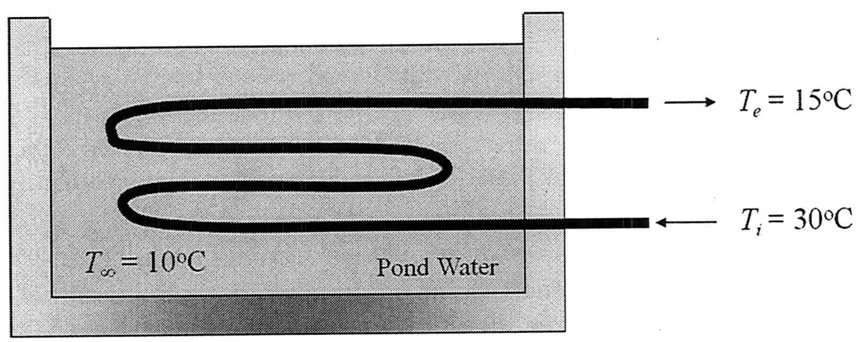
Exam Solutions

MIE 354 - Heat Transfer
Professor Rothstein
Exam #2
April 14, 2011

You will have 1.25 hours to complete the following 2 problems. The problems are given equal weight so make sure to use your time wisely. Please write carefully and clearly and make sure that you justify any assumption that you make. Place all answers in the boxes given. The exam is open book and open notes. Don't forget to print your name on the exam. Good luck.

1. Heat Pump Operation

Heat pumps can be used to heat a house in the winter and when run in reverse, they can cool a house in the summer. As shown in the figure below, the evaporator section of a heat pump can be connected to a pond or a lake which can be used as a heat source in the winter and used for air conditioning in the summer. Consider summer cooling operation where air is passed through a series of copper pipes submerged in the bath. You can ignore the thermal resistance of the copper pipes and assume that the inside surface of the pipe is fixed at the temperature of the pond water at $T_s = 10^\circ\text{C}$. The air in the pipe enters the pond with an initial temperature of $T_i = 30^\circ\text{C}$ and leaves with a desired exit temperature of $T_f = 15^\circ\text{C}$. For a pipe with an inner diameter of $D = 50\text{mm}$ and a mass flow rate of $\dot{m} = 0.001$ kg/s, the goal is to determine what length of pipe is needed to obtain the desired cooling.



Thermophysical Properties

Air	Water
$k = 0.026\,\text{W/m K}$	$k = 0.569\,\text{W/m K}$
$c_p = 1.00\,\text{kJ/kg K}$	$c_p = 4.2\,\text{kJ/kg K}$
$\rho = 1.16\,\text{kg/m}^3$	$\rho = 1000\,\text{kg/m}^3$
$Pr = 0.707$	$Pr = 12.99$
$\nu = 15.89\times10^{-6}\,\text{m}^2/\text{s}$	

a) Is the flow within the pipe turbulent or laminar? Justify your answer.

$\dot{m} = \rho A_c u$

$u = \frac{\dot{m}}{\rho A_c} = \frac{\dot{m}}{\rho(\frac{\pi D^2}{4})} = 0.439\ m/s$

$Re_D = \frac{uD}{\nu} = 1382 \rightarrow$ Laminar

Laminar
$Re = 1382 < 2000$

b) What is the average Nusselt number and heat transfer coefficient in the pipe? You can assume that the flow is fully developed

$Nu = 3.66$ for constant temperature wall and laminar flow

$h = \frac{Nu\, k_s}{D} = 1.9\ W/m^2K$

$h = 1.9\ W/m^2 K$
$Nu = 3.66$

c) What length of pipe is required to achieve the desired exit temperature?

$(T_s - T_{m,o}) = (T_s - T_{m,i}) \exp\left(\frac{PL}{\dot{m} C_p} \bar{h}\right)$

$L = \ln\left[\frac{(T_s - T_o)}{(T_s - T_i)}\right] \frac{\dot{m} C_p}{P \bar{h}} = 4.637\ m$

$L = 4.6\ m$

d) Was your assumption of fully developed flow justified? Explain why or why not. How much error did this assumption introduce into your calculation of Nusselt Number? Do you need more or less pipe to achieve the desired exit temperature?

Not fully developed
11% error.

$\frac{X_e}{L} = \frac{Re\, Pr}{20} \rightarrow X_e = 2.4\ m$

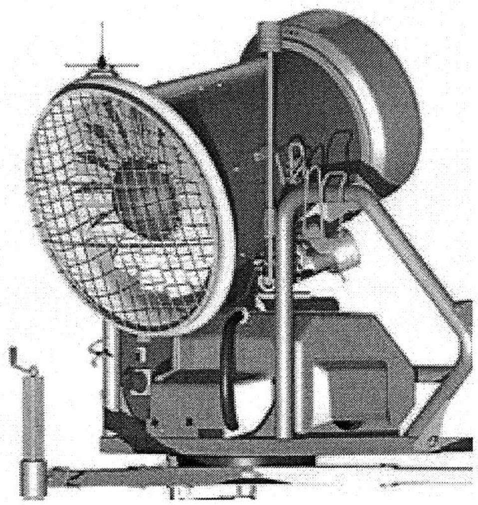
- Since this is more than 50% of the pipe, it is not fully developed.

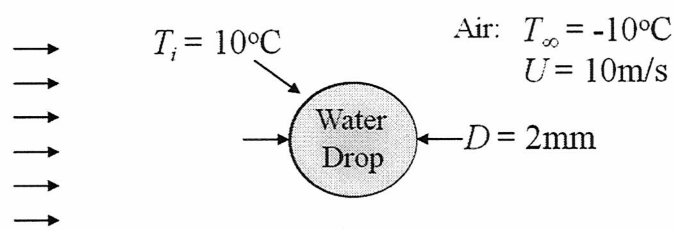
$Nu_D = 3.66 + \frac{0.0668 (D/L) Re_D Pr}{1 + 0.04 [(D/L) Re_D Pr]^{2/3}} = 4.11$

$\% \text{ error} = \frac{4.11 - 3.66}{4.11} = 11\% \Rightarrow$ Need shorter stretch of pipe.

2. Snow Making

The newer style snowmaking guns use high pressure nozzles used to atomize water into drops and large fans to blow the water droplets out onto the ski hill. See the illustration below. The water droplets that are formed are typically on the order of D= 2mm and the fan can produce a breeze with a velocity of 10m/s. If the outside air temperature is $T_\infty = -10^\circ\text{C}$ and the water droplets are atomized at an initial temperature of $T_i = 10^\circ\text{C}$ we want to know how long it will take for the droplets to begin freezing. To make life easier we won't worry about phase change to start, but will determine the time it takes for the surface of the water drop to reach $T_f = 0^\circ\text{C}$.





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$\nu = \nu_s = 15.89 \times 10^{-6}\,\text{m}^2/\text{s}$	

a) What is the Reynolds number of the flow?

$$Re_D = \frac{UD}{\nu} = 1260$$

$Re = 1260$

b) What is the Nusselt number?

$$\overline{Nu}_D = 2 + \left[0.4 Re_D^{1/2} + 0.6 Re_D^{2/3}\right] Pr^{0.4} \left(\frac{\mu}{\mu_s}\right)$$

$\overline{Nu}_D = 20.5$

$$\frac{\mu}{\mu_s} = 1 \rightarrow \overline{Nu}_D = 20.5$$

c) What is the heat transfer coefficient?

$$h = \frac{Nu_D D}{k_f} = 266\ W/m^2K$$

$h = 266\ W/m^2K$

d) You can assume that the water in the droplet does not move and so the heat transfer in the droplet is purely conductive. Can the droplet be considered lumped system? Justify your answer.

$$Bi = \frac{hR}{k_s} = 0.45 > 1/6$$

$Bi > 1/6$

Not lumped

Alternatively: $Bi = \frac{hR/3}{k_s} = 0.15 > 0.10$ → still not lumped

e) How long will it take for the outer surface of the water droplet to reach $T_f = 0^\circ C$ and begin to freeze?

Using approximate solution for sphere

$t = 3.34\ s$

$$\Theta = \frac{T - T_\infty}{T_i - T_\infty} = C_1 \exp(-\zeta^2 Fo) \frac{1}{\zeta r^*} \sin(\zeta_1 r^*)$$

$$r^* = \frac{r}{r_o} = 1 \qquad \Theta = 0.5$$

Form table 5.1 $\zeta_1 = 1.1$ and $C_1 = 1.13$

$$Fo = \frac{\alpha t}{r_o^2} = \ln\left(\frac{\Theta \zeta_1 r^*}{C_1 \sin(\zeta_1 r^*)}\right) \cdot \frac{1}{-\zeta^2} = 0.5$$

$$t = 3.34\ s$$

If $Bi = \frac{hR/3}{k} = 0.15$, $\zeta_1 = 0.66$, $C = 1.04$

$Fo = 1.5$ and $t = 10s$