Autonomous System for Multistage Cylindrical Grinding

An optimization strategy is presented for cylindrical plunge grinding operations. The optimization strategy is designed to minimize cycle time while satisfying production constraints. Monotonicity analysis together with local linearization are used to simplify the non-linear optimization problem and determine the process variables for the optimal cycle. At the end of each cycle, the uncertain parameters of the process are estimated from sensory data so as to provide a more accurate estimation of the optimal process variables for the subsequent cycle. The optimization strategy is validated both in simulation and for actual grinding tests.

1 Introduction

Grinding is commonly utilized as a final machining operation in the production of precision components requiring smooth surfaces and fine tolerances. Although grinding operations are usually the most costly of all manufacturing processes, relatively little effort has been directed in industry toward their optimization. Optimization analyses for grinding, analogous to those for other types of machining operations, have been developed which are designed to minimize either cycle time or cost (Saljé et al., 1980; Peters and Acerus, 1980; Peters, 1984). However, the prospects for their implementation are limited because indicated “optimal” conditions often violate production constraints (Malkin, 1989). Furthermore, these analyses cannot cope with model uncertainty and process variability.

Optimal operating conditions for grinding are usually bounded by production constraints related either to the machine capability or the workpiece quality (Peters, 1984; Malkin, 1989). In recognition of the critical role of constraints, a practical constraint-limited optimization strategy was developed (Malkin, 1987) which utilizes an iterative process of increasing the removal rate (decreasing cycle time) until a constraint (bottleneck) is encountered, relaxing that constraint (if possible), and thereafter continuing to increase the removal rate until another constraint is encountered. This cycle time reduction strategy continually drives the cycle faster until either of the constraints (e.g., “burning power” or surface roughness) becomes tight, and then relaxes the tight constraint by changing the dressing parameters. This process is repeated until both of the constraints simultaneously become tight. While effective in coping with modeling uncertainty and process variability, this type of strategy is applicable only to those cases with two constraints which can each be relaxed by changing the dressing conditions in the opposite way. Moreover, the implementation of this iterative approach is generally slow and tedious, and therefore not suitable for computer control.

In the present paper, a new optimization strategy is presented for minimizing grinding cycle time while satisfying production constraints. The strategy utilizes parameter estimation to cope with modeling uncertainty and process variability by continually updating the estimated optimal conditions using parameters estimated from the preceding grinding cycle. Efficient estimation of the optimal point is based upon a simplified model of the process using a monotonicity analysis (Papalambros and Wilde, 1988) and local linearization. This optimization strategy provides the practical basis for control of the grinding cycle whereby the system is smart enough to achieve its optimal operating point in response to sensory data.

2 Grinding Cycle Basics

This section presents some background information about cylindrical plunge grinding cycles as a basis for developing the optimization strategy. Material removal in cylindrical grinding occurs by radially infeeding the wheel into the workpiece with a programmed infeed velocity of the cross-slide. This is illustrated in Fig. 1 for internal cylindrical grinding and would also
Fig. 2. Illustration of a grinding cycle consisting of roughing, finishing and spark-out stages

apply to external cylindrical grinding. The grinding infeed cycle (see Fig. 2) is typically divided into three stages each characterized by a discrete programmed infeed velocity \( u(t) \): (1) roughing with a fast infeed velocity \( u_1 \), (2) finishing with a slower infeed velocity \( u_2 \), and (3) spark-out at zero infeed velocity \( u_3 = 0 \). This is followed by rapid retraction to disengage the wheel from the workpiece. Specification of the grinding cycle requires not only values for \( u_1 \) and \( u_2 \), but also the time durations for each of the three sequential stages \( (t_1, t_2, \text{and } t_3) \), the peripheral workspeed \( u_w \), and the dressing parameters (radial dressing depth \( a_r \) and dressing lead \( s_d )

In response to the programmed infeed \( p(t) \) in Fig. 2, the actual radial size reduction of the workpiece follows the curve \( r(t) \) with a slope corresponding to the actual infeed velocity \( u_f(t) \) shown in Fig. 1. The transient in \( r(t) \) at the beginning of each stage is attributed mainly to the elastic deflection of the system, and also to radial wear of the grinding wheel. This transient behavior can be approximated by a first-order system characterized by a time constant \( r \) (Malkin, 1987).

In addition to specifying the parameters for controlling the infeed, it is also necessary to specify the dressing parameters associated with periodically shaping and conditioning the wheel. For example, with fixed point diamond dressing on conventional abrasive wheels, the parameters to be specified are the radial dressing depth \( a_r \), the dressing lead \( s_d \), and also the frequency of dressing (i.e., workpieces per dress). Finer dressing (smaller dressing lead and/or depth) results in a duller wheel, thereby causing higher grinding power, higher temperatures, and a greater tendency for thermal damage to the workpiece, but the resulting surface finish is smoother due to more even protrusion of the abrasive cutting points from the wheel surface (smoother wheel). This implies the existence of an optimal dressing condition which balances the need to satisfy a maximum surface roughness requirement against the need for maintaining the power below a critical value so as to avoid thermal damage to the workpiece (Malkin, 1989).

Machine control begins with gap elimination at the maximum infeed rate available. Other than machine limitations (e.g., available spindle power and part chucking capability), the infeed rate \( u_f \) for the subsequent roughing stage is constrained by workpiece quality requirements including thermal damage and maximum surface roughness. Two thermal damage constraint scenarios are considered: (1) no thermal damage at all during the cycle (no-burn constraint) in which case it is necessary to keep the grinding zone temperature below a critical value throughout the cycle, and (2) thermal damage restricted to a certain depth during the roughing stage which is completely removed during the subsequent finishing stage (burning constraint). The grinding power can be expected to reach but not exceed the corresponding allowable burning power limit at the end of the roughing stage for the no-burn constraint, and exceed the burning power limit when the burning constraint is active. The burning constraint is less restrictive than the no-burn constraint and should therefore lead to a shorter optimal cycle.

The roughing stage is followed by an intermediate finishing stage (see Fig. 2) which may be required to (1) remove a thermally damaged layer from the previous roughing stage [2], (2) control size, (3) improve surface finish, and (4) control roundness. If the burning constraint applies, burning from the roughing stage will persist into the finishing stage until the actual infeed rate is sufficiently reduced during the transient. From the finishing stage the process proceeds to spark-out with zero programmed infeed rate, during which the workpiece is rounded up and the surface finish improved at a diminishing rate as the elastic deflection of the machine is recovered. The cycle is terminated and the wheel rapidly retracted when the final size is reached as indicated by the size gage measurement. It should be noted that the final size can be reached only if the deflection at the end of the prior finishing stage exceeds the radial allowance to be removed during spark-out.

An autonomous grinding system should be able to adjust its own operating parameters to minimize cycle time subject to production and workpiece quality constraints in response to measurements from sensors. As a practical matter, the only in-process sensors which can be reliably utilized in a harsh production environment are a power monitor which measures the wheel spindle power, and a size gage which measures the workpiece diameter (Rao and Malkin, 1990). The power monitor output can be used to identify initial wheel-workpiece

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**Nomenclature**

\( a \) = depth of cut  
\( a_d \) = dressing depth  
\( A_0 \) = constant  
\( A_{d_0} \) = effective dullness  
\( d_e \) = equivalent diameter  
\( d_p \) = diameter of part  
\( d_w \) = diameter of wheel  
\( f \) = modified coefficient  
\( G \) = grinding ratio  
\( k \) = thermal conductivity of part  
\( l_c \) = contact length  
\( m \) = constant  
\( n_w \) = rotational speed of workpiece  
\( P \) = grinding power  
\( P_b \) = burning power  
\( q_i \) = actual infeed for the \( i \)th stage  
\( r \) = actual value of out-of-roundness  
\( r_0 \) = constant  
\( R_a \) = measured surface finish  
\( R_0 \) = constant  
\( s_d \) = dressing lead  
\( t_i \) = grinding time for the \( i \)th stage  
\( u \) = specific energy  
\( u_{ch} \) = specific energy for chip formation  
\( u_i \) = programmed infeed rate  
\( u \) = effective programmed infeed rate  
\( u_f \) = actual infeed rate  
\( v_i \) = actual infeed rate for the \( i \)th stage  
\( v_w \) = velocity of wheel  
\( x \) = constant  
\( y \) = constant  
\( z \) = depth of burn  
\( \alpha \) = thermal diffusivity of part  
\( \beta \) = fraction of finishing stage with burning  
\( \delta \) = equivalent dressing infeed angle  
\( \gamma \) = constant  
\( \theta_{mb} \) = critical temperature for burning  
\( \tau \) = time constant
Table 1  Gridding conditions used for nonlinear programming examples

<table>
<thead>
<tr>
<th>Gridding conditions</th>
<th>Case 1</th>
<th>Case 2</th>
<th>Case 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wheel spec.</td>
<td>32A80L8</td>
<td>32A80M6</td>
<td>32A8007</td>
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<tr>
<td>Wheel dia. (mm)</td>
<td>50</td>
<td>50</td>
<td>25</td>
</tr>
<tr>
<td>Wheel speed (m/s)</td>
<td>57</td>
<td>45</td>
<td>50</td>
</tr>
<tr>
<td>Work material</td>
<td>AISI 52100</td>
<td>AISI 52100</td>
<td>AISI 52100</td>
</tr>
<tr>
<td>Hardness (Rc)</td>
<td>60</td>
<td>60</td>
<td>60</td>
</tr>
<tr>
<td>Width (mm)</td>
<td>9</td>
<td>9</td>
<td>9</td>
</tr>
<tr>
<td>Grinding width (mm)</td>
<td>0.25</td>
<td>0.25</td>
<td>0.25</td>
</tr>
<tr>
<td>u1 (mm/s)</td>
<td>0.002</td>
<td>0.002</td>
<td>0.002</td>
</tr>
<tr>
<td>u2 (mm/s)</td>
<td>0.035</td>
<td>0.025</td>
<td>0.045</td>
</tr>
<tr>
<td>Rmax (mm)</td>
<td>0.66</td>
<td>0.66</td>
<td>0.66</td>
</tr>
<tr>
<td>rmax (μm)</td>
<td>1.4</td>
<td>1.4</td>
<td>1.7</td>
</tr>
</tbody>
</table>

engagement for gap elimination, to estimate the effective wheel dullness Aeff, and to indicate whether thermal damage is occurring and, if so, to estimate the depth of the damaged layer z1 on the workpiece. The output from the size gage, which directly indicates the remaining radial allowance to be removed, can also be used to estimate the elastic deflection of the system and the radial wheel wear (or grinding ratio C). The information derived from these two in-process sensors, together with post-process measurement (inspection) of part quality (e.g., finish and roundness), will be used to estimate the optimal process variables for control of the cycle.

3 Optimization Analysis

Based on the description of the process presented above, specification of the grinding operation requires selection of ten independent variables: two infeed velocities u1 and u2, time durations for the three stages t1, t2, and t3, dressing depth ar, dressing lead aL, dressing frequency vD, wheel velocity vn, and work velocity vw. The optimization problem can be greatly simplified if the number of the independent variables is reduced. In practice, the dressing depth ar is usually held constant and the dressing lead aL varied to alter the dressing severity. Furthermore, it can be shown that automation of the dressing operation so that the associated non-productive time becomes an insignificant portion of the total cycle would indicate an optimal dressing frequency of once per part (Malkin, 1987). The peripheral velocity vn is usually limited by safety considerations, so it too is held constant. Simulation results indicate that the peripheral work velocity vw has a secondary influence on the cycle time, so it is not included as an independent variable in this problem.

In view of the above considerations, the number of independent variables can be reduced to six. The optimization problem can now be defined mathematically as:

Minimize cycle time: Φ = t1 + t2 + t3
with respect to: u1, u2, t1, t2, t3, x

subject to the constraints:

\[ g_1 = P_m(u_1, A_{eff}) - P_f(u_1) \leq 0 \] (no-burn constraint)  \( g_2 = z_1(u_1, A_{eff}) - q_2(u_1, u_2, t_1, t_2) \leq 0 \) (burning constraint)  \( g_3 = R_{max} - R_{min} \leq 0 \) (surface finish constraint)  \( g_4 = r(u_1, u_2, t_1, t_2) - R_{min} \leq 0 \) (out-of-roundness constraint)  \( g_5 = u_1 t_1 + u_2 t_2 - \Delta r \leq 0 \) (size constraint)  \( g_6 = u_1 - u_2 \leq 0 \)  (lower infeed constraint)  \( g_7 = u_1 - u_2 \leq 0 \)  (upper infeed constraint)

In the above model, the inequality gi defines the power for the no-burn constraint, where Pm is the maximum power during the cycle, Pf represents the burning power limit, and Aeff denotes the effective wheel dullness which depends on the wheel, workpiece material, and dressing conditions. The inequality g6 defines the depth of workpiece burn for the burning constraint, where z1 represents the depth of workpiece burn produced during the roughing stage and q2 the depth of removal in the subsequent finishing stage. In the limit with z1 = 0, this constraint reverts to the no-burn constraint. The inequality g3 defines the surface finish requirement where Rmax denotes the actual surface roughness and Rmin its maximum allowable value. The inequality g4 defines the out-of-roundness requirement where r is the actual value of out-of-roundness (mainly dependent on t1 and t2), and rmax is its maximum allowable value. The equality g5 defines the size requirement where Δr denotes the radial workpiece allowance. For formulating the mathematical problem, the size constraint in Eq. (6) indicates zero tolerance and neglects wheel wear. However, these effects are actually taken into account by direct size measurements at the end of the spark-out stage. The last two constraints (Eqs. (7) and (8)) represent limitations on the machine infeed rates — u1 represents the lower bound and u2 the upper bound. The detailed relationships among the process variables, process parameters, and part constraints are included in the Appendix.

In order to study the characteristic of the optimal point, the optimization problem of Eq. (1) was solved using the nonlinear programming package OPTDES (Parkinson and Wilson, 1988). The optimal process variables obtained for the conditions listed in Table 1 are included in Table 2. The results in Table 2 indicate that the optimal infeed rate during the finishing stage is equal to its lower bound (i.e., u2 = u1), and that all the constraints g2, g3, g4, g5, and g6 are tight at the optimal point. The constraint g2 prevails over g1 because a shorter cycle time can be achieved with the burning constraint than with the no-burn constraint. This will be considered in more detail later in the paper.

4 Monotonicity Analysis

In the previous section, the optimal grinding conditions were obtained using nonlinear programming. However, in order to implement an autonomous grinding system, whereby the optimal operating variables are found in response to sensory data, a closed-form solution to the optimization problem is needed. For this purpose, the problem can be simplified through monotonicity analysis (Papalambros and Wilde, 1988) as follows:

- The objective function (see Eq. (1)) is a monotonically increasing function of t2 and t3 (i.e., \( \Phi(t_2) \) and \( \Phi(t_3) \)), and g2 and g4 are the only monotonically decreasing functions of t1 and t2, respectively (i.e., \( g_2(t_1) \) and \( g_4(t_2) \)). Therefore, according to the First Monotonicity Principle (MP1)1 both of the constraints g2 and g4 are critical and

1In a well-constrained objective function every increasing (decreasing) variable is bounded below (above) by at least one active constraint (Papalambros and Wilde, 1988).
active. As such, they can be converted to equality constraints (i.e., \( g_2 = 0 \) and \( g_4 = 0 \)), and \( t_2 \) and \( t_3 \) can be defined in terms of the other process variables to eliminate them from the objective function.

- The reduced model obtained by elimination of \( t_2 \) and \( t_3 \) indicates that the objective function is a monotonically decreasing function of \( s_2 \) (i.e., \( \Phi(s_2) \)). Since \( g_5 \) is the only monotonically increasing function of \( s_2 \) (i.e., \( g_5(s_2') \)), according to the MP1, \( g_5 \) is also critical and active (i.e., \( g_5 = 0 \) at the optimal point).

- The monotonicity of the partial derivative of the objective function with respect to \( u_2 \) should be studied for the burning constraint, where the cycle time is expected to be minimum. In order to quantify the duration of burning during the cycle, a parameter \( 0 \leq \beta < 1 \) is defined

\[
\beta = \frac{(t_i - t_1)}{t_2}
\]  

which denotes the fraction of the finishing stage during which burning persists. (The equality \( \beta = 0 \) would apply to the no-burn constraint and \( 0 < \beta < 1 \) to the burning constraint.) Introducing \( \beta \) into the objective function and taking its derivative with respect to \( u_2 \) yields

\[
\frac{\partial \Phi}{\partial u_2} = \frac{t_i [\exp(\beta t_2/\tau) - 1] - t_2}{u_1}
\]  

which is positive for

\[
\beta > \frac{\tau}{t_2} \ln \left( \frac{1 + t_2}{t_1} \right)
\]  

In this case the objective function is a regionally monotonically increasing function of \( u_2 \) (i.e., \( \Phi(u_2') \)), and since the constraint \( g_6 \) is a monotonically decreasing function of \( u_2 \) (i.e., \( g_6(u_2') \)), according to the MP1, it is critical and active in the region defined by Eq. (11). The condition of Eq. (11) is generally found to be satisfied at the optimal point. However, in cases where the validity of Eq. (11) is questionable, the parameter \( \beta \) can be included as a design variable and Eq. (11) as an additional constraint in the optimization problem.

Based upon the monotonicity analysis presented above, the constraints \( g_2, g_3, g_4, \) and \( g_6 \) are tight at the optimal point, so they can be written as equalities.

5 Cycle Time Optimization Strategy

The optimization problem defined in Eqs. (1)-(8) contains six variables: \( u_1, u_2, t_1, t_2, t_3, \) and \( s_2 \). One approach to solve for these optimal variables might be to combine the equality constraints \( g_2, g_3, g_4, \) and \( g_6 \) together with the equality \( g_5 \) to obtain the variables \( t_1', t_2', t_3', u_1', u_2' \) and \( s_2' \), which could then be eliminated from the objective function to obtain \( u_1' \) (i.e., \( u_1' = \text{arg}(\min \Phi) \)). But due to the nonlinearity of these equations, this does not provide a closed-form solution. A partial solution, however, can be obtained by defining the variables \( u_1', t_1', t_2' \) and \( t_3' \) in terms of the other three variables \( u_1', t_1', \) and \( s_2' \) as:

\[
u_1^* = u_1 \quad \text{(from } g_6 = 0)\]

\[
t_1^* = \frac{\Delta r - u_1 t_2^*}{u_1} \quad \text{(from } g_3 = 0)\]

\[
t_2' = r_0 \ln \left( \frac{u_1}{\text{max}} \right) \quad \text{(from } g_4 = 0)\]

where \( r_0 \) in Eq. (14) denotes a constant. Since the three variables \( u_1', t_1', \) and \( s_2' \) cannot be computed from the corresponding set of nonlinear equations \( g_2 = 0, g_3 = 0 \) and \( \frac{\partial \Phi}{\partial u_2} = 0 \) (\( H = 0 \)), they need to be estimated. For this purpose, the set of equations \( H = 0 \) are expanded through Taylor series as

\[
H |_{x = x, t_1} = \left[ \frac{\partial H}{\partial x} \right] |_{x = x_0} \Delta X
\]

where \( X \) denotes the vector of variables \( u_1, t_1, \) and \( s_2 \), \( \Delta X \) represents the incremental modification of the process variables, \( i \) represents the iteration step, and \( \frac{\partial H}{\partial x} \Delta X \) denotes the Jacobian matrix. Based on Eq. (15), the solution for \( \Delta X \) is obtained such that

\[
H |_{x = x_{i+1}} = 0
\]

to provide the estimation algorithm for \( \Delta X \) as

\[
\Delta X = \left[ \frac{\partial H}{\partial x} \right]^{-1} \left( -H |_{x = x_i} \right)
\]

Note that accurate estimation of the optimal variables \( u_1, u_2, t_1', t_2', t_3' \), and \( s_2' \) depends upon the validity of the mathematical model of the process (e.g., see the Appendix). As such, any modeling inaccuracy could cause the algorithm to overshoot the target \( H = 0 \) and damage the workpiece. To avoid this situation, (1) the uncertain parameters of the model are estimated and updated at the end of each cycle, and (2) a margin of modeling error \( \epsilon \) associated with \( H \) is used which yields

\[
\Delta X = \left[ \frac{\partial H}{\partial x} \right]^{-1} (\epsilon - H |_{x = x_i})
\]

The margin of modeling error \( \epsilon \) denotes the inaccuracy of the model in representing

\[
H = \left( \begin{array}{c} z_i - q_i \\ R_s - R_{\text{max}} \\ \frac{\partial \Phi}{\partial u_i} \\ \end{array} \right)
\]

and can be specified based on our confidence in the model to represent the parameters \( z_i \) and \( R_s \). Note that the margin of modeling error associated with \( \frac{\partial \Phi}{\partial u_i} \) is zero.

The estimation algorithm presented above requires values for some parameters, namely, the time constant of the equivalent first order system \( \tau \), the grinding ratio \( G \), and the effective wheel dullness \( A_{\text{eff}} \), which are not known a priori. The time constant \( \tau \) is estimated based on the infeed transient at the beginning of the roughing stage using least square parameter estimation techniques (Ljung, 1987). The grinding ratio \( G \), which is the volumetric ratio of material removal to wheel wear, is estimated from the difference between the controlled infeed velocity \( u(t) \) and the slope of the actual size reduction curve \( r(t) \) (see Fig. 2). Linear regression is used to estimate the slope of actual infeed rate after the initial transient in the roughing stage. The effective wheel dullness \( A_{\text{eff}} \) is estimated at the end of each cycle by matching the measured power against its computed values throughout the cycle (see Eq. (26) in the Appendix) using the least square criterion.

Based on the above analysis, a cycle time optimization strategy is formulated as follows: The first cycle is run conservatively with the process variables \( u_1', u_2', t_1', t_2', t_3', s_2' \) so as to estimate the parameters \( \tau, G, \) and \( A_{\text{eff}} \). After the first cycle, the optimal process variables for an intermediate cycle with the no-burn constraint (\( \beta = 0 \)) is found. The process variables \( u_1' \) and \( s_2' \) for this intermediate cycle are estimated using the constraint \( g_1 = 0 \) in place of \( g_2 = 0 \) in Eq. (17) with \( g_3 = 0 \). The parameter \( t_1' \) is set equal to the approximate transient time \( 3\tau \), and the other variables \( u_1, t_1, \) and \( t_3 \) are obtained using Eqs. (12)-(14). At the end of the second cycle, the estimation algorithm defined by Eq. (17) along with Eqs. (12)-(14) can then be used to estimate the optimal process variables for the third cycle with the burning constraint (\( 0 < \beta < 1 \)). This is achieved by estimating the incremental modification \( \Delta X \) for each of the process variables \( u_1, u_2, t_1, t_2, t_3, s_2 \) from Eq. (17) to yield the optimal variables \( u_1', u_2', t_1', t_2', t_3', \) and \( s_2' \).
Table 3  Estimated parameters, process variables dictated by the cycle-time optimization strategy, and the values of the constraints for the three simulated cycles

<table>
<thead>
<tr>
<th>Estimated parameters</th>
<th>Cycle 1</th>
<th>Cycle 2</th>
<th>Cycle 3</th>
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<tr>
<td>$i$ (s)</td>
<td>1.58</td>
<td>1.4</td>
<td>1.34</td>
</tr>
<tr>
<td>$G$</td>
<td>21.5</td>
<td>15.8</td>
<td>7.2</td>
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<tr>
<td>$A_{ef}$</td>
<td>0.006</td>
<td>0.015</td>
<td>0.022</td>
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<table>
<thead>
<tr>
<th>Grinding variables</th>
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<tr>
<td>$u_1$ (mm/s)</td>
<td>0.012</td>
<td>0.017</td>
<td>0.0254</td>
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<td>$u_2$ (mm)</td>
<td>0.004</td>
<td>0.002</td>
<td>0.002</td>
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<td>$t_1$ (s)</td>
<td>16.8</td>
<td>14.2</td>
<td>9.5</td>
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<td>$t_2$ (s)</td>
<td>10</td>
<td>4.7</td>
<td>3.8</td>
</tr>
<tr>
<td>$t_3$ (s)</td>
<td>5</td>
<td>4.4</td>
<td>3.7</td>
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<tr>
<td>$s_y$ (mm)</td>
<td>0.137</td>
<td>0.104</td>
<td>0.066</td>
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<table>
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<th>Constraint values</th>
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<td>$g_1$ (kW)</td>
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<td>$g_2$ (mm)</td>
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<td>$R_1$ (mm)</td>
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<td>$R_2$ (mm)</td>
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</tr>
<tr>
<td>$g_3$ (mm)</td>
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<td>0</td>
</tr>
<tr>
<td>$g_4$ (mm/s)</td>
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<td>0</td>
</tr>
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</table>

| Cycle time (s)       | 31.8    | 23.3    | 17.0    |

6 Verification in Simulation

Prior to implementation on an actual system, the proposed cycle time optimization strategy was investigated in simulation using the grinding simulation software ALFRED (Chiu, 1989) developed at the University of Massachusetts. This simulation is based upon the same models in the Appendix as for the optimization strategy. The grinding conditions used for simulation were those for Case 1 of Table 1. For these conditions, the estimated process parameters, the values of process variables, and the values of constraints at the end of each cycle are listed in Table 3. The results indicate that the proposed cycle time optimization strategy reduces the cycle time from 31.8 seconds in Cycle 1 down to 23.3 seconds in Cycle 2 (no-burn constraint) and 17.0 seconds in Cycle 3 (burning constraint). The simulation results for the infeed, depth of burn, and power during Cycle 3 with the burning constraint are shown in Fig. 3. During most of the roughing stage and part of the finishing stage up to time $t_3$, the grinding power exceeds the corresponding burning power $P_b$.

The optimal variables obtained from the proposed cycle-time optimization strategy with the burning constraint must satisfy the condition that thermal damage from the roughing stage is completely removed in the finishing stage. This condition is satisfied if:

$$u_1 + \frac{\partial z}{\partial t} \bigg|_{t=t_3} \leq 0 \quad (18)$$

Simulation results generally show that this condition is automatically satisfied by the algorithm of Eq. (17). However, if this condition is not satisfied, Eq. (18) can then be used as an equality constraint to determine $u_1$.

7 Experimental Implementation

The proposed cycle-time optimization strategy was experimentally implemented on the grinding system illustrated in Fig. 4. This system consists of a Bryant Model 1116 internal grinder modified by the addition of a stepper motor infeed drive, an electrical workpiece drive in place of the original hydraulic motor for computer control, a power monitor (A.F. Green T2), and a diametral size gage (Marposs Micrometer and 5995030 amplifier) with a personal computer for data acquisition and control (Rao and Malkin, 1990). The grinding conditions used in the experiments were the same as those used in the simulation (Case 1 of Table 1). No instrument was available to measure out-of-roundness, so this is not given in Table 1.

---

Fig. 3  Infeed, depth of burn, power and burning power during the optimal cycle of Case 1 in Table 1

Fig. 4 Illustration of the internal cylindrical grinding system

The estimated parameters, the process variables from the optimization strategy, and values of the constraints at the end of each cycle are included in Table 4, and the programmed and actual infeeds for the three cycles in Table 4 are shown in Fig. 5. These results are remarkably close to those obtained for the simulation, which reflects the validity of the grinding model. The nonlinear nature of the process model is apparent from the variation of the values for $G$, $A_{ef}$, estimated at the end of each cycle. The cycle time optimization strategy reduced the cycle time from 31.8 seconds in Cycle 1 down to 22.7 seconds in Cycle 2 (no-burn constraint), and 16.6 seconds in Cycle 3 (burning constraint). For Cycle 1, the constraints $g_1$ and $g_2$ are much smaller than zero indicating that the grinding power is less than the burning power at the end of the roughing stage ($P_{gr}/P_b = 0.87$) and the surface roughness ($R_s = 0.63 \mu m$) is less than its maximum allowed value ($R_{max} = 0.70 \mu m$). For Cycle 2, the roughing infeed $u_1$ is raised so that $g_1 = 0$ and the dressing condition is adjusted so that $g_2 = 0$. It should be noted that the actual grinding power and surface roughness values obtained in Cycle 2 were almost identical to their intended constrained values ($P_{gr}/P_b = 0.99$ and $R_s = 0.70 \mu m$).
Table 4  Estimated parameters, process variables, and constraint values experimentally obtained for the three consecutive cycles

<table>
<thead>
<tr>
<th>Estimated parameters</th>
<th>Cycle 1</th>
<th>Cycle 2</th>
<th>Cycle 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tau$ (s)</td>
<td>1.60</td>
<td>1.32</td>
<td>1.31</td>
</tr>
<tr>
<td>$G$</td>
<td>22.6</td>
<td>14.7</td>
<td>8.2</td>
</tr>
<tr>
<td>$A_{eff}$</td>
<td>0.01</td>
<td>0.0162</td>
<td>0.021</td>
</tr>
</tbody>
</table>

Grinding variables

<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$u_1$ (mm/s)</td>
<td>0.012</td>
<td>0.018</td>
<td>0.027</td>
</tr>
<tr>
<td>$u_2$ (mm/s)</td>
<td>0.004</td>
<td>0.002</td>
<td>0.002</td>
</tr>
<tr>
<td>$t_1$ (s)</td>
<td>16.8</td>
<td>13.5</td>
<td>9.0</td>
</tr>
<tr>
<td>$t_2$ (s)</td>
<td>10.0</td>
<td>4.8</td>
<td>3.9</td>
</tr>
<tr>
<td>$t_3$ (s)</td>
<td>5.0</td>
<td>4.4</td>
<td>3.7</td>
</tr>
<tr>
<td>$x_0$ (mm)</td>
<td>0.137</td>
<td>0.102</td>
<td>0.064</td>
</tr>
</tbody>
</table>

Constraint values

<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$g_1$ (KW)</td>
<td>-0.48</td>
<td>0</td>
<td>NA</td>
</tr>
<tr>
<td>$g_2$ (mm)</td>
<td>NA</td>
<td>NA</td>
<td>NA</td>
</tr>
<tr>
<td>$g_3$ (mm)</td>
<td>-0.07</td>
<td>0</td>
<td>-0.01</td>
</tr>
<tr>
<td>$g_4$ (mm)</td>
<td>-0.12</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$g_5$ (mm/s)</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$g_6$ (mm/s)</td>
<td>-0.002</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Others

<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
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</tr>
</thead>
<tbody>
<tr>
<td>$R_e$ (mm)</td>
<td>0.63</td>
<td>0.70</td>
<td>0.69</td>
</tr>
<tr>
<td>$P_{eq}/P_0$</td>
<td>0.87</td>
<td>0.99</td>
<td>1.22</td>
</tr>
</tbody>
</table>

Cycle time (s) 31.8  22.7  16.6

Fig. 5  Programmed and actual infeeds for the three successive cycles using the cycle-time optimization strategy

In Cycle 3, the constraint $g_2$ is activated to further reduce the cycle time. In this case, the grinding power at the end of the roughing stage exceeds the corresponding estimated burning power ($P_{eq}/P_0 = 1.22$) as expected, and the final roughness ($R_e = 0.60 \mu m$) is again very close to its maximum allowed value. Although not included in Table 4, the cycle-time optimization strategy also gave virtually identical optimal results for Cycle 2 and Cycle 3 when using different initial grinding conditions for Cycle 1.

Acknowledgment

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References


Parkinson, A., and Wilson, M., 1988, OPTDES.BYU, Technical Report, Brigham Young University, Salt Lake City, Utah.

A P P E N D I X

The following equations obtained from grinding theory are used for calculating and estimating the process parameters (Malkin, 1989):

\[ a = v_j/n_w \] (19)
\[ n_w = U/d_a \] (20)
\[ d_e = (d_1d_2)/(d_2 - d_1) \] (21)
\[ v_j = u_j - (u_j - v_{j-1})\exp\left(-\frac{t_j}{\tau}\right) \] (22)
\[ q_i = u_i t_i + \tau \left[(u_i - v_{i-1})\exp\left(-\frac{t_i}{\tau}\right) - (u_i - v_{i-1})\right] \; i = 1,2,3 \] (23)
\[ u'_i = u_i/f \] (24)
\[ f = 1 + d_w/(d_0) \] (25)
\[ P = 0.0138\pi d_w v_j + 9.62 \times 10^{-7} v_i \]
\[ + \{8.55 \times 10^{-6} + 2.10 v_{w}\} (v_j d_2)^{0.5} \] (26)
\[ P_b = 0.00617\pi d_w v_j + 0.0072(\pi d_w v_j n_w) \] (27)
\[ z = -1.449 \left(\frac{v_j d_2}{4\alpha}\right)^{0.37} \left(\frac{2a_1}{v_w}\right) \ln \left[\pi k_{a,b} v_w \right] \] (28)
\[ \times \left[\frac{\pi k_{a,b} v_w}{6.22 \pi d_w v_j (v_j d_2)^{0.5} (u_0 - 0.45u_{ch})}\right] \] (29)
\[ u = P/(\pi d_w v_j) \] (30)
\[ l_c = (\pi d_0 d_w v_j)^{0.5} \] (31)
\[ r = \pi d_0 d_w v_j / n_w \] (32)
\[ R_e = R_0 S a_d^\gamma / v_j \] (33)
\[ A_{eff} = -0.008 A_0 \ln(1.4 \times 10^6 m) \] (34)
\[ \delta = 1.1 \times 10^{-11} a_d^{0.75} d_i^{1.75} \] (35)
\[ r = (u_i t_i - f q_i)/u_i \] (36)
\[ G = (\pi n j d_1)/0.54 d_2 = (v_j d_2)/(u_1 - v_1) d_1 \] (37)

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