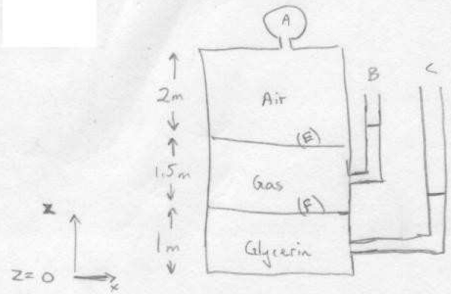
MIE 340
Prof. Perot

Solutions for HW2 - HW4

2.1 see solution
2.2 (a) 0.226 m
2.3 Force on section AB. water force: 78320 N, Glycerin force -54384 N
water position: -0.0833 m below CG. Glycerin position -0.1515 m
2.4 see solution
2.5 (a) 59 kPa (b) 0.44 m
2.6 see class notes. 98 MN right 154 MN down.

3.1 see solution
3.2 (a) rho_cold = 1.185 kg/m3, rho_hot=1.030 kg/m3, D=11.8 m
3.3 see solution
3.4 26.8 m/s, 1 atm.
3.5 (a) P=Weight/Projected Area = 1770 Pa, (b) 54.2 m/s

4.1 (a) 0.00318 m3/s (b) out
4.2 V = Q2/A1
4.3 See solution
4.4 (a) 156 kg/s (b) 102 kN
4.5 see solution
4.6 2600 W



$P_A = 1.5$ kPa (gage)

P_B and P_C are atmospheric or 0 kPa (gage)

$$\boxed{P_2 - P_1 = -\rho g (z_2 - z_1)} \quad [\text{where } z \text{ is `up'}]$$

The air is a short distance so the incompressible formula above is probably OK.

$$P_E - P_A = -\rho_{air}\, g\, (z_E - z_A)$$

$$P_E = P_A - \left(12.0\ \tfrac{N}{m^3}\right)(2.5m - 4.5m)$$

$$P_E = 1500 \text{ Pa (gage)} + (24 \text{ Pa}) = 1524 \text{ Pa (gage)}$$

$$P_B - P_E = -\rho_{gas}\, g\, (z_B - z_E)$$

$$P_B - P_E = -\left(6670\ \tfrac{N}{m^3}\right)(z_B - 2.5m)$$

$$\frac{0 \text{ kPa (gage)} - 1524 \text{ Pa (gage)}}{-6670\ \frac{N}{m^3}} + 2.5m = z_B = 2.73 \text{ m}$$

Note : (1) Converting everything to absolute pressure will also work but will introduce some round off error due to the large value of $P_{ATM} = 101$ kPa

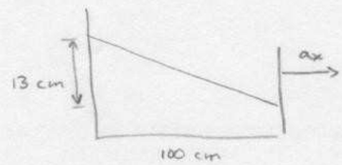
(2) Watch kPa and Pa → convert to Pa before calculating h

$$P_F - P_E = -\rho_{gas}\, g\, (z_F - z_E)$$

$$P_F \text{ (gage)} = 1524 \text{ Pa (gage)} - \left(6670\ \tfrac{N}{m^3}\right)(1m - 2.5m) = 11.53 \text{ kPa (gage)}$$

$$P_C - P_F = -\rho_{gly}\, g\, (z_C - z_F)$$

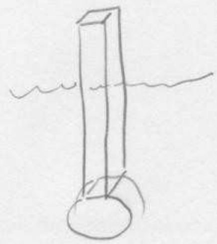
$$(0 - 11{,}530 \text{ Pa}) / (-12360\ N/m^3) + 1m = z_C = 1.93 \text{ m}$$



$$\frac{a_x}{g+a_z} = \tan\theta$$

(a) $a_x = g\tan\theta = \left(9.81\ \frac{m}{s^2}\right)\left(\frac{13}{100}\right) = 1.275\ \frac{m}{s^2}$

(b) The slope depends only on the acceleration - not the fluid.
So same result for Mercury.



F_{up} = Bouyancy = F_{down} = weight

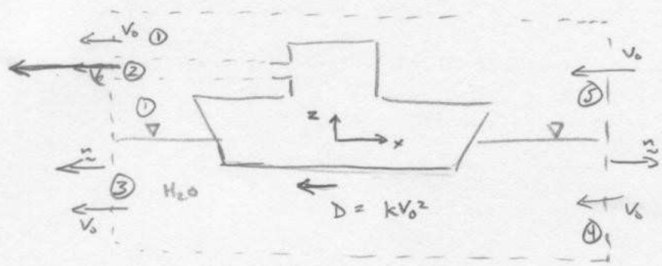
$$(V_{lead} + V_{wood\text{-}wet})\,\rho_{H2O}\,\cancel{g} = V_{lead}\,\rho_{lead}\,\cancel{g} + V_{wood}\,\rho_{wood}\,\cancel{g}$$

$$V_{lead}\,(\cancel{\rho_{H2O}} - 11.4\,\cancel{\rho_{H2O}}) = V_{wood}\,(.65)\,\cancel{\rho_{H2O}} - V_{wood\text{-}wet}\,\cancel{\rho_{H2O}}$$

$$V_{lead} = \frac{V_{wood}\,(.65) - V_{wood\text{-}wet}}{(1.0 - 11.4)} = .000\,113\ m^3$$

so $W_{lead} = V_{lead}\,11.4\,\rho_{H2O}\,g = 12.6\ N$

Problem in boat reference frame



Assumptions :

Incompressible

steady

water is flat

constant speed - V_0

V_e = 343 m/s relative to boat

Momentum Eqn :

$$\int_{CS} \rho u_x (\underline{v}\cdot\underline{n})\, dA = -\int_{CS} p n_x\, dA - kV_0^2$$

$$\rho_{air}(-V_0)\,V_0\,(A_5 - A_2) + \rho_{air}(-V_e)(V_e)\,A_2 + \cancel{\rho_{H_2O}(-V_0)(+V_0)\,A_3}$$

cancels   cancel   cancel

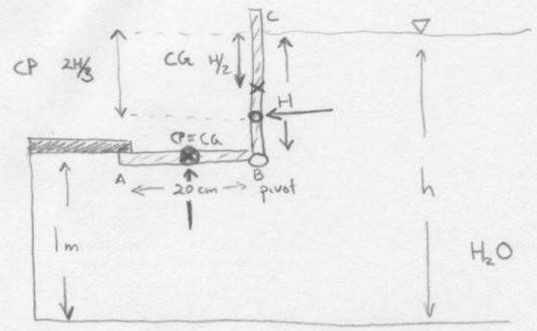
$$+ \rho_{air}(-V_0)(-V_0)\,A_5 + \cancel{\rho_{H_2O}(-V_0)(-V_0)\,A_4} = -(-1)\int_{③} p\,dA - (+1)\int_{④} p\,dA - kV_0^2$$

$A_3 = A_4$

$$-\rho_{air}A_2\left(V_e^2 - V_0^2\right) = -kV_0^2$$

assume small compared to V_e^2 → ✓ correct ↓

$$V_0 = \left[\frac{\rho_{air}\,\frac{\pi}{4}(.03)^2\,(343\ m/s)^2}{k}\right]^{1/2} = \boxed{2.29\ \frac{m}{s}}$$



$$F_{CB} = P_{CG} A = (\rho_{H_2O} g h_{CG}) A = \left(\rho_{H_2O} g \frac{H}{2}\right)(H w)$$

$$F_{AB} = P_{CG} A = (\rho_{H_2O} g h_{CG}) A = (\rho_{H_2O} g H)(.2m\, w)$$

For face BC:

$$\hat{y}_{CP} = \frac{-I_{xx} \sin\theta}{h_{CG} A} = -\frac{\left(\frac{w H^3}{12}\right)(1)}{\frac{H}{2}\; wH} = -\frac{H}{6}$$

$$\sum M_B$$

$$F_{CB}\left(\frac{H}{3}\right) = F_{AB}(.1\ m)$$

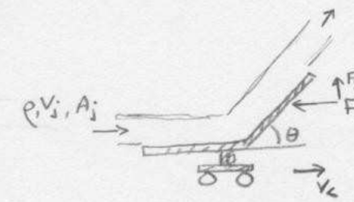
$$\rho g \frac{H^2}{2} w \frac{H}{3} = \rho g H w (.1\ m)(.2\ m)$$

$$H^2 = .12\ m^2 \qquad \boxed{H = 0.346\ m}$$

Then ...

$$H = h - 1$$

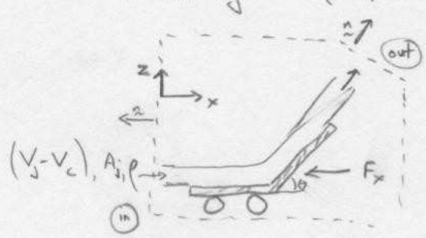
So $\boxed{h = 1.346\ m}$



Assumptions:

- Steady
- Incompressible
- no gravity
- 1D at exit/entrance
- CS ⊥ flow
- frictionless cart

⇓ new system (cart fixed)



$$V_{out} = (V_j - V_c)$$

Mass:

$$A_{out} = A_{in}$$

Momentum

$$\rho (V_j - V_c)(-(V_j - V_c)) A_{in} + \rho \overbrace{(V_j - V_c)\cos\theta}^{V_x} \overbrace{(V_j - V_c)}^{V \cdot \hat{n}} = -F_x - \int \overbrace{\cancel{p}}^{0 \text{ gage}} n_x \, dA$$

so $$\boxed{F_x = \rho (V_j - V_c)^2 A \,(1 - \cos\theta)}$$

Power delivered ⇒ force × velocity $$\boxed{P = V_c F_x = \rho V_c (V_j - V_c)^2 A\,(1 - \cos\theta)}$$

Maximum Force: $\boxed{V_c = -\infty}$ but physically $0 < V_c < V_j$ so $\boxed{V_c = 0}$

Maximum Power: $\dfrac{dP}{dV_c} = 0$

$$\rho A (1 - \cos\theta) \left[V_c \, 2(V_j - V_c)(-1) + (V_j - V_c)^2 \right] = 0$$

$$(V_j - V_c) - 2V_c = 0$$

$$\boxed{V_c = \frac{V_j}{3}}$$

no power if $V_c \to 0$
no force if $V_c \to V_j$

so somewhere in between. ✓