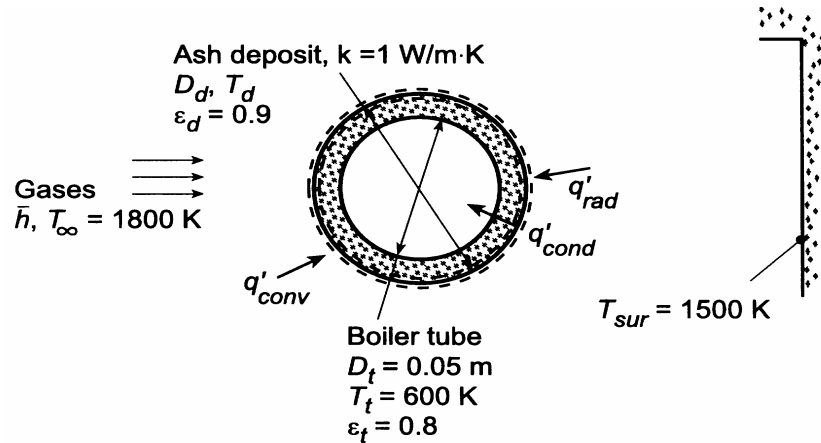


### PROBLEM 13.94

**KNOWN:** Diameter, temperature and emissivity of boiler tube. Thermal conductivity and emissivity of ash deposit. Convection coefficient and temperature of gas flow over the tube. Temperature of surroundings.

**FIND:** (a) Rate of heat transfer to tube without ash deposit, (b) Rate of heat transfer with an ash deposit of diameter  $D_d = 0.06$  m, (c) Effect of deposit diameter and convection coefficient on heat rate and contributions due to convection and radiation.

**SCHEMATIC:**



**ASSUMPTIONS:** (1) Diffuse/gray surface behavior, (2) Surroundings form a large enclosure about the tube and may be approximated as a blackbody, (3) One-dimensional conduction in ash, (4) Steady-state.

**ANALYSIS:** (a) Without an ash deposit, the heat rate per unit tube length may be calculated directly.

$$q' = \bar{h}\pi D_t (T_\infty - T_t) + \varepsilon_t \sigma \pi D_t (T_{sur}^4 - T_t^4)$$

$$q' = 100 \text{ W/m}^2 \cdot \text{K} (\pi) 0.05 \text{ m} (1800 - 600) \text{ K} + 0.8 \times 5.67 \times 10^{-8} \text{ W/m}^2 \cdot \text{K}^4 (\pi) (0.05 \text{ m}) (1500^4 - 600^4) \text{ K}^4$$

$$q' = (18,850 + 35,150) \text{ W/m} = 54,000 \text{ W/m} \quad <$$

(b) Performing an energy balance for a control surface about the outer surface of the ash deposit,

$$q'_{conv} + q'_{rad} = q'_{cond}, \text{ or}$$

$$\bar{h}\pi D_d (T_\infty - T_d) + \varepsilon_d \sigma \pi D_d (T_{sur}^4 - T_d^4) = \frac{2\pi k (T_d - T_t)}{\ln(D_d/D_t)}$$

Hence, canceling  $\pi$  and considering an ash deposit for which  $D_d = 0.06$  m,

$$100 \text{ W/m}^2 \cdot \text{K} (0.06 \text{ m}) (1800 - T_d) \text{ K} + 0.9 \times 5.67 \times 10^{-8} \text{ W/m}^2 \cdot \text{K}^4 (0.06 \text{ m}) (1500^4 - T_d^4) \text{ K}^4 = \frac{2(1 \text{ W/m} \cdot \text{K})(T_d - 600) \text{ K}}{\ln(0.06/0.05)}$$

A trial-and-error solution yields  $T_d \approx 1346$  K, from which it follows that

$$q' = \bar{h}\pi D_d (T_\infty - T_d) + \varepsilon_d \sigma \pi D_d (T_{sur}^4 - T_d^4)$$

$$q' = 100 \text{ W/m}^2 \cdot \text{K} (\pi) 0.06 \text{ m} (1800 - 1346) \text{ K} + 0.9 \times 5.67 \times 10^{-8} \text{ W/m}^2 \cdot \text{K}^4 (\pi) 0.06 \text{ m} (1500^4 - 1346^4) \text{ K}^4$$

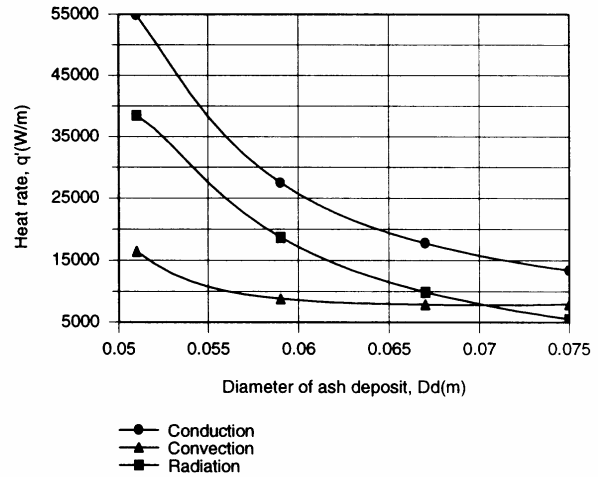
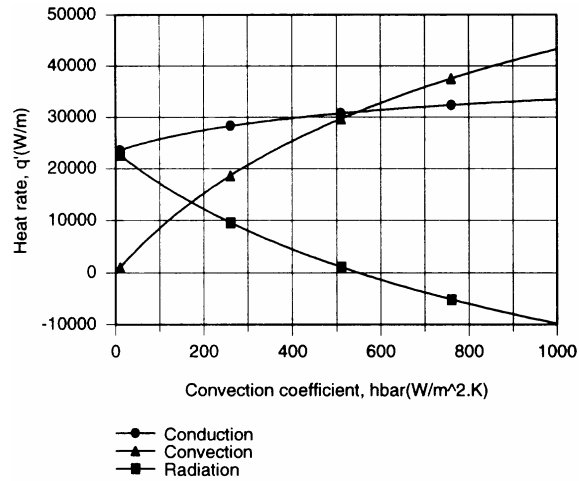
Continued .....

### PROBLEM 13.94 (Cont.)

$$q' = (8560 + 17,140) \text{ W/m} = 25,700 \text{ W/m}$$

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(c) The foregoing energy balance was entered into the *IHT* workspace and parametric calculations were performed to explore the effects of  $\bar{h}$  and  $D_d$  on the heat rates.



For  $D_d = 0.06$  m and  $10 \leq \bar{h} \leq 1000$  W/m<sup>2</sup>·K, the heat rate to the tube,  $q'_{\text{cond}}$ , as well as the contribution due to convection,  $q'_{\text{conv}}$ , increase with increasing  $\bar{h}$ . However, because the outer surface temperature  $T_d$  also increases with  $\bar{h}$ , the contribution due to radiation decreases and becomes negative (heat transfer from the surface) when  $T_d$  exceeds 1500 K at  $\bar{h} = 540$  W/m<sup>2</sup>·K. Both the convection and radiation heat rates, and hence the conduction heat rate, increase with decreasing  $D_d$ , as  $T_d$  decreases and approaches  $T_t = 600$  K. However, even for  $D_d = 0.051$  m (a deposit thickness of 0.5 mm),  $T_d = 773$  K and the ash provides a significant resistance to heat transfer.

**COMMENTS:** Boiler operation in an energy efficient manner dictates that ash deposits be removed periodically.