9.29

9.29 A plate is oriented parallel to the free stream as is indicated in Fig. 9.29. If the boundary layer flow is laminar, determine the ratio of the drag for case (a) to that for case (b). Explain your answer physically.

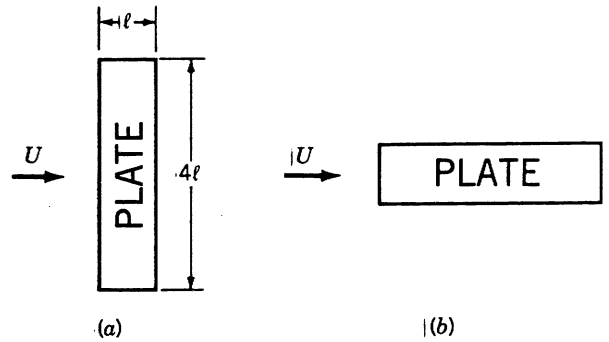

FIGURE P9.29

For case (a):

$$\mathcal{D}_{fa} = \tfrac{1}{2}\rho U^2 C_{Df} A \quad \text{where} \quad C_{Df} = \frac{1.328}{\sqrt{Re_\ell}} = \frac{1.328}{\sqrt{\frac{U\ell}{\nu}}} \quad \text{and} \quad A = 4\ell^2$$

Thus,

$$\mathcal{D}_{fa} = \tfrac{1}{2}\rho U^2 \frac{1.328\sqrt{\nu}}{\sqrt{U\ell}}(4\ell^2) = 2.56\,\rho U^{3/2}\sqrt{\nu}\,\ell^{3/2} \qquad (1)$$

For case (b)

$$\mathcal{D}_{fb} = \tfrac{1}{2}\rho U^2 C_{Df} A \quad \text{where} \quad C_{Df} = \frac{1.328}{\sqrt{\frac{U(4\ell)}{\nu}}} \quad \text{and} \quad A = 4\ell^2$$

Thus,

$$\mathcal{D}_{fb} = \tfrac{1}{2}\rho U^2 \frac{1.328\sqrt{\nu}}{\sqrt{4U\ell}}(4\ell^2) = \tfrac{1}{2}\left(2.56\,\rho U^{3/2}\sqrt{\nu}\,\ell^{3/2}\right) \qquad (2)$$

From Eqs. (1) and (2) we see that

$$\frac{\mathcal{D}_{fa}}{\mathcal{D}_{fb}} = \underline{\underline{2}}$$

The shear stress decreases with distance from the leading edge of the plate (i.e., the thickening of the boundary layer). Thus, even though the plate area is the same for case (a) or (b), the average shear stress (and the drag) is greater for case (a).