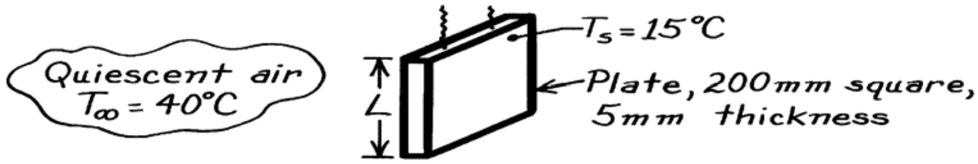
## PROBLEM 9.8

**KNOWN:** Square aluminum plate at 15°C suspended in quiescent air at 40°C.

**FIND:** Average heat transfer coefficient by two methods – using results of boundary layer similarity and results from an empirical correlation.

**SCHEMATIC:**




**ASSUMPTIONS:** (1) Uniform plate surface temperature, (2) Quiescent room air, (3) Surface radiation exchange with surroundings negligible, (4) Perfect gas behavior for air, $\beta = 1/T_f$.

**PROPERTIES:** *Table A-4*, Air ($T_f = (T_s + T_\infty)/2 = (40 + 15)°C/2 = 300K$, 1 atm): $\nu = 15.89 \times 10^{-6}$ $m^2/s$, k = 0.0263 W/m·K, $\alpha = 22.5 \times 10^{-6}$ $m^2/s$, Pr = 0.707.

**ANALYSIS:** Calculate the Rayleigh number to determine the boundary layer flow conditions,

$$Ra_L = g\,\beta\,\Delta T\,L^3 / \nu\,\alpha$$

$$Ra_L = 9.8\ \text{m/s}^2\,(1/300\text{K})\,(40-15)°\text{C}\,(0.2\text{m})^3 / \left(15.89\times10^{-6}\text{m}^2/\text{s}\right)\left(22.5\times10^{-6}\text{m}^2/\text{s}\right) = 1.827\times10^7$$

where $\beta = 1/T_f$ and $\Delta T = T_\infty - T_s$. Since $Ra_L < 10^9$, the flow is laminar and the *similarity solution* of Section 9.4 is applicable. From Eqs. 9.21 and 9.20,

$$\overline{Nu}_L = \frac{\bar{h}_L L}{k} = \frac{4}{3}\left(Gr_L/4\right)^{1/4}\,g(Pr)$$

$$g(Pr) = \frac{0.75\,Pr^{1/2}}{\left[0.609 + 1.221\,Pr^{1/2} + 1.238\,Pr\right]^{1/4}}$$

and substituting numerical values with $Gr_L = Ra_L/Pr$, find

$$g(Pr) = 0.75(0.707)^{1/2} / \left[0.609 + 1.22(0.707)^{1/2} + 1.238\times0.707\right]^{1/4} = 0.501$$

$$\bar{h}_L = \left(\frac{0.0263\ \text{W/m}\cdot\text{K}}{0.20\text{m}}\right)\times\frac{4}{3}\left(\frac{1.827\times10^7/0.707}{4}\right)^{1/4}\times0.501 = 4.42\ \text{W/m}^2\cdot\text{K}. \quad <$$

The appropriate empirical correlation for estimating $\bar{h}_L$ is given by Eq. 9.27,

$$\overline{Nu}_L = \frac{\bar{h}_L L}{k} = 0.68 + \frac{0.670\,Ra_L^{1/4}}{\left[1 + (0.492/Pr)^{9/16}\right]^{4/9}}$$

$$\bar{h}_L = (0.0263\ \text{W/m}\cdot\text{K}/0.20\text{m})\left[0.68 + 0.670\left(1.827\times10^7\right)^{1/4} / \left[1 + (0.492/0.707)^{9/16}\right]^{4/9}\right]$$

$$\bar{h}_L = 4.51\ \text{W/m}^2\bullet\text{K}. \quad <$$

**COMMENTS:** The agreement of $\bar{h}_L$ calculated by these two methods is excellent. Using the Churchill-Chu correlation, Eq. 9.26, find $\bar{h}_L = 4.87$ W/m·K. This relation is not the most accurate for the laminar regime, but is suitable for both laminar and turbulent regions.