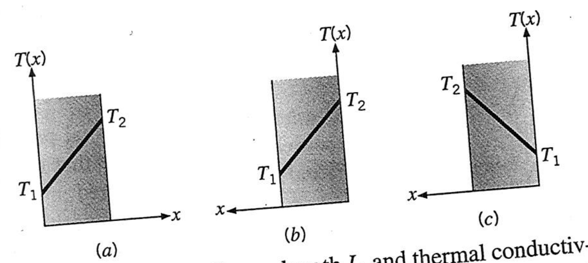
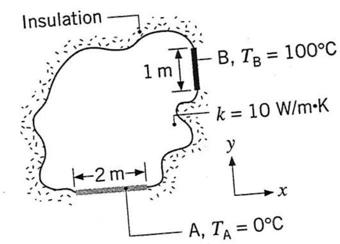


2.10 A cylinder of radius r_o, length L, and thermal conductivity k is immersed in a fluid of convection coefficient h and unknown temperature T_∞. At a certain instant the temperature distribution in the cylinder is $T(r) = a + br^2$, where a and b are constants. Obtain expressions for the heat transfer rate at r_o and the fluid temperature.

2.11 In the two-dimensional body illustrated, the gradient at surface A is found to be $\partial T/\partial y = 30$ K/m. What are $\partial T/\partial y$ and $\partial T/\partial x$ at surface B?

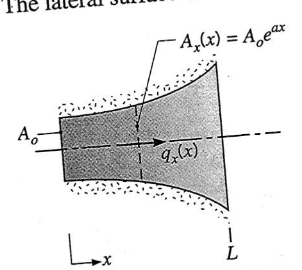


2.12 Sections of the trans-Alaska pipeline run above the ground and are supported by vertical steel shafts ($k = 25$ W/m · K) that are 1 m long and have a cross-sectional area of 0.005 m^2. Under normal operating conditions, the temperature variation along the length of a shaft is known to be governed by an expression of the form

$$T = 100 - 150x + 10x^2$$

where T and x have units of °C and meters, respectively. Temperature variations are small over the shaft cross section. Evaluate the temperature and conduction heat rate at the shaft–pipeline joint ($x = 0$) and at the shaft–ground interface ($x = 1$ m). Explain the difference in the heat rates.

2.13 Steady-state, one-dimensional conduction occurs in a rod of constant thermal conductivity k and variable cross-sectional area $A_x(x) = A_o e^{ax}$, where A_o and a are constants. The lateral surface of the rod is well insulated.



(a) Write an expression for the conduction heat rate, $q_x(x)$. Use this expression to determine the temperature distribution $T(x)$ and qualitatively sketch the distribution for $T(0) > T(L)$.

(b) Now consider conditions for which thermal energy is generated in the rod at a volumetric rate $\dot{q} = \dot{q}_o \exp(-ax)$, where $\dot{q}_o$ is a constant. Obtain an expression for $q_x(x)$ when the left face ($x = 0$) is well insulated.

Thermophysical Properties

2.14 Consider a 300 mm × 300 mm window in an aircraft. For a temperature difference of 80°C from the inner to the outer surface of the window, calculate the heat loss through L = 10-mm-thick polycarbonate, soda lime glass, and aerogel windows, respectively. The therma conductivities of the aerogel and polycarbonate ar k_{ag} = 0.014 W/m · K and k_{pc} = 0.21 W/m · K, respec tively. Evaluate the thermal conductivity of the soc lime glass at 300 K. If the aircraft has 130 windows ar the cost to heat the cabin air is \$1/kW · h, compare t costs associated with the heat loss through the windo for an 8-hour intercontinental flight.

2.15 Gold is commonly used in semiconductor packaging form interconnections (also known as interconne that carry electrical signals between different device the package. In addition to being a good electrical c ductor, gold interconnects are also effective at protec the heat-generating devices to which they are atta by conducting thermal energy away from the devic surrounding, cooler regions. Consider a thin film of that has a cross section of 60 nm × 250 nm.

(a) For an applied temperature difference of 20°C termine the energy conducted along a 1-μm thin-film interconnect. Evaluate properties at

(b) Plot the lengthwise (in the 1-μm directio spanwise (in the thinnest direction) therma ductivities of the gold film as a function of t thickness, L, for $30 \leq L \leq 140$ nm.

2.16 A TV advertisement by a well-known insulation m turer states: it isn't the thickness of the insulating that counts, it's the R-value. The ad shows that t an R-value of 19, you need 18 ft of rock, 15 in. or just 6 in. of the manufacturer's insulation. Is th tisement technically reasonable? If you are like viewers, you don't know the R-value is define where L (in.) is the thickness of the insulation ar in./hr · ft^2 · °F) is the thermal conductivity of the

2.17 An apparatus for measuring thermal conduc ploys an electrical heater sandwiched bet

(b) What is the volumetric rate of heat generation $\dot{q}$ in the wall?

(c) Determine the surface heat fluxes, $q''_x(-L)$ and $q''_x(+L)$. How are these fluxes related to the heat generation rate?

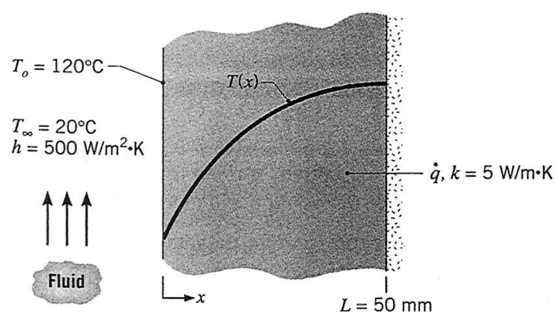
(d) What are the convection coefficients for the surfaces at $x = -L$ and $x = +L$?

(e) Obtain an expression for the heat flux distribution, $q''_x(x)$. Is the heat flux zero at any location? Explain any significant features of the distribution.

(f) If the source of the heat generation is suddenly deactivated ($\dot{q} = 0$), what is the rate of change of energy stored in the wall at this instant?

(g) What temperature will the wall eventually reach with $\dot{q} = 0$? How much energy must be removed by the fluid per unit area of the wall (J/m^2) to reach this state? The density and specific heat of the wall material are 2600 kg/m^3 and 800 $J/kg \cdot K$, respectively.

2.26 One-dimensional, steady-state conduction with uniform internal energy generation occurs in a plane wall with a thickness of 50 mm and a constant thermal conductivity of 5 $W/m \cdot K$. For these conditions, the temperature distribution has the form, $T(x) = a + bx + cx^2$. The surface at $x = 0$ has a temperature of $T(0) \equiv T_o = 120°C$ and experiences convection with a fluid for which $T_\infty = 20°C$ and $h = 500\ W/m^2 \cdot K$. The surface at $x = L$ is well insulated.



(a) Applying an overall energy balance to the wall, calculate the internal energy generation rate, $\dot{q}$.

(b) Determine the coefficients a, b, and c by applying the boundary conditions to the prescribed temperature distribution. Use the results to calculate and plot the temperature distribution.

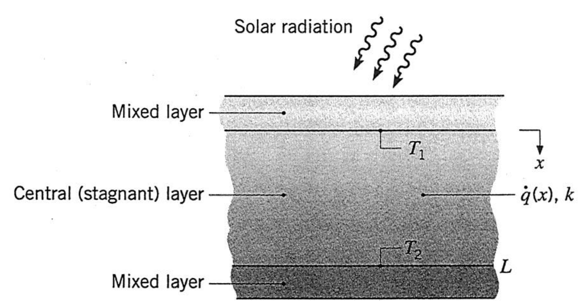
(c) Consider conditions for which the convection coefficient is halved, but the internal energy generation rate remains unchanged. Determine the new values of a, b, and c, and use the results to plot the temperature distribution. *Hint:* recognize that $T(0)$ is no longer 120°C.

(d) Under conditions for which the internal energy generation rate is doubled, and the convection coefficient remains unchanged ($h = 500\ W/m^2 \cdot K$), determine the new values of a, b, and c and plot the corresponding temperature distribution. Referring to the results of parts (b), (c), and (d) as Cases 1, 2, and 3, respectively, compare the temperature distributions for the three cases and discuss the effects of h and $\dot{q}$ on the distributions.

2.27 A salt-gradient solar pond is a shallow body of water that consists of three distinct fluid layers and is used to collect solar energy. The upper- and lower-most layers are well mixed and serve to maintain the upper and lower surfaces of the central layer at uniform temperatures T_1 and T_2, where $T_2 > T_1$. Although there is bulk fluid motion in the mixed layers, there is no such motion in the central layer. Consider conditions for which solar radiation absorption in the central layer provides nonuniform heat generation of the form $\dot{q} = Ae^{-ax}$, and the temperature distribution in the central layer is

$$T(x) = -\frac{A}{ka^2}e^{-ax} + Bx + C$$

The quantities A (W/m^3), a (1/m), B (K/m), and C (K) are known constants having the prescribed units, and k is the thermal conductivity, which is also constant.



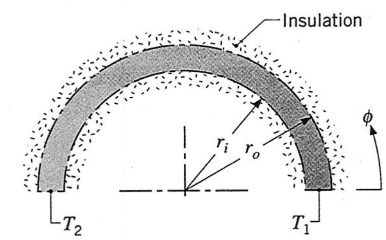
(a) Obtain expressions for the rate at which heat is transferred per unit area from the lower mixed layer to the central layer and from the central layer to the upper mixed layer.

(b) Determine whether conditions are steady or transient.

(c) Obtain an expression for the rate at which thermal energy is generated in the entire central layer, per unit surface area.

2.28 The steady-state temperature distribution in a semitransparent material of thermal conductivity k and thickness L exposed to laser irradiation is of the form

$$T(x) = -\frac{A}{ka^2}e^{-ax} + Bx + C$$



(a) Beginning with a properly defined control volume and considering energy generation and storage effects, derive the differential equation that prescribes the variation in temperature with the angular coordinate ϕ. Compare your result with Equation 2.24.

(b) For steady-state conditions with no internal heat generation and constant properties, determine the temperature distribution $T(\phi)$ in terms of the constants T_1, T_2, r_i, and r_o. Is this distribution linear in ϕ?

(c) For the conditions of part (b) write the expression for the heat rate q_ϕ.

2.33 Beginning with a differential control volume in the form of a cylindrical shell, derive the heat diffusion equation for a one-dimensional, cylindrical, radial coordinate system with internal heat generation. Compare your result with Equation 2.24.

2.34 Beginning with a differential control volume in the form of a spherical shell, derive the heat diffusion equation for a one-dimensional, spherical, radial coordinate system with internal heat generation. Compare your result with Equation 2.27.

2.35 Derive the heat diffusion equation, Equation 2.24, for cylindrical coordinates beginning with the differential control volume shown in Figure 2.12.

2.36 Derive the heat diffusion equation, Equation 2.27, for spherical coordinates beginning with the differential control volume shown in Figure 2.13.

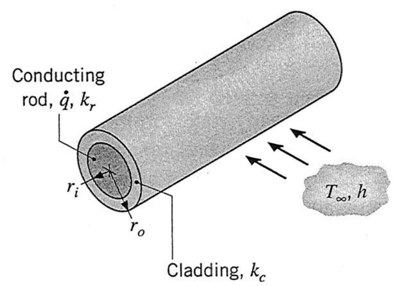
2.37 A steam pipe is wrapped with insulation of inner and outer radii, r_i and r_o, respectively. At a particular instant the temperature distribution in the insulation is known to be of the form

$$T(r) = C_1 \ln\left(\frac{r}{r_o}\right) + C_2$$

Are conditions steady-state or transient? How do the heat flux and heat rate vary with radius?

2.38 For a long circular tube of inner and outer radii r_1 and r_2, respectively, uniform temperatures T_1 and T_2 are maintained at the inner and outer surfaces, while thermal energy generation is occurring within the tube wall ($r_1 < r < r_2$). Consider steady-state conditions for which $T_1 > T_2$. Is it possible to maintain a *linear* radial temperature distribution in the wall? If so, what special conditions must exist?

2.39 Passage of an electric current through a long conducting rod of radius r_i and thermal conductivity k_r results in uniform volumetric heating at a rate of $\dot{q}$. The conducting rod is wrapped in an electrically nonconducting cladding material of outer radius r_o and thermal conductivity k_c, and convection cooling is provided by an adjoining fluid.



For steady-state conditions, write appropriate forms of the heat equations for the rod and cladding. Express appropriate boundary conditions for the solution of these equations.

2.40 Two-dimensional, steady-state conduction occurs in a hollow cylindrical solid of thermal conductivity $k = 16\ \text{W/m}\cdot\text{K}$, outer radius $r_o = 1$ m, and overall length $2z_o = 5$ m, where the origin of the coordinate system is located at the midpoint of the centerline. The inner surface of the cylinder is insulated, and the temperature distribution within the cylinder has the form $T(r, z) = a + br^2 + c \ln r + dz^2$, where $a = 20°\text{C}$, $b = 150°\text{C/m}^2$, $c = -12°\text{C}$, $d = -300°\text{C/m}^2$ and r and z are in meters.

(a) Determine the inner radius r_i of the cylinder.

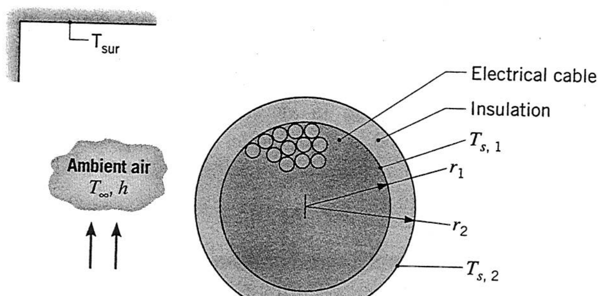
(b) Obtain an expression for the volumetric rate of heat generation, $\dot{q}(\text{W/m}^3)$.

(c) Determine the axial distribution of the heat flux at the outer surface, $q''_r(r_o, z)$. What is the heat rate at the outer surface? Is it into or out of the cylinder?

(d) Determine the radial distribution of the heat flux at the end faces of the cylinder, $q''_z(r, +z_o)$ and $q''_z(r, -z_o)$. What are the corresponding heat rates? Are they into or out of the cylinder?

(e) Verify that your results are consistent with an overall energy balance on the cylinder.

2.41 An electric cable of radius r_1 and thermal conductivity k_c is enclosed by an insulating sleeve whose outer surface is of radius r_2 and experiences convection heat transfer and radiation exchange with the adjoining air and large surroundings, respectively. When electric current passes through the cable, thermal energy is generated within the cable at a volumetric rate $\dot{q}$.



(a) Write the steady-state forms of the heat diffusion equation for the insulation and the cable. Verify that these equations are satisfied by the following temperature distributions:

$$\textit{Insulation:}\quad T(r) = T_{s,2} + (T_{s,1} - T_{s,2})\frac{\ln(r/r_2)}{\ln(r_1/r_2)}$$

$$\textit{Cable:}\quad T(r) = T_{s,1} + \frac{\dot{q}r_1^2}{4k_c}\left(1 - \frac{r^2}{r_1^2}\right)$$

Sketch the temperature distribution, $T(r)$, in the cable and the sleeve, labeling key features.

(b) Applying Fourier's law, show that the rate of conduction heat transfer per unit length through the sleeve may be expressed as

$$q_r' = \frac{2\pi k_s(T_{s,1} - T_{s,2})}{\ln(r_2/r_1)}$$

Applying an energy balance to a control surface placed around the cable, obtain an alternative expression for q_r', expressing your result in terms of $\dot{q}$ and r_1.

(c) Applying an energy balance to a control surface placed around the outer surface of the sleeve, obtain an expression from which $T_{s,2}$ may be determined as a function of $\dot{q}$, r_1, h, T_∞, ε, and T_{sur}.

(d) Consider conditions for which 250 A are passing through a cable having an electric resistance per unit length of $R_e' = 0.005\ \Omega/\text{m}$, a radius of $r_1 = 15$ mm, and a thermal conductivity of $k_c = 200\ \text{W/m} \cdot \text{K}$. For $k_s = 0.15\ \text{W/m} \cdot \text{K}$, $r_2 = 15.5$ mm, $h = 25\ \text{W/m}^2 \cdot \text{K}$, $\varepsilon = 0.9$, $T_\infty = 25°\text{C}$, and $T_{sur} = 35°\text{C}$, evaluate the surface temperatures, $T_{s,1}$ and $T_{s,2}$, as well as the temperature T_o at the centerline of the cable.

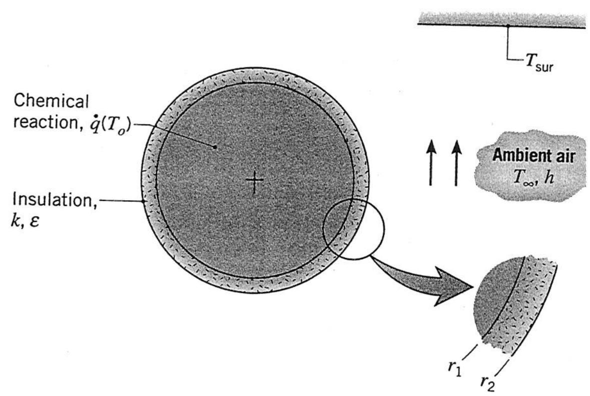
(e) With all other conditions remaining the same, compute and plot T_o, $T_{s,1}$, and $T_{s,2}$ as a function of r_2 for $15.5 \le r_2 \le 20$ mm.

2.42 A spherical shell of inner and outer radii r_i and r_o, spectively, contains heat-dissipating components, anc a particular instant the temperature distribution in shell is known to be of the form

$$T(r) = \frac{C_1}{r} + C_2$$

Are conditions steady-state or transient? How do t heat flux and heat rate vary with radius?

2.43 A chemically reacting mixture is stored in a thin-wall spherical container of radius $r_1 = 200$ mm, and the exoth mic reaction generates heat at a uniform, but temperatu dependent volumetric rate of $\dot{q} = \dot{q}_o \exp(-A/T_o)$, whe $\dot{q}_o = 5000\ \text{W/m}^3$, $A = 75$ K, and T_o is the mixture tempe ature in kelvins. The vessel is enclosed by an insulati material of outer radius r_2, thermal conductivity k, ar emissivity ε. The outer surface of the insulation expe ences convection heat transfer and net radiation exchan with the adjoining air and large surroundings, respectivel



(a) Write the steady-state form of the heat diffusion equation for the insulation. Verify that this equation is satisfied by the temperature distribution

$$T(r) = T_{s,1} - (T_{s,1} - T_{s,2})\left[\frac{1 - (r_1/r)}{1 - (r_1/r_2)}\right]$$

Sketch the temperature distribution, $T(r)$, labeling key features.

(b) Applying Fourier's law, show that the rate of heat transfer by conduction through the insulation may be expressed as

$$q_r = \frac{4\pi k(T_{s,1} - T_{s,2})}{(1/r_1) - (1/r_2)}$$

Applying an energy balance to a control surface about the container, obtain an alternative expression for q_r, expressing your result in terms of $\dot{q}$ and r_1.