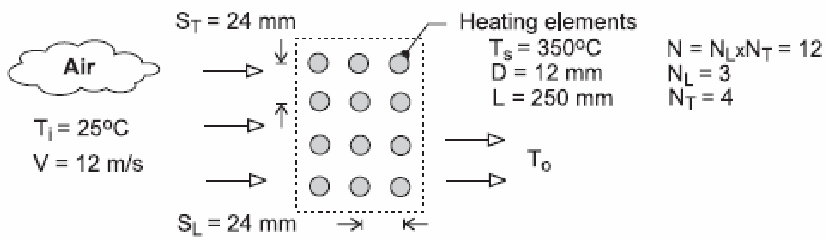
PROBLEM 7.87

KNOWN: An air duct heater consists of an aligned arrangement of electrical heating elements with $S_L = S_T = 24$ mm, $N_L = 3$ and $N_T = 4$. Atmospheric air with an upstream velocity of 12 m/s and temperature of 25°C moves in cross flow over the elements with a diameter of 12 mm and length of 250 mm maintained at a surface temperature of 350°C.

FIND: (a) The total heat transfer to the air and the temperature of the air leaving the duct heater, (b) The pressure drop across the element bank and the fan power requirement, (c) Compare the average convection coefficient obtained in part (a) with the value for an isolated (single) element; explain the relative difference between the results; (d) What effect would increasing the longitudinal and transverse pitches to 30 mm have on the exit temperature of the air, the total heat rate, and the pressure drop?

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Negligible radiation effects, (3) Negligible effect of change in air temperature across tube bank on air properties.

PROPERTIES: *Table A-4*, Air (T_i = 298, 1 atm): $\rho = 1.171$ kg/m^3, $c_p = 1007$ J/kg·K; Air ($T_m = (T_i + T_o)/2 = 309$ K, 1 atm): $\rho = 1.130$ kg/m^3, $c_p = 1007$ J/kg·K, $\mu = 1.89 \times 10^{-5}$ N·s/m^2, k = 0.02699 W/m·K, Pr = 0.7057; Air (T_s = 623 K, 1 atm): $Pr_s = 0.687$; Air ($T_f = (T_i + T_o)/2 = 461$ K, 1 atm): $\nu = 3.373 \times 10^{-5}$ m^2/s, k = 0.03801 W/m·K, Pr = 0.686.

ANALYSIS: (a) The total heat transfer to the air is determined from the rate equation, Eq. 7.68,

$$q = N\left(\bar{h}_D \pi D \, \Delta T_{\ell m}\right) \tag{1}$$

where the log mean temperature difference, Eq. 7.66, is

$$\Delta T_{\ell m} = \frac{T_s - T_i}{T_s - T_o} \bigg/ \ln \frac{(T_s - T_i)}{(T_s - T_o)} \tag{2}$$

and from the overall energy balance, Eq. 7.67,

$$\frac{T_s - T_o}{T_s - T_i} = \exp\left(\frac{\pi D N \bar{h}_D}{\rho V N_T S_T c_p}\right) \tag{3}$$

The properties ρ and c_p in Eq. (3) are evaluated at the inlet temperature T_i. The average convection coefficient using the Zukauskus correlation, Eq. 7.64 and 7.65,

$$\overline{Nu}_D = \frac{\bar{h}_D}{k} = C\ Re_{D,max}^m\ Pr^{0.36}\left(Pr/Pr_s\right)^{1/4} \tag{4}$$

where C = 0.27, m = 0.63 are determined from Table 7.7 for the *aligned* configuration with $S_T/S_L = 1 > 0.7$ and $10^3 < Re_{D,max} \le 10^5$. All properties except Pr_s are evaluated at the arithmetic mean temperature $T_m = (T_i + T_o)/2$. The maximum Reynolds number, Eq. 7.59, is

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$$Re_{D,max} = \rho V_{max} D / \mu \qquad (5)$$

where for the *aligned* arrangement, the maximum velocity occurs at the transverse plane, Eq. 7.63,

$$V_{max} = \frac{S_T}{S_T - D} V \qquad (6)$$

The results of the analyses for $S_T = S_L = 24$ mm are tabulated below.

V_{max} (m/s)	$Re_{D,max}$	$\overline{Nu}_D$	$\bar{h}_D$ (W/m^2·K)	$\Delta T_{\ell m}$ (°C)	q (W)	T_o (°C)	
24	1.723×104	96.2	216	314	7671	47.6	<

(b) The pressure drop across the tube bundle follows from Eq. 7.69,

$$\Delta p = N_L \chi \left(\rho V_{max}^2 / 2 \right) f \qquad (7)$$

where the friction factor, f, and correction factor, χ, are determined from Fig. 7.13 using $Re_{D,max} = 1.723 \times 10^4$,

$$f = 0.2 \qquad \chi = 1$$

Substituting numerical values,

$$\Delta p = 3 \times 1 \left[1.171 \text{ kg/m}^3 \times (24 \text{ m/s})^2 / 2 \right] \times 0.2$$

$$\Delta p = 195 \text{ N/m}^2 \qquad <$$

The fan power requirement is

$$P = \forall \Delta p = V N_T S_T L \Delta p \qquad (8)$$

$$P = 12 \text{ m/s} \times 4 \times 0.024 \text{ m} \times 0.250 \text{ m} \times 195 \text{ N/m}^2$$

$$P = 56 \text{ W} \qquad <$$

where $\forall$ is the volumetric flow rate. For this calculation, ρ in Eq. (7) was evaluated at T_m.

(c) For a single element in cross flow, the average convection coefficient can be estimated using the Churchill-Bernstein correlation, Eq. 7.54,

$$\overline{Nu}_D = \frac{\bar{h}_D D}{k} = 0.3 + \frac{0.62\, Re_D^{1/2}\, Pr^{1/3}}{\left[1 + (0.4/Pr)^{2/3}\right]^{1/4}} \left[1 + \left(\frac{Re_D}{282{,}000}\right)^{5/8}\right]^{4/5} \qquad (9)$$

where all properties are evaluated at the film temperature, $T_f = (T_i + T_o)/2$. The results of the calculations are

$$Re_D = 4269 \qquad \overline{Nu}_{D,1} = 33.4 \qquad \bar{h}_{D,1} = 106 \text{ W/m}^2 \cdot \text{K} \qquad <$$

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PROBLEM 7.87 (Cont.)

For the isolated element, $\overline{h}_{D,1} = 106\ W/m^2 \cdot K$, compared to the average value for the array,

$\overline{h}_D = 216\ W/m^2 \cdot K$. Because the first row of the array acts as a turbulence grid, the heat transfer coefficient for the second and third rows will be larger than for the first row. Here, the array value is twice that for the isolated element.

(d) The effect of increasing the longitudinal and transverse pitches to 30 mm, should be to reduce the outlet temperature, heat rate, and pressure drop. The effect can be explained by recognizing that the maximum Reynolds number will be decreased, which in turn will result in lower values for the convection coefficient and pressure drop. Repeating the calculations of part (a) for $S_L = S_T = 30$ mm, find

V_{max} (m/s)	$Re_{D,max}$	$\overline{Nu}_D$	$\overline{h}_D$ (W/m²·K)	$\Delta T_{\ell m}$ (°C)	q (W)	T_o (°C)
12	1.46×10^4	86.7	193	317	6925	41.3

and part (b) for the pressure drop and fan power, find

$$f = 0.18 \qquad \chi = 1 \qquad \Delta p = 122\ N/m^2 \qquad P = 44\ W$$