MIE 340
Prof. Perot
Handout #10

Flawless Dimensional Analysis
(The Secret Recipe)

(1) List all the Parameters involved. (n = number of parameters)

If you forget an important parameter your result will be incorrect.
This is the most likely place you will make a mistake in real life.
If you pick a parameter that is not really involved it will involve more work
– but the answer will still be correct.

(2) Select fundamental (primary) dimensions. (m = number of dimensions)

[M],[L],[T] (mass, length, time) are almost always used. (m=3)
[Θ] (temperature) might also be used in this class. (m=4)
Need other dimensions for problems in electromagnetics (charge), etc.
Dimensions *not* Units. [T] is time *not* Temperature.

(3) Find dimensions of all parameters in terms of primary dimensions.

For example: $\mathbf{v}=\left[\frac{L}{T}\right]$ $\rho=\left[\frac{M}{L^3}\right]$ $D=[L]$ $\mu=\left[\frac{M}{LT}\right]$ $A=[L^2]$

(4) Select m <u>independent</u> parameters from the List in step (1).

Don’t select the parameter you are most interested in (such as drag in the example)
Don’t include two (or more) parameters with essentially the same dimensions (such as D & A above)
Do make sure that each primary dimension is represented at least once.
Do try to pick parameters with simple dimensions.

(5) Form Dimensionless Groups with the remaining parameters.

One group for each remaining parameter (so n - m dimensionless groups)
The dimensions of the group should cancel out (so that it is dimensionless).
See the example on the reverse side.

(6) Check that all groups are dimensionless. (*Always check* – I mean it).

(7) Interpret your results. <u>What does it mean</u>. See the example.

One dimensionless group is always a function of all the other dimensionless groups.
If there is only one dimensionless group then it is a function of nothing (= constant).

Example:

The drag on a sphere (tennis or golf ball) depends on many variables. How will you organize you data (or someone else's) so that it is clear what the dependencies are.

(1) List parameters: D(Drag Force), ρ(density), μ(viscosity), V(velocity), R(radius) [n=5]

(2) Select primary dimensions: [M],[L],[T] (no temperature dependence here) [m=3]

(3) Find Dimensions: $D=[ML/T^2]$, $\rho=[M/L^3]$, $\mu=[M/(LT)]$, $V=[L/T]$, $R=[L]$

(4) Select m (= 3 in this case) parameters: ρ, V, R
(which collectively include all primary dimensions - [M],[L],[T],
AND are simple, AND do not include the quantity of interest – Drag in this case)

(5) Nondimensionalize <u>all</u> the remaining parameters (which are D, and μ) [2 = n-m]

(i) $G_D = D\rho^a V^b R^c = [ML/T^2]^1[M/L^3]^a[L/T]^b[L]^c = [M]^0[L]^0[T]^0$

Find a,b,c such that this holds true – then G_D is dimensionless.

M: 1+a=0
L: 1-3a+b+c=0
T: -2-b=0

So a = -1, b = -2, c= -2 therefore $G_D = D/\rho V^2R^2$

(ii) $G_\mu = \mu\rho^a V^b R^c = [M/LT]^1[M/L^3]^a[L/T]^b[L]^c = [M]^0[L]^0[T]^0$

Find a,b,c such that this holds true – then G_μ is dimensionless.

M: 1+a=0
L: -1-3a+b+c=0
T: -1-b=0

So a = -1, b = -1, c= -1 therefore $G_\mu = \mu/\rho VR$

(6) Check answer.

$G_D = [ML/T^2]^1[M/L^3]^{-1}[L/T]^{-2}[L]^{-2} = [1]$
$G_\mu = [M/LT]^1[M/L^3]^{-1}[L/T]^{-1}[L]^{-1} = [1]$

(7) Interpret.

The most general relationship for this problem is that G_D is some function of G_μ (or vice versa), so … $D/\rho V^2R^2$ = function($\mu/\rho VR$)
Or… The drag coefficient is solely a function of the Reynolds number.
Dynamic similarity occurs when all dimensionless variables are equal.

Note: Any dimensionless group can be inverted and still be dimensionless.

- *The Reynolds number* ($Re = \rho VR/\mu$) *is a very common dimensionless group.*