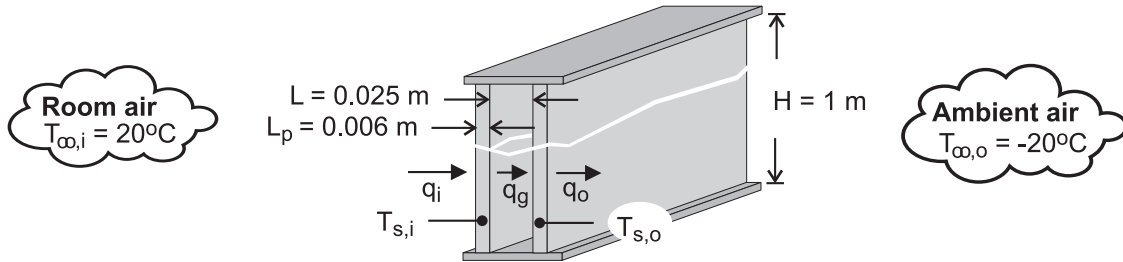


PROBLEM 9.97

KNOWN: Dimensions of double pane window. Thickness of air gap. Temperatures of room and ambient air.

FIND: (a) Temperatures of glass panes and heat rate through window, (b) Resistance of glass pane relative to smallest convection resistance.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state, (2) Negligible glass pane thermal resistance, (3) Constant properties.

PROPERTIES: Table A-3, Plate glass: $k_p = 1.4 \text{ W/m}\cdot\text{K}$. Table A-4, Air ($p = 1 \text{ atm}$). $T_{f,i} = 287.6\text{K}$: $\nu_i = 14.8 \times 10^{-6} \text{ m}^2/\text{s}$, $k_i = 0.0253 \text{ W/m}\cdot\text{K}$, $\alpha_i = 20.9 \times 10^{-6} \text{ m}^2/\text{s}$, $\text{Pr}_i = 0.710$, $\beta_i = 0.00348 \text{ K}^{-1}$. $\bar{T} = (T_{s,i} + T_{s,o})/2 = 272.8\text{K}$: $\nu = 13.49 \times 10^{-6} \text{ m}^2/\text{s}$, $k = 0.0241 \text{ W/m}\cdot\text{K}$, $\alpha = 18.9 \times 10^{-6} \text{ m}^2/\text{s}$, $\text{Pr} = 0.714$, $\beta = 0.00367 \text{ K}^{-1}$. $T_{f,o} = 258.2\text{K}$: $\nu_o = 12.2 \times 10^{-6} \text{ m}^2/\text{s}$, $k_o = 0.0230 \text{ W/m}\cdot\text{K}$, $\alpha = 17.0 \times 10^{-6} \text{ m}^2/\text{s}$, $\text{Pr} = 0.718$, $\beta_o = 0.00387 \text{ K}^{-1}$.

ANALYSIS: (a) The heat rate may be expressed as

$$q = q_o = \bar{h}_o H^2 (T_{s,o} - T_{\infty,o}) \quad (1)$$

$$q = q_g = \bar{h}_g H^2 (T_{s,i} - T_{s,o}) \quad (2)$$

$$q = q_i = \bar{h}_i H^2 (T_{\infty,i} - T_{s,i}) \quad (3)$$

where \bar{h}_o and \bar{h}_i may be obtained from Eq. (9.26),

$$\bar{\text{Nu}}_H = \left\{ 0.825 + \frac{0.387 \text{Ra}_H^{1/6}}{\left[1 + (0.492/\text{Pr})^{9/16} \right]^{8/27}} \right\}^2$$

with $\text{Ra}_H = g\beta_o (T_{s,o} - T_{\infty,o}) H^3 / \alpha_o \nu_o$ and $\text{Ra}_H = g\beta_i (T_{\infty,i} - T_{s,i}) H^3 / \alpha_i \nu_i$, respectively. Assuming $10^4 < \text{Ra}_L < 10^7$, \bar{h}_g is obtained from

$$\bar{\text{Nu}}_L = 0.42 \text{Ra}_L^{1/4} \text{Pr}^{0.012} (H/L)^{-0.3}$$

where $\text{Ra}_L = g\beta (T_{s,i} - T_{s,o}) L^3 / \alpha \nu$. A simultaneous solution to Eqs. (1) – (3) for the three unknowns yields

Continued

PROBLEM 9.97 (Cont.)

$$T_{s,i} = 9.1^\circ\text{C}, \quad T_{s,o} = -9.6^\circ\text{C}, \quad q = 35.7 \text{ W} \quad <$$

where $\bar{h}_i = 3.29 \text{ W/m}^2 \cdot \text{K}$, $\bar{h}_o = 3.45 \text{ W/m}^2 \cdot \text{K}$ and $\bar{h}_g = 1.90 \text{ W/m}^2 \cdot \text{K}$.

(b) The unit conduction resistance of a glass pane is $R''_{\text{cond}} = L_p / k_p = 0.00429 \text{ m}^2 \cdot \text{K/W}$, and the smallest convection resistance is $R''_{\text{conv},o} = (1/\bar{h}_o) = 0.290 \text{ m}^2 \cdot \text{K/W}$. Hence,

$$R''_{\text{cond}} \ll R''_{\text{conv},\text{min}} \quad <$$

and it is reasonable to neglect the thermal resistance of the glass.

COMMENTS: (1) Assuming a heat flux of 35.7 W/m^2 through a glass pane, the corresponding temperature difference across the pane is $\Delta T = q''(L_p / k_p) = 0.15^\circ\text{C}$. Hence, the assumption of an isothermal pane is good. (2) Equations (1) – (3) were solved using the IHT workspace and the temperature-dependent air properties provided by the software. The property values provided in the PROPERTIES section of this solution were obtained from the software.