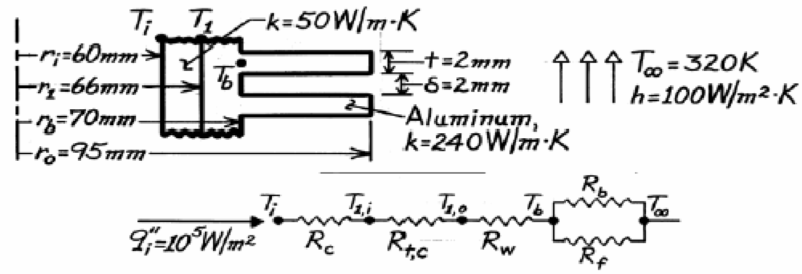
PROBLEM 3.146

KNOWN: Dimensions and materials of a finned (annular) cylinder wall. Heat flux and ambient air conditions. Contact resistance.

FIND: Surface and interface temperatures (a) without and (b) with an interface contact resistance.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional, steady-state conditions, (2) Constant properties, (3) Uniform h over surfaces, (4) Negligible radiation.

ANALYSIS: The analysis may be performed per unit length of cylinder or for a 4 mm long section. The following calculations are based on a unit length. The inner surface temperature may be obtained from

$$q' = \frac{T_i - T_\infty}{R'_{tot}} = q''_i(2\pi\, r_i) = 10^5\ \text{W/m}^2 \times 2\pi \times 0.06\ \text{m} = 37,700\ \text{W/m}$$

where $R'_{tot} = R'_c + R'_{t,c} + R'_w + R'_{equiv}$; $\quad R'_{equiv} = \left(1/R'_f + 1/R'_b\right)^{-1}$.

R'_c, *Conduction resistance of cylinder wall:*

$$R'_c = \frac{\ln(r_1/r_i)}{2\pi\, k} = \frac{\ln(66/60)}{2\pi(50\ \text{W/m}\cdot\text{K})} = 3.034\times10^{-4}\ \text{m}\cdot\text{K/W}$$

$R'_{t,c}$, *Contact resistance:*

$$R'_{t,c} = R''_{t,c}/2\pi\, r_1 = 10^{-4}\ \text{m}^2\cdot\text{K/W}/2\pi\times0.066\ \text{m} = 2.411\times10^{-4}\ \text{m}\cdot\text{K/W}$$

R'_w, *Conduction resistance of aluminum base:*

$$R'_w = \frac{\ln(r_b/r_1)}{2\pi\, k} = \frac{\ln(70/66)}{2\pi\times240\ \text{W/m}\cdot\text{K}} = 3.902\times10^{-5}\ \text{m}\cdot\text{K/W}$$

R'_b, *Resistance of prime or unfinned surface:*

$$R'_b = \frac{1}{hA'_b} = \frac{1}{100\ \text{W/m}^2\cdot\text{K}\times0.5\times2\pi(0.07\ \text{m})} = 454.7\times10^{-4}\ \text{m}\cdot\text{K/W}$$

R'_f, *Resistance of fins:* The fin resistance may be determined from

$$R'_f = \frac{T_b - T_\infty}{q'_f} = \frac{1}{\eta_f h A'_f}$$

The fin efficiency may be obtained from Fig. 3.19,

$$r_{2c} = r_o + t/2 = 0.096\ \text{m} \qquad L_c = L + t/2 = 0.026\ \text{m}$$

Continued …..

PROBLEM 3.146 (Cont.)

$$A_p = L_c t = 5.2\times10^{-5}\ m^2 \qquad r_{2c}/r_1 = 1.45 \qquad L_c^{3/2}\left(h/kA_p\right)^{1/2} = 0.375$$

Fig. 3.19 $\rightarrow \eta_f \approx 0.88$.

The total fin surface area per meter length

$$A'_f = 250\left[\pi\left(r_o^2 - r_b^2\right)\times 2\right] = 250\ m^{-1}\left[2\pi\left(0.096^2 - 0.07^2\right)\right]m^2 = 6.78\ m.$$

Hence $$R'_f = \left[0.88\times100\ W/m^2\cdot K\times6.78\ m\right]^{-1} = 16.8\times10^{-4}\ m\cdot K/W$$

$$1/R'_{equiv} = \left(1/16.8\times10^{-4} + 1/454.7\times10^{-4}\right)W/m\cdot K = 617.2\ W/m\cdot K$$

$$R'_{equiv} = 16.2\times10^{-4}\ m\cdot K/W.$$

Neglecting the *contact resistance*,

$$R'_{tot} = (3.034 + 0.390 + 16.2)10^{-4}\ m\cdot K/W = 19.6\times10^{-4}\ m\cdot K/W$$

$$T_i = q'R'_{tot} + T_\infty = 37,700\ W/m\times19.6\times10^{-4}\ m\cdot K/W + 320\ K = 393.9\ K$$ <

$$T_1 = T_i - q'R'_w = 393.9\ K - 37,700\ W/m\times3.034\times10^{-4}\ m\cdot K/W = 382.5\ K$$ <

$$T_b = T_1 - q'R'_b = 382.5\ K - 37,700\ W/m\times3.902\times10^{-5}\ m\cdot K/W = 381.0\ K.$$ <

Including the *contact resistance*,

$$R'_{tot} = \left(19.6\times10^{-4} + 2.411\times10^{-4}\right)m\cdot K/W = 22.0\times10^{-4}\ m\cdot K/W$$

$$T_i = 37,700\ W/m\times22.0\times10^{-4}\ m\cdot K/W + 320\ K = 402.9\ K$$ <

$$T_{1,i} = 402.9\ K - 37,700\ W/m\times3.034\times10^{-4}\ m\cdot K/W = 391.5\ K$$ <

$$T_{1,o} = 391.5\ K - 37,700\ W/m\times2.411\times10^{-4}\ m\cdot K/W = 382.4\ K$$ <

$$T_b = 382.4\ K - 37,700\ W/m\times3.902\times10^{-5}\ m\cdot K/W = 380.9\ K.$$ <

COMMENTS: (1) The effect of the contact resistance is small.

(2) The effect of including the aluminum fins may be determined by computing T_i without the fins. In this case $R'_{tot} = R'_c + R'_{conv}$, where

$$R'_{conv} = \frac{1}{h2\pi r_1} = \frac{1}{100\ W/m^2\cdot K\ 2\pi(0.066\ m)} = 241.1\times10^{-4}\ m\cdot K/W.$$

Hence, $R_{tot} = 244.1\times10^{-4}\ m\cdot K/W$, and

$$T_i = q'R'_{tot} + T_\infty = 37,700\ W/m\times244.1\times10^{-4}\ m\cdot K/W + 320\ K = 1240\ K.$$

Hence, the fins have a significant effect on reducing the cylinder temperature.

(3) The overall surface efficiency is

$$\eta_o = 1 - \left(A'_f/A'_t\right)\left(1-\eta_f\right) = 1 - 6.78\ m/7.00\ m(1-0.88) = 0.884.$$

It follows that $q' = \eta_o h_o A'_t \theta_b = 37,700\ W/m$, which agrees with the prescribed value.