

7.59 Wind blowing past a flag causes it to "flutter in the breeze." The frequency of this fluttering,  $\omega$ , is assumed to be a function of the wind speed,  $V$ , the air density,  $\rho$ , the acceleration of gravity,  $g$ , the length of the flag,  $\ell$ , and the "area density,"  $\rho_A$ , (with dimensions of  $ML^{-2}$ ) of the flag material. It is desired to predict the flutter frequency of a large  $\ell = 40$  ft flag in a  $V = 30$  ft/s wind. To do this a model flag with  $\ell = 4$  ft is to be tested in a wind tunnel. (a) Determine the required area density of the model flag material if the large flag has  $\rho_A = 0.006$  slugs/ft<sup>2</sup>. (b) What wind tunnel velocity is required for testing the model? (c) If the model flag flutters at 6 Hz, predict the frequency for the large flag.

$$\omega = f(V, \rho, g, \ell, \rho_A)$$

$$\omega \doteq T^{-1} \quad V \doteq LT^{-1} \quad \rho \doteq ML^{-3} \quad g \doteq LT^{-2} \quad \ell \doteq L \quad \rho_A \doteq ML^{-2}$$

From the pi Theorem,  $6-3 = 3$  pi terms required, and a dimensional analysis yields

$$\omega \sqrt{\frac{\ell}{g}} = \phi\left(\frac{V}{\sqrt{g\ell}}, \frac{\rho_A}{\rho\ell}\right)$$

(a) For similarity

$$\frac{\rho_{Am}}{\rho_m \ell_m} = \frac{\rho_A}{\rho\ell}$$

and since  $\rho_m = \rho$

$$\rho_{Am} = \frac{\ell_m}{\ell} \rho_A = \left(\frac{4 \text{ ft}}{40 \text{ ft}}\right) (0.006 \frac{\text{slugs}}{\text{ft}^2}) = \underline{\underline{0.0006 \frac{\text{slugs}}{\text{ft}^2}}}$$

(b) For similarity

$$\frac{V_m}{\sqrt{g_m \ell_m}} = \frac{V}{\sqrt{g\ell}}$$

and with  $g_m = g$

$$V_m = \sqrt{\frac{\ell_m}{\ell}} V = \sqrt{\frac{4 \text{ ft}}{40 \text{ ft}}} (30 \frac{\text{ft}}{\text{s}}) = \underline{\underline{9.49 \frac{\text{ft}}{\text{s}}}}$$

(c) With the similarity requirements satisfied the prediction equation is

$$\omega \sqrt{\frac{\ell}{g}} = \omega_m \sqrt{\frac{\ell_m}{g_m}}$$

so that

$$\omega = \sqrt{\frac{g}{g_m}} \sqrt{\frac{\ell_m}{\ell}} \omega_m = \sqrt{\frac{4 \text{ ft}}{40 \text{ ft}}} (6 \text{ Hz}) = \underline{\underline{1.90 \text{ Hz}}}$$