Impact of Consumer Returns on Supply Chain Coordination

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Abstract

The aim of this paper is to understand the impact of consumer returns on the decision making process of a two-echelon supply chain. Currently, in practice, consumer returns are largely ignored when managers make decisions regarding order quantities and prices, as evidenced in the literature and in our interaction with industry. Consumer returns, however, have been reported to be considerably high, often in the range of 6% to 15%, and as high as 35%; they are predicted to exceed a record $220 billion in 2008. How does accounting for this growing factor affect ordering quantities, profits and supply chain coordination? Should all companies consider returns in their production plans? We seek to answer these questions for a buyer-vendor supply chain in which all consumer returns are inspected and salvaged by the vendor. We model both a price-only contract and a buy-back contract between the parties, and analytically study the properties of the optimal decisions. Buy-back contracts are known to fully coordinate the supply chain; we extend this result to the case where consumer returns are present, and find a bound on the minimum coordinating buy-back price. Further managerial insights are obtained through extensive computational testing. The results are counterintuitive: (1) higher profits and better coordination can be achieved when the players acting in a decentralized fashion do not consider any information about consumer returns as they make their pricing and ordering decisions, (2) retailer, manufacturer and total supply chain profits increase as the retailer faces a larger share of the logistics costs associated with consumer returns. We also show that although buy-back contracts coordinate the supply chain when consumer returns are considered, they may be detrimental to supply chain coordination when returns are ignored in the planning process.

1 Introduction

In an effort to attract buyers in a highly competitive marketplace, consumer return policies have been drastically relaxed in the last decade. Most mass merchandisers offer full refunds within $15-90$ days of purchase; no questions asked. As a result, return rates from consumers to manufacturers or retailers are high, often in the range of $6-11\%$ for mass merchandisers (Gentry (1999)). For electronic retailers, they are reported to be between $11-20\%$ (Lawton (2008)); for mail order companies and e-tailers, they can be as high as 35% (Gentry (1999)). Mostard and Teunter (2006) study the case of a mail order company where the return rates are generally around $35-40\%$ and can be as high as 75%. Retailers are expecting to receive almost \$220 billion in returns in 2008, the largest amount in record (Joseph Larocca, National

Retail Federation, NPR's Marketplace 11/13/08.) Previous reports show that the value of products returned in the United States exceeds $100 billion per year (Stock, Speh and Spear (2002)). Just in the electronics industry, around $13.8 billion was spent in 2007 to repackage, re-stock and re-sell returned products (Lawton (2008)). Processing returns is estimated to cost 2-3 times more than their outbound shipment, and to amount to $30-35$ per return for items purchased on the internet (Stock, Speh and Spear (2006)). Managing consumer returns effectively thus becomes essential to business profits. This is one of the goals of the new and growing field of Reverse Logistics (Rogers and Tibben-Lembke (1999); see Fleischmann, Bloemhof-Ruwaard, Dekker, van der Laan, van Nunen and van Wassenhove (1997) and Dekker, Fleischmann, Inderfruth and van Wassenhove (2004) for a review of quantitative research on Reverse Logistics). In practice, consumer returns are often handled on an ad hoc basis (Stock et al. (2006)), or by third party logistics providers (Hindo (2007)) since the operations involved suppose a major departure from the core manufacturing and forward logistics activities of the firms. As a result, they are typically not included in the forward supply chain planning activities.

In retail industries, a returned item is handled differently depending on the condition of the product and the relationship between the retailer and the manufacturer/vendor. If the item is in good condition, with no apparent damage, it will often go back to the shelf. However, if the manufacturer desires to keep high standards, the item will not return to the retail shelf until the manufacturer inspects the product. This may be a necessary step for products with high risk of liability, such as welding equipment. HP and Bosch follow this policy (Ferguson, Guide and Souza (2006)). In general, the returned product may follow different paths depending on multiple factors, such as shipping cost, risk of obsolescence, revenue margins, etc. A determinant factor is which player has more power along the supply chain. Large retailers normally transfer part or even all of the costs associated with the returned item to the manufacturer/vendor. In some cases, the item physically returns back to the manufacturer, who incurs most of the logistics costs associated with the return, and the retailer obtains a brand new item. In other cases, the item is not physically returned to the manufacturer; instead, the following two practices are commonplace: (1) the retailer decides what to do with the returned product and reports the cost incurred to the manufacturer, (2) the manufacturer gives the retailer a certain amount of return allowance credit and guidelines to properly dispose of the product (Corbett and Savaskan (2001)). According to our conversations with industry analysts, most vendors in the retail sector are willing to offer between 3% to 5% of the annual sales to the retailer as damage compensation in order not to lose good will. For this case where the retailer is fully in charge of the disposition of the returned product, Ruiz-Benitez and Muriel (2008b) model a return allowance credit contract and find that the supplier is better off offering a discounted wholesale price on the entire order quantity rather than a credit allowance proportional to the units sold. In the current paper, we focus on the vendor-buyer relationship in the former case, where returns go back to the manufacturer for repair, quality inspection, or simply for resale to liquidators and outlet stores (which typically purchase the product at only $10-20\%$ of their original value). As a result, vendor and buyer share the costs associated with returns.

We consider a two-echelon supply chain with a single manufacturer and a single retailer that faces stochastic demand and a certain percentage of consumer returns of a single product in a single period. The manufacturer sets a wholesale price for the product and may also set a repurchase price at which she will buy the product left at the retailer at the end of the selling season. We will refer to the return of unsold product from retailer to manufacturer at the end of the season as buy-back to avoid confusion with the central issue of consumer returns, which make their way back to the manufacturer via the retailer throughout the season. The retailer determines the order quantity at the start of the season, having no additional replenishment opportunity. Consumers return a fraction of the sold units to the retailer, who in turn sends them back to the manufacturer. These consumer returns do not make their way back to the supply stream during the sales period under consideration; they may be sold to off-price retailers or offshore buyers that reach a different market segment. Logistics costs related to consumer returns are incurred at both the retailer and manufacturer sites. Typically, a small percentage of this cost, corresponding to the handling of such returns, is incurred by the retailer. Meanwhile, the manufacturer faces the larger share of the cost, including transportation, inspection and remanufacturing or disposal of the product.

Our objective is to assess the impact that consumer returns have on the decision making process and on the coordination of the supply chain. For that purpose, we compare two decision policies: one in which consumer returns are considered prior to the calculation of the optimal decision variables, and a second one in which those consumer returns are ignored in the planning process but later observed with the corresponding impact on profits. This latter situation is common in practice as described by the study done by Bernon and Cullen (2007) on the management of product returns by UK high street retailers. As one can expect, the lower marginal profit and higher costs associated with consumer returns lead to lower ordering quantities. Regarding the coordination of the supply chain, however, the results obtained are counterintuitive: In general, higher profits and better coordination can be achieved when the players acting in a decentralized fashion do not consider any information about consumer returns as they make their pricing and ordering decisions. In extensive computational testing, this result holds true throughout, except in cases where profit margins are very low - close to the break even point.

The remainder of the paper is organized as follows. In Section 2, we review the existent literature and put our work in perspective. Section 3 introduces the model and assumptions. In Section 4, we analyze the centralized system where the supply chain is operated by a central planner and thus fully coordinated. When vendor and buyer act independently to maximize their own profits, we investigate the effect that consumer returns have on the coordination of the supply chain, first under no incentive schemes, analytically in Section 5 and computationally in Section 7.1, and then through buy-back contracts, analytically in Section 6 and computationally in Section 7.2. An extensive computational study, summarized in Section 7 and detailed in the Appendix B, includes sensitivity analysis with respect to the coefficient of variation of demand, the marginal revenues, the rate of consumer returns and the associated logistic costs, and shows the robustness of our results. Finally, Section 8, summarizes our findings and points out future research directions. The proofs of analytical results are detailed in the Appendix, as well as a full description of the computational study and results.

2 Literature Review

Supply chains require the collaboration of independent players who seek to maximize their own profits. The presence of returns adds one more dimension to the relation between vendor and buyer and underscores the need for coordination. Some incentives to coordinate, or at least improve, the supply chain can be offered through contracts. Supply chain contracts have become a growing field of study, in which special attention has been paid to whether or not the contract can coordinate the system. Extensive reviews on supply chain contracts and coordination literature can be found in Lariviere (1999), Tsay, Nahmias and Agrawal (1999) and Cachon (2003). Furthermore, Corbett and Savaskan (2001) review the state-of-the-art in supply chain contracts and coordination, and examine how the lessons learned change in the presence of a reverse supply chain.

Buy-back contracts as a tool to achieve supply chain coordination have been studied by several authors. Pasternack (1985) shows that neither a policy that does not allow for buy-backs nor one that allows for unlimited buy-back at full credit could be optimal. He also demonstrates that a policy that allows for unlimited buy-back at a partial credit will coordinate the supply chain when appropriate wholesale and repurchase prices are chosen. In this paper, we extend this result to the case where consumer returns are an important factor.

In the case of stochastic and price-dependent demand, Emmons and Gilbert (1998) show that for a given wholesale price, the offer to buy back the remaining inventory at the end of the season tends to increase the total combined profits of the retailer and manufacturer. Granot and Yin (2005) obtain closed form expressions for the optimal wholesale and repurchase prices for different expected demand functions: linear, negative power and exponential. However, Bernstein and Federgruen (2005) show that buy-back contracts cannot coordinate the supply chain when demand depends on retailer price. Similarly, when consumer returns are present in the system and considered in the decision making process, Ruiz-Benitez and Muriel (2008a) show that although buy-back contracts may improve supply chain coordination, they cannot fully coordinate the supply chain under price-dependent demand.

Coordination in supply chains with consumer returns has also been addressed in different settings. A review of the early work, along with interesting open research questions, can be found in Debo, Savaskan and Van Wassenhove (2003) and Corbett and Savaskan (2001). Savaskan and Van Wassenhove (2001) consider a supplier that sells to competing retailers and show that the flexibility of offering different compensation for returns to each retailer enables supply chain coordination in the case of non-identical retail markets. Ferguson et al. (2006) propose a target rebate contract to reduce false failure returns (i.e. returns of non-defective products) by increasing the retailer's sales effort. The profit improvement to both parties and the overall supply chain is substantial. More recently, Shulman, Coughlan and Savaskan (2008a) study the magnitude and effect of restocking fees in a competitive environment, and Shulman, Coughlan and Savaskan (2008b) explore the impact of channel structure (whether retailer or manufacturer salvage the returns) on restocking fees and profits in a bilateral monopoly. Su (2008) demonstrates that full returns (i.e., consumers obtain a full refund of the selling price) fail to optimize supply chain performance. He also studies the interaction between consumer returns and supplier buy-backs; he finds that buy-back contracts may induce retailers to implement excessively generous returns policies and recommends differentiating between unsold items and returned items, which is the setting we consider in our paper.

The centralized problem and the retailer problem within the decentralized supply chain that we study are simple single-stage newsvendor problems. The newsvendor problem taking into account product returns has been previously studied by Vlachos and Dekker (2003) and Mostard and Teunter (2006). The former consider that a constant proportion of products sold is returned and the returned products are either sold in secondary markets or resold only once during the selling season; they obtain optimal order quantities for the different return handling options. The latter consider the problem where each product has a certain probability of being returned and the returned products can be resold any number of times; they derive a closed-form expression to determine the optimal order quantity. The extension to multiple stages under centralized system management has also been thoroughly addressed in the literature. De Croix, Song and Zipkin (2005) consider a centralized multi-stage series system with returns (of both end products and components) under stationary stochastic demand over an infinite horizon. They show that a stationary echelon base-stock policy is optimal and, not surprisingly, returns lower the optimal base-stock level. Furthermore, De Croix and Zipkin (2005) show that an assembly system with only end product returns can be transformed to a series system, but this is not true in general when individual components may be returned/recovered. In a very different setting, using a single-stage, single-period model, Grasas, Alptekinoglu, and Akcali (2008) explore the interaction between resalable product returns and the product assortment decision of a retailer under both make-to-stock and make-to-order operational environments.

Along this paper, we will adopt the convention, as in Cachon (2003), that the upstream firm, in this case the manufacturer, is female and the accepting firm, in this case the retailer, is male.

3 Model and Assumptions

We consider the classical decentralized newsvendor model with a simple two-stage supply chain composed of a single manufacturer and retailer, and a single product. The manufacturer has unlimited production capacity and incurs a production cost of c. The retailer faces stochastic demand y with density distribution function $f(\cdot)$ and cumulative distribution function $F(\cdot)$, and has a single inventory replenishment opportunity. The selling price r is exogenously given and remains constant during the selling season. Unmet demand is lost, resulting in lost revenue; the model ignores costs associated with customer goodwill. The product that has not been sold by the end of the season has a salvage value of v.

We incorporate consumer returns into this classical model as follows. We assume that a certain percentage α of total units sold are returned to the retailer who gives the consumer a full refund and incurs a handling cost l_2 per unit returned. The manufacturer, in turn, gives the retailer a full refund of w and is in charge of the shipping, inspection, possibly remanufacturing and final disposition of the returned units at an average logistic cost of l_1 per unit. An average salvage value of v_r is expected for each of the returned units. This quantity could be estimated as the mean revenue obtained when appropriately disposing of the products returned by the consumer, which will of course vary greatly from unit to unit depending on its condition. Thus, the total cost per unit of product returned can be written

r :	exogenous product price;
c :	production cost;
v :	salvage value for unsold product at end of season;
w :	wholesale price;
s :	buy-back price;
Q :	retailer's order quantity;
α :	proportion of product returns;
l_1 :	logistics costs per product returned faced by manufacturer;
l_2 :	logistics costs per product returned faced by retailer;
l :	$= l_1 + l_2$;
v_r :	average salvage value per unit returned by consumers;
y :	product demand with density $f(\cdot)$ and cumulative distribution $F(\cdot)$.

Table 1: Notation Summary

as $r + l - v_r$, where $l = l_1 + l_2$ is the total logistic cost associated with the return of the product. The manufacturer and retailer shares of the cost associated with each returned unit are, respectively, $w + l_1 - v_r$ and $r - w + l_2$. We also consider the possibility for the manufacturer to buy back the unsold items at the retailer at the end of the selling season for a price s, in which case the manufacturer obtains the salvage value v for each unsold item. If no buy-back option is considered, the salvage value v is kept by the retailer. The full notation is summarized in Table 1.

The sequence of events is as follows: (1) Prior to the selling season the manufacturer sets the product wholesale price w, and under a buy-back contract also a repurchase price s for which she will buy any product left over at the retailer at the end of the selling season. (2) The retailer determines an order quantity Q. (3) Demand uncertainty y is realized, and sales $S = \min(Q, y)$, consumer returns $R = \alpha S$ and profits are observed.

Figure 1 depicts the financial flows in this simple two-stage supply chain.

To have a meaningful problem, we assume, as in the previous contracts literature, that $r > w > c$ and $w > s > v \geq v_r$. In addition, when taking consumer returns into account, the restrictions

$$(1 - \alpha)r > (1 - \alpha)w + \alpha l_2 \tag{1}$$

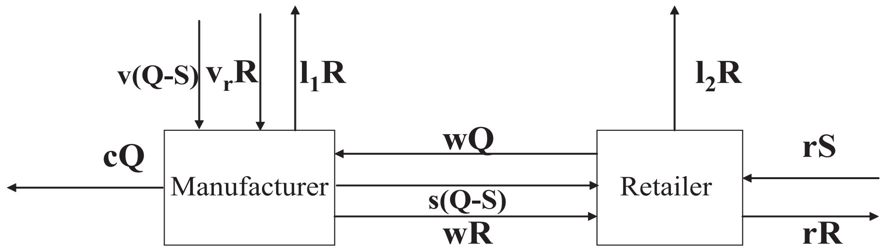

Figure 1: Financial Flows in the Manufacturer-Retailer System

$$(1-\alpha)w > c + \alpha(l_1 - v_r) \tag{2}$$

$$(1-\alpha)r > c + \alpha(l - v_r) \tag{3}$$

are needed to ensure positive profits for retailer, manufacturer and centralized system respectively.

We consider two different decision rules:

- **Policy IR**: Consumer returns are ignored in the calculation of the optimal decision variables for both, manufacturer and retailer, and only considered a posteriori to calculate respective profits;

- **Policy CR**: Consumer returns are included in the optimization model.

Policy IR represents the current situation in most companies, whereby planning occurs without regard to the fact that consumer returns will happen and these are handled later as an afterthought. Along this paper, we assume that both, manufacturer and retailer, follow the same policy, i.e., both consider returns in the decision making process or both ignore returns in the decision making process. The extension of our results to the asymmetric case, where one may consider and the other ignore returns, can be found in Schmid(2008); asymmetry achieves some but not all of the gains in supply chain coordination associated with ignoring returns, and results in a prisoner's dilemma for manufacturer and retailer.

Policy IR has been widely studied in the literature (see Pasternack (1985), Emmons and Gilbert (1998), Tsay(1998)). We will borrow the appropriate results and simply substract the costs associated with returns to compare its performance with that of a policy that explicitly considers consumer returns. In what follows, we focus our attention to the derivation of optimal decision variables and profits under Policy CR, and compare them to those under Policy IR. We use superscripts ^{CR} and ^{IR} to indicate the policy under consideration.

4 Centralized system

We first consider the centralized system, where the supply chain is operated by one central planner that seeks to maximize expected profits. Thus, the supply chain expected profit is:

$$\Pi^{CR}(Q) = (r - v - \alpha(r + l - v_r))E[S] - (c - v)Q \tag{4}$$

where $E[S]$ represents the expected sales ($E[S] = \left[\int_0^Q xf(x)dx + \int_Q^\infty Qf(x)dx\right]$).

Taking derivative of $\Pi^{CR}(Q)$ with respect to Q and setting this amount equal to zero we obtain the optimal order quantity:

$$Q^{CR*} = F^{-1}\Big(\frac{r - c - \alpha(r + l - v_r)}{r - v - \alpha(r + l - v_r)}\Big) \tag{5}$$

where $F^{-1}()$ is the inverse of the cumulative distribution function. This is a global maximum, since the second derivative with respect to Q is non-positive:

$$\frac{\partial^2 \Pi^{CR}(Q)}{\partial Q^2} = -f(Q)(r - v - \alpha(r + l - v_r)) \leq 0 \tag{6}$$

Furthermore, the second derivative with respect to Q is strictly negative if $f(\cdot) > 0$ in its domain, since $r - v - \alpha(r + l - v_r) > 0$ ($v < c$ and condition (3)); thus, the solution is unique.

In the optimization model, the marginal profit per unit of product sold is lower when taking into account consumer returns, i.e., under policy CR, than when ignoring them under policy IR. It is thus intuitive that the system will decrease the order quantity in order to maximize system-wide profits, as shown in the following result.

Proposition 4.1 *Under centralized management of the supply chain, the optimal order quantity weakly decreases when consumer returns are taken into account; that is,* $Q^{IR*} \geq Q^{CR*}$. *Furthermore, it strictly decreases for any demand distribution with strictly increasing cumulative distribution function.*

5 Decentralized system

We now turn our attention to the decentralized supply chain, where channel members are rational, act in their own best interests and seek to maximize their own respective expected profits. We present here the general model with buy-back incentives offered from manufacturer to retailer. The model with no incentives corresponds to setting the repurchase price s equal to the salvage value of leftover items, v. The retailer's expected profit is:

$$\Pi_R^{CR}(Q) = ((r - s - \alpha(r + l_2 - w))E[S] - (w - s)Q \tag{7}$$

where, as in the previous section, $E[S]$ represents the expected sales ($E[S] = \Big[\int_0^Q xf(x)dx + \int_Q^\infty Qf(x)dx\Big]$).

To ensure positive profits to the retailer we need condition (1). Taking derivative with respect to Q we obtain the first order condition:

$$\frac{\delta\Pi_R^{CR}}{\delta Q} = ((r - s - \alpha(r + l_2 - w)) \int_Q^\infty f(x)dx - w + s = 0 \tag{8}$$

Thus, the optimal order quantity is:

$$Q^{CR*} = F^{-1}\Big(\frac{r - w - \alpha(r + l_2 - w)}{r - s - \alpha(r + l_2 - w)}\Big) \tag{9}$$

Again, this quantity is a global maximum since the second derivative with respect to Q is non-positive:

$$\frac{\partial^2 \Pi_R^{CR}(Q)}{\partial Q^2} = -f(Q)((1-\alpha)r - \alpha(l_2 - w) - s) \leq 0 \tag{10}$$

In particular, the second derivative with respect to Q is strictly negative if $f(\cdot) > 0$ in its domain, since $(r - s - \alpha(r + l_2 - w)) > 0$ ($s < w$ and condition (1)).

As in the centralized system, we have the following result.

Proposition 5.1 *Given decentralized management of the system and a fixed wholesale price w, the optimal order quantity weakly decreases when consumer returns are taken into account; that is, $Q^{IR*} \geq Q^{CR*}$. Furthermore, it strictly decreases for any demand distribution such that the cumulative distribution function is strictly increasing.*

The proof is identical to the one of Proposition 4.1.

Observe, however, that the wholesale price is not an exogenous parameter. It will be appropriately set by the manufacturer to maximize her own profits and differ depending on whether or not returns are considered. The manufacturer's expected profit function under consumer returns is:

$$\Pi_M^{CR}(w, s; Q) = (w - c - s + v)Q - (\alpha(w + l_1 - v_r) - s + v)E[S]. \tag{11}$$

We focus on an equivalent formulation for the manufacturer, considering that she will correctly anticipate the retailer's order for any wholesale price using (9). Thus, the manufacturer faces the inverse demand curve: $w^{CR}(Q) = \frac{((1-\alpha)r-\alpha l_2)\bar{F}(Q)+sF(Q)}{1-\alpha\bar{F}(Q)}$, where $\bar{F}(x) = 1 - F(x)$. This generalizes the result in Lariviere and Porteus (2001) that shows that the inverse demand curve for the manufacturer is $w(Q) = r\bar{F}(Q)$ in the absence of consumer returns and buy-back incentives.

Corollary 5.1 *To achieve a certain order quantity, Q, the wholesale price offered by the manufacturer when consumer returns are considered is lower than when consumer returns are ignored, that is, $w^{CR}(Q) < w^{IR}(Q)$*

This result can be easily proven using Proposition 5.1 and the fact that the order quantities Q^{CR} and Q^{IR} are decreasing in w.

Intuitively, if both manufacturer and retailer consider consumer returns in the decision making process, the manufacturer has to charge a lower wholesale price in order to induce the retailer to order the desired value Q, compared to when they both ignore consumer returns, to compensate for the retailer's return costs.

When both manufacturer and retailer maximize their respective expected profit functions the relationship between both wholesale prices, for policy CR and policy IR, is not that clear. In fact, our computational study shows that the optimal wholesale price when consumer returns are considered may be higher or lower than when consumer returns are ignored, depending on different factors, such as logistics costs and marginal revenues of both parties. The optimal retailer order quantity, on the other hand, is always found to be lower under consumer returns. Unfortunately, accounting for consumer returns makes the joint determination of Q and w in closed form intractable, even in the simple case of uniform demand distribution[1].

[1]Please note the added complexity brought about by the fact that the wholesale price (1) appears both in the numerator and denominator of the expression for the optimal ordering quantity, and (2) multiplies the expected sales in the manufacturer's objective function under consumer returns.

Observe that the "double marginalization" effect (see the seminal work of Spengler (1950) and Pasternack (1985)) that drives the decentralized solution away from the system optimal is milder when consumer returns are considered: as in the original model, the retailer tends to order less because he perceives a higher cost, $w > c$, but now, as the retailer sees lower costs associated with returns, $r + l_2 - w < r + l - v_r$, the order quantity is closer to the one that maximizes system-wide profits. In the following section, we study how buy-back contracts can coordinate the supply chain in the presence of consumer returns. In Section 7 we resort to numerical experiments to analyze the manufacturer's decision and the induced supply chain performance.

6 Supply Chain Coordination through Buy-Back Contracts

As in Pasternack (1985), we study whether there exist transfer prices w and buy-back rebates s so that the supply chain is coordinated; that is, so that the ordering quantity chosen by the retailer Q satisfies both retailer profit maximization, $F(Q) = \frac{r-w-\alpha(r+l_2-w)}{r-s-\alpha(r+l_2-w)}$, and system-wide profit maximization, $F(Q) = \frac{r-c-\alpha(r+l-v_r)}{r-v-\alpha(r+l-v_r)}$. The following theorem extends the results in Pasternack (1985) and shows that unlimited buy-back at partial credit, s, leads to supply chain coordination under consumer returns for appropriate values of w and s. Interestingly, we find that there is a threshold value for the credit s offered by the manufacturer to the retailer below which no coordination is possible.

Theorem 6.1 *Under consumer returns and decentralized management of the two-stage supply chain, a policy that allows for unlimited buy-back at a partial credit s will lead to supply chain coordination for appropriate values of s and w. In particular, $\frac{\alpha}{1-\alpha}(c + l_1 - v_r) + v < s < r - \frac{\alpha}{(1-\alpha)}l_2$.*

The theorem implies that the policy of a manufacturer allowing no buy-back is system suboptimal, since coordination requires $s > \frac{\alpha}{1-\alpha}(c + l_1 - v_r) + v > v$. Unlimited buy-back for full credit, on the other hand, would lead the retailer to place orders for infinite amounts and not provide coordination either.

The different choices of $(w(s), s)$, with $w(s)$ given in (14) and $\dfrac{\alpha}{1-\alpha}(c+l_1-v_r)+v < s < r - \dfrac{\alpha}{(1-\alpha)} l_2$, represent the share of supply chain profits between manufacturer and retailer.

Note that the pair $s = \frac{\alpha}{1-\alpha}(c + l_1 - v_r) + v$ and $w = \dfrac{\alpha(l_1 - v_r) + c}{1-\alpha}$ corresponds to the case in which the manufacturer obtains zero profits and the retailer keeps all the supply chain profits. The other extreme, $s = w = r - \frac{\alpha}{(1-\alpha)} l_2$, corresponds to the case where the manufacturer fully assumes the risk and reaps all the profits of the supply chain. As pointed out by Pasternack, as s increases, so does w and the expected profits of the manufacturer.

Finally, observe that the set of coordinating contracts $(w(s), s)$ does not depend on the demand distribution, but the associated profits for retailer and manufacturer do.

7 Computational Study

The main objective of our computational work is to evaluate the impact of consumer returns on the optimal supply chain profits, ordering quantities, wholesale prices and buy-back rebates, as compared against those in the classical setting in which returns are overlooked in the decision making process.

We have performed an extensive computational study assuming normal demand distributions, as well as gamma demand distributions with a variety of parameters to cover a wide range of possible demand settings. Both, the gamma distribution and the normal distribution have an increasing generalized failure rate; thus, as proven by Lariviere and Porteus (2001), the manufacturer's profit function is well behaved and an optimum can be found.

For illustrative purposes, we will only explicitly show in this paper the results obtained for the normal demand distribution; the results obtained for the gamma demand distribution are consistent with those and can be found in the appendix.

In what follows, we assume a normally distributed demand, $y \sim N(\mu, \sigma)$, and for the base case, consider $\mu = 3$, and $\sigma = 0.75$ given a retail price of $r = 4$. The production cost is $c = 1$ and we set the salvage values v and v_r to zero. The manufacturer's and retailer's logistic costs per unit returned by the consumer are, respectively, $l_1 = 1.9$ and $l_2 = 0.1$. The manufacturer faces the largest share of the logistic cost, in this case 95% of the total logistic cost, and the retailer only a small percentage of it, 5%, corresponding to the handling of returns at the store level. In general, we refer to the retailer's percent share of the logistics cost as β, and will vary it from 5% to 95%; see Section 7.2.3.

As discussed in the introduction, the return rates from consumer to retailer are often in the range of 6% to 15% and they could exceed 35% in mail order companies and e-tailers. For illustrative purposes, we will assume the percent of returns $\alpha = 0.2, (20\%)$. Results are consistent for different values of α, but less accentuated for lower values of α.

In this section we report the results for the base case and some interesting sensitivity analysis. Unless otherwise specified, the parameters are set to the described base case in each of the sections below. Further computational experiments have been carried out for different parameter values and also support the qualitative conclusions presented below (for more details see Ruiz-Benitez (2007)).

To study coordination issues, we establish the Centralized System as the benchmark for the best performance that the supply chain may achieve. In this case, total supply chain profits obviously increase when considering consumer returns a priori in making the ordering decision. The magnitude of profit improvement as a function of the volume of consumer returns, α, and the total logistics cost associated, l is illustrated in Figure 2.

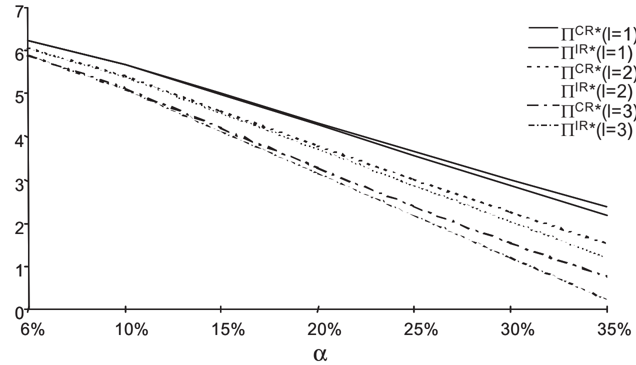

Figure 2: Expected profits in the Centralized System as a function of the returns volume (α) for different levels of the logistics cost (l), under policies CR and IR.

Under decentralized management of the system, however, the effect of incorporating consumer returns in the decision making process is not as predictable. In what follows, we focus on the decentralized case under both wholesale price and buy-back contracts.

7.1 Wholesale Price Contracts

We first consider the case where the manufacturer does not offer to buy back leftover inventory at the retailer, that is, the parameter s is set to v. For different wholesale prices, the retailer calculates the order quantity in order to maximize his own expected profit. Figure 3 shows the optimal decision variables and the expected manufacturer, retailer, and system-wide profits, respectively, for both policies, CR and IR, as a function of the wholesale price.

We observe that the optimal ordering quantity for each wholesale price is lower when consumer returns are taken into account in the decision making process, as proven in Proposition 5.1.

For low values of w, the manufacturer's profits are negative and considering returns, i.e., policy CR, may slightly enhance the manufacturer's profits. After a certain threshold however, the manufacturer is significantly better off when the retailer ignores consumer

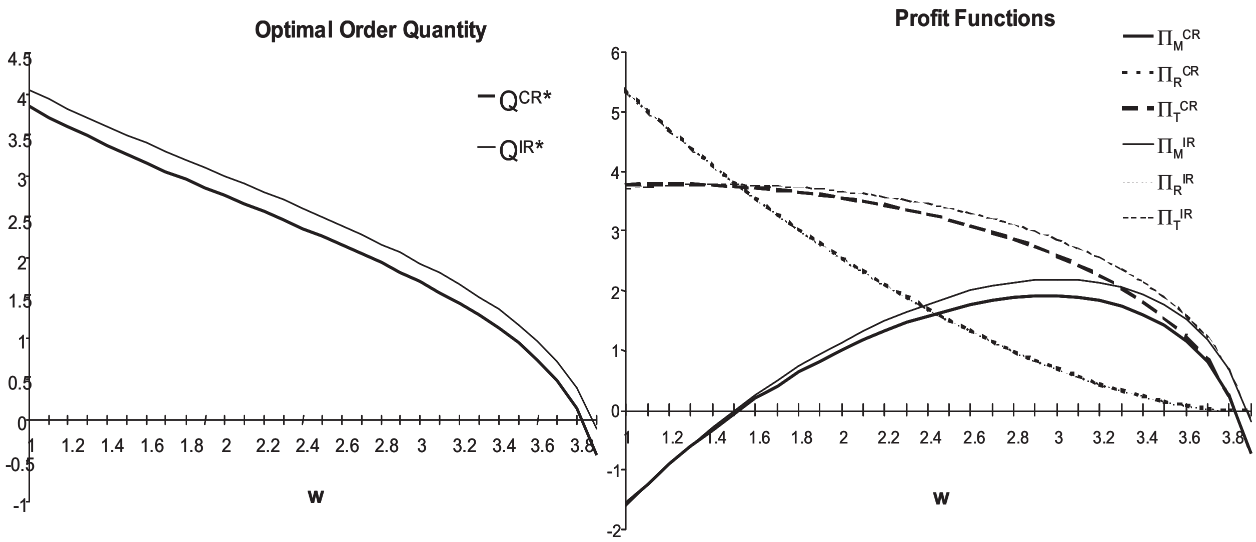

Figure 3: Optimal decision variables and division of profit when no repurchase price is offered

returns in making his ordering decision. On the other hand, given a certain wholesale price w, the retailer will always be better off by including consumer returns and optimizing the more accurate representation of the situation he faces. The difference in retailer profits is very small, though. As a result, the system-wide expected profits are generally higher when overlooking consumer returns in the decision making process. This is because *the effect of double marginalization is reduced* as the retailer sees a higher profit margin and consequently orders a higher quantity when consumer returns are ignored.

The optimal profits and order quantities under both centralized and decentralized management of the system for the base case parameters are presented in Table 2 for each of the policies, that is, considering and ignoring consumer returns in the decision making process. In the decentralized case, the transfer or wholesale price is also reported. Interestingly, once the manufacturer optimizes her wholesale price, *both the manufacturer and the retailer are better off by ignoring consumer returns.* We observe that under both centralized and decentralized management of the system the optimal ordering quantity decreases when consumer

		Policy (CR)	Policy (IR)
Centr.	Q^*	3.27	3.5
	Π	4.61	4.58
Decentr.	Q^*	1.97	2.16
	w^*	3.56	3.47
	Π_R	0.54	0.68
	Π_M	2.92	3.07
	Π_T	3.46	3.75

Table 2: Optimal solution in the base case

returns are taken into account. This result was proven theoretically under centralized management of the system in Proposition 4.1, but it is difficult to extend to the decentralized case. Note that the optimal wholesale price is not the same when considering versus ignoring consumer returns. Furthermore, our computational study shows that the optimal wholesale price may be greater or lower when consumer returns are considered versus ignored, depending on parameters such as the logistics costs and the ratio between the production cost and the retailer price (see Table 4). The optimal ordering quantity, however, is always lower when considering returns in the wide range of computational settings tested, as one would expect, and we conjecture that this result is true in general.

In conclusion, we arrive to a counterintuitive result for the decentralized system: *in the base case, the supply chain is better coordinated and all parties better off when consumer returns are overlooked in the decision making process.*

7.2 Sensitivity Analysis

The natural question to address at this point is whether the counterintuitive result found in analyzing the base case is true in general and/or determine under what conditions it holds. For that purpose, we analyze the sensitivity of the optimal decision variables and profits with respect to demand uncertainty, marginal revenues, share of logistics costs incurred by retailer and manufacturer, and return volumes. Unless noted otherwise, the parameters used for the following computational studies are those of the base case.

7.2.1 Impact of the Coefficient of Variation

First, we study how the coefficient of variation ($CV = \sigma/\mu$) of demand impacts the optimal decision variables and the respective expected profits. Lariviere (1999) (Theorem 5, pg.244) proves that if demand is normally distributed, the optimal wholesale price is determined by the coefficient of variation and the manufacturer charges more the smaller the CV is. We observe that the same result holds when consumer returns are taken into account.

In Table 3 we show the optimal wholesale price, order quantity and expected profits for increasing variance, σ^2. We also report the percent difference between policies CR and IR. For high values of σ^2 we use a truncated normal distribution to avoid negative values of the demand.

We observe that, for both policies, the manufacturer decreases the wholesale price as the variance increases. This result is consistent with the results in Lariviere and Porteus (2001), where consumer returns are not present. The expected profits for both agents, retailer and manufacturer, increase since the actual mean of the truncated normal demand, μ_T, increases significantly as the variance increases and the actual variance of the truncated normal, σ_T^2, increases slower than the original variance, σ. The mean and variance of the truncated normal demand are calculated as follows:

$$\mu_T = \mu + \sigma \frac{\exp(-\mu^2/2\sigma^2)}{(1-F(0))\sqrt{2\pi}} \tag{12}$$

$$\sigma_T^2 = \mu^2 + \sigma^2 + \mu\sigma \frac{\exp(-\mu^2/2\sigma^2)}{(1-F(0))\sqrt{2\pi}} - \mu_T^2 \tag{13}$$

Interestingly, the higher the coefficient of variation, the higher the profit gains associated with ignoring consumer returns, for all parties. The manufacturer observes up to 14.92%

	Policy CR					Policy IR					% Diff.		
σ/μ	w*	Q*	Π_M	Π_R	Π_T	w*	Q*	Π_M	Π_R	Π_T	Π_M	Π_R	Π_T
0.25	3.56	1.97	2.93	0.54	3.47	3.47	2.16	3.07	0.68	3.76	4.79	20.57	7.65
0.50	3.08	1.78	2.05	0.80	2.85	2.95	2.12	2.28	0.98	3.25	10.03	18.30	12.51
0.75	2.88	1.84	1.88	0.90	2.79	2.73	2.31	2.15	1.12	3.27	12.62	19.09	14.83
1.00	2.79	2.02	1.94	1.03	2.97	2.63	2.60	2.25	1.28	3.53	13.69	19.59	15.83
1.25	2.75	2.23	2.09	1.15	3.24	2.58	2.92	2.44	1.44	3.88	14.29	20.50	16.60
1.50	2.73	2.46	2.27	1.27	3.54	2.55	3.26	2.66	1.62	4.28	14.66	21.53	17.25
1.75	2.71	2.72	2.47	1.41	3.89	2.53	3.62	2.91	1.80	4.70	14.89	21.38	17.37
2.00	2.70	2.97	2.69	1.55	4.23	2.51	4.00	3.16	1.99	5.15	14.92	22.43	17.83

Table 3: Optimal decisions and profits as the variance, σ^2, of the normal demand distribution $N(\mu = 3, \sigma^2)$ truncated at zero increases

profit improvement, the retailer up to 22.43% and the supply chain 17.83%, when we vary the standard deviation of the original normal distribution with mean 3 from 0.75 to 6.

7.2.2 Impact of Marginal Revenues

In this section we evaluate the extent to which the marginal revenues of the supply chain affect its performance. In particular, we carry out computational studies for different values of the cost to price ratio, c/r, and the logistics costs, $l = \{0.5, 1, 2\}$. In the computational study we set $c = 1$ and vary $r = \{20, 10, 4, 2, r_{min}\}$. Observe that the retailer price r must be above a certain threshold, r_{min}, in order to obtain positive supply chain expected profits (condition (3)), and this threshold depends on the logistics cost, l; for instance, in the case of $l = 2$, we need $r > 1.75$. In this example we consider $r_{min} = \frac{c+\alpha(l-v_r)}{1-\alpha} + 0.01$. The results obtained for policy CR, policy IR and centralized system when consumer returns are considered are presented in Table 4.

Observe that it is only when the profit margin is very large that the manufacturer lowers her transfer price when considering returns, despite facing higher costs, to keep the retailer's order quantity high and not miss her fat profit margin per unit (observe that the manufacturer keeps the larger share of the profit margin by far). Still, the order quantity and total

	Policy CR					Policy IR					CENTR.	
c/r	w*	Q*	Π_M	Π_R	Π_T	w*	Q*	Π_M	Π_R	Π_T	Q	Π
Logistics costs = 2												
1/20	16.54	2.19	26.25	4.98	31.23	16.56	2.29	27.43	4.92	32.35	4.14	42.3
1/10	8.43	2.14	11.62	2.19	13.81	8.38	2.26	12.16	2.26	14.43	3.84	18.5
1/4	3.56	1.97	2.92	0.54	3.46	3.47	2.16	3.07	0.68	3.75	3.27	4.61
1/2	1.93	1.43	0.23	0.042	0.28	1.83	1.97	0.18	0.17	0.35	2.27	0.37
1/1.76	1.73	0.75	0.004	0.001	0.005	1.63	1.91	-0.12	0.11	-0.01	1.19	0.01
Logistics costs = 1												
1/20	16.51	2.20	26.6	5.06	31.74	16.56	2.29	27.8	4.94	32.8	4.15	42.9
1/10	8.4	2.16	12.0	2.26	14.30	8.38	2.26	12.5	2.28	14.8	3.85	19.1
1/4	3.53	2.01	3.31	0.60	3.925	3.47	2.16	3.47	0.70	4.17	3.32	5.18
1/2	1.9	1.64	0.55	0.09	0.643	1.83	1.97	0.55	0.19	0.74	2.58	0.84
1/1.51	1.49	1.01	0.002	0.004	0.007	1.42	1.83	-0.09	0.08	-0.005	1.19	0.007
Logistics costs = 0.5												
1/20	16.49	2.20	26.8	5.10	31.9	16.56	2.29	28.0	4.95	33.0	4.15	43.2
1/10	8.38	2.16	12.2	2.31	14.5	8.38	2.26	12.7	2.29	15.0	3.86	19.4
1/4	3.51	2.03	3.51	0.65	4.16	3.47	2.16	3.67	0.71	4.38	3.35	5.46
1/2	1.88	1.73	0.71	0.13	0.84	1.83	1.97	0.74	0.20	0.94	2.68	1.09
1/1.38	1.37	0.68	0.002	0.0007	0.002	1.31	1.74	-0.07	0.06	-0.008	1.01	0.003

Table 4: Optimal decision variables and profits for different ratios c/r and logistics costs

supply chain profits are higher when ignoring returns. In further computational studies (see Ruiz-Benitez (2007)) we find that, in addition, the manufacturer's cost of returns must be sufficiently low (i.e., α and l_1 relatively low) for the manufacturer's transfer price to decrease as the volume of consumer returns increases. In the case where the retailer bears most of the logistics cost (i.e. $\beta = 0.95\%$, as studied in the next section), however, the transfer price w always decreases as the volume of returns increases to offset the higher costs faced by the retailer. It is thus always lower under policy CR in this case. In all other cases, the manufacturer needs to raise the wholesale price in order to account for her larger return costs. When profit margins are very low, the manufacturer's profits when ignoring returns become negative, since the ignored costs of return are very high relative to her profits; although the retailer still benefits when all ignore returns, his portion of profits are rather small and not sufficient to bring the supply chain profits above those achieved when considering returns.

We now compare the performance of policy CR versus IR. Table 5 reports the percent difference in expected profits between the two policies; that is, $\frac{(greatervalue)-(smallervalue)}{(greatervalue)} * 100$. The coordination gap is also reported in the table. We calculate such quantity as follows:

$$\frac{(\text{Benchmark}) - (\text{Total expected profits for policy IR or CR})}{(\text{Benchmark})} * 100$$

where (Benchmark) corresponds to the expected profits for the centralized system considering consumer returns, that is, the best the supply chain can do. For most of the cases, the expected profits under policy IR are greater than those under policy CR. But there are some instances where policy CR results in greater expected profits, i.e., better coordination, than policy IR and those instances are highlighted in bold. The values that are omitted in the table correspond to the instances where the expected profit under policy IR is negative. We observe different behavior for the intermediate versus the extreme marginal revenue instances, which leads us to think that the optimal use of policy CR versus policy IR will depend on the marginal revenues for the industry in question. We draw the following conclusions:

- The percent difference between policies increases as the marginal revenue decreases.
- When marginal revenue is very low, ignoring returns may lead to negative profits for the manufacturer and for the entire system. In contrast, the lower the revenues, the higher the profit improvement the retailer obtains from ignoring returns.
- When the marginal revenue is high enough, however, the retailer is the one that benefits from considering returns. Intuitively, this results from the fact that when the system margins are high, the manufacturer margins are also high and she can set a lower wholesale price under policy CR (versus IR) to offset the higher costs observed by the retailer.
- For the remaining intermediate instances, retailer and manufacturer are better off

	Difference IR vs. CR (%)			Coordination Gap(%)		
c/r	Π_T	Π_M	Π_R	IR	CR	Diff.
Logistics costs = 2						
1/20	3.464	4.314	**1.257**	23.56	26.21	2.64
1/10	4.296	4.496	3.221	22.35	25.69	3.33
1/4	7.65	4.789	20.57	18.66	24.88	6.22
1/2	21.88	**21.53**	75.68	4.276	25.22	20.94
1/1.76	**-**	**-**	98.9	-	23.8	**-**
Logistics costs = 1						
1/20	3.218	4.234	**2.445**	23.58	26.04	2.45
1/10	3.82	4.34	0.957	22.45	25.41	2.96
1/4	6.035	4.563	13.33	19.37	24.24	4.86
1/2	13.88	1.256	50.61	11.35	23.65	12.30
1/1.51	**-**	**-**	94.5	-	5.178	**-**
Logistics costs = 0.5						
1/20	3.109	4.193	**2.941**	23.6	25.97	2.37
1/10	3.528	4.26	**0.542**	22.5	25.23	2.73
1/4	5.087	4.396	8.654	19.69	23.78	4.08
1/2	10.01	3.146	35.3	13.69	22.33	8.63
1/1.38	**-**	**-**	98.92	-	18.56	**-**

Table 5: Percent difference between policies and coordination gap

ignoring returns and thus, the coordination of the supply chain is improved when consumer returns are ignored.

- As the marginal revenue decreases, the coordination gap decreases mildly when considering returns, but very steeply when ignoring them.

In summary, for moderate marginal revenues, ignoring returns results in profit improvements for all parties. When the marginal revenues are very high, however, the retailer is better off considering returns, since the high-premium manufacturer would lower the wholesale price to offset the retailer's return costs. When marginal revenues are very low, on the other hand, the manufacturer cannot afford to ignore the costs associated with returns, since it would result in negative profits.

7.2.3 Impact of Share of Logistics Costs and Return Volumes

Clearly, the impact of consumer returns is driven by the volume of returns, α, and their associated cost, l. In addition, how this cost is shared between manufacturer and retailer, that is, the relative magnitude of l_2 versus l_1, will play a key role in the optimal decisions of each party and thus in supply chain coordination. To evaluate these effects, we now analyze the sensitivity of our results to the percent share of returns logistics costs faced by the retailer, which we denote by $\beta = \frac{l_2}{l_1+l_2} \times 100$, and to the percent volume of consumer returns, α. For the instances studied in this section we will focus on the cases of intermediate marginal revenues. In particular, we will consider the base case parameters: $c = 1$, $r = 4$ and $l = l_1 + l_2 = 2$.

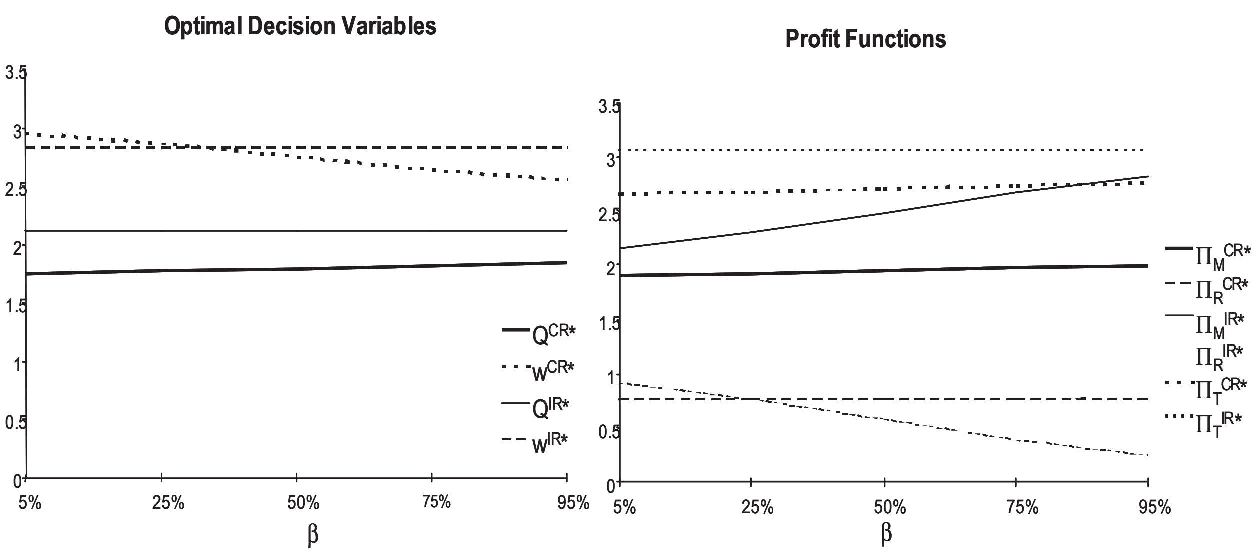

Figure 4: Optimal Order Quantity and Respective Profits for different percentage of logistics costs associated with consumer returns faced by the retailer

The optimal transfer prices and ordering quantities remain constant as we change β when consumer returns are ignored. Counter to intuition, *when consumer returns are considered, the optimal order quantity increases as the retailer faces higher returns logistics costs*; see Figure 4. The reason is that the manufacturer more than compensates for the increase in logistics costs by offering a lower wholesale price. Consequently, *the manufacturer ends up bearing the lion share of the returns logistics cost regardless of where they physically occur.* Interestingly, *the total supply chain profits improve (slightly) as the share of logistics costs born by the retailer increases*: The supplier observes lower costs and offers a significantly lower wholesale price. The retailer sees a lower purchase cost and orders a larger quantity. As a result, the supply chain is better coordinated and both parties slightly improve their profits. Observe, however, that the supply chain is better coordinated if both parties ignore returns in all the cases.

Figure 5 shows manufacturer, retailer, and system-wide profits as a function of the percent of consumer returns α for the extreme cases $\beta = 0.05$ and $\beta = 0.95$ for the base case under consideration. We observe that:

- If the manufacturer bears most of the logistics costs associated with the consumer returns ($\beta = 5\%$), then both manufacturer and retailer are better off when ignoring consumer returns.

- If the retailer bears most of the logistics costs associated with consumer returns ($\beta = 95\%$), the manufacturer is better off if both parties ignore consumer returns in the decision making process, but the retailer is better off if both parties consider consumer returns in the optimization process.

- System-wide profits are always enhanced when ignoring consumer returns.

7.3 Buy-Back Contracts

As mentioned previously, Pasternack (1985) proves that there exists a family of buy back contracts for which the system is coordinated. These contracts allow for unlimited returns at partial credit s and are defined by the parameters (w, s), where each of the values of the pair

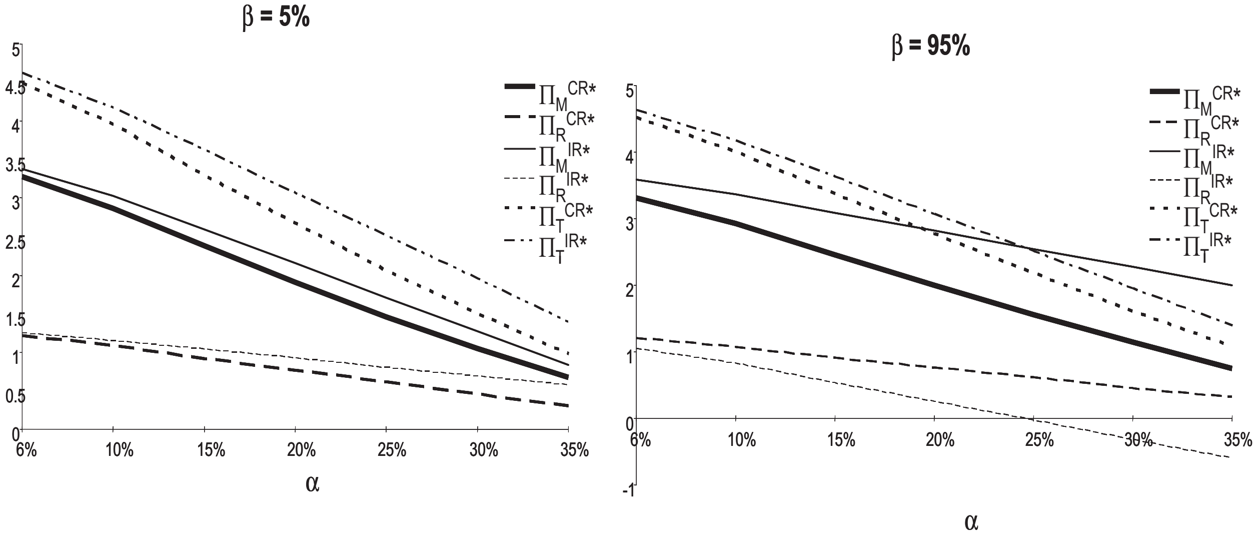

Figure 5: Profit functions for different percentages of consumer returns and for $\beta = 5\%, 95\%$

(w, s) leads to a different division of the total supply chain profits between manufacturer and retailer. When consumer returns are considered, this result still holds and, in particular, we have proven in Theorem 6.1 a condition over the repurchase price s to ensure coordination.

	Policy CR	Policy IR
w	2.6	2.5
s	2	2
Π_R	2.82	3.09
Π_M	1.79	1.48
Π_T	4.61	4.58

Table 6: Coordinating buy-back contracts

In table 6 we compare, for a fixed repurchase price s, the wholesale price that coordinates the supply chain and the associated profits under policies CR and IR, for the base case. Not surprisingly, Policy CR leads to higher system-wide profits under buy-back contracts, since the supply chain, including all existing costs, is coordinated for these values of (w, s).

Next, we evaluate the performance of buy-back contracts when ignoring returns. For that purpose, we compare the supply chain profits when using a wholesale contract versus a coordinating buy-back contract in a decentralized setting under policy IR. Note that the centralized solution using policy IR provides the total supply chain profits for coordinating contracts, such as the buy-back contract. If the centralized IR solution leads to lower total profits, there exist values of w and s in which both manufacturer and retailer are worse off. Table 7 shows that when consumer returns are significant in volume and/or cost, that is, $\alpha(r+l-v_r)$ is high relative to the profit margin of the product, a buy-back contract designed without consideration to these costs will be detrimental to supply chain coordination and result in lower supply chain profits (after the costs of returns are incurred).

8 Conclusions

We have studied supply chain coordination in a vendor-buyer system facing stochastic demand and consumer returns. Retail price is fixed exogenously, as would be the case under perfect competition. We have compared two decision policies, one that considers consumer returns in the calculation of the decision variables of both manufacturer and retailer, policy CR, and a second one that represents the current practice of ignoring consumer returns in the optimization model, but observing them a posteriori with the corresponding impact on profits, namely policy IR.

Our numerical analysis leads to counterintuitive results:

1. Better coordination of the supply chain can be achieved when the players acting in a decentralized fashion do not consider any information about consumer returns as they make their pricing and ordering decisions.

2. When consumer returns are accounted for, retailer, manufacturer and total supply chain profits increase as the retailer faces a larger share of the logistics costs associated with consumer returns.

	Policy IR						
	Wholesale price contract					Coordinated Sol.	
c/r	w^*	Q^*	Π_M	Π_R	Π_T	Q^*	Π_T
$\alpha = 20\%$							
Logistics costs=2							
1/4	3.47	2.164	3.075	0.681	3.755	3.506	4.581
1/3	2.65	2.106	1.597	0.423	2.021	3.323	2.347
1/2	1.83	1.971	0.187	0.172	0.359	3	0.241
Logistics costs=1.5							
1/4	3.47	2.164	3.275	0.691	3.967	3.506	4.870
1/3	2.65	2.106	1.793	0.433	2.227	3.323	2.630
1/2	1.83	1.971	0.372	0.181	0.553	3	0.511
Logistics costs=1							
1/4	3.47	2.164	3.476	0.702	4.178	3.506	5.159
1/3	2.65	2.106	1.990	0.444	2.433	3.323	2.914
1/2	1.83	1.971	0.556	0.191	0.747	3	0.781
$\alpha = 10\%$							
Logistics costs=2							
1/4	3.47	2.164	4.210	0.814	5.024	3.506	6.314
1/3	2.65	2.106	2.536	0.516	3.052	3.323	3.764
1/2	1.83	1.971	0.912	0.224	1.136	3	1.321
Logistics costs=1.5							
1/4	3.47	2.164	4.310	0.819	5.129	3.506	6.4582
1/3	2.65	2.106	2.634	0.521	3.155	3.323	3.906
1/2	1.83	1.971	1.004	0.229	1.233	3	1.456
Logistics costs=1							
1/4	3.47	2.164	4.411	0.824	5.235	3.506	6.6026
1/3	2.65	2.1061	2.732	0.526	3.259	3.323	4.048
1/2	1.83	1.9708	1.096	0.234	1.330	3	1.591

Table 7: Wholesale price contract vs. coordinated solution under Policy IR, for $\beta = 5\%$ and different returns logistics costs, l, volumes of returns, α, and ratios, c/r.

For a fixed wholesale price, the retailer is clearly better off when considering consumer returns in the decision making process, but once the manufacturer also sets her wholesale value to maximize profits, both parties tend to be better off by ignoring them in their optimization. This is because the double marginalization effect gets reduced, as the order quantity is always larger when consumer returns are ignored. The optimal manufacturer's wholesale price, on the other hand, may be higher or lower for one policy versus the other depending on the logistics costs, the volume of returns and the ratio between the production cost and the retailer price. Interestingly, when the share of the returns logistics costs faced by the retailer increases, the optimal strategy for the manufacturer is to significantly reduce the wholesale price; this results in better coordination and higher profits for all parties.

We have also proven that there exists a set of buy-back contracts that achieve coordination of the supply chain when consumer returns are considered in the decision making process and have obtained a condition for the manufacturer's repurchase price to ensure coordination. If product returns are ignored in the planning process, however, introducing a buy-back contract may result in lower profits to all parties, since the retailer order quantity may grow well beyond its system-optimal level.

Further research is being conducted to understand how consumer returns impact the decentralized supply chain under price-dependent demand, where the pricing decision may be taken before or after observing the random variable (see Ruiz-Benitez and Muriel (2008a)). An interesting further extension considers the expected proportion of consumer returns to be a function of price (see Schmid (2008)). Finally, the common practice of offering credit allowances to the retailer in settings where the retailer is fully in charge of the disposal of the returned product also deserves attention to ensure coordination of the supply chain in the presence of consumer returns (see Ruiz-Benitez and Muriel (2008b)).

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10 Appendix A

This appendix details the mathematical proofs of the propositions and theorems presented in the paper.

Proof of Proposition 4.1. The optimal order quantity for each of the policies considered can be expressed as:

$$Q^{IR*} = F^{-1}\Big(\frac{r-c}{r-v}\Big) \text{ and } Q^{CR*} = F^{-1}\Big(\frac{r-c-\alpha(r+l-v_r)}{r-v-\alpha(r+l-v_r)}\Big)$$

where F^{-1} is the inverse of the cumulative distribution function. Clearly,

$$\frac{r-c}{r-v} > \frac{r-c-\alpha(r+l-v_r)}{r-v-\alpha(r+l-v_r)}$$

Thus, since F is (strictly) increasing, F^{-1} is (strictly) increasing, and therefore, $Q^{IR*} \geq Q^{CR*}$ ($Q^{IR*} > Q^{CR*}$). ∎

Proof of Corollary 5.1. For the case with buy-back incentives we have $w^{IR}(Q) = r\bar{F}(Q) + sF(Q)$.

Therefore, for a fixed Q,

$$\frac{((1-\alpha)r - \alpha l_2)\bar{F}(Q) + sF(Q)}{1-\alpha\bar{F}(Q)} < r\bar{F}(Q) + sF(Q) \qquad \Leftrightarrow^{\bar{F}(Q)>0}$$
$$((1-\alpha)r - \alpha l_2)\bar{F}(Q) + sF(Q) < (1-\alpha\bar{F}(Q))(r\bar{F}(Q) + sF(Q))$$

After some calculations and simplifications, and since $F(Q) > 0$ and $\bar{F}(Q) > 0$, the expression above is equivalent to:

$$(r-s)\alpha F(Q) > -l_2\alpha$$

Since this inequality is always satisfied, we conclude $w^{CR}(Q) < w^{IR}(Q)$. ∎

Proof of Theorem 6.1. Substituting the expression of the optimal order quantity for the centralized system (5), in the derivative with respect to Q of the retailer's expected profit, equation (8), we get:

$$((1-\alpha)r - \alpha(l_2 - w) - s)\Big(1 - \frac{(1-\alpha)r - \alpha(l - v_r) - c}{(1-\alpha)r - \alpha(l - v_r) - v}\Big) - w + s = 0$$

Doing some algebraic transformations, we find that a wholesale price w that coordinates the system must satisfy:

$$w = \frac{(r(1-\alpha) - l_2\alpha)(c - v) + s((1-\alpha)r - \alpha(l - v_r) - c)}{(1-\alpha)r - \alpha(l - v_r + c - v) - v} \tag{14}$$

At the same time, conditions (2) and (1) are necessary to ensure positive profits to the manufacturer and retailer, respectively. Substituting the value of w obtained in (14) into these conditions we get that:

$$\begin{aligned} &\alpha c^2 + (\alpha^2(r + l - v_r) - \alpha r - (1-\alpha)(s - v) + \alpha(l_1 - v_r))c \\ &+((1-\alpha)(s - v) - \alpha(l_1 - v_r))((1-\alpha)r - \alpha(l - v_r)) > 0 \end{aligned} \tag{15}$$

$$((1-\alpha)r - (1-\alpha)s\alpha l_2)c < ((1-\alpha)r - \alpha(l - v_r))((1-\alpha)r - (1-\alpha)s - \alpha l_2) \tag{16}$$

Solving the first equation in c and simplifying, we obtain the following conditions:

$$(c - (1-\alpha)r + \alpha(l - v_r))\Big(c - \frac{(1-\alpha)(s - v) - \alpha(l_1 - v_r)}{\alpha}\Big) > 0, \tag{17}$$

$$c - (1-\alpha)r + \alpha(l - v_r) < 0. \tag{18}$$

Observe that the latter condition is always satisfied by our parameters in order for the supply chain to turn any profits; see (3). Then the first condition reduces to $c < \dfrac{(1-\alpha)(s - v) - \alpha(l_1 - v_r)}{\alpha}$, or equivalently, $s > \dfrac{\alpha}{1-\alpha}(c + l_1 - v_r) + v$.

There is a set of values $(w(s), s)$ satisfying

$$s > \frac{\alpha}{1-\alpha}(c + l_1 - v_r) + v \text{ and } w = \frac{(r(1-\alpha) - l_2\alpha)(c - v) + s((1-\alpha)r - \alpha(l - v_r) - c)}{(1-\alpha)r - \alpha(l - v_r + c - v) - v}$$

that achieve supply chain coordination. Observe that $w > s$ must also be satisfied; otherwise the retailer would obtain profits for every unit ordered, even those not sold. This condition is equivalent to $s < r - \frac{\alpha}{1-\alpha} l_2$. ∎

11 Appendix B

This appendix presents further computational tests to show the generality of the findings reported in the paper. In particular, the first section details the optimal quantities, transfer prices and profits as the share of logistics costs born manufacturer shifts, the profit margin and the logistics costs decrease. The second section, replicates the study done for normal demand distributions using gamma distributions with a variety of parameters; this results in a wide range of demand distribution shapes reinforcing the generality of the findings.

11.1 Sensitivity Analysis for β

Table 8, Table 9 and Table 10 below present the results obtained for different values of the percentage of logistics costs associated with consumer returns faced by the retailer (β) and profit margin ratios c/r, for logistics costs $l = 2, 1, 0.5$, respectively.

	CR					IR					Centralized	
β	w^*	Q^*	Π_M	Π_R	Π_T	w^*	Q^*	Π_M	Π_R	Π_T	Q^*	Π
c/r = 1/20												
5%	16.54	2.198	26.253	4.986	31.239	16.56	2.29	27.44	4.92	32.36	4.14	42.33
25%	16.44	2.201	26.278	4.991	31.269	16.56	2.29	27.61	4.75	32.36	4.14	42.33
50%	16.31	2.204	26.309	5.007	31.316	16.56	2.29	27.84	4.52	32.36	4.14	42.33
75%	16.19	2.207	26.341	5.005	31.346	16.56	2.29	28.06	4.30	32.36	4.14	42.33
95%	16.09	2.209	26.367	5.011	31.377	16.56	2.29	28.24	4.12	32.36	4.14	42.33
c/r = 1/10												
5%	8.43	2.148	11.619	2.192	13.811	8.38	2.26	12.17	2.26	14.43	3.84	18.58
25%	8.33	2.152	11.641	2.197	13.837	8.38	2.26	12.34	2.09	14.43	3.84	18.58
50%	8.21	2.157	11.669	2.194	13.863	8.38	2.26	12.56	1.87	14.43	3.84	18.58
75%	8.09	2.162	11.697	2.192	13.889	8.38	2.26	12.78	1.65	14.43	3.84	18.58
95%	7.99	2.167	11.720	2.197	13.917	8.38	2.26	12.96	1.47	14.43	3.84	18.58
c/r = 1/4												
5%	3.56	1.972	2.927	0.541	3.468	3.47	2.16	3.07	0.68	3.76	3.27	4.62
25%	3.46	1.983	2.941	0.544	3.485	3.47	2.16	3.24	0.51	3.76	3.27	4.62
50%	3.34	1.992	2.959	0.539	3.498	3.47	2.16	3.46	0.30	3.76	3.27	4.62
75%	3.22	2.001	2.977	0.535	3.512	3.47	2.16	3.67	0.09	3.76	3.27	4.62
95%	3.12	2.013	2.992	0.539	3.531	3.47	2.16	3.84	-0.08	3.76	3.27	4.62
c/r = 1/2												
5%	1.93	1.433	0.239	0.042	0.281	1.83	1.97	0.19	0.17	0.36	2.27	0.38
25%	1.83	1.449	0.241	0.042	0.283	1.83	1.97	0.34	0.02	0.36	2.27	0.38
50%	1.70	1.503	0.244	0.049	0.293	1.83	1.97	0.54	-0.18	0.36	2.27	0.38
75%	1.58	1.493	0.248	0.044	0.291	1.83	1.97	0.73	-0.37	0.36	2.27	0.38
95%	1.47	1.579	0.249	0.057	0.306	1.83	1.97	0.89	-0.53	0.36	2.27	0.38
c/r = 1/1.76												
5%	1.73	0.875	0.004	0.003	0.006	1.63	1.91	-0.13	0.12	-0.01	1.19	0.01
25%	1.63	0.890	0.004	0.003	0.007	1.63	1.91	0.02	-0.03	-0.01	1.19	0.01
50%	1.50	0.909	0.004	0.003	0.007	1.63	1.91	0.21	-0.22	-0.01	1.19	0.01
75%	1.38	0.930	0.004	0.003	0.007	1.63	1.91	0.40	-0.41	-0.01	1.19	0.01
95%	1.28	0.948	0.004	0.003	0.007	1.63	1.91	0.55	-0.56	-0.01	1.19	0.01

Table 8: Sensitivity Analysis of β, for different values of c/r, and for $l = 2$

	CR					IR					Centralized	
β	w^*	Q^*	Π_M	Π_R	Π_T	w^*	Q^*	Π_M	Π_R	Π_T	Q^*	Π
c/r = 1/20												
5%	16.51	2.204	26.679	5.060	31.739	16.56	2.29	27.86	4.95	32.80	4.15	42.93
25%	16.46	2.205	26.692	5.063	31.755	16.56	2.29	27.95	4.86	32.80	4.15	42.93
50%	16.39	2.208	26.708	5.080	31.787	16.56	2.29	28.06	4.75	32.80	4.15	42.93
75%	16.33	2.209	26.724	5.079	31.803	16.56	2.29	28.17	4.63	32.80	4.15	42.93
95%	16.28	2.210	26.736	5.082	31.818	16.56	2.29	28.26	4.55	32.80	4.15	42.93
c/r = 1/10												
5%	8.40	2.160	12.037	2.265	14.302	8.38	2.26	12.58	2.29	14.87	3.85	19.17
25%	8.35	2.162	12.048	2.267	14.316	8.38	2.26	12.67	2.20	14.87	3.85	19.17
50%	8.29	2.165	12.062	2.266	14.329	8.38	2.26	12.78	2.09	14.87	3.85	19.17
75%	8.23	2.167	12.077	2.265	14.342	8.38	2.26	12.89	1.98	14.87	3.85	19.17
95%	8.18	2.169	12.089	2.267	14.356	8.38	2.26	12.98	1.89	14.87	3.85	19.17
c/r = 1/4												
5%	3.53	2.012	3.318	0.608	3.926	3.47	2.16	3.48	0.70	4.18	3.32	5.18
25%	3.48	2.018	3.325	0.610	3.935	3.47	2.16	3.56	0.62	4.18	3.32	5.18
50%	3.42	2.022	3.335	0.608	3.943	3.47	2.16	3.67	0.51	4.18	3.32	5.18
75%	3.36	2.027	3.345	0.606	3.951	3.47	2.16	3.77	0.41	4.18	3.32	5.18
95%	3.31	2.033	3.353	0.608	3.961	3.47	2.16	3.86	0.32	4.18	3.32	5.18
c/r = 1/2												
5%	1.90	1.646	0.549	0.094	0.644	1.83	1.97	0.56	0.19	0.75	2.58	0.84
25%	1.85	1.655	0.552	0.095	0.647	1.83	1.97	0.63	0.11	0.75	2.58	0.84
50%	1.78	1.695	0.555	0.106	0.661	1.83	1.97	0.73	0.02	0.75	2.58	0.84
75%	1.72	1.698	0.558	0.103	0.662	1.83	1.97	0.83	-0.08	0.75	2.58	0.84
95%	1.67	1.708	0.561	0.104	0.665	1.83	1.97	0.91	-0.16	0.75	2.58	0.84
c/r = 1/1.51												
5%	1.495	0.748	0.005	0.001	0.0057	1.42	1.83	-0.09	0.09	0.00	1.19	0.01
25%	1.445	0.755	0.005	0.001	0.0057	1.42	1.83	-0.02	0.01	0.00	1.19	0.01
50%	1.3825	0.766	0.005	0.001	0.0058	1.42	1.83	0.07	-0.08	0.00	1.19	0.01
75%	1.3201	0.767	0.005	0.001	0.0058	1.42	1.83	0.16	-0.17	0.00	1.19	0.01
95%	1.2701	0.776	0.005	0.001	0.0059	1.42	1.83	0.24	-0.24	0.00	1.19	0.01

Table 9: Sensitivity Analysis of β, for different values of c/r, and for $l = 1$

	CR					IR					Centralized	
β	w^*	Q^*	Π_M	Π_R	Π_T	w^*	Q^*	Π_M	Π_R	Π_T	Q^*	Π
c/r = 1/20												
5%	16.49	2.207	26.892	5.107	31.999	16.56	2.29	28.07	4.96	33.03	4.15	43.23
25%	16.46	2.209	26.899	5.117	32.016	16.56	2.29	28.11	4.91	33.03	4.15	43.23
50%	16.43	2.209	26.907	5.116	32.023	16.56	2.29	28.17	4.86	33.03	4.15	43.23
75%	16.40	2.210	26.915	5.116	32.031	16.56	2.29	28.22	4.80	33.03	4.15	43.23
95%	16.38	2.210	26.921	5.114	32.035	16.56	2.29	28.27	4.76	33.03	4.15	43.23
c/r = 1/10												
5%	8.38	2.167	12.247	2.310	14.557	8.38	2.26	12.79	2.30	15.09	3.86	19.47
25%	8.36	2.167	12.252	2.303	14.555	8.38	2.26	12.84	2.25	15.09	3.86	19.47
50%	8.33	2.168	12.260	2.302	14.562	8.38	2.26	12.89	2.20	15.09	3.86	19.47
75%	8.30	2.169	12.267	2.302	14.569	8.38	2.26	12.95	2.14	15.09	3.86	19.47
95%	8.27	2.172	12.273	2.312	14.584	8.38	2.26	12.99	2.10	15.09	3.86	19.47
c/r = 1/4												
5%	3.51	2.036	3.516	0.651	4.166	3.47	2.16	3.68	0.71	4.39	3.35	5.47
25%	3.49	2.034	3.520	0.643	4.163	3.47	2.16	3.72	0.67	4.39	3.35	5.47
50%	3.46	2.036	3.525	0.642	4.167	3.47	2.16	3.77	0.62	4.39	3.35	5.47
75%	3.43	2.039	3.530	0.641	4.171	3.47	2.16	3.83	0.56	4.39	3.35	5.47
95%	3.40	2.046	3.534	0.651	4.185	3.47	2.16	3.87	0.52	4.39	3.35	5.47
c/r = 1/2												
5%	1.88	1.738	0.717	0.130	0.847	1.83	1.97	0.74	0.20	0.94	2.68	1.09
25%	1.86	1.726	0.719	0.123	0.842	1.83	1.97	0.78	0.16	0.94	2.68	1.09
50%	1.83	1.728	0.721	0.122	0.843	1.83	1.97	0.83	0.11	0.94	2.68	1.09
75%	1.79	1.762	0.723	0.135	0.858	1.83	1.97	0.88	0.06	0.94	2.68	1.09
95%	1.77	1.751	0.725	0.128	0.853	1.83	1.97	0.92	0.03	0.94	2.68	1.09
c/r = 1/1.38												
5%	1.373	0.504	0.002	0.000	0.0019	1.31	1.77	-0.08	0.07	-0.01	1.01	0.00
25%	1.348	0.508	0.002	0.000	0.0019	1.31	1.77	-0.04	0.03	-0.01	1.01	0.00
50%	1.316	0.662	0.002	0.001	0.0025	1.31	1.77	0.00	-0.01	-0.01	1.01	0.00
75%	1.284	0.758	0.002	0.001	0.0027	1.31	1.77	0.05	-0.05	-0.01	1.01	0.00
95%	1.258	0.849	0.001	0.002	0.0029	1.31	1.77	0.08	-0.09	-0.01	1.01	0.00

Table 10: Sensitivity Analysis of β, for different values of c/r, and for $l = 0.5$

11.2 Computational study for the Gamma Distribution Demand

The gamma distribution models very different shapes of the demand distribution for the different values of its shape parameter, γ. For γ equal to one the distribution corresponds to an exponential distribution. In our study, we set the scale parameter to 1 and vary the shape parameter (γ). In Figure 6 we represent the different shapes of the gamma distribution demand considered in our computational study.

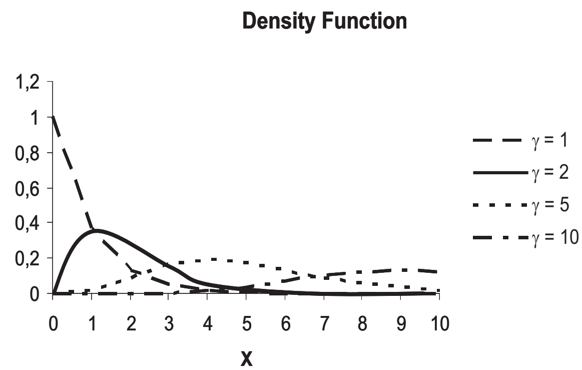

Figure 6: Density function of the gamma distribution for different values of γ

In Figures 7, 8, 9 and 10 we present the optimal order quantity and profit functions for different wholesale prices and for the cases of $\gamma = 1, \gamma = 2, \gamma = 5$ and $\gamma = 10$ respectively. In Figures 11, 12, and 13 we present the optimal decision variables and respective profits for different percentage of logistics costs associated with consumer returns faced by the retailer and for the cases of $\gamma = 1, \gamma = 5$ and $\gamma = 10$ respectively. Table 11 contains the results at optimality for the base case under different values of the parameter γ. Finally, in Tables 12, 13 and 14 we present the optimal decision variables and profits for different logistics costs and different ratios c/r for $\gamma = 1$, $\gamma = 5$ and $\gamma = 10$, respectively.

		Centralized		Decentralized				
γ	Policy	Q^*	Π	Q^*	w^*	Π_R	Π_M	Π_T
1	CR	1.030	0.770	0.376	2.53	0.204	0.298	0.502
	IR	1.386	0.714	0.562	2.28	0.282	0.360	0.642
2	CR	2.189	2.097	0.885	2.93	0.455	0.926	1.381
	IR	2.693	2.018	1.166	2.7	0.622	1.050	1.672
5	CR	5.502	6.601	2.651	3.35	1.018	3.530	4.548
	IR	6.274	6.480	3.118	3.18	1.382	3.790	5.172
10	CR	10.849	14.614	5.990	3.57	1.643	8.924	10.567
	IR	11.914	14.449	6.668	3.45	2.218	9.363	11.581

Table 11: Base case for different values of γ

	Policy CR					Policy IR					CENTR.	
c/r	w*	Q*	Π_M	Π_R	Π_T	w*	Q*	Π_M	Π_R	Π_T	Q	Π
Logistics costs = 2												
1/20	7.86	0.80	4.43	3.38	7.81	8.3	0.88	5.23	3.02	8.25	2.75	11.85
1/10	4.56	0.67	1.75	1.29	3.03	4.58	0.78	2.09	1.25	3.34	2.03	4.57
1/4	2.53	0.38	0.30	0.20	0.50	2.28	0.56	0.36	0.28	0.64	1.03	0.77
1/2	1.82	0.07	0.01	0.00	0.01	1.46	0.31	-0.04	0.05	0.01	0.18	0.02
1/1.76	1.73	0.00	0.00	0.00	0.00	1.35	0.27	-0.06	0.03	-0.03	0.01	0.00
Logistics costs = 1												
1/20	7.77	0.81	4.54	3.45	7.99	8.3	0.88	5.34	3.03	8.36	2.76	12.04
1/10	4.47	0.69	1.84	1.34	3.19	4.58	0.78	2.20	1.25	3.45	2.05	4.75
1/4	2.42	0.42	0.36	0.24	0.61	2.28	0.56	0.44	0.29	0.73	1.10	0.90
1/2	1.68	0.14	0.03	0.02	0.04	1.46	0.31	0.01	0.05	0.06	0.34	0.06
1/1.51	1.49	0.00	0.00	0.00	0.00	1.24	0.20	-0.03	0.01	-0.02	0.01	0.00
Logistics costs = 0.5												
1/20	7.72	0.82	4.60	3.48	8.08	8.3	0.88	5.39	3.03	8.42	2.77	12.13
1/10	4.42	0.70	1.89	1.38	3.27	4.58	0.78	2.25	1.25	3.50	2.07	4.83
1/4	2.36	0.44	0.40	0.27	0.66	2.28	0.56	0.48	0.29	0.77	1.13	0.97
1/2	1.62	0.17	0.04	0.02	0.06	1.46	0.31	0.04	0.05	0.09	0.41	0.09
1/1.38	1.37	0.00	0.00	0.00	0.00	1.18	0.16	-0.02	0.01	-0.01	0.00	0.00

Table 12: Optimal decision variables and profits for different ratios c/r and logistics costs for $\gamma = 1$

	Policy CR					Policy IR					CENTR.	
c/r	w*	Q*	Π_M	Π_R	Π_T	w*	Q*	Π_M	Π_R	Π_T	Q	Π
Logistics costs = 2												
1/20	14.4	3.30	34.05	11.09	45.15	14.4	3.53	36.62	10.89	47.51	8.75	67.71
1/10	7.51	3.14	14.81	4.65	19.47	7.39	3.43	15.96	4.92	20.88	7.51	28.85
1/4	3.35	2.65	3.53	1.02	4.55	3.18	3.12	3.79	1.38	5.17	5.50	6.60
1/2	1.92	1.58	0.06	0.25	0.31	1.75	2.62	0.11	0.33	0.43	2.89	0.44
1/1.76	1.73	0.90	0.00	0.00	0.01	1.57	2.49	-0.27	0.22	-0.04	1.21	0.01
Logistics costs = 1												
1/20	14.3	3.32	34.67	11.31	45.98	14.4	3.53	37.24	10.92	48.17	8.77	68.69
1/10	7.44	3.18	15.41	4.86	20.27	7.39	3.43	16.57	4.95	21.52	7.55	29.81
1/4	3.29	2.75	4.05	1.17	5.22	3.18	3.12	4.35	1.41	5.76	5.66	7.47
1/2	1.87	1.95	0.61	0.14	0.75	1.75	2.62	0.59	0.35	0.94	3.56	1.05
1/1.51	1.49	1.02	0.00	0.00	0.01	1.39	2.26	-0.16	0.14	-0.02	1.21	0.01
Logistics costs = 0.5												
1/20	14.3	3.33	34.98	11.41	46.39	14.4	3.53	37.55	10.97	48.52	8.78	69.18
1/10	7.41	3.19	15.70	4.96	20.66	7.39	3.43	16.87	4.96	21.84	7.58	30.29
1/4	3.26	2.80	4.31	1.26	5.57	3.18	3.12	4.63	1.43	6.06	5.73	7.91
1/2	1.84	2.11	0.81	0.20	1.01	1.75	2.62	0.83	0.36	1.20	3.81	1.39
1/1.38	1.37	0.89	0.00	0.00	0.00	1.29	2.13	-0.12	0.10	-0.02	1.02	0.00

Table 13: Optimal decision variables and profits for different ratios c/r and logistics costs for $\gamma = 5$

	Policy CR					Policy IR					CENTR.	
c/r	w*	Q*	Π_M	Π_R	Π_T	w*	Q*	Π_M	Π_R	Π_T	Q	Π
Logistics costs = 2												
1/20	16.3	6.82	80.29	16.62	96.91	16.3	7.17	84.20	16.91	101.11	15.18	138.87
1/10	8.37	6.62	35.48	7.11	42.59	8.25	7.05	37.25	7.71	44.95	13.57	60.33
1/4	3.57	5.99	8.92	1.64	10.57	3.45	6.67	9.36	2.22	11.58	10.85	14.61
1/2	1.94	4.37	0.76	0.11	0.86	1.83	6.02	0.56	0.54	1.11	6.97	1.16
1/1.76	1.73	3.32	0.01	0.01	0.03	1.63	5.85	-0.40	0.38	-0.02	3.99	0.03
Logistics costs = 1												
1/20	16.3	6.84	81.61	16.91	98.52	16.3	7.17	85.52	16.98	102.50	15.21	140.85
1/10	8.32	6.68	36.77	7.44	44.21	8.25	7.05	38.55	7.77	46.32	13.63	62.27
1/4	3.53	6.14	10.11	1.90	12.01	3.45	6.67	10.60	2.28	12.88	11.07	16.44
1/2	1.9	5.10	1.70	0.31	2.00	1.83	6.02	1.69	0.60	2.30	8.02	2.60
1/1.51	1.49	3.60	0.01	0.02	0.03	1.43	5.48	-0.23	0.23	0.00	3.99	0.03
Logistics costs = 0.5												
1/20	16.3	6.86	82.27	17.11	99.38	16.3	7.17	86.17	17.01	103.19	15.22	141.84
1/10	8.3	6.70	37.42	7.58	45.00	8.25	7.05	39.20	7.81	47.01	13.66	63.24
1/4	3.51	6.21	10.72	2.03	12.75	3.45	6.67	11.22	2.32	13.54	11.17	17.36
1/2	1.89	5.26	2.21	0.37	2.58	1.83	6.02	2.26	0.63	2.89	8.39	3.38
1/1.38	1.37	3.30	0.00	0.01	0.01	1.32	5.29	-0.19	0.18	-0.01	3.59	0.01

Table 14: Optimal decision variables and profits for different ratios c/r and logistics costs for $\gamma = 10$

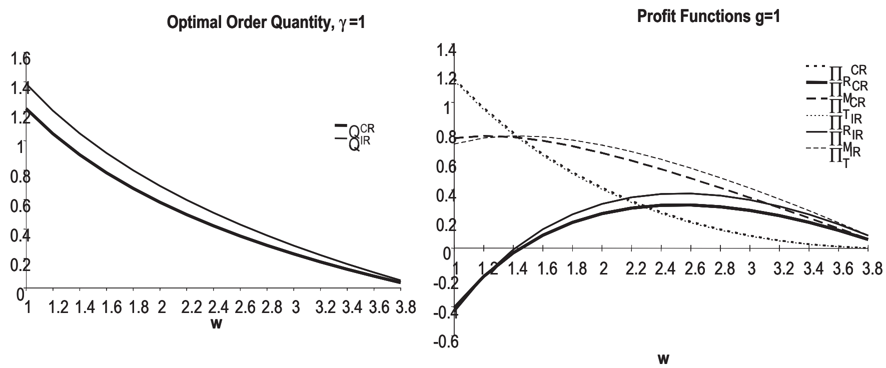

Figure 7: Optimal order quantity and Profit functions for the base case and $\gamma = 1$

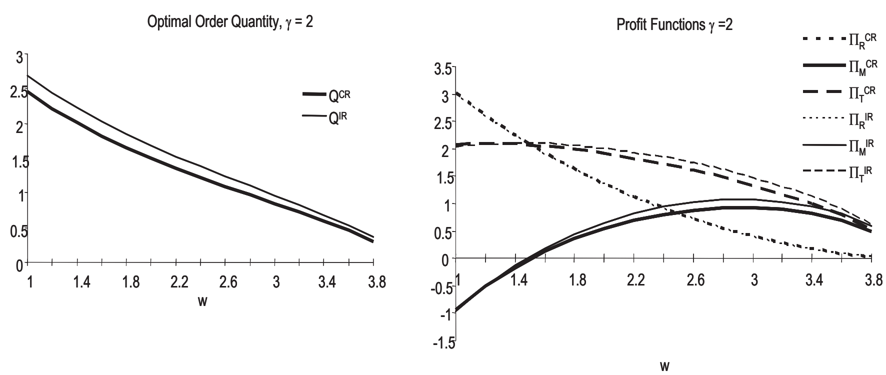

Figure 8: Optimal order quantity and Profit functions for the base case and $\gamma = 2$

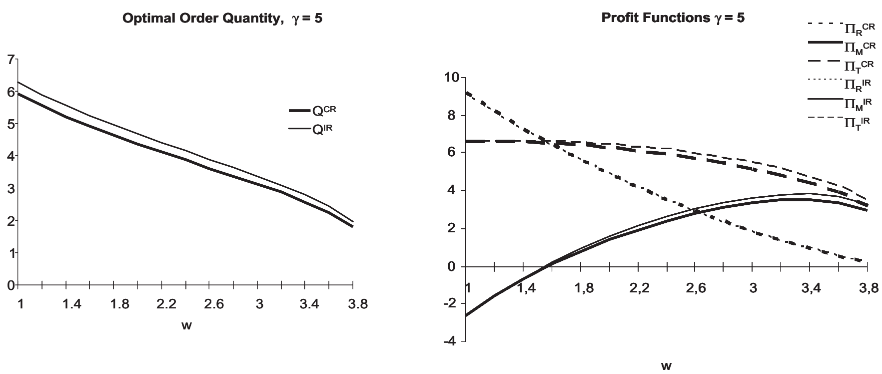

Figure 9: Optimal order quantity and Profit functions for the base case and $\gamma = 5$

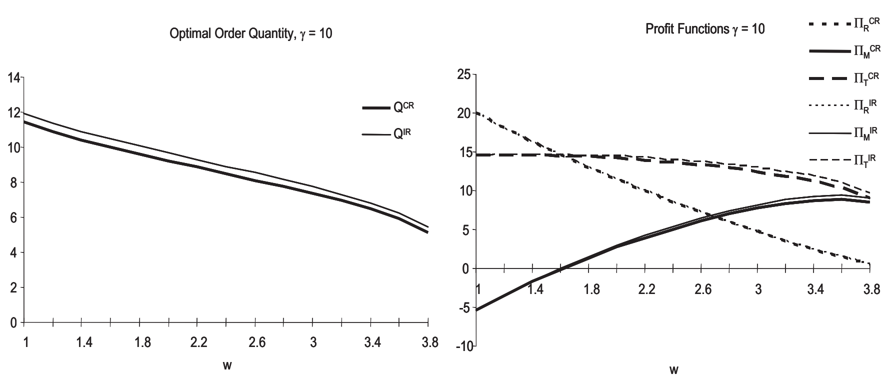

Figure 10: Optimal order quantity and Profit functions for the base case and $\gamma = 10$

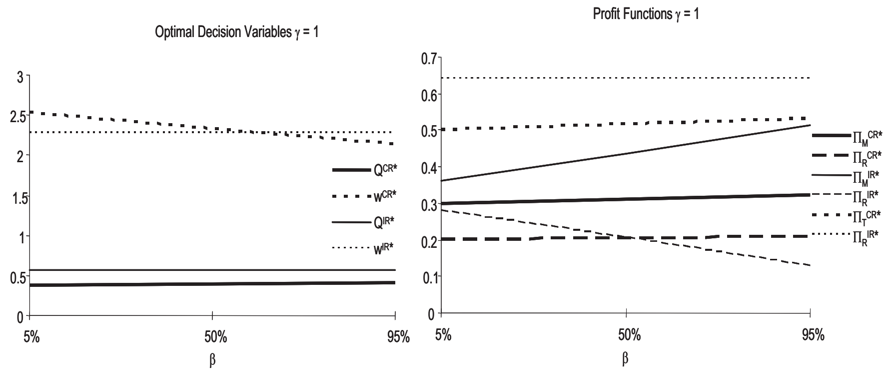

Figure 11: Optimal decision variables and respective profits for different percentage of logistics costs associated with consumer returns faced by the retailer for $\gamma = 1$

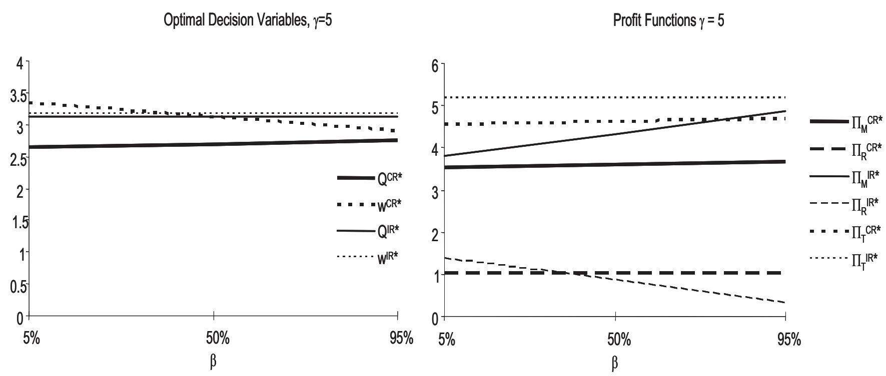

Figure 12: Optimal decision variables and respective profits for different percentage of logistics costs associated with consumer returns faced by the retailer for $\gamma = 5$

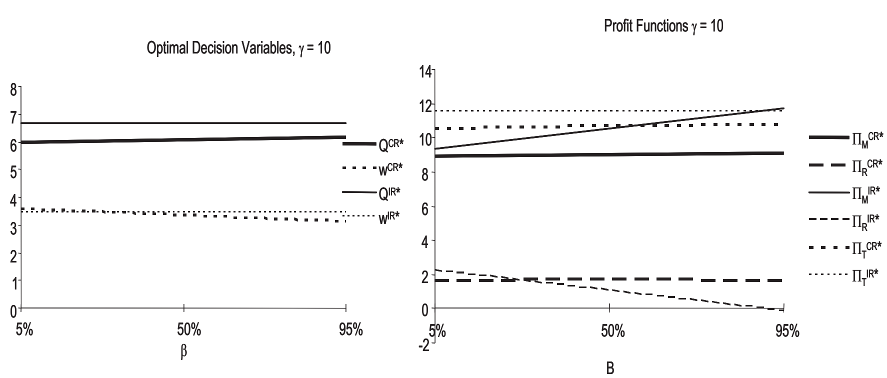

Figure 13: Optimal decision variables and respective profits for different percentage of logistics costs associated with consumer returns faced by the retailer for $\gamma = 10$