Impact of Price Postponement on Capacity and Flexibility Investment Decisions

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Investments in dedicated and flexible capacity have traditionally been based on demand forecasts obtained under the assumption of a predetermined product price. However, the impact on revenue of poor capacity and flexibility decisions can be mitigated by appropriately changing prices. While investment decisions need to be made years before demand is realized, pricing decisions can easily be postponed until product launch, when more accurate demand information is available. We study the effect of this price decision delay on the optimal investments on dedicated and flexible capacity. Computational experiments show that considering price postponement at the planning stage leads to a large reduction in capacity investments, especially in the more expensive flexible capacity, and a significant increase in profits. Its impact depends on demand correlation, elasticity and diversion, ratio of fixed to variable capacity costs, and uncertainty remaining at the times the pricing and production decisions are made.

Keywords: Dedicated and Flexible Capacity Investments, Pricing, Stochastic Programming

1. Introduction

Auto manufacturers are rapidly increasing their manufacturing flexibility. The goal, as pointed out by Gary Convis, head of manufacturing operations of Toyota (Wall Street Journal Sept. 10 2001), is to refit assembly lines around the world to have common assembly equipment and layouts. A variety of issues contribute to this trend. Increased competition and product proliferation have led to a wide variety of low volume models built in shared plants, since typical plants have a production of 200-400K vehicles per year. Demand is highly uncertain at the time of the capacity investment and product-mix flexibility has proven to be extremely useful to meet demand and achieve high capacity utilization, as illustrated in the examples below. Finally, as product life cycles are shortening, new product introductions are facilitated by the use of existing plants and flexible equipment. This can be a key competitive advantage to move swiftly into new markets and is being heavily rewarded by the financial community. Dedicated plants, however, are still appropriate in some situations. The high demand volume for some pick-up truck models justifies one or more dedicated assembly plants, plus possibly some flexible plants to offer volume-mix flexibility.

In this context, it is important to distinguish between two types of flexibility: 1) Time-spanning or multi-generation flexibility that requires heavy investments in flexible machinery and equipment, along with some common features in the engineering design and build processes of the various vehicle models across generations, to allow for capacity reuse and the speed up of new product launches. 2) Product-mix flexibility that allows for changes in the volume mix of the different products in a plant and is constrained mainly by supplier tooling capacity. The tooling specific to each particular vehicle model is provided to suppliers by the automaker and is thus a significant investment for the OEM (Original Equipment Manufacturer), which needs to be made 6 months to 2 years in advance. This product or volume-mix flexibility (along with price postponement) is the focus of this paper.

As emphasized in a recent Prudential Financial Research Report (Bruynestein, Tomlinson and Katayama (2002)), “market share, profitability and capacity utilization can be maximized when flexible plants are available to capture demand that would otherwise have been unmet.” The dangers of lacking flexibility are well illustrated by Chrysler’s Neon-based PT Cruiser. As it turned out to be a very fashionable model in 2000 and 2001, the dedicated plant in Toluca, Mexico, was not able to keep up with its demand. At the same time, the plant making the Neon in Belvidere, Illinois, was underutilized but not configured to build the PT Cruiser. The estimated loss was of $240M in pretax profit and another 0.5 points of market share in each of those years. Price flexibility was not an effective lever in this case because the product price had been announced before demand spiked. Product-mix flexibility, on the other hand, was extremely successful in the case of the Pontiac Aztek and Buick Rendezvous, both built in GM’s Ramos, Mexico, plant. While demand for the Aztek was well less than forecasted, the Rendezvous became very popular. Product-mix flexibility was key to meeting demand and enhancing profitability in this situation. Nonetheless, price flexibility was also an important lever in balancing supply and demand, as significant discounts were offered to spark demand for the Aztek.

Traditional strategic capacity investment decisions are usually based on point demand forecasts. More sophisticated planning methods account for a certain forecast error through demand distributions around the point forecast or a set of possible future scenarios. To mitigate the impact of forecast errors, flexible capacity (with respect to product mix) is often installed in multi-product settings as a hedge against the high uncertainty faced at the investment stage. While these decisions need to be made years before production and sales are realized, the pricing decision can easily be postponed until product launch, when more accurate demand information is available. As the product launch gets closer, the firm can change the pricing of the products, using market research, to bring demand closer to the available supply. Figure 1 illustrates the timeline of decisions.

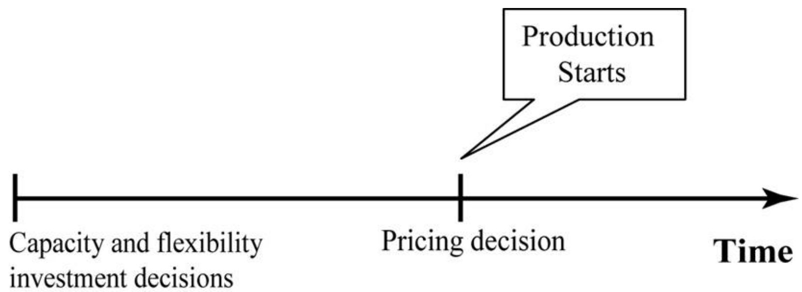

Figure 1: Timeline of decisions.

However, while price adjustments are commonly exercised before the production and sales season starts, the capacity decisions are typically made under the assumption of a predetermined fixed price. The question is how this assumption affects the firm's profits and its investments on flexible and dedicated capacities. How much is it going to affect the bottom line? Should the firm install more or less capacity given that it can later discount or increase prices to make better use of it? Should it invest in as much flexible capacity knowing that the pricing lever is also there to hedge against demand uncertainty?

Price postponement and flexible capacity are both effective mechanisms to mitigate the effects of demand uncertainty; see for instance Jordan and Graves (1995), Fine and Freund (1986, 1990), Van Mieghem (1998), Van Mieghem and Dada (1999), Bruynesteyn, Tomlinson and Katayama (2002). As a result, the optimal investment in flexible capacity will decrease with postponed pricing. The effect on dedicated capacity investments, however, is less intuitive and, as we shall see, depends on the relative cost of fixed (capacity) versus variable (production) costs. Price postponement lowers the financial risk associated with the capacity investment and makes the optimal capacity investment less sensitive to its cost.

Observe, however, that the two mechanisms, price postponement and flexible capacity, act in very different ways to hedge against demand uncertainty. Flexible capacity can be used successfully mainly by taking advantage of risk pooling effects in the demand of the various products. Postponed pricing, on the other hand, is a valuable lever for each product to better match its demand with available supply. Consequently, their benefits as the correlation in the demand of the various products grows will be vastly different.

In recent years, manufacturing flexibility and the benefits of investing in flexible capacity have become of significant interest in both industries and academia; see Beach et al. (2000), Bordoloi, Cooper, and Matsuo (1999), De Toni and Tonchia (1998), de Groote (1994), Upton (1994), Kouvelis (1992) and Sethi and Sethi (1990) for extensive reviews and perspective on this topic. Several studies have focused on the joint optimization of prices and capacities. Most relevant to our work is that of Fine and Freund (1986, 1990). In their model, capacity acquisition and variable production costs are linear, while the revenue function is concave in the amount produced. Production costs are independent of the technology, flexible or dedicated, used. They characterize the optimal solutions of the resulting two-stage stochastic program and use it to analyze the tradeoff between the acquisition costs of flexible capacity and the benefits of the increased capability to hedge against future uncertainty in demand. The assumption of a concave revenue function generalizes our postponed pricing model formulation. Our goal, however, is to compare the performance of the postponed pricing policy with that of the traditional fixed policy in flexible production environments.

Van Mieghem and Dada (1999) conduct a comprehensive study of postponement strategies for a single-product firm deciding on the optimal capacity levels, production quantities and price. The strategies studied differ in the timing of making the three decisions relative to the realization of uncertainty in customer demands. They assume a simple linear demand model with additive uncertainty, identical to the one considered in the current paper. Their analysis helps managers understand how these different strategies perform in different marketing settings, where uncertainty in demand or capacity cost may vary largely. Since they work with a single product, flexible capacity is not an issue. In our work, we focus on the impact of price postponement on the flexible and dedicated capacity requirements.

Birge, Drogosz and Duenyas (1998) consider the joint capacity and price decision problem for a single-firm, single-period, two-product case with substitutable demands, where the substitutability is only due to the price differential but not to shortages in either of the products. This assumption made sense in the automobile industry at the time of the study because immediate availability was not the major concern for most new-car shoppers, relative to other factors such as quality, price or brand loyalty. They explore several cases where at least one or two of the four decision variables (prices and capacities of the two products) are fixed while trying to optimize the rest of the variables. Flexible capacity is not available.

Although not jointly optimizing for prices and capacities, the work by Van Mieghem (1995) analyzes the relationship between the optimal capacity and flexibility investment decisions and product prices in a two-product setting. He shows that contribution margin differentials have a strong effect on the capacity investment decisions. Furthermore, it may be optimal to invest in flexible capacity even if the products are perfectly positively correlated, but have different contribution margins. This work has been extended recently by Bish and Wang (2004) to incorporate prices as endogenous variables that are fixed after demand uncertainty is realized (i.e. responsive pricing or price postponement). Chod and Rudi (2002) study a very similar model but consider investments in either flexible or dedicated resources exclusively. They characterize the benefits of flexibility assuming the investment costs for the flexible and dedicated technologies are identical. Finally, the work of Goyal and Netessine (2003) generalizes the previous model to account for different technology costs and to allow competition between two firms.

Considering price postponement at the investment stage differs significantly from the operational price changes proposed in the revenue management literature. In some industries, such as the airline, rental car and hospitality industries, revenue management techniques where prices are continuously changed according to market and inventory conditions have proven to be very successful. Yield management techniques were first developed for the airline industry in 1960 (McGill and van Ryzin (1999)), see Littlewood (1972) and Rothstein (1971) for the early work. The last decade has seen a lot of research in this field as a proactive response to the need of seeking out higher profit in an already rather efficient business process. These yield management, also known as revenue management, techniques have now been applied successfully in various industries characterized by the perishability of inventories, such as hospitality, car rental agencies, and fashion clothing. Furthermore, the application of these operational price changes to general manufacturing industries is being the subject of much recent research (Swann (1999b), Federgruen and Heching (1999), Chan, Simchi-Levi and Swann (2000 and 2001), Chen and Simchi-Levi (2002a and 2002b)). In traditional manufacturing industries, however, the resulting price uncertainty could alienate customers and be hard to implement, especially for high-ticket items such as automobiles. While it many not be practical to change prices on a day-to-day basis, i.e. at the operational level, it would be of interest to fine-tune prices right before the sales season.

The remainder of the paper is organized as follows. In the following section, we describe the mathematical models considered and state the assumptions made. In the computational section we compare the profits and capacity decisions obtained with and without price postponement at the planning stage and perform sensitivity analysis with respect to the correlation among products, the magnitude of forecast error, the elasticity and profit margins of the products considered, the ratio of fixed to variable costs, the costs associated with flexible capacity, and the amount of uncertainty remaining at the time production and pricing decisions are made. We conclude with a summary of the managerial insights obtained and a discussion of extensions to our model.

2. Model and Assumptions

We consider the strategic capacity investment decision faced by a manufacturer whose future demand for n products is uncertain and price dependent. Capacity of each product i, i=1,2,…,n, can be installed at a cost of c_i per unit. In addition, flexible capacity, which can be used to produce any mix of the n products, can be purchased at a higher cost, c_f, to hedge against demand uncertainty. Production decisions are made after demand has been realized, unless otherwise specified. Product i can be manufactured at a cost of r_i when using the dedicated production technology and at a slightly higher cost, r_{fi}, when using the flexible technology. This higher cost may be used to approximate the increased setup costs in the flexible environment.

The manufacturer needs to determine the amount of dedicated and flexible capacity to install, at a time when little information on product demand is available. At this strategic level, the average forecast error in the automotive industry has been found to be around 40% (Jordan and Graves (1995)). We assume that the uncertain demand can be represented as a linear downward sloping function of price (see Figure 1) with known slope but random y-intercept. That is, the demand for product i, D_i, i=1,2,…,n, is given by,

$$D_i = \xi_i - \alpha_i P_i$$

where the parameter $\alpha_i > 0$ is derived from the elasticity of product *i*, P_i is the price of the product, and ξ_i is a non-negative random variable with mean μ_i and standard deviation σ_i that depicts the market size (*i* = 1, 2,…,n). Elasticity is defined as the percent decrease in demand volume when price is increased by 1%, or $-\frac{\text{\% change in quantity}}{\text{\%change in price}} = -\frac{\Delta\text{Volume / Volume}}{\Delta\text{Price / Price}}$ and the parameter can be calculated as $\alpha = \frac{\text{Elasticity} \times \text{Base Volume}}{\text{Base Price}}$, given a base or initial price-demand forecast. We let ε_i represent a realization of ξ_i. This demand model is commonly used in the pricing literature (e.g., Van Mieghem and Dada (1999)).

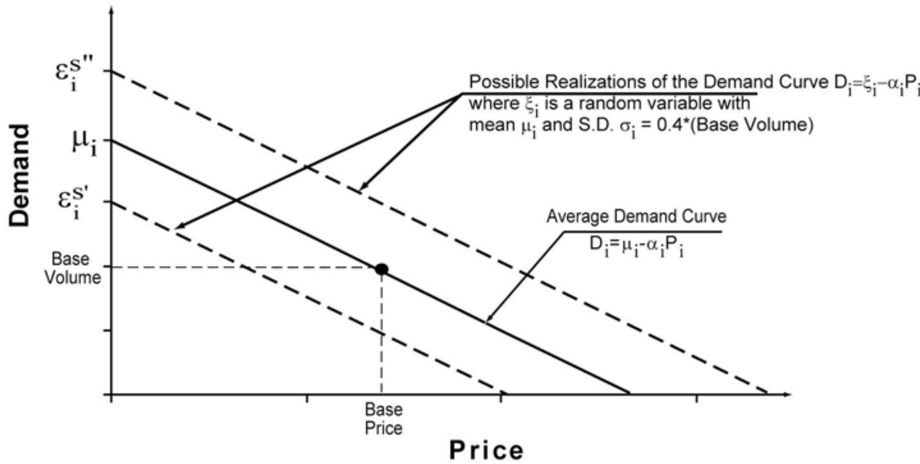

Figure 2: Linear demand curve

We consider S possible demand scenarios: $(\xi_1, \xi_2, ..., \xi_n) = (\varepsilon_1^s, \varepsilon_2^s, ..., \varepsilon_n^s)$ with probability q_s, s=1,2,…,S. Observe that the demand curves of the different products may be correlated (i.e., ξ_i, i=1,2,…n, are correlated), but we ignore the effect of product substitution, which would occur when customers switch their product preference as prices change or their original choice is no longer available. The extension of our models to incorporate product substitution is considered in Section 5.

The firm may choose to make the capacity decisions assuming that either product prices are fixed at the strategic stage (*fixed pricing model*) or postponed to right before product launch (*postponed pricing model*), when more information on product demand is available. As an approximation at the strategic level, we consider that demand is known with certainty at this time and study the impact of pricing under partial information only in Section 3.7.

In order to study the impact of the assumed pricing strategies on the optimal levels of manufacturing capacity and flexibility at the planning stage, we compare the decisions generated when using a fixed pricing policy versus a flexible (or postponed) pricing policy in a single-period production model that considers the possible future demand realizations and their associated probabilities. This can be done mathematically by solving the stochastic programming problems described below.

We use the following notation in our formulation:

Variables:

P_i^s	price of product *i* in scenario *s* (for postponed pricing model)
y_i^s	production quantity of product *i* in scenario *s* using dedicated capacity
z_i^s	production quantity of product *i* in scenario *s* using flexible capacity
K_i	capacity dedicated to product *i*
K_f	flexible capacity that can be used for producing both of the products

Parameters:

ε_i^s	the intercept of the demand curve of product *i* in scenario *s* (refer to figure 2)
α_i	slope of the price-demand curve of product *i*
P_i	price of product *i* fixed over each scenario (for fixed pricing model)
q_s	Probability of scenario *s*
c_i	unit cost of dedicated capacity for product *i*
c_f	unit cost of flexible capacity

r_i unit production cost of product *i* using dedicated capacity

r_{fi} unit production cost of product *i* using flexible capacity

Under price postponement, the objective is to jointly determine the optimal dedicated and flexible capacity investment together with the prices and production levels for each scenario, to maximize expected profits. We refer to the resulting stochastic program as the *Postponed Pricing Model.*

$$Max \quad \sum_{s=1}^{S}\sum_{i=1}^{n} q_s\left[P_i^s\left(\varepsilon_i^s-\alpha_i P_i^s\right)-r_i y_i^s-r_{fi}z_i^s\right]-\sum_{i=1}^{n}c_i K_i - c_f K_f$$

subject to :

$$y_i^s \le K_i \qquad \forall i,s \qquad (1)$$

$$\sum_{i=1}^{n} z_i^s \le K_f \qquad \forall s \qquad (2)$$

$$y_i^s + z_i^s = \varepsilon_i^s - \alpha_i P_i^s \qquad \forall i,s \qquad (3)$$

$$P_i^s, K_i, K_f, y_i^s, z_i^s \ge 0 \qquad \forall i,s$$

Constraints (1) ensure that the production quantity using dedicated resources should be less than the dedicated capacity installed for each product. Constraints (2) require that the total flexible production of the n products does not exceed the flexible capacity available. Constraints (3) state that the total production of each product should equal the demand for that product. This must be true at optimality, since profit would increase by raising the price to clear the available inventory. Observe that the price and production variables change from scenario to scenario, while capacities remain the same; that is, capacity decisions are made at the planning stage, while pricing and production decisions are made after demand is realized. The postponed pricing model is a concave (over the feasible region) maximization quadratic problem; therefore, it is easy to determine the optimal prices and capacities. Furthermore, the optimal solution is unique for the two-product case (Fine and Freund (1986)).

When the prices, P_i, for all the products are given in advance, however, the dedicated and flexible capacities maximizing expected profits can be found by solving the following stochastic program, which we call the *Fixed Pricing Model*.

$$Max \quad \sum_{s=1}^{S}\sum_{i=1}^{n} q_s \left[P_i \left(y_i^s + z_i^s \right) - r_i y_i^s - r_{fi} z_i^s \right] - \sum_{i=1}^{n} c_i K_i - c_f K_f$$

subject to:

$$y_i^s \le K_i \qquad \forall i, s \qquad (4)$$

$$\sum_{i=1}^{n} z_i^s \le K_f \qquad \forall s \qquad (5)$$

$$y_i^s + z_i^s \le \varepsilon_i^s - \alpha_i P_i \qquad \forall i, s \qquad (6)$$

$$K_i, K_f, y_i^s, z_i^s \ge 0 \qquad \forall i, s$$

In this case, production quantities do not always equal the demand (as they did in Constraints (3) in the postponed pricing model). This is because for some scenarios the fixed prices might be too low and there may not be enough capacity to satisfy the resulting high demand.

This model represents the traditional capacity decision framework. At the strategic planning stage, prices are assumed fixed in advance based on demand forecasts, before the capacity investment decision is addressed. To reflect this practice in our computational experiments, we need to find an appropriate set of fixed prices. For that purpose, we solve the postponement pricing problem over a single demand scenario corresponding to the average demand curve (which represents the point forecast). Alternatively, one could solve a joint pricing and capacity model, identical to the above fixed pricing model except that the prices, P_i, are nonnegative variables rather than given parameters. This results in a difficult to optimize non-concave maximization problem. For the cases tested, the joint optimization problem yielded comparable profits, prices and capacities to those reported for the above pricing model.

In addition, since the fixed prices assumed at the planning stage will later be changed to reflect market conditions and make better use of the installed capacity, the actual expected profits will differ considerably from those projected by the Fixed Pricing Model. To calculate the actual expected profits, we solve a new stochastic program, identical to the Postponed Pricing Model except that the flexible and dedicated capacities are parameters set to the values dictated by the Fixed Pricing Model. We refer to this as the *Actual Fixed Pricing Profit Model.*

Finally, observe that a feasible solution K, y, z for the fixed pricing model is always feasible and achieves a higher revenue, and thus a higher profit, in the postponed pricing model. This is because in the cases of demand higher than production a higher premium will be charged, while in the cases of low demand more volume can be sold by lowering prices if needed.

3. Computational Study

The objective of the computational study is to compare the capacity and flexibility investment decisions under fixed versus postponed pricing. Note, however, that when we assume fixed prices at the planning stage, the *projected* expected profits at that time will significantly differ from the *actual* expected profits that will be observed after prices are adjusted before product launch. Hence, the investment decisions will be made based on projected profits, but the actual impact on the firm's bottom line will be buffered by the later price adjustments. Note that it is of interest to compare both quantities with the expected profits generated when price postponement is accounted for at the planning stage (in this case projected and actual would be identical).

To incorporate this behavior, the study of the fixed pricing assumption requires three steps. First, initial prices for each of the products are found by solving a postponed pricing model with a single scenario (S=1), representing the expected demand curve. Second, flexible and dedicated capacity decisions are made by solving the fixed pricing model over a number of scenarios using those prices. Third, the actual profits associated with those decisions are calculated through a postponed pricing model where prices are variables but capacities are set to the levels previously determined. The fixed pricing model is a simple linear program. The postponed pricing model is a concave quadratic program that can be efficiently solved by mathematical programming optimizers, such as CPLEX. We use the barrier optimizer in CPLEX version 6.5 to solve the quadratic model and the primal optimizer to solve the linear model.

We consider only two products (n=2) in the computational study. For each instance tested, we generate the set of future demand scenarios as follows. We assume that the random variables, ξ_i, i=1,2, are normally distributed with mean μ_i and standard deviation σ_i, respectively. Based on historical estimates in the auto industry, the forecasting error of demand is assumed to be 40%; i.e. the standard deviation of demand is 40% of the expected demand (point demand forecast) associated with a certain base price. In our model, this needs to be translated to a standard deviation of the market size random variables, ξ_i. This is straightforward since we have a linear demand model (refer to figure 2). Demand for both products may be correlated, with a correlation coefficient of ρ. We discretize the normal distributions by equally dividing the range $(-2\sigma_i, 2\sigma_i)$ into ten intervals. Using the midpoints of these intervals as the values for each scenario, we calculate the 100 joint probabilities of the correlated ξ_i's using an Excel add-in developed by Numerical Technologies.

We start by considering a base case with two representative products whose demands are uncorrelated, and compare the projected and actual profits, flexible and dedicated capacity decisions, average prices and production levels, and expected capacity utilization under the two pricing assumptions. Next, we study the sensitivity of these results to changes in (1) the demand correlation coefficient, (2) the level of uncertainty at the investment stage, (3) the product characteristics, in particular, demand elasticity and profit margins, (4) the ratio of capacity to production costs, c_i / r_i, (5) the cost of installing and using flexible capacity, and (6) the amount of information available at the time production and pricing decisions are made.

3.1 Base Case

Product related parameters are described in Table 1. These parameters are all hypothetical, but representative of various vehicle models in their market segment. This set is referred to as product group A and will be used throughout the computational study unless otherwise specified. It is composed of two Midsize vehicles, which have relatively low profit margins and high demand elasticity. Both products have demand elasticity of 4.3.

		Base Volume	**Base Price**	α_i	c_i	r_i	r_{fi}	c_f
Group A	**Product$_1$**	40,000	$15,000	11	$500	$10,000	$10,100	$525
	Product$_2$	160,000	$14,000	49	$450	$9,000	$9,090	

Table 1: Product parameters.

We start by analyzing the pricing/investment problem for product group A under the assumption of independent demands for the two products, i.e. $\rho = 0$. The results are summarized in Tables 2 through 5.

Table 2 shows that while the projected expected profit is 6.5% larger when considering price postponement, the optimal capacity investment is greatly reduced. Clearly, this constitutes a significant improvement for the business case as both profits and ROI increase and the risk exposure of the company decreases.

Even more striking is the reduction of 69% in the amount of flexible capacity required. When pricing flexibility is used to balance demand with the available supply at the tactical level, the need for flexible capacity is greatly reduced. Postponed pricing is another effective lever to mitigate demand uncertainty and thus reduces the value of manufacturing flexibility. They are not substitutes, however, and their effect on system performance is quite different. Incorporating price postponement not only reduces the amount of optimal flexible capacity in our example, but also that of total dedicated capacity in the system. As we shall see in Section 3.5, when capacity costs per unit are relatively low, the financial risk of purchasing plenty of spare capacity to cover the wider variability in optimal production levels in the fixed pricing case is well justified. If the later price adjustment is considered, however, a premium can be charged when capacity is scarce and higher profits can be made with a lower capacity investment.

Observe that the overinvestment in capacity as a result of the use of fixed prices at the planning stage reduces actual expected profits by 2.64%. Note that the increase in expected profits ($24M) is lower than the reduction in the capacity investment ($33M) since the revenues obtained with looser capacity constraints are higher

Policy	**Projected Profit**	**Actual Profit**	**Flexible Capacity**	**Capacity$_1$**	**Capacity$_2$**	**Total Capacity**	**Capacity Investment**
Fixed Pricing	$883M	$916M	38,400	36,183	243,014	317,596	$148M
Postponed Pricing	$940M	$940M	11,835	40,861	196,531	249,227	$115M
Change	**6.49%**	**2.64%**	**-69.18%**	**12.93%**	**-19.13%**	**-21.53%**	**-22.03%**

Table 2: Impact of pricing strategy on profit and optimal capacity investments.

Another interesting observation is that while the total dedicated capacity in the system decreases with price postponement, that of product 1 increases. This is because the demand for product 1 is much lower than that of product 2 (base demand of 40,000 versus 160,000) and its fixed unit capacity cost higher ($500 vs $450, while flexible capacity cost is $525). As a result, in the fixed pricing solution, the amount of dedicated capacity of product 1 installed is low, less than its base volume demand, and a good part of its production is covered through the slightly pricier flexible capacity. Plenty of capacity of Product 2 is installed (over 50% more than its base volume demand), however, since its demand is much higher (product 1 would never use that much flex capacity given its low demand) and its dedicated capacity costs less.

The optimal production quantities of each of the two products using the dedicated and flexible resources are analyzed in Table 3. The percent change in production quantities projected (not actual) by the fixed and postponed pricing models is also given. We observe that variability in production quantities decreases significantly under the postponed pricing policy relative to the fixed pricing policy. This is yet another advantage of considering price postponement at the strategic level. The smaller range of production levels over the possible demand scenarios translates into a better prediction of future component needs. In many industries, such as the automobile industry, supply capacity must be acquired (or secured) long before production starts. It often requires the manufacturer to commit to a minimum purchase quantity, and the supplier to install enough capacity to provide a certain maximum quantity on demand. The lower variability in the predicted demand for components has a significant impact in decreasing the risk to the supplier and hence the price to the manufacturer.

Projected Production Quantity	Policy	Product 1		Product 2	
		Dedicated	Flexible	Dedicated	Flexible
Average	Fixed Pricing	34,081	11,702	185,942	3,796
	Actual Fixed	36,118	11,977	201,990	663
	Post. Pricing	40,403	5,380	187,980	3,833
	Change	**18.55%**	**-54.02%**	**1.10%**	**0.99%**
SD	Fixed Pricing	4,739	11,327	48,127	9,067
	Actual Fixed	348	6,910	26,772	2,767
	Post. Pricing	1,413	4,605	14,082	4,827
	Change	**-70.18%**	**-59.35%**	**-70.74%**	**-46.77%**

Table 3: Optimal projected production quantities.

It is also of interest to compare the average prices and production (or equivalently demand or sales) associated with the postponed and fixed pricing policies. Total demand is of great concern to many industries in their fight for market share under competitive pressures. Table 4 reports the average prices and production levels for each product across the 100 scenarios. The first row contains the results of the fixed pricing model, the second contains the actual average prices and production levels that are expected once prices are changed before production starts but capacities have been set under the assumption of fixed prices and the third contains the solution of the postponed pricing model.

Policy	Product 1		Product 2	
	Price	Production	Price	Production
Fixed Pricing	14,491	45,783	13,353	186,374
Actual Fixed	14,290	48,095	13,133	202,653
Post. Pricing	14,491	45,782	13,353	191,813

Table 4: Average price and production levels.

We observe that overcapacity leads to discounts and thus a lower average price in the actual fixed pricing model. The optimal prices under the different scenarios stay well within an acceptable range (if not, the model would need to incorporate bounds on the allowed price change; see extensions in the following section). Interestingly, the average prices generated by the postponed pricing model are almost identical (they vary in a few decimal points) to those initially fixed when solving the fixed pricing model. However, the projected average total production is higher even if much less capacity is installed. This is because when prices are fixed, much of the installed capacity goes idle in low demand scenarios resulting in the lower average production. Of course, this relationship is reversed once prices are allowed to change and the higher capacity levels installed are optimally used (see the "Actual Fixed" row in Table 4).

Intuitively, one expects much higher capacity utilization under the postponed pricing policy. Table 5 reports the average capacity utilization resulting from both fixed and postponed pricing models. Observe that the flexible resources have lower utilization as a result of their higher variable cost. Under the fixed pricing assumption with subsequent adjustment policy the more expensive production using flexible capacity is further reduced.

Policy	Capacity1	Capacity2	FlexCap
Fixed Pricing	94%	77%	40%
Actual Fixed	100%	83%	33%
Post. Pricing	99%	96%	78%

Table 5: Capacity utilization.

3.2 Impact of Demand Correlation

Table 6 presents the results of running the same instance (product group A) under various levels of demand correlation. The study indicates that when the demands of the two products are positively correlated the postponement pricing policy becomes even more desirable. The larger the correlation coefficient, the higher the profit increases and the capacity reduction. The reason behind this effect is that when demands are positively correlated, there is an increased level of competition for capacity between the two products and the opportunity to change pricing before product launch proves to be very effective in making the most out of the capacity available. When demands are negatively correlated, however, risk pooling effects make flexible capacity a very valuable lever to hedge against demand variability, which results in the observed lower impact of the postponement pricing strategy.

Correlation	Policy	Projected Profit	Actual Profit	Flexible Capacity	Capacity1	Capacity2	Total Capacity	Capacity Investment
-0.99	**Fixed Pricing**	$892M	$924M	57,600	16,983	217,414	291,996	$137M
	Post. Pricing	$942M	$942M	19,909	34,262	190,041	244,213	$113M
	Change	**5.58%**	**1.95%**	**-65.44%**	**101.75%**	**-12.59%**	**-16.36%**	**-17.18%**
-0.75	**Fixed Pricing**	$889M	$919M	51,200	23,383	230,214	304,796	$142M
	Post. Pricing	$941M	$941M	17,234	36,425	192,171	245,829	$114M
	Change	**5.78%**	**2.40%**	**-66.34%**	**55.78%**	**-16.52%**	**-19.35%**	**-20.00%**
-0.5	**Fixed Pricing**	$887M	$917M	44,800	29,783	236,614	311,196	$145M
	Post. Pricing	$940M	$940M	15,077	38,165	194,224	247,466	$114M
	Change	**6.07%**	**2.53%**	**-66.35%**	**28.15%**	**-17.92%**	**-20.48%**	**-21.04%**
-0.25	**Fixed Pricing**	$884M	$915M	38,400	36,183	243,014	317,596	$148M
	Post. Pricing	$940M	$940M	13,409	39,602	195,508	248,519	$115M
	Change	**6.32%**	**2.69%**	**-65.08%**	**9.45%**	**-19.55%**	**-21.75%**	**-22.21%**
0	**Fixed Pricing**	$883M	$916M	38,400	36,183	243,014	317,596	$148M
	Post. Pricing	$940M	$940M	11,835	40,861	196,531	249,227	$115M
	Change	**6.49%**	**2.64%**	**-69.18%**	**12.93%**	**-19.13%**	**-21.53%**	**-22.03%**
0.25	**Fixed Pricing**	$881M	$913M	32,000	42,583	249,414	323,996	$150M
	Post. Pricing	$939M	$939M	9,534	42,583	198,074	250,191	$115M
	Change	**6.60%**	**2.84%**	**-70.21%**	**0.00%**	**-20.58%**	**-22.78%**	**-23.21%**
0.5	**Fixed Pricing**	$880M	$913M	25,600	42,583	255,814	323,996	$150M
	Post. Pricing	$939M	$939M	7,070	44,116	199,626	250,812	$116M
	Change	**6.66%**	**2.77%**	**-72.38%**	**3.60%**	**-21.96%**	**-22.59%**	**-22.85%**
0.75	**Fixed Pricing**	$879M	$914M	19,200	48,983	255,814	323,996	$150M
	Post. Pricing	$938M	$938M	3,438	46,533	201,607	251,578	$116M
	Change	**6.71%**	**2.70%**	**-82.09%**	**-5.00%**	**-21.19%**	**-22.35%**	**-22.64%**
0.99	**Fixed Pricing**	$877M	$918M	0	61,783	255,814	317,596	$146M
	Post. Pricing	$938M	$938M	0	48,720	203,978	252,698	$116M
	Change	**6.98%**	**2.21%**	**N/A**	**-21.14%**	**-20.26%**	**-20.43%**	**-20.45%**

Table 6: Profit and optimal capacity investment under different demand correlations.

Note that the flexible capacity needed is zero (see Table 6) when the two demands are highly positively correlated ($\rho = 0.99$). In this case, flexible capacity can be beneficial to allocate more capacity to the more profitable product when capacities are tight (Van Mieghem (1995)). In our study, however, the marginal profits of the two products are very close, which eliminates the need for keeping the more expensive flexible capacity.

3.3 Impact of Forecast Uncertainty

Next, we study the impact of higher forecast uncertainty. For that purpose, we consider a standard deviation of 60% around the point demand forecast associated with the base price.

Table 7 presents the computational results for the various levels of the demand correlation coefficient. As expected, the benefits of postponed pricing in profits and reduced capacity requirements increases with the higher forecast error (Figure 3). Nonetheless, we note that the resulting profit increase and capacity decrease as the correlation level grows follows the same pattern as in the previous cases (see Figure 3).

Correlation	**Projected Profit**	**Actual Profit**	**Flexible Capacity**	**Total Capacity**	**Capacity Investment**
-0.99	10.03%	2.78%	-65.44%	-16.36%	-21.92%
-0.75	10.26%	3.44%	-66.34%	-19.35%	-25.35%
-0.5	10.75%	3.61%	-66.35%	-20.48%	-26.33%
-0.25	11.16%	3.83%	-65.08%	-21.75%	-27.44%
0	11.43%	3.73%	-69.18%	-21.53%	-26.93%
0.25	11.58%	4.04%	-70.21%	-22.78%	-28.33%
0.5	11.64%	3.92%	-72.38%	-22.59%	-27.80%
0.75	11.67%	3.80%	-82.09%	-22.35%	-27.70%
0.99	12.12%	3.04%	N/A	-20.43%	-24.79%

Table 7: Changes in profit and capacity investment with 60% forecast error.

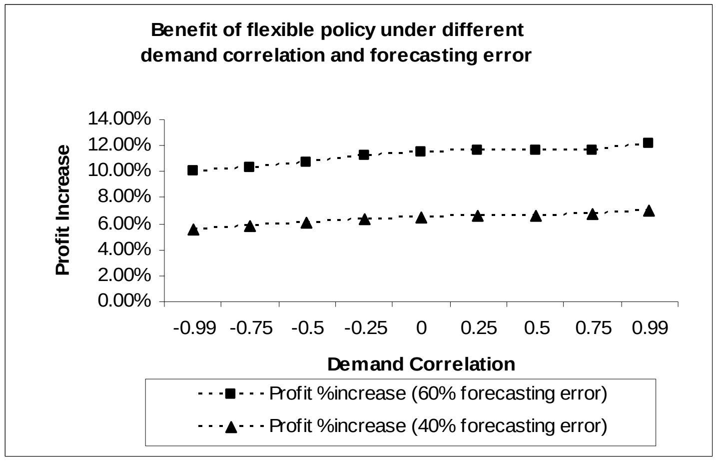

Figure 3: Sensitivity to forecasting error.

3.4 Sensitivity to Product Characteristics

All the results presented thus far are based on a single group of products with certain demand and price characteristics. Three additional product groups described in Table 8 will now be used to test the sensitivity of our results to these product characteristics. For that purpose, we selected products with parameters that are as disparate as possible from those in group A. In particular, group B represents vehicles in the luxury segment, which command much higher prices and exhibit fairly low demand elasticity. Group C represents the low end of the market with high elasticity and low margins. Finally, Group D represents a mixed scenario, where the profit margin of the first product is much higher than that of the second. Here again the forecasting error of demands is assumed to be 40% and product demands independent.

		Base Volume	Base Price	E	α_i	c_i	r_i	r_{fi}	c_f
Group A	**Product$_1$**	40,000	$15,000	4.3	11	$500	$10,000	$10,100	$525
	Product$_2$	160,000	$14,000	4.3	49	$450	$9,000	$9,090	
Group B	**Product$_1$**	21,000	$32,000	3.3	2	$1,200	$24,000	$24,240	$1,260
	Product$_2$	11,000	$30,000	3.3	1	$1,050	$21,000	$21,210	
Group C	**Product$_1$**	170,000	$12,000	3.3	47	$400	$8,000	$8,080	$473
	Product$_2$	160,000	$14,000	4.3	49	$450	$9,000	$9,090	
Group D	**Product$_1$**	11,000	$30,000	3.3	1	$1,050	$21,000	$21,210	$1,103
	Product$_2$	45,000	$18,000	2.3	6	$650	$13,000	$13,130	

Table 8: Input parameters. E stands for demand elasticity.

The results presented in Table 9 suggest that the benefits of postponed pricing in larger profits and lower capacity investments are higher when the elasticity parameters are low. This may seem counterintuitive, but can be explained as follows. When demand is fairly inelastic, a fat premium can be charged in high demand scenarios to balance demand and supply. Overcapacity, on the other hand, is hard to fight since prices would decline fast to increase demand. Thus, it makes sense to further reduce the capacity investment, especially in the more expensive flexible capacity, and rake a larger percent increase in expected profits with the high premiums.

Product	Policy	Projected Profit	Actual Profit	Flexible Capacity	Capacity$_1$	Capacity$_2$	Total Capacity	Capacity Investment
Group B	**Fixed Pricing**	$216M	$230M	8,640	20928	9590	39,158	$46M
	Flex Pricing	$238M	$238M	2,772	17724	9575	30,070	$35M
	Change	**10.44%**	**3.57%**	**-67.92%**	**-15.31%**	**-0.16%**	**-23.21%**	**-24.43%**
Group C	**Fixed Pricing**	$1291M	$1350M	76,800	187550	204614	468,964	$203M
	Flex Pricing	$1378M	$1378M	31,622	165453	182760	379,835	$163M
	Change	**6.71%**	**2.07%**	**-58.82%**	**-11.78%**	**-10.68%**	**-19.01%**	**-19.68%**
Group D	**Fixed Pricing**	$278M	$297M	7,200	9270	53006	69,476	$52M
	Flex Pricing	$308M	$308M	2,302	9532	38698	50,532	$38M
	Change	**10.73%**	**3.74%**	**-68.03%**	**2.83%**	**-26.99%**	**-27.27%**	**-27.67%**

Table 9: Profit and optimal capacity investments for product groups B, C and D.

3.5 Sensitivity to Fixed-Variable Cost Ratio

The steep reduction in capacity investment under postponed pricing has been pervasive throughout the above computational results. A natural question at this point is the generality of this result. Will postponed pricing always result in lower optimal capacity requirements?

The answer is no. It depends on the cost of installing capacity, see Figure 4. As the ratio of fixed to variable costs increases, the optimal capacity investments under the fixed pricing assumption drop sharply. When considering postponed pricing, however, they remain at similar levels.

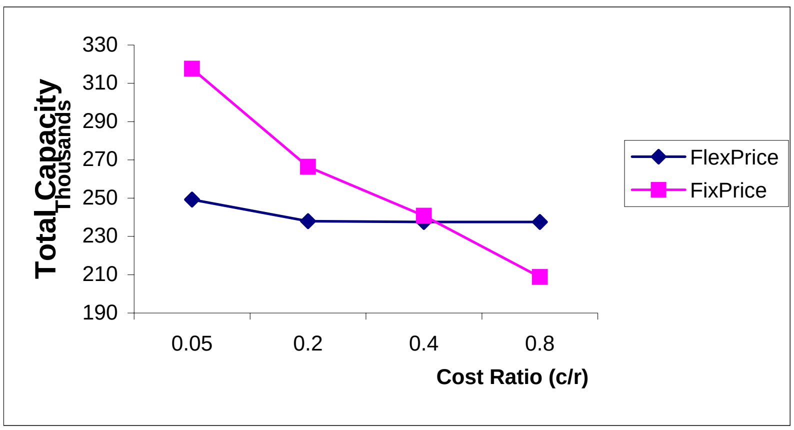

Figure 4: Total capacity as a function of the ratio of fixed versus variable costs (c/r)

When capacity costs per unit are low there is little financial risk when purchasing spare capacity to cover the wider variability in optimal production levels in the fixed pricing case. If the later price adjustment is considered, however, a premium can be charged when capacity is scarce and higher profits can be made with a lower capacity investment. On the other hand, when capacity costs are high, there is a high risk associated with purchasing capacity and much less will be purchased under the fixed pricing assumption. Meanwhile, when accounting for postponed pricing, the risk is reduced by the ability to change prices and profits can be increased by purchasing more capacity and reducing prices, if necessary, in the scenarios where it would not be used. The following table shows this effect as a function of the ratio of fixed (capacity) versus variable (production) costs per unit, while the total cost per unit remains fixed. The results shown are for independent product demands. Similar results were observed for various levels of correlation. We also observe that the projected profit disparity at the investment stage when considering or not postponed pricing increases sharply as fixed costs constitute a larger portion of the total product costs. While the amount of capacity and resulting profits are sharply reduced as fixed costs increase when assuming fixed prices, the total capacity and profits remain almost identical under postponed prices. The investment in the more expensive flexible capacity, however, is reduced greatly under postponed pricing and very mildly under fixed pricing.

Cost Ratio (c/r)	Policy	Projected Profit	Actual Profit	FlexCap	Capacity1	Capacity2	TotalCap
0.05	**FixPrice**	882,516,798	915609094.1	38,400	36,183	243,014	317,596
	FlexPrice	939,762,617	939,762,617	11,835	40,861	196,531	249,227
	Change	6.49%	2.64%	-69.18%	12.93%	-19.13%	**-21.53%**
0.2	**FixPrice**	799,051,466	917551638	32,000	29,783	204,614	266,396
	FlexPrice	933,367,469	933,367,469	10,016	39,997	187,950	237,963
	Change	16.81%	1.72%	-68.70%	34.30%	-8.14%	**-10.67%**
0.4	**FixPrice**	733,568,037	923676533.4	32,000	29,783	179,014	240,796
	FlexPrice	932,067,846	932,067,846	5,292	42,526	189,778	237,596
	Change	27.06%	0.91%	-83.46%	42.79%	6.01%	**-1.33%**
0.8	**FixPrice**	656,156,652	900137427.8	32,000	23,383	153,414	208,796
	FlexPrice	931,467,203	931,467,203	0	45,782	191,813	237,596
	Change	41.96%	3.48%	-100.00%	95.80%	25.03%	**13.79%**

Table 10: Profits and capacities as a function of the ratio of fixed versus variable costs (c/r).

In addition, Table 10 also helps in understanding the value of shifting costs from fixed to variable. As a larger portion of the product is outsourced, i.e. the ratio of fixed to variable cost decreases, it might be reasonable to purchase and install more capacity to maximize profits.

3.6 Sensitivity to the Costs of Flexible Capacity and Production

In our previous results, we have observed a marked reduction in the optimal amount of flexible capacity required, a large reduction in the overall capacity investment and significant gains in projected profits when price postponement is incorporated in the planning analysis. How sensitive are these effects to the cost of installing and using flexible capacity? To answer this question, we lower the cost of flexible capacity, c_f, and the cost of producing each product in the flexible environment, r_{fi} for i=1,2, as follows. Let $c_f = C \max\{c_1,c_2\}$ and $r_{fi} = A\, r_i$. We vary C in the set {1.05, 1.03, 1.01} and A in {1.01,1}, where C=1.05, A=1.01 is our original base case.

We observe that the effects of price postponement are fairly robust to the cost of flexible capacity. The reduction in the optimal levels of flexible capacity, system capacity and capacity investment are well over 50%, 20% and 15%, respectively, even when the flexible technology is only slightly more expensive and involves no extra production cost as compared to the dedicated technology. Nonetheless, as the costs of installing and using flexible capacity increase, the effect of considering postponed pricing on projected profits and optimal capacity levels increases, as one would expect.

		Profit	Flexible Capacity	capacity1 c_1=500	capacity2 c_2=450	Total Capacity	Capacity Investment
Base Case	**Fixed Pricing**	$883M	38,400	36,183	243,014	317,596	$148M
r_{fi}=1.01r_i	**Flexible Pricing**	$940M	11,835	40,861	196,532	249,228	$115M
c_f=525	**change**	**6.49%**	**-69.18%**	**12.93%**	**-19.13%**	**-21.53%**	**-22.03%**
Case 2	**Fixed Pricing**	$883M	38,400	36,183	243,014	317,596	$147M
r_{fi}=1.01r_i	**Flexible Pricing**	$940M	12,482	40,447	196,274	249,203	$115M
c_f=515	**change**	**6.45%**	**-67.49%**	**11.79%**	**-19.23%**	**-21.53%**	**-21.90%**
Case3	**Fixed Pricing**	$883M	38,400	36,183	243,014	317,596	$147M
r_{fi}=1.01r_i	**Flexible Pricing**	$940M	13,101	40,131	196,019	249,251	$115M
c_f=505	**change**	**6.42%**	**-65.88%**	**10.91%**	**-19.34%**	**-21.52%**	**-21.76%**
Case 4	**Fixed Pricing**	$884M	44,800	29,783	243,014	317,596	$124M
r_{fi}=r_i	**Flexible Pricing**	$941M	20,064	34,625	193,581	248,270	$104M
c_f=525	**change**	**6.43%**	**-55.22%**	**16.26%**	**-20.34%**	**-21.83%**	**-15.95%**
Case 5	**Fixed Pricing**	$885M	44,800	29,783	243,014	317,596	$124M
r_{fi}=r_i	**Flexible Pricing**	$941M	21,810	33,308	193,091	248,209	$104M
c_f=515	**change**	**6.40%**	**-51.32%**	**11.84%**	**-20.54%**	**-21.85%**	**-16.66%**
Case 6	**Fixed Pricing**	$885M	51,200	23,383	243,014	317,596	$121M
r_{fi}=r_i	**Flexible Pricing**	$941M	24,664	30,885	192,607	248,156	$102M
c_f=505	**change**	**6.37%**	**-51.83%**	**32.08%**	**-20.74%**	**-21.86%**	**-15.64%**

Table 11: Profits and capacities as a the cost of installing and using flexible capacity decreases.

3.7 Uncertainty After Pricing and Production Decisions

Our basic postponed pricing model assumes that demand is known exactly at the time prices and production quantities are fixed. This is rarely the case in practice. Market research and early demand information may have been gathered by the time these decisions are made, resulting in a better understanding of demand and thus a great reduction in forecast error. Actual demand, however, has not yet been realized at that point. In this section, we evaluate the impact of the residual uncertainty surrounding demand after prices have been announced and/or production decisions have been made.

Capacity decisions typically need to be made early on under high demand uncertainty. The timing of production and pricing decisions, however, depends on the particular business situation under consideration. While a manufacturer that sells through catalogue will set prices in advance and has little flexibility to change them after the catalogues have been printed, an internet retailer can potentially change prices constantly as he/she observes demand (as airlines do). Production, similarly, has to be fixed months in advance in industries, such as fashion clothing, which are ruled by long supply lead times. Meanwhile, make-to-order manufacturers plan their production quantities as they receive orders from customers. As a result, there are different cases to consider regarding how much demand uncertainty is faced at the time these decisions are made. For that purpose, we split the uncertainty faced at the capacity investment stage into two parts. The first one will be observed at some middle stage and the second will not become known until demand is realized. Then we consider the different situations that may arise depending on how much information is available at the time of making each of the decisions, see Figure 5.

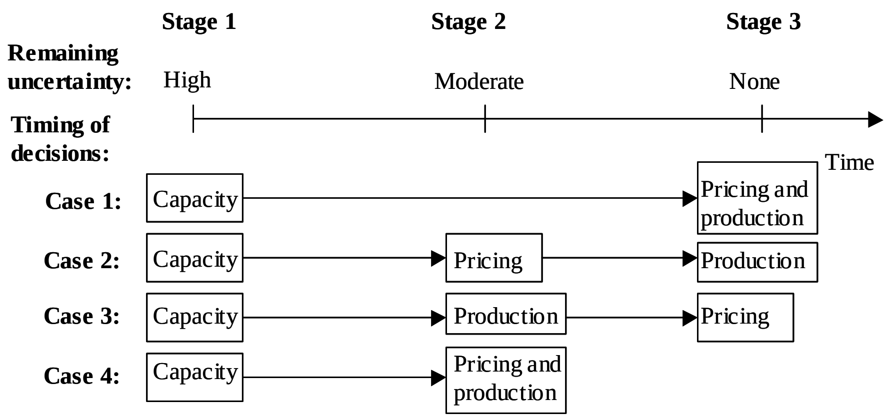

Figure 5: Timeline of decisions.

Observe that Case 1 is the one studied in depth in this paper. Each case needs to be compared with the fixed pricing alternative, where only two cases need to be considered: production under full information, as in the previous sections, and production under partial information. In what follows we investigate the impact of price postponement in these new cases where production and/or pricing decisions are made under partial information. For that purpose, we first model the problems mathematically and then perform an extensive computational study.

3.7.1 Mathematical Models

We keep the notation introduced in Section 2, except that now each probability scenario s has associated a number, T_s, of possible subscenarios. The scenarios represent the acquisition of some information and the subscenarios correspond to the actual demand realizations. We let t be the index of the subscenarios and u_{st}, $t=1,2,...,T_S$, be the probability of subscenario t happenning once scenario s has been realized. The decisions made under moderate uncertainty are made for each scenario s and thus indexed in s, while the decisions made after demand is realized are indexed in s and t since a potentially different one may be taken under each scenario and subscenario. The final demand for product i, given that scenario s was observed and subscenario t subsequently realized, is $\varepsilon_i^{st} - \alpha_i \mathrm{Price}_i$, where Price_i is the price of product i selected, i.e. P_i, P_i^s or P_i^{st} depending on whether the pricing decision is made under high, moderate or no uncertainty, respectively. We are now ready to formulate each of the cases described in Figure 5.

Case 2: Pricing under partial information – Production under full information.

Production quantities depend on both the scenario and subscenario taking place. We thus denote the production quantities of product i under scenario s and subscenario t as y_i^{st} and z_i^{st}, for the dedicated and flexible resources, respectively.

$$
\begin{aligned}
&Max \qquad \sum_{s=1}^{S}\sum_{i=1}^{n} q_s \left[\sum_{t=1}^{T_S} u_{st} \left(P_i^s \left(y_i^{st} + z_i^{st} \right) - r_i y_i^{st} - r_{fi} z_i^{st} \right) \right] - \sum_{i=1}^{n} c_i K_i - c_f K_f \\
&\text{subject to :} \\
&\qquad y_i^{st} \le K_i \qquad \forall i,s,t \\
&\qquad \sum_{i=1}^{n} z_i^{st} \le K_f \qquad \forall s,t \\
&\qquad y_i^{st} + z_i^{st} \le \varepsilon_i^{st} - \alpha_i P_i^s \qquad \forall i,s,t \\
&\qquad P_i^s, K_i, K_f, y_i^{st}, z_i^{st} \ge 0 \qquad \forall i,s,t
\end{aligned}
$$

Remaining cases: Production under partial information.

Since production no longer will necessarily coincide with sales, we need to introduce a new variable, V_i^{st}, that represents the number of units of product i sold under scenario s and subscenario t. We again use the variable Price_i to represent P_i, P_i^s or P_i^{st} depending on whether the pricing decision is made under high, moderate or no uncertainty, respectively. Observe that we have a non-concave maximization problem for each case. We solve these models using a non-linear optimization software with various initial values, including the solution to the original postponed pricing and fixed pricing models. Although we cannot guarantee a global optimal solution, the results illustrate how the value of postponed pricing decreases as more uncertainty in demand needs to be resolved after prices are fixed, and the effect of fixing production under partial information.

$$
\begin{aligned}
&Max \qquad \sum_{s=1}^{S}\sum_{i=1}^{n} q_s\left[\sum_{t=1}^{T_S} u_{st}\mathrm{Price}_{\mathrm{i}} V_i^{st} - r_i y_i^s - r_{fi} z_i^s\right] - \sum_{i=1}^{n} c_i K_i - c_f K_f \\
&\text{subject to :} \\
&\qquad y_i^s \le K_i \qquad\qquad \forall i,s \\
&\qquad \sum_{i=1}^{n} z_i^s \le K_f \qquad\qquad \forall s \\
&\qquad V_i^{st} \le \varepsilon_i^{st} - \alpha_i \mathrm{Price}_{\mathrm{i}} \qquad\qquad \forall i,s,t \\
&\qquad V_i^{st} \le y_i^s + z_i^s \qquad\qquad \forall i,s,t \\
&\qquad \mathrm{Price}_{\mathrm{i}}, K_i, K_f, y_i^s, z_i^s, V_i^{st} \ge 0 \qquad \forall i,s,t
\end{aligned}
$$

3.7.2 Computational Results

To make the experiments comparable with the previous ones, we fix the expected demand value and the forecast error observed at stage 1 (that is, the mean and standard deviation of the full set of demand subscenarios considered) to those in our original base case for Group A. We then consider various levels of information by varying the percentage of that uncertainty that is resolved in the second stage in the set {50%, 70%, 90%} as follows. First, we generate normally distributed scenarios with the same expected value but standard deviation equal to the corresponding percentage of the original one. These scenarios represent the updated demand forecasts available under partial information, i.e. at stage 2 under moderate remaining uncertainty. For each of the scenarios, we generate the subscenarios that will be realized in stage 3 using a normal distribution with expected value equal to the scenario value and a coefficient of variation that results in an overall forecast error identical to that in the original case (see Appendix for details). In both cases, we discretize the normal distributions as explained in the introduction of Section 3.

Case 2: Pricing under partial information – Production under full information.

Observe that the original fixed pricing and postponed pricing models considered in the paper correspond to the cases where 0% and 100% of the uncertainty, respectively, is realized at the time the pricing decision is made, while production decisions are made under full information. Thus, we can easily compare the new results with the previous ones. Table 12 below shows how profits get reduced while the amounts of total and flexible capacity required grow, as more uncertainty remains at the time the pricing decision is made. The value of price postponement gets progressively reduced as the amount of information acquired by the pricing stage decreases. The price adjustments made under imperfect information are not as effective in balancing supply and demand. As a result, more flexible capacity needs to be purchased to meet demand as uncertainty grows. Since production decisions are made under full information, capacity can be used more efficiently than pricing in this case. Observe, nonetheless, that even in the case where only 50% of the initial forecast uncertainty is realized before prices are set, the optimal amounts of flexible and total capacity under postponed pricing are still almost 18% and 3%, respectively, lower than under fixed pricing (case 0-100 in the table).

Uncertainty split btw stages 2-3	Profit	Flexible Capacity	capacity0	capacity1	Total Capacity	Capacity Investment
100-0	$940M	11,835	40,861	196,532	249,228	$115M
90-10	$919M	22,519	39,676	217,455	279,650	$130M
70-30	$902M	29,016	38,635	230,779	298,430	$138M
50-50	$892M	31,675	38,675	238,231	308,582	$143M
0-100	$883M	38,400	36,183	243,014	317,596	$148M

Table 12: Profits and capacities as the amount of information when prices are decided decreases.

Case 3: Production under partial information – Pricing under full information.

Table 13 below shows how profits, along with the amount of total and flexible capacity required, decrease as more uncertainty remains at the time the production decision is made while prices are adjusted after demand is observed. The same pattern holds true when prices are fixed at stage 1. The flexible and dedicated capacity installed is not used as effectively when production levels are fixed under uncertainty. As a result, the extra capacity is not as valuable and the optimal levels decrease with the level of uncertainty. Observe that the reduction in flexible capacity resulting from price postponement tends to grow with uncertainty. The reduction in the amount of total capacity purchased, however, decreases significantly as it becomes financially less risky to have extra capacity and production when prices can be adjusted later on. The value of price postponement to enhance profits increases drastically as the amount of information available at the time of production decisions are made diminishes. Adjusting prices under full demand information becomes increasingly important in order to improve the poor capacity and production decisions made under uncertainty.

	Uncertainty split btw stages 2-3	Profit	Flexible Capacity	capacity0	capacity1	Total Capacity	Capacity Investment
Postponed Pricing	100-0	$940M	11,835	40,861	196,532	249,228	$115M
	90-10	$938M	10,127	41,476	195,683	247,286	$114M
	70-30	$934M	6,894	42,719	193,675	243,288	$112M
	50-50	$932M	3,831	44,032	192,152	240,014	$110M
	0-100	$931M	-	45,783	191,813	237,596	$109M
Fixed Pricing	100-0	$883M	38,400	36,183	243,014	317,596	$148M
	90-10	$753M	28,233	36,013	222,103	286,349	$133M
	70-30	$677M	21,014	37,060	202,758	260,832	$121M
	50-50	$639M	14,612	36,404	191,795	242,810	$112M
	0-100	$615M	-	42,114	175,905	218,019	$100M
Change	100-0	6.49%	-69.18%	12.93%	-19.13%	-21.53%	-22.03%
	90-10	24.54%	-64.13%	15.17%	-11.90%	-13.64%	-14.06%
	70-30	38.01%	-67.19%	15.27%	-4.48%	-6.73%	-7.18%
	50-50	45.99%	-73.78%	20.95%	0.19%	-1.15%	-1.50%
	0-100	51.55%	0.00%	8.71%	9.04%	8.98%	8.97%

Table 13: Profits and capacities as the amount of information when production is decided decreases. Case 0-100 is an extreme case where the production decision is made at stage 1.

Case 4: Pricing and production under partial information.

Table 14 below shows how profits get markedly reduced while the amount of total capacity required decreases moderately as more uncertainty remains at the time both production and pricing decisions are made. The fixed pricing setting is identical to that in Case 3. The optimal flexible and dedicated capacities under price postponement are slightly lower than in that case since prices are adjusted only at the second stage, under uncertainty, and thus not as much profit can be made of the available capacity.

Again we observe that the amount of flexible capacity needed is drastically reduced as uncertainty grows, since it cannot be used as efficiently when production decisions are taken under uncertainty. As in Case 2 above, the benefits of postponed pricing in profits are significantly reduced as the amount of uncertainty under which the pricing decisions are made grows. Observe, however, that postponed pricing is more beneficial in this case where production decisions are made under partial information since the performance of the fixed pricing strategy also deteriorates as uncertainty increases.

	Uncertainty split btw stages 2-3	Profit	Flexible Capacity	capacity0	capacity1	Total Capacity	Capacity Investment
Postponed Pricing	**100-0**	$940M	11,835	40,861	196,532	249,228	$115M
	90-10	$796M	9,897	39,925	189,052	238,874	$110M
	70-30	$705M	6,614	40,159	182,809	229,582	$106M
	50-50	$656M	3,596	40,879	179,144	223,619	$103M
	0-100	$615M	-	42,114	175,905	218,019	$100M
Fixed Pricing	**100-0**	$883M	38,400	36,183	243,014	317,596	$148M
	90-10	$753M	28,233	36,013	222,103	286,349	$133M
	70-30	$677M	21,014	37,060	202,758	260,832	$121M
	50-50	$639M	14,612	36,404	191,795	242,810	$112M
	0-100	$615M	-	42,114	175,905	218,019	$100M
Change	**100-0**	6.49%	-69.18%	12.93%	-19.13%	-21.53%	-22.03%
	90-10	5.70%	-64.95%	10.86%	-14.88%	-16.58%	-16.98%
	70-30	4.14%	-68.53%	8.36%	-9.84%	-11.98%	-12.41%
	50-50	2.76%	-75.39%	12.29%	-6.60%	-7.90%	-8.24%
	0-100	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%

Table 14: Profits and capacities as the amount of information when both pricing and production are decided decreases. Case 0-100 is an extreme case where the production and pricing decisions are made at stage 1.

4. Conclusions

The study of the impact of pricing and demand forecast assumptions on dedicated and flexible capacity investment decisions at the planning stage has led us to the following conclusions:

By considering price postponement at the planning stage, the *projected* profit increases (6-10%) while the total capacity investment cost is greatly reduced (17-24%) in representative scenarios, assuming capacity investment costs increase linearly, i.e., no economies of scale are considered. The *actual* expected profit of the associated capacity decisions, accounting for appropriate price changes before product launch, increases by well over 2%. These effects are less pronounced as the costs of installing and using flexible capacity decrease.

Since price postponement can be used to balance demand with available supply at the tactical level, the benefits of flexible capacity to hedge against demand uncertainty are significantly reduced. Both the total capacity and the comparatively more expensive flexible capacity required decrease significantly. Clearly, this constitutes a significant improvement on RONA (*Return on Net Assets)*.

The overcapacity installed under the fixed pricing assumption results in discounts during the sales season and leads to lower expected prices, capacity utilization and profits.

The optimal capacity levels increase with the demand elasticity of the products. When demands are fairly inelastic, overcapacity would result in heavy discounting and/or idle capacity; if capacity is scarce, however, high premiums can be charged.

As the ratio of fixed to variable costs per unit increases: (i) expected profits decay slightly under postponed pricing and sharply under fixed pricing, (ii) the optimal investments in total capacity decrease, barely under postponed pricing and very steeply (over 30%) under fixed pricing, and (iii) the investment in the more expensive flexible capacity is reduced, greatly under postponed pricing and very mildly under fixed pricing. As a result, when capacity costs are very high, price postponement leads to increased investments in total capacity. This is because the flexibility to adjust prices lowers the risk of installing the costly capacity.

Price postponement is even more desirable when demands are positively correlated. The resulting profit gains decrease as correlation decreases since risk pooling effects make production flexibility a very effective lever to hedge against demand uncertainty and thus reduce the impact of postponing the pricing decision.

Price postponement results in much lower variability of the optimal production levels over the set of demand scenarios considered at the planning stage, and thus a better prediction of future production levels and component requirements. This should result in significant improvements in component capacity acquisition.

Finally, we observe that the benefits of postponed pricing are even higher when production is made under partial information but prices are adjusted under full information, since there is greater need for the pricing lever to balance supply and demand. The benefits of postponed pricing, however, decrease rapidly as the level of uncertainty under which the pricing decision is made grows. In that setting, the gains associated with price flexibility are actually higher when production is also determined under uncertainty because the performance of the fixed pricing strategy deteriorates further.

This research shows that the capacity investment decision is very sensitive to the pricing assumptions at the capacity planning stage. Accounting for the future price flexibility during the strategic planning stage results in higher projected profit, reduced investments in dedicated and flexible capacity (for representative scenarios) and lower production variability.

An interesting research question that arises from these results is whether or not there exists a set of fixed prices that would lead to the same capacity decisions, and consequently identical actual performance, as the postponed pricing model. The properties of such set of fixed prices for the current models and related models that may consider the firm as a price-taker at the pricing stage deserve further investigation.

5. Extensions

In this section, we address possible extensions to our models to incorporate demand diversions, uncertain price elasticities, price bounds, capacity updates and demand seasonality.

In our current models, consumer purchasing behavior is linearly affected by the price of the product. In practice, however, changing the prices of various products simultaneously necessitates much more complex consumer choice models with demand diversions being a major influence. Demand diversion is said to occur when a customer opts to buy an alternate product on finding that his first choice is unavailable or the utility of the alternative(s) is higher than the original choice. Unfortunately, incorporating demand diversion into our models results in losing the concavity of the profit function and the linearity of the demand constraints, and hence, in a much harder optimization problem (see Hanson and Martin (1996)). The difficulty of correctly modeling customer purchasing behavior and demand diversions is evidenced in research in this area (McFadden (1986), Berry, Levinson and Pakes (1995) and Swann (1999a)).

A simple linear model that includes an additional factor in the demand function to reflect customers diverting when the price of the other product is increased (i.e. $D_i = \varepsilon_i - \alpha_i P_i + \beta_j P_j$, as in Birge, Drogosz and Duenyas (1998)) enlarges the entire market size and has the effect that products become less sensitive to increases in both product prices. It thus results in unrealistic higher optimal prices and volumes for both products. This problem can be partially fixed by making sure that the demand volume at base prices remains constant (e.g. using $D_i = \varepsilon_i - \alpha_i P_i + \beta_j (P_j - P_j^{base})$). Observe, however, that the number of customers diverting from product i to j is independent of the price of j an d this effect still drives prices up. To avoid this pitfall, we could fix the overall market share lost when increasing each price and consider that an additional portion of customers will divert to the cheaper product. For instance, take $D_i = \varepsilon_i - \alpha_i P_i + \beta(P_j - aP_i)$, where a is the ratio between the base prices of the two products. The overall market share is $D_1 + D_2 = \varepsilon_1 + \varepsilon_2 - \alpha_1 P_1 - \alpha_2 P_2$, irrespective of the value of the crosselasticity factor β. The associated optimization model behaves well, and shows that profits would decrease as additional customers divert to the cheaper product.

While these simple models are by no means perfect representations of reality, they do help us draw one conclusion. A common result in the different models we have tested is the reduction of optimal flexible capacity investments when diversions are considered. Since demand can be shifted from one product to another through pricing, the need for flexible capacity diminishes.

We have considered the sensitivity of customers to prices as known, i.e. the demand slope, α, for each product is fixed, while the overall market size, ξ, for that product is a random variable. Uncertainty in the demand slope could be easily included in our models. However, this would lead to a wild increase in the number of scenarios, S, to consider within each model and would complicate the interpretation of the computational results. Since our conclusions are robust under a wide range of values of α, we chose not to consider this case.

In the proposed postponed pricing model, optimal prices may vary widely to make the best use of capacity over the different scenarios. In practice, however, competitive pressures would preclude us from setting prices too high and other considerations, such as customer's perception of quality, may deter us from setting it too low. Furthermore, the linear demand model does not appropriately reflect consumer preferences for very high or low prices; it is only a good representation of customer behavior around the base price. Thus, the postponed pricing model could be improved by adding appropriate upper and lower bounds on product prices. Unfortunately, this results again in a non-concave program. The unbounded problem studied here provides an upper bound on the gains obtained by using a postponed pricing policy under these additional constraints. We must point out that the deviations from the base price that we observed in our base case were below 15%.

In practice, a company may be able to react to a poor capacity decision by installing additional capacity at a later time, when demand forecasts are more accurate. However, increasing capacity at this later stage can only be done at a significantly higher cost. Partial postponement of the capacity investment may then be feasible and advantageous. The tradeoff between waiting to collect more demand information versus incurring lower capacity acquisition cost at the earlier stage needs to be studied. How much will the initial investment decisions be affected by considering the later expansion opportunity?

Finally, we would like to mention that our postponed pricing model can also be used to consider seasonality of demand. While it may not be practical to change prices on a day-to-day basis i.e. at the operational level, it would be of interest to fine-tune prices at the tactical level according to the seasonal demands. In that case, each scenario would represent a possible future demand curve associated with a particular season. The model then maximizes the expected profit across seasons within a modeling year, using delayed pricing to hedge against uncertainty as well as against the fluctuation in product demand from one season to another

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Appendix: Splitting variability to account for remaining uncertainty (Section 3.7).

We have two demand random variables: S which gets realized at the second stage and T which gets realized at the third stage. They are significantly correlated as S provides much information, i.e. resolves much of the uncertainty.

We would like the final demand scenarios, represented by T, to have the same expected value and variance over all the cases that we generate with different levels of informativeness of S. Observe that

$$\mathrm{Var}[T] = E[\mathrm{Var}[T|S]] + \mathrm{Var}[E[T|S]]$$

Let the coefficient of variation of the original (two-stage) model, where S had all the information, be denoted by c and the new reduced coefficient of variation for S, when some uncertainty remains, be denoted by c_1. We would like the coefficient of variation of T|S=s to be constant for every s with a certain value c_2 while keeping the coefficient of variation of T equal to c. Under these conditions, we have that:

$$\begin{aligned}
&\mathrm{Var}[T]= c^2\mu^2 \\
&E[\mathrm{Var}[T|S]]= E[c_2^2 S^2] = c_2^2 E[S^2] = c_2^2(\mu^2 + c_1^2\mu^2) \\
&\mathrm{Var}[E[T|S]]=\mathrm{Var}[S]=c_1^2\mu^2
\end{aligned}$$

Thus, c_2 must be such that,

$$\begin{aligned}
&c^2\mu^2 = c_2^2(\mu^2 + c_1^2\mu^2) + c_1^2\mu^2 \\
&\Rightarrow c^2 = c_2^2 + c_2^2 c_1^2 + c_1^2
\end{aligned}$$