ON THE EFFECTIVENESS OF ZERO-INVENTORY-ORDERING POLICIES FOR THE ECONOMIC LOT-SIZING MODEL WITH A CLASS OF PIECEWISE LINEAR COST STRUCTURES

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We consider an economic lot-sizing problem with a special class of piecewise linear ordering costs, which we refer to as the class of modified all-unit discount cost functions. Such an ordering cost function represents transportation costs charged by many less-than-truckload carriers. We show that even special cases of the lot-sizing problem are NP-hard and therefore analyze the effectiveness of easily implementable policies. In particular, we demonstrate that there exists a zero-inventory-ordering (ZIO) policy, i.e., a policy in which an order is placed only when the inventory level drops to zero, whose total inventory and ordering cost is no more than 4/3 times the optimal cost. Furthermore, if the ordering cost function does not vary over time, then the cost of the best ZIO policy is no more than $\frac{5.6}{4.6}$ times the optimal cost. These results hold for any transportation and holding cost functions that satisfy the following properties: (i) they are nondecreasing functions, and (ii) the associated cost per unit is nonincreasing. Finally, we report on a numerical study that shows the effectiveness of ZIO policies on a set of test problems.

1. INTRODUCTION

The *single-item economic lot-sizing problem* can be stated as follows: A facility, possibly a warehouse or a retail outlet, faces known demands over a finite planning horizon. At each period, the order cost function and the holding cost function are given and they can be different from period to period. There are no constraints on the quantity ordered each period and backlogging is not allowed. The objective is to decide when and how many units to order so as to minimize total ordering and holding costs over the finite horizon without any shortages.

As in the classical economic lot-sizing problem, we analyze a deterministic model in which the planner has forecast demand for the next few time periods, say 10 to 12 weeks. Of course, forecast demand is not enough to determine an effective inventory policy; uncertainty in demand also needs to be incorporated in the analysis. In practice, this is typically done by decomposing the problem into two parts: The first is identifying an inventory policy that balances holding and fixed cost assuming forecast demand over a given planning horizon. The second is determining safety stock levels and incorporating these in the inventory level that should be maintained at the beginning of each period. Indeed, this is precisely the approach used in a number of decision-support tools we are familiar with. Thus, the model analyzed in this paper helps optimize inventory decisions associated with the first part of the decomposition approach used in practice.

Typically, the facility relies on external parties for the transportation of goods. For instance, in the retail, grocery, and electronic industries, the most frequently used mode of transportation is the LTL (less-than-truckload) mode, which is attractive when shipment sizes are less than truck capacity. Indeed, in these industries, most items are of small size and the number of products required by a specific retail outlet or grocery store at any given time is fairly small.

Our focus is on practical situations in which the LTL carrier offers an *all-unit discount* to encourage large size shipments. Such a cost function, described in Figure 1, implies that if the facility orders Q units, the transportation

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