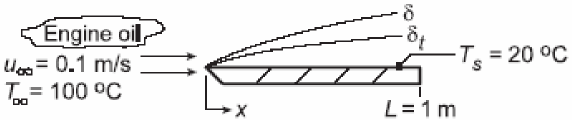
PROBLEM 7.2

KNOWN: Temperature and velocity of engine oil. Temperature and length of flat plate.

FIND: (a) Velocity and thermal boundary layer thickness at trailing edge, (b) Heat flux and surface shear stress at trailing edge, (c) Total drag force and heat transfer per unit plate width, and (d) Plot the boundary layer thickness and local values of the shear stress, convection coefficient, and heat flux as a function of x for 0 ≤ x ≤ 1 m.

SCHEMATIC:



ASSUMPTIONS: (1) Critical Reynolds number is 5×10^5, (2) Flow over top and bottom surfaces.

PROPERTIES: *Table A.5*, Engine Oil (T_f = 333 K): $\rho = 864\ \text{kg/m}^3$, $\nu = 86.1 \times 10^{-6}\ \text{m}^2/\text{s}$, k = 0.140 W/m·K, Pr = 1081.

ANALYSIS: (a) Calculate the Reynolds number to determine nature of the flow,

$$\text{Re}_L = \frac{u_\infty L}{\nu} = \frac{0.1\,\text{m/s} \times 1\,\text{m}}{86.1\times10^{-6}\ \text{m}^2/\text{s}} = 1161$$

Hence the flow is laminar at x = L. From Eqs. 7.19 and 7.24,

$$\delta = 5L\,\text{Re}_L^{-1/2} = 5(1\text{m})(1161)^{-1/2} = 0.147\,\text{m}$$ <

$$\delta_t = \delta\,\text{Pr}^{-1/3} = 0.147\,\text{m}(1081)^{-1/3} = 0.0143\,\text{m}$$ <

(b) The local convection coefficient, Eq. 7.23, and heat flux at x = L are

$$h_L = \frac{k}{L} 0.332\,\text{Re}_L^{1/2}\,\text{Pr}^{1/3} = \frac{0.140\ \text{W/m}\cdot\text{K}}{1\text{m}} 0.332(1161)^{1/2}(1081)^{1/3} = 16.25\ \text{W/m}^2\cdot\text{K}$$

$$q''_x = h_L\left(T_s - T_\infty\right) = 16.25\ \text{W/m}^2\cdot\text{K}(20-100)^\circ\text{C} = -1300\ \text{W/m}^2$$ <

Also, the local shear stress is, from Eq. 7.20,

$$\tau_{s,L} = \frac{\rho u_\infty^2}{2} 0.664\,\text{Re}_L^{-1/2} = \frac{864\,\text{kg/m}^3}{2}(0.1\text{m/s})^2\,0.664(1161)^{-1/2}$$

$$\tau_{s,L} = 0.0842\,\text{kg/m}\cdot\text{s}^2 = 0.0842\,\text{N/m}^2$$ <

(c) With the drag force per unit width given by $D' = 2L\overline{\tau}_{s,L}$ where the factor of 2 is included to account for both sides of the plate, it follows from Eq. 7.29 that

$$D' = 2L\left(\rho u_\infty^2/2\right)1.328\,\text{Re}_L^{-1/2} = (1\text{m})864\,\text{kg/m}^3\,(0.1\text{m/s})^2\,1.328(1161)^{-1/2} = 0.337\,\text{N/m}$$ <

For laminar flow, the average value $\overline{h}_L$ over the distance 0 to L is twice the local value, h_L,

$$\overline{h}_L = 2h_L = 32.5\ \text{W/m}^2\cdot\text{K}$$

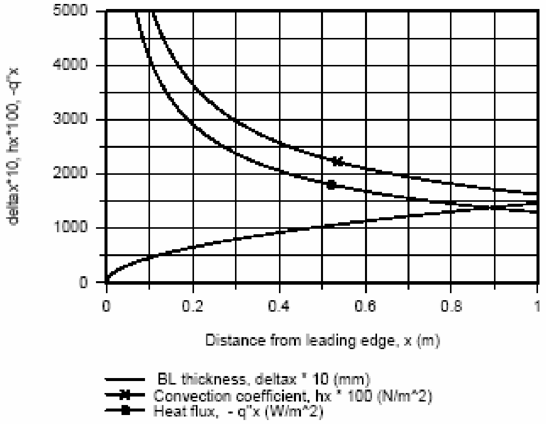
The total heat transfer rate per unit width of the plate is

$$q' = 2L\overline{h}_L\left(T_s - T_\infty\right) = 2(1\text{m})32.5\ \text{W/m}^2\cdot\text{K}(20-100)^\circ\text{C} = -5200\ \text{W/m}$$ <

Continued...

PROBLEM 7.2 (Cont.)

(c) Using IHT with the foregoing equations, the boundary layer thickness, and local values of the convection coefficient and heat flux were calculated and plotted as a function of x.



COMMENTS: (1) Note that since $Pr \gg 1$, $\delta \gg \delta_t$. That is, for the high Prandtl liquids, the velocity boundary layer will be much thicker than the thermal boundary layer.

(2) A copy of the *IHT Workspace* used to generate the above plot is shown below.

```
// Boundary layer thickness, delta
delta = 5 * x * Rex ^-0.5
delta_mm = delta * 1000
delta_plot = delta_mm * 10          // Scaling parameter for convenience in  plotting

// Convection coefficient and heat flux, q"x
q"x = hx * (Ts - Tinf)
Nux = 0.332 * Rex^0.5 * Pr^(1/3)
Nux = hx * x / k
hx_plot = 100 * hx                  // Scaling parameter for convenience in plotting
q"x_plot = ( -1 ) * q"x             // Scaling parameter for convenience in plotting

// Reynolds number
Rex = uinf * x / nu

// Properties Tool: Engine oil
// Engine Oil property functions : From Table A.5
// Units: T(K)
rho = rho_T("Engine Oil",Tf)             // Density, kg/m^3
cp = cp_T("Engine Oil",Tf)               // Specific heat, J/kg·K
nu = nu_T("Engine Oil",Tf)               // Kinematic viscosity, m^2/s
k = k_T("Engine Oil",Tf)                 // Thermal conductivity, W/m·K
Pr = Pr_T("Engine Oil",Tf)               // Prandtl number


// Assigned variables
Tf = (Ts + Tinf) / 2                     // Film temperature, K
Tinf = 100 + 273                         // Freestream temperature, K
Ts = 20 + 273                            // Surface temperature, K
uinf = 0.1                               // Freestream velocity, m/s
x = 1                                    // Plate length, m
```