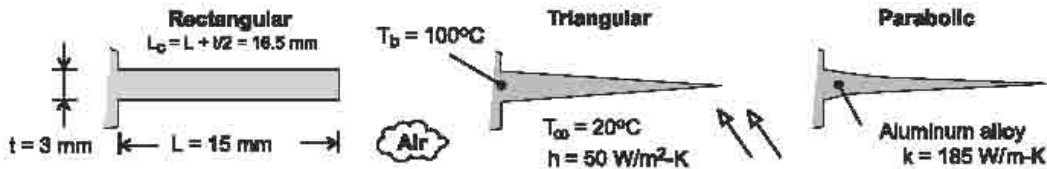


PROBLEM 3.123

KNOWN: Length, thickness and temperature of straight fins of rectangular, triangular and parabolic profiles. Ambient air temperature and convection coefficient.

FIND: Heat rate per unit width, efficiency and volume of each fin.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state, (2) One-dimensional conduction, (3) Constant properties, (4) Negligible radiation, (5) Uniform convection coefficient.

ANALYSIS: For each fin,

$$q'_f = q'_{\max} = \eta_f h A'_f (T_b - T_\infty), \quad V' = A_p L$$

where η_f depends on the value of $m = (2h/kt)^{1/2} = (100 \text{ W/m}^2 \cdot \text{K} / 185 \text{ W/m} \cdot \text{K} \times 0.003 \text{ m})^{1/2} = 13.4 \text{ m}^{-1}$ and the product $mL = 13.4 \text{ m}^{-1} \times 0.015 \text{ m} = 0.201$ or $mL_c = 0.222$. Expressions for η_f , A'_f and A_p are obtained from Table 3-5.

Rectangular Fin:

$$\eta_f = \frac{\tanh mL_c}{mL_c} = \frac{0.218}{0.222} = 0.982, \quad A'_f = 2L_c = 0.033 \text{ m} \quad <$$

$$q'_f = 0.982 (50 \text{ W/m}^2 \cdot \text{K}) (0.033 \text{ m}) (80^\circ\text{C}) = 129.6 \text{ W/m}, \quad V' = tL = 4.5 \times 10^{-5} \text{ m}^2 \quad <$$

Triangular Fin:

$$\eta_f = \frac{1 - I_0(2mL)}{mL I_0(2mL)} = \frac{0.205}{(0.201)1.042} = 0.978, \quad A'_f = 2 \left[L^2 + (t/2)^2 \right]^{1/2} = 0.030 \text{ m} \quad <$$

$$q'_f = 0.978 (50 \text{ W/m}^2 \cdot \text{K}) (0.030 \text{ m}) (80^\circ\text{C}) = 117.3 \text{ W/m}, \quad V' = (t/2)L = 2.25 \times 10^{-5} \text{ m}^2 \quad <$$

Parabolic Fin:

$$\eta_f = \frac{2}{\left[4(mL)^2 + 1 \right]^{1/2} + 1} = 0.963, \quad A'_f = \left[C_1 L + \left(L^2 + t \right) \ln(t/L + C_1) \right] = 0.030 \text{ m} \quad <$$

$$q'_f = 0.963 (50 \text{ W/m}^2 \cdot \text{K}) (0.030 \text{ m}) (80^\circ\text{C}) = 115.6 \text{ W/m}, \quad V' = (t/3)L = 1.5 \times 10^{-5} \text{ m}^2 \quad <$$

COMMENTS: Although the heat rate is slightly larger (~10%) for the rectangular fin than for the triangular or parabolic fins, the heat rate per unit volume (or mass) is larger and largest for the triangular and parabolic fins, respectively.