MIE 340
Prof. Perot
Handout #6

Pressure

(1) Pressure has the same units as energy/volume and force/area. Learn to use it both ways.

(2) Bernoulli's Equation (Conservation of Energy)

$$\left(p+\rho\tfrac{1}{2}\vec{\mathbf{v}}\cdot\vec{\mathbf{v}}+\rho gz\right)_2=\left(p+\rho\tfrac{1}{2}\vec{\mathbf{v}}\cdot\vec{\mathbf{v}}+\rho gz\right)_1+\tfrac{E}{V}\big|_{pump}$$

(i) Works for statics (v=0) <u>and</u> dynamics.
(ii) Energy losses and inputs must be explicitly added in (like the last term).
(iii) Remember this applies for a *single* <u>*incompressible*</u> fluid.
(iii) For dynamics ($\mathbf{v}\neq 0$) you also need a *steady* flow and a *streamline* (always draw your streamline so you know you have one).

Hints:

Get the right density.
Use a single coordinate system.
The z-direction is always up.
If you have multiple fluids you apply this equation multiple times.

(3) The real equation for pressure (so far) is:

$$\vec{\mathbf{a}}=-\frac{1}{\rho}\nabla p+\vec{\mathbf{g}}$$

Where **g** is the gravity vector and **a** is the possible acceleration of the (non-inertial) coordinate system. This equation says that pressure always arranges itself to counteract forces (density times accelerations).

(i) You use this equation if the density is not constant, or you have a nonzero acceleration, or if the acceleration varies (for example very far from earth – the acceleration of gravity changes with distance).

(4) <u>Pressure can acts as a distributed **FORCE** on a surface</u>.

(i) What you would like to do is figure out the *equivalent point force*.

(ii) You need to know the equivalent point force's:
Direction
Magnitude
Effective location

Forces on Planar Surfaces (due to linear pressure fields)

(1) The **direction** is normal (perpendicular) to the surface and outwards.
(2) The **magnitude** is equal to $|F| = p^{CG} A_{surface}$.
(3) The **location** is down (the surface) from the CG by an amount $\Delta y = \frac{\rho g \sin\theta \hat{I}_{xx}}{|F|}$

Note:
(i) Air is a fluid and it exerts a force – so use gage pressure to make the air force = 0.
(ii) Don't let the center of gravity CG trick you. It **is** the location we use to calculate the force magnitude (by a trick). It **is usually not** the location of the force.

Curved Surfaces

To analyze curved surfaces use a free body diagram. Then figure out all the forces the fluid is exerting on all the surfaces (real and imaginary) and on the earth (due to gravity).

Side ways forces on curved surfaces are easy (and end up using the projected sideways area).

Vertical forces a slightly trickier because the weight of the water between the curved surface and the flat surface comes into play (and the gravity force – of the fluid on the earth – is upwards).

Often the horizontal and vertical forces on a curved surface do not act at the same location on that curved surface. Get the locations by using the fact that the sum of the moments is zero in a static object (and the trick is picking the point to take moments about that eliminates all the unknown forces).

Buoyancy Forces

The buoyancy force is the net force on the curved top (less pressure) and the curved bottom (more pressure) of an object. After all the math - it turns out that this net vertical force on the two curved surfaces is always *the weight of the fluid that is displaced (i.e the weight of the fluid that is not there due to the object being there).*

This means that every object in a fluid has an upwards force equal to the weight of the fluid displaced. For partially submerged objects - sometimes figuring out the fluid displaced is not so obvious – then go back to evaluating the forces as a set of curved surfaces.