system variability will be product dependent. As mentioned previously, we consider, without loss of generality, that product 1 has a higher priority than product 2, and analyze the resulting system variability of both products. Recall that the profit-based allocation policy works as follows:

$$\begin{aligned} X_{11}^{ProfitP} &= \min\{D_1, C\}, & X_{12}^{ProfitP} &= \min\{(D_1 - C)^+, C\} \\ X_{21}^{ProfitP} &= \min\{(C - D_1)^+, (D_2 - C)^+\}, & X_{22}^{ProfitP} &= \min\{D_2, C, (2C - D_1)^+\} \end{aligned}$$

We start with product 1. Observe that

$$\begin{aligned} Var(System_1^{ProfitP}) &= Var(X_{11}^{ProfitP} + X_{12}^{ProfitP}) \\ &= Var(\min\{D_1, C\} + \min\{(D_1 - C)^+, C\}) \\ &= Var(\min\{D_1, C\} + (D_1 - C)^+), \quad \text{since } P(D_1 > 2C) \approx 0 \\ &= Var(D_1) \\ &= \sigma_D^2 \end{aligned}$$

We now derive the system variability for product 2. For that purpose, we again need to determine the covariance term,

$$Cov(X_{21}^{ProfitP}, X_{22}^{ProfitP}) = E[X_{21}^{ProfitP} \times X_{22}^{ProfitP}] - E[X_{21}^{ProfitP}]\ E[X_{22}^{ProfitP}].$$

We first derive $E[X_{21}^{ProfitP} \times X_{22}^{ProfitP}]$, where

$$X_{21}^{ProfitP} \times X_{22}^{ProfitP} = \begin{cases} C \times \min\{(C - D_1),\ (D_2 - C)\}, & \text{if } D_1 < C,\ D_2 > C \\ 0, & \text{otherwise.} \end{cases}$$

Thus,

$$\begin{aligned} Cov[X_{21}^{ProfitP}, X_{22}^{ProfitP}] &= CE[\min\{(C - D_1), (D_2 - C) \mid D_1 < C \text{ and } D_2 > C\ \}\ ] \times \\ &\quad P(D_1 < C \text{ and } D_2 > C) - E[X_{21}^{ProfitP}]\ E[X_{22}^{ProfitP}] \\ &= C\ E[X_{21}^{ProfitP}] - E[X_{21}^{ProfitP}]\ E[X_{22}^{ProfitP}], \quad \text{by definition} \\ &= E[X_{21}^{ProfitP}]\ (\ C - E[X_{22}^{ProfitP}]\ ) \\ &= [\frac{1}{\sqrt{2\pi}} - \frac{1}{2\sqrt{\pi}}\ ]\ [\frac{(\sqrt{2}+1)}{2\sqrt{\pi}}]\ \sigma_D^2, \quad \text{by Equations (34) and (43)} \\ &= \frac{1}{4\pi}\ \sigma_D^2 \end{aligned} \tag{59}$$

Thus, using Theorem 5.5 along with (59) in the following, we obtain:

$$\begin{aligned} Var(System_2^{ProfitP}) &= Var(X_{21}^{ProfitP}) + Var(X_{22}^{ProfitP}) + \\ &\quad + 2Cov(X_{21}^{ProfitP}, X_{22}^{ProfitP}) = [1 - \frac{1}{\pi}]\ \sigma_D^2 \end{aligned}$$

D Simulation Results: Impact of Inventory Constraints

In this section of the Appendix, we present the simulation results discussed in Section 5.1.4 for demand coefficients of variation of both 0.15 and 0.25. This allows the reader to assess the impact of higher period-to-period demand variability.

For each performance measure, theoretical results are compared with simulation results, which consider inventory constraints. In addition, to assess the accuracy of the simulation values, we have included a column under the sales section, "Simulation-NoInv", that reports the average sales under no inventory constraints and should thus coincide with the theoretical values. Indeed we observe that the results are very close. The slightly lower values are due to the average of the simulated demands turning out to be slightly less than 100.

$C = 100,\ \frac{\sigma_{D_i}}{\mu_{D_i}} = 0.15,\ L = 2$ weeks, $z = 1.64$									
	Ave Sales			Ave Inventory		Supplier Variability(SD)			
	theor.	simulation		theor.	simul.	Product 1		Product 2	
		no-inv	with-inv			theo	simu	theo	simu
Dedicated	188.03		187.94	117.95	118.22	8.76	8.87	8.76	8.85
SymP	191.54	191.41	190.84	139.04	140.09	10.73	10.39	10.73	10.36
NewSymP	191.54	191.41	191.26	154.44	154.86	10.73	10.72	10.73	10.69
SymD	191.54	191.41	191.10	159.28	159.94	12.29	11.91	12.28	11.88
ProfitP	191.54	191.41	191.06	158.14	158.86	15.00	14.33	12.38	11.38
ProfitD	191.54	191.41	191.09	159.28	159.94	15.00	14.53	12.38	11.69

Table 8: Simulation versus theoretical results for sales, inventory and supplier variability for a demand coefficient of variation of 15%

$C = 100,\ \frac{\sigma_{D_i}}{\mu_{D_i}} = 0.15,\ L = 2$ weeks, $z = 1.64$					
	%stockout				Stock-out Value
	I_{11}	I_{22}	I_{21}	I_{12}	
Dedicated	N\A	N\A	N\A	N\A	N\A
SymP	0.00	0.00	0.08	0.08	0.57
NewSymP	0.00	0.00	0.02	0.02	0.15
SymD	0.04	0.04	0.04	0.05	0.32
ProfitP	0.00	0.00	0.08	0.06	0.35
ProfitD	0.04	0.04	0.04	0.04	0.32

Table 9: Stockout percentage and stockout values under the different allocation policies for a demand coefficient of variation of 15%

$C = 100,\ \frac{\sigma_{D_i}}{\mu_{D_i}} = 0.25,\ \ L = 2$ weeks, $\ z = 1.64$									
	Ave Sales			Ave Inventory		Supplier Variability(SD)			
	theor.	simulation		theor.	simul.	Product 1		Product 2	
		no-inv	with-inv			theo	simu	theo	simu
Dedicated	180.05		179.79	129.92	130.31	14.60	14.79	14.60	14.76
SymP	185.90	185.68	184.73	165.06	166.81	17.88	17.32	17.88	17.27
NewSymP	185.90	185.68	185.43	190.74	191.44	17.88	17.87	17.88	17.82
SymD	185.90	185.68	185.16	198.80	199.90	20.47	19.85	20.47	19.80
ProfitP	185.90	185.68	185.10	196.91	198.10	25.00	23.88	20.64	18.96
ProfitD	185.90	185.68	185.16	198.80	199.90	25.00	24.21	20.64	19.48

Table 10: Simulation versus theoretical results for sales, inventory and supplier variability for a demand coefficient of variation of 25%

$C = 100,\ \frac{\sigma_{D_i}}{\mu_{D_i}} = 0.25,\ \ L = 2$ weeks, $\ z = 1.64$					
	%stockout				Stock-out Value
	I_{11}	I_{22}	I_{21}	I_{12}	
Dedicated	0	N\A	0	N\A	N\A
SymP	0.00	0.00	0.08	0.08	0.96
NewSymP	0.00	0.00	0.02	0.02	0.26
SymD	0.04	0.04	0.04	0.05	0.53
ProfitP	0.00	0.00	0.08	0.06	0.59
ProfitD	0.04	0.04	0.04	0.04	0.53

Table 11: Stockout percentage and stockout values under the different allocation policies for a demand coefficient of variation of 25%

Finally, observe in Table 9 that the frequency of stock-outs is between 4 and 5% for the distributed-type policies (in accordance to the selected safety factor $z = 1.64$ or 95% service level, since the production levels are close to normally distributed) but quite higher than that in the prioritized-type policies. The modified order-up-to calculation using the mean and variance of production levels leads to frequent stock-outs in the prioritized allocation policy due to the skewedness of the distribution of production of the secondary products. In fact, simulation results show that the distribution of the positive values is very well approximated by the right half of a normal distribution centered at 0. Based on this, we introduce a new policy, NewPrioritized (NewSymP), with allocation identical to that of the prioritized but more adequate inventory levels. We calculate the inventory value assuming a normal distribution with mean 0 and a variance of:

$$\sigma^2 = E((X_{ij}^{SymP})^2 | X_{ij}^{SymP} > 0) = E((X_{ij}^{SymP})^2 | D_i > C \text{ and } D_j < C)) = [\ 1 - \frac{2}{\pi}\ ]\sigma_D^2 \approx 0.36\sigma_D^2.$$

This overestimates the safety stock, since production of the secondary product is zero 75% of the time and thus the normal curve only comes into play one forth of the time. The new policy leads to the highest average sales value among all the policies in our simulation, with the lowest inventory levels (except, of course, those of the original prioritized policy). Moreover, stock-outs occur in only 2% of the periods under the new inventory policy as compared with around 8% for the original prioritized and over 4% for most of the other policies. Sales are increased by over 0.1% and 0.14% for coefficients of variation of 0.15 and 0.25, respectively, relative to any other policy. The new inventory rule could be applied to the profit-based prioritized policy as well to lower its stock-out probability.

E Analysis of Outbound Distribution

E.1 Derivation of $E[Z_{k(1)}]$ and $E[Z_{k(2)}]$ in Section 5.3

As defined in Section 5.3, $Z_{kj} = |X_k - X_{oj}| + |Y_k - Y_{oj}|$ is the rectilinear distance between customer k and plant j. Without loss of generality, we assume that all locations are distributed in a square of unit length. We let $X = |X_k - X_{oj}|$ and $Y = |Y_k - Y_{oj}|$. X and Y are iid, with the following pdf (see Larson and Odoni (1981)):

$$f(x) = \begin{cases} 2(1-x), & \text{for } 0 \le x \le 1, \\ 0, & \text{otherwise.} \end{cases}$$

We let g(.) and G(.) denote the pdf and CDF of Z_{kj}, respectively. We can write

$$g(z) = \begin{cases} \int_0^z f(z-a)f(a)da = \int_0^z 2(1-z+a)\ 2(1-a)\ da, & \text{for } 0 \le z \le 1 \\ \int_{z-1}^1 f(z-a)f(a)da = \int_{z-1}^1 2(1-z+a)\ 2(1-a)\ da, & \text{for } 1 \le z \le 2. \end{cases}$$

Solving the above integrals, we obtain:

$$g(z) = \begin{cases} \frac{2}{3}z^3 - 4z^2 + 4z, & \text{for } 0 \le z \le 1 \\ -\frac{2}{3}z^3 + 4z^2 - 8z + \frac{16}{3}, & \text{for } 1 \le z \le 2. \end{cases} \tag{60}$$

Given the pdf, we obtain the CDF, $G(.)$, as:

$$G(z) = \begin{cases} \frac{1}{6}z^4 - \frac{4}{3}z^3 + 2z^2, & \text{for } 0 \le z \le 1 \\ \frac{5}{6} - \frac{1}{6}(z^4-1) + \frac{4}{3}(z^3-1) - 4(z^2-1) + \frac{16}{3}(z-1), & \text{for } 1 \le z \le 2. \end{cases} \tag{61}$$

As defined in the body of the paper, $Z_{k(1)} = \min_j\{Z_{kj}\}$ and $Z_{k(2)} = \max_j\{Z_{kj}\}$. Using Equations (20) and (21), we can write

$$E[Z_{k(1)}] = 2\int_0^{+\infty} z\ [1 - G(z)]\ g(z)\ dz, \qquad E[Z_{k(2)}] = 2\int_0^{+\infty} z\ G(z)\ g(z)\ dz$$

Substituting the pdf and CDF given in (60) and (61) in the above integrals and solving them leads to the desired results.

E.2 Derivation of $E(X_{j,\text{“}in\text{”}}^{SymD})$ and the Upper Bound on $E(X_{j,\text{“}out\text{”}}^{SymD})$:

As defined in Section 5.3, $X_{j,\text{“}in\text{”}}^{SymD} = \min\{N_j,\ C\}$. Thus, we obtain $E(X_{j,\text{“}in\text{”}}^{SymD})$ by using N_j and $\beta = 1$ in Equation (9).

By definition, $X_{j,\text{“}out\text{”}}^{SymD} = \min\{\ (C - N_j)^+,\ (N_{j^c} - C)^+\ \}$, where N_j and N_{j^c} are approximately normally distributed with means μ_{N_j} and $\mu_{N_{j^c}}$, standard deviations σ_{N_j} and $\sigma_{N_{j^c}}$, and CDFs $F_{N_j}(.)$

and $F_{N_{j^c}}(.)$, respectively, $j, j^c \in 1, 2$ and $j \neq j^c$. We can write:

$$
\begin{aligned}
E\,[X^{SymD}_{j,\text{``}out\text{''}}] &= E[\min\,\{(C - N_j)^+,\ (N_{j^c} - C)^+\}\,] \\
&\leq \min\{\ E[(C - N_j)^+],\ E[(N_{j^c} - C)^+]\ \} \\
&= \min\{\ E[C - N_j | N_j < C]\ P(N_j < C),\ E[N_{j^c} - C | N_{j^c} > C]\ P(N_{j^c} > C)\ \} \\
&= \min\{\ (C - E[N_j | N_j < C])\ P(N_j < C),\ (E[N_{j^c} | N_{j^c} > C] - C)\ P(N_{j^c} > C)\ \} \\
&= \min\{\ [\ C - \mu_{N_j} + \frac{e_{N_j}(C)}{F_{N_j}(C)\ \sqrt{2\pi}}\ \sigma_{N_j}\ ]\ F_{N_j}(C), \\
&\quad [\ -C + \mu_{N_{j^c}} + \frac{e_{N_{j^c}}(C)}{(1 - F_{N_{j^c}}(C))\ \sqrt{2\pi}}\ \sigma_{N_{j^c}}\ ]\ [1 - F_{N_{j^c}}(C)]\ \} \quad \text{by Equations (10) and (11)}
\end{aligned}
$$

This completes the derivation.

E.3 Lower Bound on Outbound Transportation Costs

Next, we analyze the impact of "demand unbalance" (i.e., demand distributions with different means) on the outbound distribution in the flexible environment. Table 14 reports *lower bounds* on the percent reduction in the expected unit shipping cost in the flexible environment for the general case with different mean product demands. Again, this is done for a demand coefficient of variation of 0.15 for both products, when each plant has a capacity of 100 units. Observe that the insights obtained for the case of identically distributed product demands continue to hold for the general case.

$C = 100$, $\frac{\sigma_{D_i}}{\mu_{D_i}} = 0.15$, $i = 1, 2$	product 1: Mean Demand (μ_{D_1})							
Reduction in Unit Shipping (%)		60	75	90	100	110	125	140
product 2: Mean Demand (μ_{D_2})	60	36.67	36.67	36.65	36.55	36.23	34.98	32.52
	75	36.67	36.65	36.45	35.99	35.08	32.65	34.70
	90	36.65	36.45	35.62	34.46	32.73	34.78	35.81
	100	36.55	35.99	34.46	32.74	34.27	35.58	36.20
	110	36.23	35.08	32.73	34.27	35.25	36.06	36.41
	125	34.98	32.65	34.78	35.58	36.06	36.42	36.57
	140	32.52	34.70	35.81	36.20	36.41	36.57	36.63

Table 12: Lower bounds on the percent reduction in the expected unit shipping cost in the flexible system, for different mean product demands.

F Selected Numerical Results

In what follows, c.v. denotes the coefficient of variation of demand.

F.1 Sales

$C = 100$, $\frac{\sigma_{D_i}}{\mu_{D_i}} = 0.15$, $i = 1, 2$ Increase in Sales (%)		product 1: Mean Demand (μ_{D_1})						
		60	**75**	**90**	**100**	**110**	**125**	**140**
product 2: Mean Demand (μ_{D_2})	**60**	0.0	**0.0**	1.1	**3.6**	7.2	**13.0**	17.7
	75	0.0	0.0	**0.6**	2.2	**4.4**	8.0	**10.6**
	90	1.1	0.6	0.0	**0.4**	1.4	**3.1**	4.5
	100	3.6	2.2	0.4	0.0	**0.3**	1.2	**2.0**
	110	7.2	4.4	1.4	0.3	0.0	**0.3**	0.7
	125	13.0	8.0	3.1	1.2	0.3	0.0	**0.1**
	140	17.7	10.6	4.5	2.0	0.7	0.1	0.0

Table 13: Percentage increase in sales of fixed-capacity versus dedicated (c.v. 15%)

$C = 100$, $\frac{\sigma_{D_i}}{\mu_{D_i}} = 0.25$, $i = 1, 2$ Increase in Sales (%)		product 1: Mean Demand (μ_{D_1})						
		60	**75**	**90**	**100**	**110**	**125**	**140**
product 2: Mean Demand (μ_{D_2})	**60**	0.0	**0.3**	2.3	**4.4**	7.0	**10.8**	14.1
	75	0.3	0.0	**0.7**	1.9	**3.3**	5.6	**7.7**
	90	2.3	0.7	0.0	**0.3**	0.9	**2.1**	3.3
	100	4.4	1.9	0.3	0.0	**0.2**	0.9	**1.7**
	110	7.0	3.3	0.9	0.2	0.0	**0.2**	0.7
	125	10.8	5.6	2.1	0.9	0.2	0.0	**0.1**
	140	14.1	7.7	3.3	1.7	0.7	0.1	0.0

Table 14: Percentage increase in sales of fixed-capacity versus dedicated (c.v. 0.25)

$C = 100$, $\frac{\sigma_{D_i}}{\mu_{D_i}} = 0.15$, $i = 1, 2$	product 1: Mean Demand (μ_{D_1})							
Increase in Sales (%)		**60**	**75**	**90**	**100**	**110**	**125**	**140**
product 2: Mean Demand (μ_{D_2})	**60**	0.0	0.0	0.1	0.2	0.6	**1.4**	**1.7**
	75	0.0	0.1	0.4	0.9	**1.5**	**1.8**	**1.6**
	90	0.1	0.4	1.2	**1.7**	**1.9**	**1.7**	1.2
	100	0.2	0.9	**1.7**	**1.9**	**1.8**	1.4	0.9
	110	0.6	**1.5**	**1.9**	**1.8**	1.5	1.0	0.6
	125	**1.4**	**1.8**	**1.7**	1.4	1.0	0.6	0.4
	140	**1.7**	**1.6**	1.2	0.9	0.6	0.4	0.2

Table 15: Percent increase in sales when using flexibility to accommodate short-term variability (i.e. percent increase in sales when using fully flexible policy minus that resulting from using fixed-capacity allocations) for demand coefficients of variation of 0.15

$C = 100$, $\frac{\sigma_{D_i}}{\mu_{D_i}} = 0.15$, $i = 1, 2$	product 1: Mean Demand (μ_{D_1})							
Increase in Sales (%)		**60**	**75**	**90**	**100**	**110**	**125**	**140**
product 2: Mean Demand (μ_{D_2})	**60**	0.0	0.2	0.8	1.4	2.0	**2.7**	**3.0**
	75	0.2	0.8	1.8	2.4	**2.8**	**3.1**	**3.1**
	90	0.8	1.8	2.7	**3.0**	**3.2**	**3.2**	2.9
	100	1.4	2.4	**3.0**	**3.2**	**3.2**	**3.0**	2.6
	110	2.0	**2.8**	**3.2**	**3.2**	3.1	2.8	2.4
	125	**2.7**	**3.1**	**3.2**	3.0	2.8	2.4	2.0
	140	**3.0**	**3.1**	2.9	2.6	2.4	2.0	1.7

Table 16: Percent increase in sales when using flexibility to accommodate short-term variability (i.e. percent increase in sales when using fully flexible policy minus that resulting from using fixed-capacity allocations over the non-flexible policy) for demands coefficients of variation of 0.25

F.2 Inventory

$C = 100$, $\frac{\sigma_{D_i}}{\mu_{D_i}} = 0.15$, $L = 2$ weeks		product 1: Mean Demand (μ_{D_1})						
		60	**75**	**90**	**100**	**110**	**125**	**140**
product 2: Mean Demand (μ_{D_2})	**60**	0.00	7.43	5.86	5.92	4.31	2.13	0.93
	75	7.43	0.00	5.95	6.16	2.46	0.38	-0.29
	90	5.86	5.95	0.00	-2.16	-2.28	-2.36	-2.39
	100	5.92	6.16	-2.16	0.00	-2.54	-2.63	-2.66
	110	4.31	2.46	-2.28	-2.54	0.00	-2.80	-2.83
	125	2.13	0.38	-2.36	-2.63	-2.80	0.00	-2.94
	140	0.93	-0.29	-2.39	-2.66	-2.83	-2.94	0.00

Table 17: Percent increase in inventory per percentage point increase in sales: Fixed-Capacity over dedicated (c.v. 0.15)

$C = 100$, $\frac{\sigma_{D_i}}{\mu_{D_i}} = 0.25$, $L = 2$ weeks		product 1: Mean Demand (μ_{D_1})						
		60	**75**	**90**	**100**	**110**	**125**	**140**
product 2: Mean Demand (μ_{D_2})	**60**	0.00	3.82	5.66	5.06	3.37	1.55	0.72
	75	3.82	0.00	9.59	2.63	0.69	-0.39	-0.83
	90	5.66	9.59	0.00	-1.91	-2.02	-2.13	-2.19
	100	5.06	2.63	-1.91	0.00	-2.20	-2.32	-2.38
	110	3.37	0.69	-2.02	-2.20	0.00	-2.46	-2.53
	125	1.55	-0.39	-2.13	-2.32	-2.46	0.00	-2.67
	140	0.72	-0.83	-2.19	-2.38	-2.53	-2.67	0.00

Table 18: Percent increase in inventory per percentage point increase in sales: Fixed-Capacity over dedicated (c.v. 0.25)

$C = 100$, $\frac{\sigma_{D_i}}{\mu_{D_i}} = 0.15$, $L = 2$ weeks		product 1: Mean Demand (μ_{D_1})						
		60	**75**	**90**	**100**	**110**	**125**	**140**
product 2: Mean Demand (μ_{D_2})	**60**	N/A	**33.2**	7.9	**5.4**	4.1	**3.0**	2.3
	75	**33.2**	33.2	**8.5**	5.5	**4.1**	3.1	**2.6**
	90	7.9	**8.5**	9.5	**7.7**	6.1	**4.9**	4.5
	100	**5.4**	5.5	**7.7**	9.6	**9.4**	8.2	**7.6**
	110	4.1	**4.1**	6.1	**9.4**	12.3	**13.2**	12.5
	125	**3.0**	3.1	**4.9**	8.2	**13.2**	21.0	**24.2**
	140	2.3	**2.6**	4.5	**7.6**	12.5	**24.2**	37.7

Table 19: Percent increase in inventory per percentage point increase in sales: symmetric prioritized over dedicated (c.v. 0.15)

$C = 100$, $\frac{\sigma_{D_i}}{\mu_{D_i}} = 0.25$, $L = 2$ weeks		product 1: Mean Demand (μ_{D_1})						
		60	**75**	**90**	**100**	**110**	**125**	**140**
product 2: Mean Demand (μ_{D_2})	**60**	48.0	**10.6**	5.5	**4.4**	3.8	**3.1**	2.7
	75	**10.6**	10.4	**6.6**	5.3	**4.5**	3.8	**3.4**
	90	5.5	**6.6**	7.6	**7.3**	6.8	**6.0**	5.6
	100	**4.4**	5.3	**7.3**	8.3	**8.6**	8.3	**7.8**
	110	3.8	**4.5**	6.8	**8.6**	9.8	**10.5**	10.4
	125	**3.1**	3.8	**6.0**	8.3	**10.5**	13.0	**14.3**
	140	2.7	**3.4**	5.6	**7.8**	10.4	**14.3**	17.1

Table 20: Percent increase in inventory per percentage point increase in sales: symmetric prioritized over dedicated (c.v. 0.25)

$C = 100$, $\frac{\sigma_{D_i}}{\mu_{D_i}} = 0.15$, $L = 2$ weeks		product 1: Mean Demand (μ_{D_1})						
		60	**75**	**90**	**100**	**110**	**125**	**140**
product 2: Mean Demand (μ_{D_2})	**60**	N/A	**11.82**	6.23	**6.24**	4.73	**3.50**	3.11
	75	**11.82**	11.23	**6.73**	6.61	**5.13**	4.17	**4.04**
	90	6.23	**6.73**	9.06	**11.02**	9.78	**8.85**	8.91
	100	**6.24**	6.61	**11.02**	19.41	**22.69**	23.03	**23.33**
	110	4.73	**5.13**	9.78	**22.69**	38.49	**52.77**	57.95
	125	**3.50**	4.17	**8.85**	23.03	**52.77**	126.83	**192.35**
	140	3.11	**4.04**	8.91	**23.33**	57.95	**192.35**	449.87

Table 21: Percent increase in inventory per percentage point increase in sales: symmetric distributed over dedicated (c.v. 0.15)

$C = 100$, $\frac{\sigma_{D_i}}{\mu_{D_i}} = 0.25$, $L = 2$ weeks		product 1: Mean Demand (μ_{D_1})						
		60	**75**	**90**	**100**	**110**	**125**	**140**
product 2: Mean Demand (μ_{D_2})	**60**	N/A	**4.8**	5.7	**5.3**	4.8	**4.3**	4.0
	75	**4.8**	5.2	**6.7**	6.6	**6.1**	5.8	**5.6**
	90	5.7	**6.7**	10.6	**12.1**	12.4	**12.3**	12.2
	100	**5.3**	6.6	**12.1**	16.6	**19.2**	20.9	**21.5**
	110	4.8	**6.1**	12.4	**19.2**	25.2	**31.3**	34.4
	125	**4.3**	5.8	**12.3**	20.9	**31.3**	46.7	**58.1**
	140	4.0	**5.6**	12.2	**21.5**	34.4	**58.1**	81.4

Table 22: Percent increase in inventory per percentage point increase in sales: symmetric distributed over dedicated (c.v. 0.25)

G Full Numerical and Simulation Tables

In this last section of the Appendix, we present (1) the full numerical results for the analysis of the effect of system and product unbalance in Section 5.2, and (2) the complete simulation results discussed in Section 5.3.3 to explore the inventory/transportaion trade-off. The first two tables report the actual sales and inventory in the non-flexible system (columns 3 and 6) along with the increase in sales and inventory resulting from using the various allocation policies considered in the flexible system, for coefficients of variation of demand of 15% and 25%, respectively. The next two tables present the standard deviation of the component orders observed by suppliers in the flexible system for each product (columns 3 and 8) followed by the increase in standard deviation induced by the flexible system under the different allocation policies, for coefficients of variation of demand of 15% and 25%, respectively. These are all numerical results. Finally, Table 27 reports the percent change in sales, inventory levels, and unit outbound cost in the flexible setting under various allocation policies relative to the non-flexible one obtained in a simulation study.

		Sales	% Increase in Sales		Inventory	%Increase in Inventory				
μ_1	μ_2	NF	Fixed	Fully Flex	NF	Fixed	Fully Flex			
							SymP	SymD	Profit	ProfitD
60	60	120.00	0.00	0.00	101.75	0.00	0.02	0.16	0.19	0.16
60	75	134.95	0.04	0.04	114.13	0.26	1.27	0.45	1.99	0.47
60	90	148.20	1.14	1.20	118.57	6.70	9.52	7.50	9.74	7.59
60	100	154.02	3.58	3.81	109.85	21.18	20.61	23.80	20.85	24.01
60	110	157.24	7.16	7.80	105.01	30.85	32.03	36.86	32.28	36.63
60	125	159.21	12.97	14.37	102.07	27.59	42.44	50.29	42.71	46.33
60	140	159.77	17.69	19.43	101.22	16.53	44.48	60.41	44.75	47.79
75	60	134.95	0.04	0.04	114.13	0.26	1.27	0.45	1.41	0.43
75	75	149.90	0.00	0.06	126.52	0.00	2.14	0.73	2.85	0.73
75	90	163.15	0.60	1.03	130.96	3.54	8.69	6.91	9.21	6.96
75	100	168.96	2.16	3.08	122.24	13.29	16.89	20.37	17.69	20.10
75	110	172.19	4.45	5.91	117.39	10.93	24.23	30.30	25.27	28.62
75	125	174.15	7.97	9.81	114.45	3.06	30.14	40.91	31.43	33.91
75	140	174.72	10.62	12.25	113.60	-3.08	32.16	49.52	33.58	35.26
90	60	148.20	1.14	1.20	118.57	6.70	9.52	7.50	9.76	7.41
90	75	163.15	0.60	1.03	130.96	3.54	8.69	6.91	9.86	6.86
90	90	176.40	0.00	1.20	135.40	0.00	11.41	10.90	15.13	10.90
90	100	182.22	0.40	2.05	126.68	-0.85	15.85	22.65	22.02	21.98
90	110	185.44	1.39	3.25	121.83	-3.18	19.79	31.83	28.06	29.20
90	125	187.41	3.13	4.79	118.89	-7.39	23.51	42.41	33.75	34.91
90	140	187.97	4.45	5.65	118.04	-10.63	25.32	50.31	36.54	37.52
100	60	154.02	3.58	3.81	109.85	21.18	20.61	23.80	21.21	23.60
100	75	168.96	2.16	3.08	122.24	13.29	16.89	20.37	19.23	20.65
100	90	182.22	0.40	2.05	126.68	-0.85	15.85	22.65	23.75	23.32
100	100	188.03	0.00	1.86	117.95	0.00	17.82	36.06	34.25	36.06
100	110	191.26	0.29	2.07	113.11	-0.72	19.49	46.91	40.78	44.88
100	125	193.22	1.18	2.55	110.17	-3.11	20.98	58.64	46.70	51.98
100	140	193.79	1.96	2.85	109.32	-5.21	21.58	66.57	49.38	54.90
110	60	157.24	7.16	7.80	105.01	30.85	32.03	36.86	33.87	37.09
110	75	172.19	4.45	5.91	117.39	10.93	24.23	30.30	29.74	31.98
110	90	185.44	1.39	3.25	121.83	-3.18	19.79	31.83	33.84	34.46
110	100	191.26	0.29	2.07	113.11	-0.72	19.49	46.91	49.42	48.94
110	110	194.49	0.00	1.53	108.27	0.00	18.79	58.81	58.45	58.81
110	125	196.45	0.29	1.34	105.33	-0.82	17.66	70.71	64.47	66.22
110	140	197.01	0.71	1.35	104.48	-2.00	16.83	78.32	66.91	68.92
125	60	159.21	12.97	14.37	102.07	27.59	42.44	50.29	52.18	54.25
125	75	174.15	7.97	9.81	114.45	3.06	30.14	40.91	46.59	47.91
125	90	187.41	3.13	4.79	118.89	-7.39	23.51	42.41	49.72	49.92
125	100	193.22	1.18	2.55	110.17	-3.11	20.98	58.64	65.97	65.30
125	110	196.45	0.29	1.34	105.33	-0.82	17.66	70.71	76.62	75.20
125	125	198.41	0.00	0.65	102.38	0.00	13.61	82.03	84.14	82.03
125	140	198.98	0.09	0.46	101.54	-0.26	11.22	89.13	86.70	84.24
140	60	159.77	17.69	19.43	101.22	16.53	44.48	60.41	72.08	73.03
140	75	174.72	10.62	12.25	113.60	-3.08	32.16	49.52	63.25	63.78
140	90	187.97	4.45	5.65	118.04	-10.63	25.32	50.31	63.25	63.10
140	100	193.79	1.96	2.85	109.32	-5.21	21.58	66.57	78.98	78.23
140	110	197.01	0.71	1.35	104.48	-2.00	16.83	78.32	89.00	87.73
140	125	198.98	0.09	0.46	101.54	-0.26	11.22	89.13	95.82	94.02
140	140	199.54	0.00	0.21	100.69	0.00	8.05	95.96	98.02	95.96

Table 23: Numerical results indicating the increase in sales and inventory under each of the flexible allocation policies for a coefficient of variation of 15%. Absolute sales and inventories are reported for the non-flexible base case (NF).

		Product 1					Product 2				
μ_1	μ_2	SD	% Increase in SD				SD	% Increase in SD			
		NF	Fixed	SymP	SymD	Profit	NF	Fixed	SymP	SymD	Profit
60	60	9.00	0.00	0.00	0.20	0.00	9.00	0.00	0.00	0.20	0.84
60	75	9.00	-0.06	0.00	0.20	0.00	11.12	1.13	1.19	1.39	4.66
60	90	9.00	-1.17	0.00	0.12	0.00	10.92	22.19	23.34	23.66	27.95
60	100	9.00	-4.15	0.00	-0.34	0.00	8.76	64.18	69.03	70.20	75.55
60	110	9.00	-10.13	0.00	-1.78	0.00	6.24	137.54	151.82	157.53	162.09
60	125	9.00	-24.45	0.00	-4.04	0.00	3.40	316.62	352.63	393.78	375.86
60	140	9.00	-41.62	0.00	2.13	0.00	1.82	574.63	636.83	797.77	692.24
75	60	11.12	1.13	1.19	1.39	1.19	9.00	-0.06	0.00	0.20	0.84
75	75	11.12	0.00	1.11	1.28	1.19	11.12	0.00	1.11	1.28	4.56
75	90	11.12	-5.63	0.73	0.19	1.19	10.92	15.32	21.10	22.26	25.72
75	100	11.12	-13.28	0.42	-2.35	1.19	8.76	46.79	59.20	64.37	66.09
75	110	11.12	-23.55	0.47	-6.12	1.19	6.24	99.71	121.89	138.64	133.57
75	125	11.12	-40.92	0.07	-8.30	1.19	3.40	221.93	273.36	333.29	301.10
75	140	11.12	-57.04	0.02	-1.83	1.19	1.82	390.57	540.96	685.52	602.11
90	60	10.92	22.19	23.34	23.66	23.65	9.00	-1.17	0.00	0.12	0.76
90	75	10.92	15.32	21.10	22.26	23.65	11.12	-5.63	0.73	0.19	3.03
90	90	10.92	0.00	13.61	16.20	23.65	10.92	0.00	13.61	16.20	17.27
90	100	10.92	-13.55	7.93	9.63	23.65	8.76	19.75	37.53	50.14	49.26
90	110	10.92	-27.81	3.99	4.39	23.65	6.24	54.32	80.43	112.03	110.34
90	125	10.92	-47.51	1.23	4.08	23.65	3.40	134.10	210.17	287.12	292.81
90	140	10.92	-63.18	0.37	11.04	23.65	1.82	244.13	479.43	633.63	654.90
100	60	8.76	64.18	69.03	70.20	71.29	9.00	-4.15	0.00	-0.34	0.18
100	75	8.76	46.79	59.20	64.37	71.29	11.12	-13.28	0.42	-2.35	0.06
100	90	8.76	19.75	37.53	50.14	71.29	10.92	-13.55	7.93	9.63	12.70
100	100	8.76	0.00	22.38	40.33	71.29	8.76	0.00	22.38	40.33	47.78
100	110	8.76	-18.72	11.48	34.92	71.29	6.24	25.43	51.25	99.57	118.03
100	125	8.76	-42.44	3.58	36.76	71.29	3.40	85.31	150.56	274.93	326.71
100	140	8.76	-60.13	1.06	45.39	71.29	1.82	168.95	366.49	630.01	729.68
110	60	6.24	137.54	151.82	157.53	164.33	9.00	-10.13	0.00	-1.78	-0.94
110	75	6.24	99.71	121.89	138.64	164.33	11.12	-23.55	0.47	-6.12	-0.69
110	90	6.24	54.32	80.43	112.03	164.33	10.92	-27.81	3.99	4.39	17.98
110	100	6.24	25.43	51.25	99.57	164.33	8.76	-18.72	11.48	34.92	60.83
110	110	6.24	0.00	27.84	95.15	164.33	6.24	0.00	27.84	95.15	141.89
110	125	6.24	-30.37	9.06	100.47	164.33	3.40	45.26	91.48	275.57	373.42
110	140	6.24	-52.06	2.71	112.48	164.33	1.82	109.55	240.36	641.70	812.33
125	60	3.40	316.62	352.63	393.78	451.42	9.00	-24.45	0.00	-4.04	8.12
125	75	3.40	221.93	273.36	333.29	451.42	11.12	-40.92	0.07	-8.30	17.26
125	90	3.40	134.10	210.17	287.12	451.42	10.92	-47.51	1.23	4.08	44.60
125	100	3.40	85.31	150.56	274.93	451.42	8.76	-42.44	3.58	36.76	95.05
125	110	3.40	45.26	91.48	275.57	451.42	6.24	-30.37	9.06	100.47	187.15
125	125	3.40	0.00	34.09	290.50	451.42	3.40	0.00	34.09	290.50	446.49
125	140	3.40	-31.16	10.89	312.31	451.42	1.82	44.25	104.01	674.06	936.82
140	60	1.82	574.63	636.83	797.77	1055.54	9.00	-41.62	0.00	2.13	47.98
140	75	1.82	390.57	540.96	685.52	1055.54	11.12	-57.04	0.02	-1.83	52.88
140	90	1.82	244.13	479.43	633.63	1055.54	10.92	-63.18	0.37	11.04	76.42
140	100	1.82	168.95	366.49	630.01	1055.54	8.76	-60.13	1.06	45.39	129.56
140	110	1.82	109.55	240.36	641.70	1055.54	6.24	-52.06	2.71	112.48	229.71
140	125	1.82	44.25	104.01	674.06	1055.54	3.40	-31.16	10.89	312.31	514.97
140	140	1.82	0.00	38.07	713.82	1055.54	1.82	0.00	38.07	713.82	1057.06

Table 24: Numerical results indicating the standard deviation (SD) of component demand observed by suppliers as the demand for the two products varies, for a coefficient of variation of 15%. For the non-flexible or base case (NF) the absolute SD values are reported. For all other cases, the percentage increase with respect to that base case is given.

		Sales	% Increase in Sales		Inventory	%Increase in Inventory				
μ_1	μ_2	NF	Fixed	Fully Flex	NF	Fixed	Fully Flex			
							SymP	SymD	Profit	ProfitD
60	60	119.96	0.00	0.03	129.32	0.00	1.38	0.27	1.58	0.27
60	75	134.19	0.35	0.58	141.87	1.32	6.08	2.78	6.51	2.79
60	90	145.13	2.26	3.08	136.93	12.77	16.89	17.65	17.30	17.47
60	100	150.01	4.44	5.87	129.62	22.46	25.94	31.29	26.45	30.32
60	110	153.29	6.98	9.02	124.69	23.52	33.94	43.06	34.56	40.09
60	125	156.23	10.79	13.51	120.29	16.69	42.21	57.78	42.94	48.90
60	140	157.78	14.12	17.09	117.97	10.13	46.41	68.81	47.22	52.59
75	60	134.19	0.35	0.58	141.87	1.32	6.08	2.78	6.50	2.77
75	75	148.41	0.00	0.84	154.41	0.00	8.80	4.36	9.85	4.36
75	90	159.36	0.69	2.47	149.48	6.59	16.21	16.61	18.71	16.16
75	100	164.23	1.85	4.24	142.17	4.87	22.28	27.92	25.69	26.24
75	110	167.52	3.33	6.17	137.24	2.28	27.58	37.82	31.80	33.62
75	125	170.45	5.63	8.78	132.84	-2.20	33.31	50.60	38.46	40.44
75	140	172.00	7.69	10.75	130.51	-6.40	36.78	60.33	42.57	43.97
90	60	145.13	2.26	3.08	136.93	12.77	16.89	17.65	18.31	17.84
90	75	159.36	0.69	2.47	149.48	6.59	16.21	16.61	19.30	17.06
90	90	170.30	0.00	2.66	144.55	0.00	20.26	28.16	30.21	28.16
90	100	175.18	0.26	3.30	137.23	-0.50	24.22	40.07	37.32	38.60
90	110	178.46	0.89	4.11	132.31	-1.80	27.74	50.82	43.54	46.59
90	125	181.40	2.11	5.27	127.91	-4.49	31.72	64.80	50.57	54.72
90	140	182.95	3.31	6.17	125.58	-7.24	34.28	75.22	55.19	59.65
100	60	150.01	4.44	5.87	129.62	22.46	25.94	31.29	29.42	32.25
100	75	164.23	1.85	4.24	142.17	4.87	22.28	27.92	28.70	29.60
100	90	175.18	0.26	3.30	137.23	-0.50	24.22	40.07	42.63	41.53
100	100	180.05	0.00	3.23	129.92	0.00	26.95	53.45	51.69	53.45
100	110	183.34	0.18	3.42	124.99	-0.41	29.35	65.59	58.67	62.77
100	125	186.27	0.86	3.88	120.60	-2.00	32.03	81.08	66.47	72.46
100	140	187.82	1.66	4.30	118.27	-3.95	33.68	92.33	71.55	78.41
110	60	153.29	6.98	9.02	124.69	23.52	33.94	43.06	41.89	46.03
110	75	167.52	3.33	6.17	137.24	2.28	27.58	37.82	39.80	42.02
110	90	178.46	0.89	4.11	132.31	-1.80	27.74	50.82	54.79	55.05
110	100	183.34	0.18	3.42	124.99	-0.41	29.35	65.59	43.25	68.40
110	110	186.62	0.00	3.13	120.07	0.00	30.66	78.86	73.61	78.86
110	125	189.56	0.25	3.05	115.67	-0.61	31.94	95.43	86.25	89.68
110	140	191.11	0.72	3.12	113.34	-1.83	32.58	107.18	92.29	96.26
125	60	156.23	10.79	13.51	120.29	16.69	42.21	57.78	62.42	66.66
125	75	170.45	5.63	8.78	132.84	-2.20	33.31	50.60	58.27	60.77
125	90	181.40	2.11	5.27	127.91	-4.49	31.72	64.80	74.21	74.88
125	100	186.27	0.86	3.88	120.60	-2.00	32.03	81.08	57.88	89.70
125	110	189.56	0.25	3.05	115.67	-0.61	31.94	95.43	92.84	101.18
125	125	192.49	0.00	2.42	111.27	0.00	31.39	112.86	117.14	112.86
125	140	194.04	0.12	2.15	108.94	-0.33	30.65	124.91	124.79	119.82
140	60	157.78	14.12	17.09	117.97	10.13	46.41	68.81	81.36	85.03
140	75	172.00	7.69	10.75	130.51	-6.40	36.78	60.33	74.49	76.69
140	90	182.95	3.31	6.17	125.58	-7.24	34.28	75.22	90.21	90.79
140	100	187.82	1.66	4.30	118.27	-3.95	33.68	92.33	88.29	106.26
140	110	191.11	0.72	3.12	113.34	-1.83	32.58	107.18	112.90	118.11
140	125	194.04	0.12	2.15	108.94	-0.33	30.65	124.91	133.80	130.01
140	140	195.59	0.00	1.68	106.62	0.00	28.87	137.00	143.72	137.00

Table 25: Numerical results indicating the increase in sales and inventory under each of the flexible allocation policies for a coefficient of variation of 25%. Absolute sales and inventories are reported for the non-flexible base case (NF).

		Product 1					Product 2				
μ_1	μ_2	SD	% Increase in SD				SD	% Increase in SD			
		NF	Fixed	SymP	SymD	Profit	NF	Fixed	SymP	SymD	Profit
60	60	14.95	0.00	0.34	0.40	0.35	14.95	0.00	0.34	0.40	1.24
60	75	14.95	-2.04	0.28	0.14	0.35	17.29	5.85	7.95	8.06	9.31
60	90	14.95	-7.46	0.18	-1.10	0.35	16.37	26.74	33.41	34.87	35.33
60	100	14.95	-13.03	0.13	-2.77	0.35	14.60	48.45	59.44	64.15	61.94
60	110	14.95	-19.72	0.09	-4.45	0.35	12.62	74.38	89.50	100.92	92.83
60	125	14.95	-30.67	0.05	-4.61	0.35	9.94	117.12	136.60	167.22	141.72
60	140	14.95	-41.41	0.03	-1.46	0.35	7.87	159.60	182.27	247.36	189.93
75	60	17.29	5.85	7.95	8.06	8.44	14.95	-2.04	0.28	0.14	0.98
75	75	17.29	0.00	6.49	6.69	8.44	17.29	0.00	6.49	6.69	7.23
75	90	17.29	-9.52	4.34	3.17	8.44	16.37	14.67	26.23	30.12	27.97
75	100	17.29	-17.13	3.09	0.13	8.44	14.60	30.90	45.73	55.32	49.11
75	110	17.29	-25.09	2.15	-2.21	8.44	12.62	50.59	68.64	87.11	74.47
75	125	17.29	-36.69	1.24	-2.51	8.44	9.94	83.49	107.76	146.12	118.76
75	140	17.29	-47.12	0.73	0.14	8.44	7.87	116.86	152.11	220.98	169.81
90	60	16.37	26.74	33.41	34.87	37.43	14.95	-7.46	0.18	-1.10	-0.52
90	75	16.37	14.67	26.23	30.12	37.43	17.29	-9.52	4.34	3.17	2.85
90	90	16.37	0.00	17.38	22.82	37.43	16.37	0.00	17.38	22.82	20.42
90	100	16.37	-10.07	12.44	18.34	37.43	14.60	12.08	30.76	45.05	40.68
90	110	16.37	-19.76	8.70	15.60	37.43	12.62	27.26	47.61	74.14	67.27
90	125	16.37	-32.97	5.06	15.63	37.43	9.94	53.28	79.65	130.42	118.16
90	140	16.37	-44.24	3.01	18.41	37.43	7.87	80.42	119.96	204.26	180.80
100	60	14.60	48.45	59.44	64.15	71.29	14.95	-13.03	0.13	-2.77	-1.56
100	75	14.60	30.90	45.73	55.32	71.29	17.29	-17.13	3.09	0.13	1.63
100	90	14.60	12.08	30.76	45.05	71.29	16.37	-10.07	12.44	18.34	20.73
100	100	14.60	0.00	22.36	39.79	71.29	14.60	0.00	22.36	39.79	43.35
100	110	14.60	-11.23	15.87	37.03	71.29	12.62	12.96	35.45	68.42	73.42
100	125	14.60	-26.10	9.40	37.58	71.29	9.94	35.60	61.80	124.79	131.22
100	140	14.60	-38.50	5.66	40.79	71.29	7.87	59.65	96.48	199.21	202.08
110	60	12.62	74.38	89.50	100.92	117.97	14.95	-19.72	0.09	-4.45	0.50
110	75	12.62	50.59	68.64	87.11	117.97	17.29	-25.09	2.15	-2.21	5.46
110	90	12.62	27.26	47.61	74.14	117.97	16.37	-19.76	8.70	15.60	27.89
110	100	12.62	12.96	35.45	68.42	117.97	14.60	-11.23	15.87	37.03	53.62
110	110	12.62	0.00	25.72	65.89	117.97	12.62	0.00	25.72	65.89	87.42
110	125	12.62	-16.81	15.62	67.29	117.97	9.94	19.94	46.47	123.14	151.73
110	140	12.62	-30.64	9.58	71.15	117.97	7.87	41.48	74.90	198.51	229.90
125	60	9.94	117.12	136.60	167.22	214.29	14.95	-30.67	0.05	-4.61	15.07
125	75	9.94	83.49	107.76	146.12	214.29	17.29	-36.69	1.24	-2.51	22.09
125	90	9.94	53.28	79.65	130.42	214.29	16.37	-32.97	5.06	15.63	48.30
125	100	9.94	35.60	61.80	124.79	214.29	14.60	-26.10	9.40	37.58	77.64
125	110	9.94	19.94	46.47	123.14	214.29	12.62	-16.81	15.62	67.29	115.81
125	125	9.94	0.00	29.47	126.27	214.29	9.94	0.00	29.47	126.27	187.90
125	140	9.94	-16.21	18.68	131.46	214.29	7.87	18.55	49.49	203.10	275.16
140	60	7.87	159.60	182.27	247.36	344.65	14.95	-41.41	0.03	-1.46	37.85
140	75	7.87	116.86	152.11	220.98	344.65	17.29	-47.12	0.73	0.14	42.79
140	90	7.87	80.42	119.96	204.26	344.65	16.37	-44.24	3.01	18.41	69.59
140	100	7.87	59.65	96.48	199.21	344.65	14.60	-38.50	5.66	40.79	100.62
140	110	7.87	41.48	74.90	198.51	344.65	12.62	-30.64	9.58	71.15	141.26
140	125	7.87	18.55	49.49	203.10	344.65	9.94	-16.21	18.68	131.46	218.27
140	140	7.87	0.00	32.43	209.18	344.65	7.87	0.00	32.43	209.18	311.70

Table 26: Numerical results indicating the standard deviation (SD) of component demand observed by suppliers as the demand for the two products varies, for a coefficient of variation of 25%. For the non-flexible or base case (NF) the absolute SD values are reported. For all other cases, the percentage increase with respect to that base case is given.

		% incr.	% reduction in unit outbound cost				% increase in inventory			
μ_{D_1}	μ_{D_2}	in sales	SymP	SymDL	ProfitDL	ProfitP	SymP	SymDL	ProfitDL	ProfitP
60	60	0.00	0.00	39.83	39.83	0.00	0.00	13.71	13.71	0.00
60	75	0.01	0.01	41.91	41.91	0.01	0.40	9.72	9.72	0.40
60	90	1.99	0.59	41.21	41.21	0.61	11.28	23.65	23.65	11.28
60	100	4.73	1.41	40.81	40.81	1.42	24.06	37.02	37.02	24.06
60	110	7.89	2.04	39.93	39.93	2.06	28.02	47.51	47.61	28.02
60	125	13.94	3.38	38.48	38.48	3.35	41.30	60.33	60.33	41.30
60	140	17.68	3.31	36.57	36.54	3.30	44.41	63.22	61.03	44.41
75	60	0.00	0.00	40.70	40.70	0.00	0.00	10.19	10.19	0.25
75	75	0.02	0.01	40.56	40.56	0.01	0.55	11.12	11.12	0.55
75	90	1.08	0.31	40.22	40.22	0.33	9.50	19.27	19.27	9.81
75	100	4.02	1.10	39.14	39.14	1.10	18.47	37.03	37.14	18.47
75	110	5.65	1.39	38.31	38.31	1.42	22.87	44.16	44.41	24.99
75	125	10.18	2.37	38.13	38.11	2.38	28.93	44.42	44.33	29.81
75	140	13.71	2.94	37.66	37.69	2.94	35.19	52.07	49.47	35.19
90	60	1.84	0.70	39.83	39.83	0.70	11.77	21.92	21.92	11.77
90	75	0.94	0.29	39.49	39.48	0.39	8.86	18.78	18.78	8.81
90	90	1.36	0.46	38.90	38.92	1.04	11.86	27.79	28.13	16.71
90	100	2.44	0.77	37.62	37.61	1.48	17.84	40.49	40.89	24.63
90	110	3.19	0.88	36.44	36.43	1.43	20.58	43.57	44.00	27.70
90	125	4.54	1.22	36.12	36.13	2.05	21.99	52.04	51.23	34.57
90	140	5.20	1.25	35.97	35.95	1.88	22.23	51.74	43.22	32.03
100	60	4.49	1.60	39.59	39.59	1.87	23.75	38.33	38.33	23.75
100	75	3.91	1.17	39.52	39.52	1.87	17.55	36.12	36.26	18.27
100	90	2.37	0.81	38.52	38.52	2.04	16.89	39.62	39.65	24.58
100	100	2.25	0.83	37.31	37.31	2.97	20.18	54.78	55.26	34.04
100	110	1.77	0.58	37.16	37.19	3.26	18.25	60.01	60.82	41.67
100	125	2.77	0.72	37.06	37.11	3.28	26.69	63.04	65.29	48.87
100	140	3.21	0.81	36.76	36.79	3.42	20.54	69.54	69.41	49.41
110	60	8.26	3.02	39.35	39.35	4.38	29.46	51.88	55.23	39.03
110	75	5.48	1.56	39.32	39.31	3.88	23.45	46.61	48.36	28.25
110	90	3.49	0.95	38.71	38.72	5.66	21.79	46.08	54.12	34.04
110	100	1.86	0.72	37.22	37.20	5.73	20.97	60.50	64.87	51.67
110	110	1.63	0.52	36.89	36.92	5.70	20.82	71.30	75.62	58.92
110	125	1.20	0.31	36.61	36.63	6.41	15.04	79.58	84.54	68.26
110	140	1.33	0.23	36.51	36.57	5.96	14.70	82.98	84.59	69.05
125	60	13.47	3.95	37.96	37.96	6.00	40.81	59.89	69.50	44.25
125	75	10.45	2.60	38.01	37.98	7.23	29.40	46.18	68.89	49.00
125	90	4.61	1.36	38.82	38.83	8.58	22.01	53.53	60.70	48.32
125	100	2.65	0.82	39.54	39.50	8.14	25.79	63.24	72.12	60.17
125	110	1.32	0.39	38.87	38.42	8.66	15.77	80.13	86.60	72.75
125	125	1.23	0.33	37.19	37.01	11.98	19.69	102.73	116.06	104.49
125	140	1.14	0.24	37.02	37.13	10.16	18.99	108.38	105.18	94.08
140	60	17.78	3.84	37.46	37.18	8.67	45.11	64.25	88.21	67.30
140	75	13.84	3.12	36.84	36.67	9.56	35.86	52.82	70.88	51.35
140	90	5.39	1.55	39.11	39.64	10.14	22.33	52.41	77.87	65.58
140	100	3.17	0.80	39.08	39.03	12.83	19.58	69.29	90.38	75.89
140	110	1.42	0.31	38.97	38.48	10.93	15.04	83.46	101.10	90.47
140	125	1.14	0.20	37.26	37.03	10.84	18.69	108.22	99.67	83.25
140	140	0.23	0.07	36.87	36.38	13.57	7.40	102.37	116.15	109.89

Table 27: Percent change in sales, inventory levels, and unit outbound cost in the flexible setting relative to the non-flexible one ($C = 100$, $\frac{\sigma_{D_i}}{\mu_{D_i}} = 0.15$, $L = 2$ weeks, $z = 1.64$).