Homework 1

Homework #1 :

(1) The velocity potential for the flow around a cylinder is given by $\phi = Ur(1+\frac{a^2}{r^2})\cos\theta$ where U is the freestream velocity and a is the cylinder radius.

(a) Take the gradient of the velocity potential to get the formula for the velocity field. Use cylindrical coordinates.

(b) Plot the velocity field (draw lots of vectors, and the cylinder, until I am sure you have a good idea what the velocity is doing). The math is easier in cylindrical coordinates but the plotting is a pain (remember v_r always points away from the origin, and v_θ is always perpendicular to it).

(c) Convert the velocity potential expression from r and θ to x and y. (Using stuff like $r^2 = x^2 + y^2$, you find $\cos\theta = ???$).

(d) Take the gradient again to find the velocity, but this time using Cartesian coordinates.

(e) Plot the vector field using Cartesian velocity field expression. How does it differ from part (b)?

(f) Take the curl of the velocity field to obtain the vorticity vector. Do it in either Cartesian of Cyclindical coordinates (your preference), but tell me the whole vector (all three coordinates).

(g) What is the vorticity at r=0 (or x=0 and y=0)?

(h) What is the formula for velocity dilatation (divergence of the velocity)?

(i) What is the dilatation at r=0 (or x=0 and y=0)? Why doesn't this matter?

(2) Munson 6.66

Hint for (c): Bernoulli's equation holds between any two points in potential flow. You do NOT need them on the same streamline. But you must show your flow is a potential flow (dilatation and vorticity are zero) to do this trick.

(3) Consider the Euler equations (6.127a, 6.127b, 6.127c with $\mu = 0$) to be a vector. Divide each equation through by the density and then take the divergence to get a single scalar equation for the dilation ($d = \nabla \cdot \mathbf{u}$). Keep manipulating until your answer looks like $\frac{Dd}{Dt} = ???$. I want to know the right hand side.