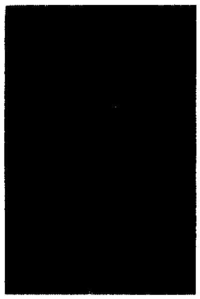
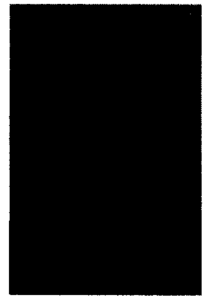
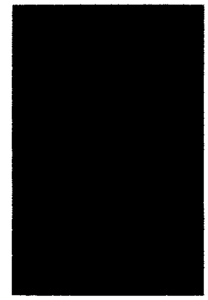
A Probability-Based Approach to Helicopter Rotor Tuning

Shengda Wang
Graduate Research Assistant

Kourosh Danai
Professor

Department of Mechanical and Industrial Engineering
University of Massachusetts Amherst

Mark Wilson
Sikorsky Aircraft
Stratford, Connecticut

A method of helicopter rotor balance is introduced that uses a probability model to maximize the likelihood of success of the selected blade adjustments. The underlying model in this method consists of two segments: a linear segment to include the sensitivity coefficients between the blade adjustments and helicopter vibration, and a stochastic segment to represent the probability densities of the vibration components. Based on this model, the blade adjustments with the maximal probability of generating acceptable vibration are selected as recommended adjustments. The effectiveness of the proposed method is evaluated in simulation using a series of neural networks trained with actual vibration data. The results indicate that the proposed method improves performance according to several criteria representing various aspects of track and balance.

Nomenclature

AI	acceptability index
a_{ij}	coefficients of the linear model representing the change in vibration
$\arg_x \max F(x)$	value of x that maximizes $F(x)$
C	coefficient matrix
$E[V_j(k)]$	expected value of the jth vibration element, ips
$e(k)$	measurement error, ips
$e_j(k)$	measurement error of the jth vibration element, ips
$\hat{e}_j(k)$	prediction error of the jth vibration element, ips
F_{cj}	estimated change between two consecutive flights of the cosine of the jth vibration element by the network, ips
F_{sj}	estimated change between two consecutive flights of the sine of the jth vibration element by the network, ips
j	index of the vibration element
k	flight number, $k = 0$ denotes the initial flight
M	number of times an adjustment set was tested in simulation
$\mathcal{N}(x, y)$	Gaussian distribution with mean x and variance y
N	dimension of the Gaussian function
$Pr[V]$	probability distribution of V
$p(\boldsymbol{x})$	joint probability density function of $\boldsymbol{x}$
S	specification region
s	limit of the specification region
s_l	Boolean index to indicate the acceptability of each simulated flight
$V_j(k)$	measured vibration of the kth flight, ips
$\hat{V}_j(k)$	prediction of the jth vibration component for the kth flight, ips
$V_{sj}(k)$	sine of the jth vibration component, ips
$V_{cj}(k)$	cosine of the jth vibration component, ips
Γ	integration region
ΔTAB	trim tab adjustments, thousandths of an inch
ΔPCR	pitch control rod adjustments, number of notches
$\Delta\boldsymbol{x}$	vector of combined blade adjustments
Δx_i	the ith element of $\Delta\boldsymbol{x}$
$\Delta V_j(k)$	change of the jth vibration component, ips
$\Delta\hat{V}_j(k)$	estimated change of the jth vibration component by the model, ips
μ	mean value
$\hat{\mu}$	estimated mean
σ^2	variance
$\hat{\sigma}^2$	estimated variance
$\boldsymbol{\Phi}$	covariance matrix of the prediction error

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Introduction

Helicopter track and balance is the process of adjusting the r blades to reduce the aircraft vibration and deviation in blade track. Tr and balance, as applied to Sikorsky's Black Hawk (UH-60) helicop is performed as follows. For initial measurements, the aircraft is flo through six different regimes during which measurements of rotor tr and vibration are recorded. Rotor track is measured by optical sens that detect the vertical position of the blades. Vibration is measure the cockpit of the helicopter at the frequency of once per blade revolu (1/rev) by two accelerometers, 'A' and 'B', which are attached to the si of the cockpit (see Fig. 1). The vibration data is vectorially combined two components: A + B, representing the vertical vibration of the aircr and A − B, representing its roll vibration. A sample of peak vibra levels for the six flight regimes, as well as the vibration phase rela to a reference blade position, are given in Table 1, along with a sam of track data. The six flight regimes in Table 1 are: ground (*fpm*), ho (*hov*), 80 knots (*80*), 120 knots (*120*), 145 knots (*145*), and maxim horizontal speed (*vh*). The track data indicate the vertical position of e blade relative to a mean position.

To bring track and 1/rev vibration within specification (usually be 0.2 inches per second (ips)), three types of adjustments can be made to rotor system: pitch control rod, trim tab, and balance weight (see Fig

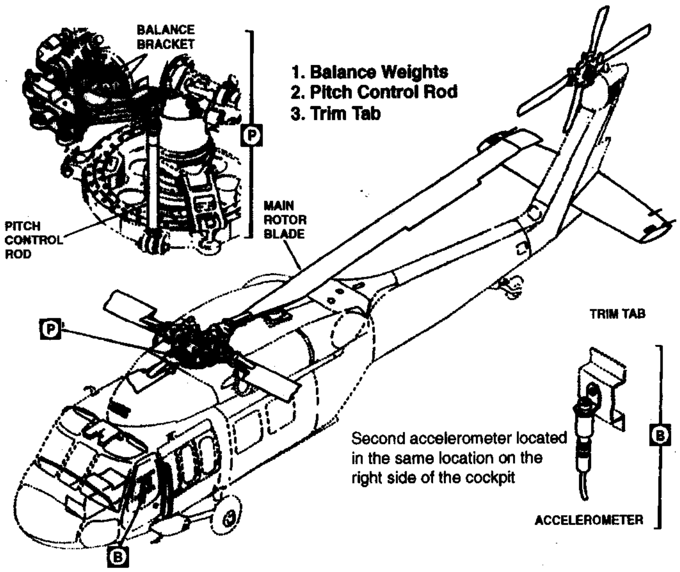

Fig. 1. Illustration of the position of accelerometers A and B on the aircraft, and the rotor blade adjustments (push control rod, trim tab and hub weights).

Table 1. Typical track and balance data recorded during a flight

	Vibration			
	A+B		A−B	
Flight regime	Mag. (ips)	Phase (deg.)	Mag. (ips)	Phase (deg.)
fpm	0.19	332	0.38	272
hov	0.07	247	0.10	217
80	0.02	86	0.04	236
120	0.04	28	0.04	333
145	0.02	104	0.07	162
vh	0.10	312	0.12	211
	Track (mm)			
	Blade #			
Flight regime	1	2	3	4
fpm	−2	3	1	−2
hov	−1	3	0	−2
80	1	11	1	−13
120	2	13	−1	−14
145	5	18	−3	−20
vh	2	13	−1	−14

Pitch control rods can be extended or contracted by a certain number of notches to alter the pitch of the rotor blades; positive push rod adjustments indicate extension. Trim tabs, which are adjustable surfaces on the trailing edge of the rotor blades, affect the aerodynamic pitch moment of the blade and consequently the overall 1/rev vibration characteristics of the rotor. Tab adjustments are measured in thousandths of an inch, with positive and negative changes representing upward and downward tabbing, respectively. Finally, balance weights can be either added to or removed from the rotor hub to tune vibrations through changes in the center of gravity of the rotor. Balance weights are measured in ounces with positive adjustments representing the addition of weight. In the case of the Sikorsky UH-60 helicopter, which has 4 main rotor blades, a total of twelve adjustments can be made to tune the rotors (i.e., three adjustments per blade and 24 outputs). Among them, balance weights mostly affect the ground vibration of the UH-60 helicopter, so they are not commonly used for in-flight tuning. Furthermore, because the symmetry of rotor blades in four-bladed aircraft produces identical effects for equal adjustment of opposite blades, the combined form of blade adjustments to each pair of opposing blades can be used as inputs. Accordingly, the input vector can be defined as:

$$\Delta x = [\Delta x_1, \Delta x_2, \Delta x_3, \Delta x_4]^T \quad (1)$$

where Δx_1 and Δx_3 denote the combined trim tab adjustments (ΔTAB) to blade combinations 1–3 and 2–4, respectively, and Δx_2 and Δx_4 represent the combined pitch control rod adjustments (ΔPCR) to blade combinations 1–3 and 2–4, respectively. The relationships between the combined and individual adjustments are in the form:

$$\Delta x_1 = \Delta TAB_3 - \Delta TAB_1 \quad (2)$$

$$\Delta x_2 = \Delta PCR_3 - \Delta PCR_1 \quad (3)$$

$$\Delta x_3 = \Delta TAB_4 - \Delta TAB_2 \quad (4)$$

$$\Delta x_4 = \Delta PCR_4 - \Delta PCR_2 \quad (5)$$

Generally the 1/rev vibration is not sufficient for rotor tuning, and additional information in the form of either blade track or vibration at higher rotation orders is required (Ref. 1). In practice, track and balance is performed by first specifying a combined set of adjustments to reduce 1/rev vibration. These adjustments are then expanded into a detailed set that best minimizes track spread (without compromising the vibration reductions). In this research, only the combined blade adjustments are determined to reduce the helicopter vibration.

Ideally, identical adjustments made to different helicopters of the same model type should result in identical changes in vibration. In reality, however, significant inconsistency in vibration changes may be present for identical adjustments to different helicopters of the same model type. This inconsistency is attributed to several factors (Ref. 2): (1) noise in vibration measurements (sensor); (2) nonuniformity of flight conditions, such as weather; (3) error in implementing blade adjustments; and (4) dissimilarities between aircraft and rotor blades. Because it is impossible to identify the source, the vibration inconsistency is treated as stochastics in this research.

Virtually all of the current systems of rotor track and balance rely on the strategy shown in Fig. 2, whereby the measurements of the flight just completed are used as the basis to search for blade adjustments. The search for blade adjustments is guided by the "Process Model" (see Fig. 2) which represents the relationship between vibration changes and blade adjustments. A difficulty of track and balance is the excess equations to degrees of freedom (4 inputs to control 24 outputs, i.e. Table 1). Another difficulty is caused by the high level of noise present in the vibration measurements.

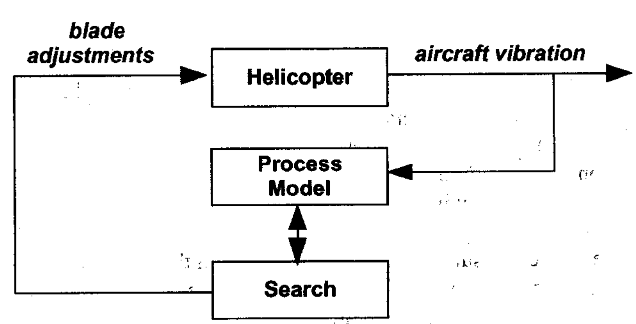

Fig. 2. Tuning strategy of the current methods.

The traditional approach to track and balance uses linear relationships to define the Process Model and uses model inversion to streamline the search. The drawback of the traditional approach, therefore, is its neglect of the stochastics of track and balance. In an attempt to cope with the stochastics of the process, Taitel et al. (Ref. 2) trained a set of neural networks with actual track and balance data to map vibration measurements to blade adjustments and to evaluate the efficacy of the solution. In effect, they developed an inverse-model to produce the solutions available in the historical track and balance data, and provided a forward-model to evaluate the solution. The potential advantage of this method is that it can interpolate among the historical solutions to account for stochastics. It is disadvantaged, however, in that it is only applicable to helicopters with extensive track and balance history and that its solutions are limited to the solutions contained in the historical data.

Another deviation from the traditional approach is by Ventres and Hayden (Ref. 1) who define the relationships between adjustments and vibration in the frequency domain, and provide an extension of these relationships to higher-order vibrations. They use an optimization method to search for the adjustments to reduce 1/rev vibration as well as higher-order vibrations. This approach has the capacity to provide a comprehensive solution, but it too neglects the stochastic relationship between the blade adjustments and aircraft vibration.

The purpose of this paper is to introduce a method of track and balance that accounts for the stochastics of track and balance. The underlying model in this method comprises two components: a deterministic component as well as a probabilistic one. It relies on the probability model to estimate the likelihood of the measured vibration satisfying the specifications as the basis of a search for blade adjustments. The likelihood measures used for blade adjustment selections are computed according to the probability distribution of vibration derived from historical track and balance data. Although the method introduced here is demonstrated with four-bladed helicopters, it can be easily extended to cope with helicopters with more than four blades.

The Proposed Method

The noted feature of the proposed method is its introduction of likelihood of success as a criterion in the search for the optimal blade adjustments. It estimates this likelihood according to the probability density of prediction error between the estimation model and vibration measurements. For selection of the blade adjustments, it quantifies the effectiveness of various adjustments sets in reducing the vibration and selects the set with the maximum probability of producing acceptable vibration.

The proposed method is best explained in the context of a simple example. If the measured vibration from the current flight is denoted by $V_j(k-1)$ and the estimated vibration change from the model is represented by $\Delta\hat{V}_j(k) = f(\Delta x)$ as a function of the blade adjustments Δx, then the predicted vibration of the next flight $\hat{V}_j(k)$ can be defined as

$$\hat{V}_j(k) = V_j(k-1) + \Delta\hat{V}_j(k) \quad (6)$$

$$V_j(k) = \hat{V}_j(k) + \hat{e}_j(k) \quad (7)$$

where $V_j(k)$ denotes the measured vibration for the next flight. In track and balance, the adjustments are selected according to the predicted vibration $\hat{V}_j(k)$, whereas the objective is defined in terms of the measured vibration. The inclusion of the probability model here is to account for the inevitable uncertainty in the actual measured vibration. In this research, the selection of adjustments is performed to maximize the probability that the future measured vibration will be within the specifications. According to Eq. (7), the mean value of the measured vibration is equal to the value of the predicted vibration plus the mean value of the prediction error. But since the predicted vibration is a deterministic entity, the probability distribution of the measured vibration is the same as that of the prediction error. Accordingly, whereas the nominal value of the measured vibration can be controlled by the blade adjustment, its optimum position within the specification region should be determined according to its probability distribution, which is the same as that of the prediction error. For a case where the prediction error $\hat{e}_j(k)$ is zero-mean normally distributed, placing the predicted vibration at the center of the specification range will be synonymous with maximizing the probability that the measured vibration will be within the range, as illustrated in Fig. 3. The likelihood of success of blade adjustments can therefore be measured by the area under the probability density function of prediction error located within the specification region. The blade adjustment set that produces the highest likelihood will be the preferred adjustment set.

However, the main difficulty with track and balance is the limited number of degrees of freedom, which precludes the ability to perfectly position the predicted vibration. This point is illustrated in Fig. 4 for a simple case where two vibration components are to be positioned at the center of the specification region with only one adjustment. If one assumes that the effect of adjustment Δx on the two vibration components $\Delta\hat{V}_j(k)$ can be represented by a linear model such as

$$\Delta\hat{V}_j(k) = a_{ij}\Delta x$$

then the movement of the two predicted vibration components will be constrained to the line L in Fig. 4. As illustrated in this figure, because it will be impossible to place the predicted vibration components at the center due to a lack of degrees of freedom, a compromised position needs to be selected. In this research, the best compromised position for

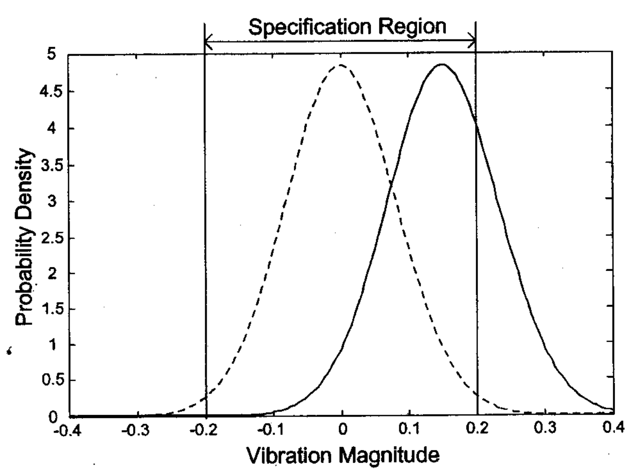

Fig. 3. Illustration of improved placement of the predicted vibration within the specification range.

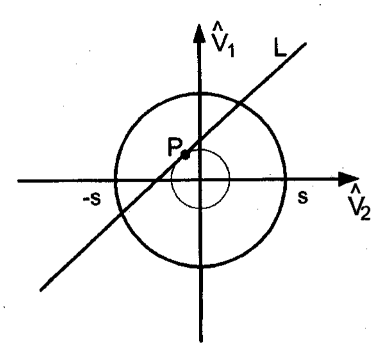

Fig. 4. Restricted placement of vibration components within the specification region for a two-dimensional case.

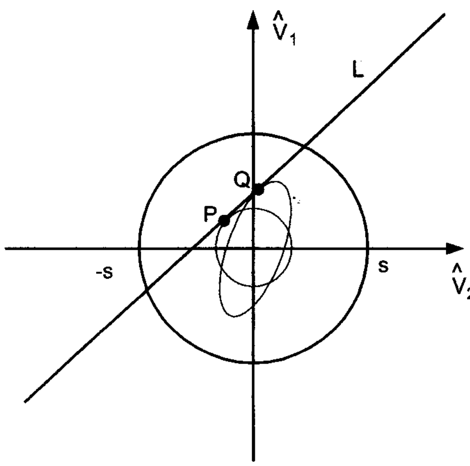

Fig. 5. The change in location of the maximal probability point when the two vibration components are dependent or have unequal distributions.

the predicted vibration is the one that renders the largest probability for the measured vibration satisfying the specifications. This position, for the two-component vibration example is the one that maximizes

$$P_r[(V_1, V_2) \in S] = \int_{(V_1, V_2) \in S} p(V_1, V_2) dV_1 dV_2 \tag{8}$$

The above formulation indicates that the placement of the predicted vibration requires knowledge of the joint probability density function $p(V_1, V_2)$ of the vibration components. In the hypothetical case of independent vibration components with equal probability distributions, the loci of the points with equal probabilities $P_r[(V_1, \ldots, V_n) \in S]$ are the surfaces of hyper-spheres. For the two-component vibration example, the loci of equal probabilities are circles centered at the origin (see Fig. 4). Therefore, the best compromised position for the constrained case is point P, which is the closest point on line L to the center of the specification circle.

Point P, however, does not represent the best position if the two vibration components are dependent or have unequal distributions. The locus of equal probabilities for this more general case is elliptical, as shown in Fig. 5, which locates point Q as the best position on line L (and on the ellipse) for placing the predicted vibration. The inadequacy of degrees of freedom is exacerbated in track and balance where 24 correlated vibration components need to be positioned within the specification region using only 4 blade adjustments.

Error analysis

As discussed earlier, the placement of the predicted vibration within the specification region will be based upon the probability density of the prediction error. Here, the probability distribution of the prediction error is determined in relation to the measurement error (see Fig. 6). The prediction error, which is defined as

$$\hat{e}_j(k) = V_j(k) - \hat{V}_j(k) \tag{9}$$

consists of three components: (1) noise (sensors, weather, etc.), (2) modeling error, and (3) adjustment error. Similarly, the measurement error $e_j(k)$ representing the difference between the measured vibration $V_j(k)$ and its expected value $E[V_j(k)]$

$$e_j(k) = V_j(k) - E[V_j(k)] \tag{10}$$

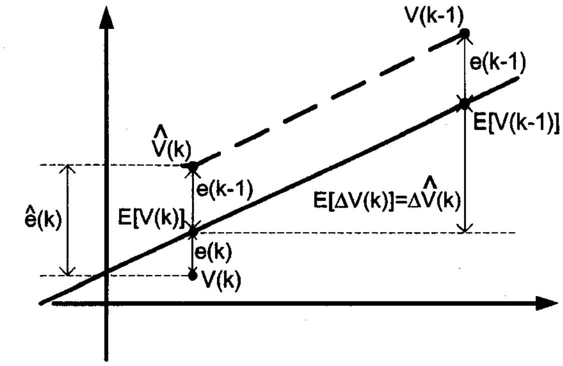

Fig. 6. Graphical representation of the relationship between the predicted and measurement errors.

comprised of measurement noise. In the absence of modeling error, one can write

$$\Delta\hat{V}_j(k) = E[\Delta V_j(k)] \tag{11}$$

and

$$E[V_j(k)] = E[V_j(k-1)] + E[\Delta V_j(k)] \tag{12}$$

Now, combining the relationship for the predicted vibration $\hat{V}_j(k)$

$$\hat{V}_j(k) = V_j(k-1) + \Delta\hat{V}_j(k) \tag{13}$$

with Eqs. (9), (10) and (12), yields

$$\begin{aligned}\hat{e}_j(k) &= V_j(k) - V_j(k-1) - \Delta\hat{V}_j(k) &(14)\\ &= E[V_j(k)] + e_j(k) - E[V_j(k-1)]\\ &\quad - e_j(k-1) - \Delta\hat{V}_j(k)\\ &= e_j(k) - e_j(k-1) &(15)\end{aligned}$$

Note that the relationship in Eq. (15) as illustrated in Fig. 6 includes the sign of $e(k)$. Equation (15) is significant in that it explicitly defines the prediction error in terms of two consecutive values of the measurement error. Assuming that the measurement error is a Gaussian random variable with the distribution $\mathcal{N}(0, \sigma^2)$, according to Eq. (15) $\hat{e}_j(k)$ is also a random variable with the distribution $\mathcal{N}(0, 2\sigma^2)$. The normality of the prediction error leads to several advantages: it (1) simplifies the definition of its probability density functions, (2) facilitates the estimation of the likelihood values for positioning the predicted vibration, and (3) defines the characteristic of noise for inclusion in the simulation model.

Deterministic component

The change in vibration is estimated by a deterministic component representing the effect of blade adjustments on the helicopter vibration. Among the various types of models, the traditionally adopted linear model is the most preferred due to its ease of use in the search process.

A total of 102 sets of vibration data were used to train and test the model. These data, which were obtained from actual flight tests at Sikorsky Aircraft in the course of rotor tuning of 39 new production UH-60 helicopters, represent vibration changes between consecutive flights caused by blade adjustments. The inputs to the model were the combined blade adjustments of push control rods and trim tabs, and its outputs were the resulting vibration changes between two consecutive flights. Because the vibration data are vector quantities that are represented by both magnitude and phase components (see Table 1), the vibration data were transformed into Cartesian coordinates, so that each vector element would denote the change in the cosine or sine component of the

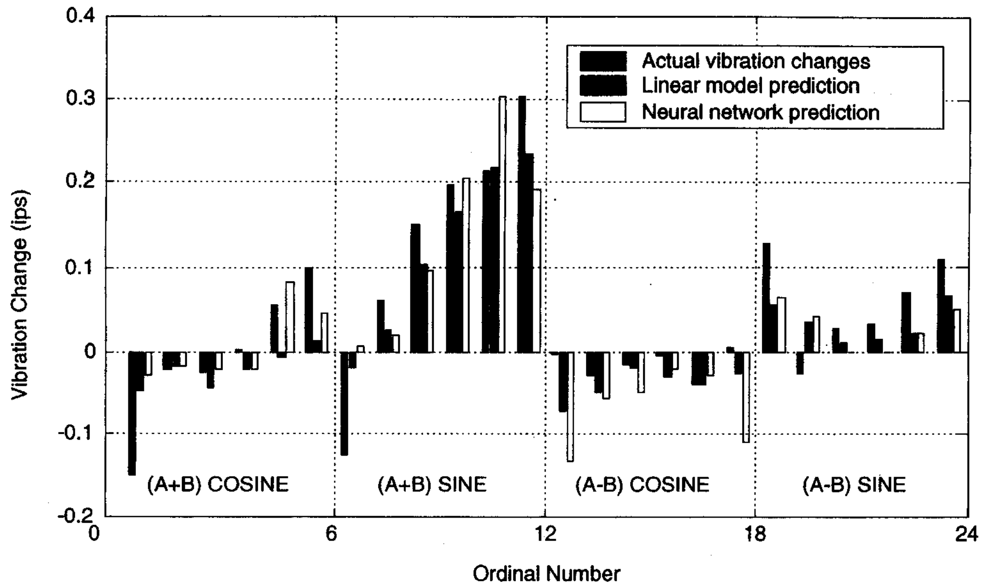

Fig. 7. The predicted vibration changes by the linear and neural network models compared with the actual vibration changes.

A + B or A − B vibration of each of the six flight regimes (see Table 1). Accordingly, the linear model consisted of four inputs and 24 outputs.

Formally, the outputs of the model, which represent the cosine and sine components of the vibration at different regimes, $V_{cj}(k)$ and $V_{sj}(k)$, respectively, are defined as:

$$\hat{V}_{sj}(k) = V_{sj}(k-1) + \Delta V_{sj}(k) \quad (16)$$

$$\hat{V}_{cj}(k) = V_{cj}(k-1) + \Delta V_{cj}(k) \quad (17)$$

$$\Delta V_{sj}(k) = F_{sj}(\Delta \boldsymbol{x}) \quad (18)$$

$$\Delta V_{cj}(k) = F_{cj}(\Delta \boldsymbol{x}) \quad (19)$$

$$\hat{V}_j(k) = \sqrt{\hat{V}_{sj}(k)^2 + \hat{V}_{cj}(k)^2} \quad (20)$$

where the input vector $\Delta\boldsymbol{x} = \{\Delta x_1, \Delta x_2, \Delta x_3, \Delta x_4\}$ denotes the set of combined blade adjustments, and each of the functionals F_{sj} or F_{cj} represent the change in vibration between two consecutive flights. Two standard model validation practices were used to check the suitability of the linear model (Ref. 3). The data were separated into two equal size sets (each including 51 input-output data pairs). For the first validation practice, two sets of linear regression models (each consisting of 24 linear regression models) were trained. For the first set of models, the first data set was used for training and the second set for testing. The second model set was trained with interchanged data sets for training and testing. The models were trained using the Least Square Error (LSE) Method (Ref. 4). The training and testing Mean Square Errors (MSEs) for the two model sets were fairly close, indicating the linear model's effectiveness in representing the effect of blade adjustments on the aircraft vibrations.

For the second model validation practice, the coefficients of the two linear regression models were compared for their similarity. The results indicated that the two sets of coefficients were quite consistent for the 24 outputs, further validating the suitability of linear regression models in estimating the changes in aircraft vibrations due to blade adjustments.

A set of neural networks were trained next to provide a basis for evaluating the suitability of the linear model. In this study, each neural network model consisted of four inputs and one output, so a total of 24 networks were trained to represent all of the vibration components. Alternatively, all of the vibration measurements can be represented by one neural network, but such a network is more difficult to train. Each of the networks consisted of two hidden layers, with 4 and 8 processing elements in the first and second layers, respectively. A logarithmic sigmoid transfer function was used for the two hidden layers and a linear transfer function was used for the output layer. Mean square error (MSE) was used to measure the network performance during training. As before, the data were divided into two equal subsets, one set to train the network and the other to test its performance. To avoid overtraining, each training iteration included two steps: in the first step the network was trained, and in the second step it was tested with the test data. The training process was stopped when the performance of the network began to deteriorate.

Samples of predicted vibration changes from the neural network and linear model are compared with the actual vibration changes in Fig. 7. The results indicate close agreement between the predicted and actual vibration changes. A comparison of the mean square error values for the two models indicates that while the neural network produces better prediction of the vibration change, its performance is not drastically better. An analysis of the prediction errors from the linear model and neural networks indicates that both errors are normally distributed with very similar mean and variance values. The cumulative probability density functions of the errors from the two models are compared in Fig. 8. The results indicate that the two density functions are very similar, and that the linear model provides a good approximation, despite the potential nonlinearity in the relation between vibration changes and blade adjustments. This provides assurance of the adequacy of the linear model in blade adjustment selection.

Probability analysis

Having identified the probability distribution of the prediction error as Gaussian significantly facilitates its estimation by narrowing the distribution parameters to the mean (μ) and variance (σ^2) values, which can be empirically estimated as

$$\hat{\mu} = \frac{1}{N}\sum_{i=1}^{N} \hat{e}_i \quad (21)$$

$$\hat{\sigma}^2 = \frac{1}{N-1}\sum_{i=1}^{N} (\hat{e}_i - \hat{\mu})^2 \quad (22)$$

The estimation results indicate that the mean values of all of the vibration components are approximately zero, as anticipated from our previous assessment. The variance values are included in Table 2.

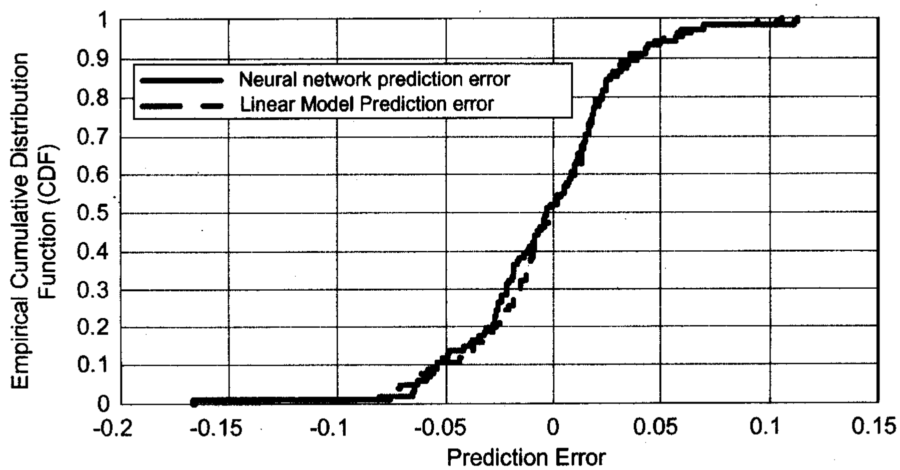

Fig. 8. Illustration of the closeness to the normal distribution of the prediction errors of the linear and neural network models.

Table 2. Estimated variance values of the 24 vibration components

	$\hat{\sigma}$			
	A + B		A − B	
Flight regime	cos	sin	cos	sin
fpm	0.0826	0.0943	0.0989	0.0873
hov	0.0380	0.0333	0.0506	0.0740
80	0.0558	0.0657	0.0388	0.0527
120	0.0761	0.0831	0.0489	0.0630
145	0.0993	0.1019	0.0521	0.0699
vh	0.1105	0.1086	0.0686	0.0916

In addition to the mean and variance values, the covariance matrix between the 24 components of the vibration is necessary for defining the probability distribution of the prediction error. The computed covariance matrix indicates that the off-diagonal elements are mostly non-zero and that the elements closer to the diagonal are larger than others, indicating a stronger correlation between the vibration of consecutive flight regimes. As an example, the cross-correlation between the prediction errors of the (A + B) cosine components at 80 knots and 120 knots is shown in Fig. 9, which indicates the close correlation between the two errors. While the results indicate similarly strong correlation between the sine components, there is little correlation between the prediction errors of cosine and sine components. Other observations are that the prediction errors of (A + B) cosine and (A − B) cosine are uncorrelated and that those of ground vibration are uncorrelated with the other five flight regimes.

As discussed earlier, the probability distribution of the prediction error is identical to that of the measured vibration. So, for the 24-component vector of measured vibration

$$V(k) = [V_{c1}(k),\ V_{s1}(k), \ldots, V_{c12}(k),\ V_{s12}(k)]^T \tag{23}$$

the joint probability density function of the measured vibration for the kth flight, $V(k)$, can be characterized as an N-dimensional Gaussian function

$$p(V(k)) = \frac{1}{(2\pi)^{\frac{N}{2}}|\Phi|^{\frac{1}{2}}} \exp\left[-\frac{1}{2}\hat{e}(k)^T\Phi^{-1}\hat{e}(k)\right] \tag{24}$$

$$\hat{e}(k) = V(k) - V(k-1) - C\Delta x(k) \tag{25}$$

where Φ represents the covariance matrix of the prediction error. Now, if $\Gamma = \{|V_j| = \sqrt{V_{cj}^2 + V_{sj}^2} \leq \alpha,\ j = 1, \ldots, 12\}$ denotes the specification

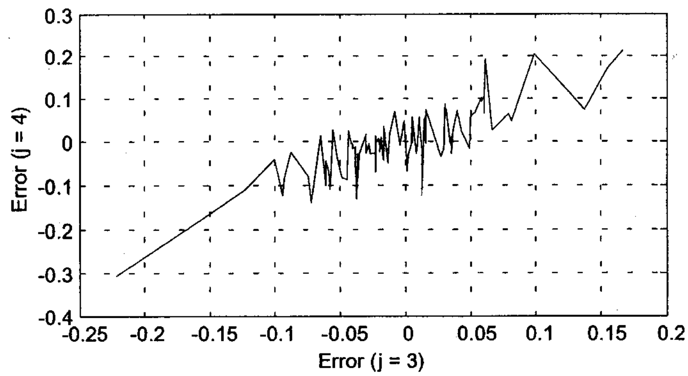

Fig. 9. Correlation between the prediction errors of (A + B) cosine components at 80 and 120 knots.

region in 24-dimensional Euclidean space, the blade adjustments Δx need to be selected such that the probability that the measured vibration is within the acceptable range is maximized. Formally,

$$\arg_{\Delta x} \max\left[Pr(V(k) \in \Gamma) = \int_{\Gamma} p(V(k))dV(k)\right] \tag{26}$$

Implementation

Ideally, the performance of the proposed method should be evaluated "side by side" against that of the traditional method. However, such an evaluation would require tuning the aircraft with one method, undoing changes, and tuning the aircraft with another. Since such testing is prohibitively costly, a compromise approach of evaluating the method in simulation is utilized. A process simulation model is therefore used to represent the "Helicopter" in Fig. 2, with the probability-based model, consisting of the deterministic and probability models, representing the "Process Model" in this figure.

Simulation model

The neural network models were used for simulation to take advantage of their better representation capacity. In addition, random numbers were added to the outputs of the networks to represent the noise in the vibration measurements. These random numbers were generated according to the

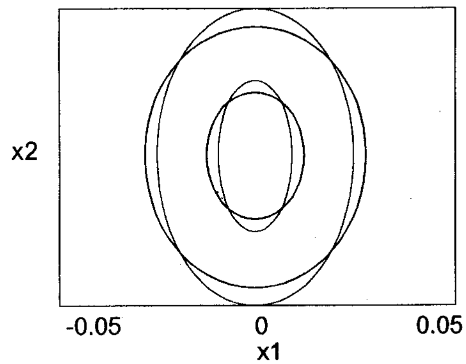

Fig. 10. The equal value contours of approximation to $Pr(V(k) \in \Gamma)$ for a two dimensional case.

Gaussian distribution $\mathcal{N}(\mu_m, \sigma_m^2)$, where μ_m and σ_m^2 were estimated from the mean and variance of the prediction error as $\hat{\mu}_m = 0$ and $\hat{\sigma}_m^2 = \hat{\sigma}^2/2$.

Selection of blade adjustments

The blade adjustments entail the solution to Eq. (26), which is a nonlinear optimization problem. Solving this problem requires that the integral $\int_\Gamma p(V(k))dV(k)$ be computed for the candidate adjustments. Among the different numerical methods to compute N-dimensional integrals (Refs. 5–10), Monte Carlo or adaptive sub-region approaches can be used to cope with the "curse of dimensionality." But even these methods require considerable computation time for integrals with more than ten variables. To reduce the computation time, the search space for the blade adjustments was narrowed to

$$\arg_{\Delta x} \max[-\hat{V}(k)^T \Phi \hat{V}(k)] \quad (27)$$

as an approximation to Eq. (26). The $\hat{V}(k) = V(k-1) + C\Delta x(k)$ in the above equation represents the predicted vibration of the kth flight using the linearly approximated change in vibration. The closeness of Eqs. (26) and (27) can be examined by comparing the contours of equal values of $-\hat{V}(k)^T \Phi \hat{V}(k)$ and $Pr(V(k) \in \Gamma)$. Such contours are shown in Fig. 10 for a two-dimensional case. The results indicate that the contours are very similar, implying the closeness of the approximate solution of Eq. (27) to that of Eq. (26). The candidate blade adjustments were, therefore, sought around the solution to Eq. (27) and their likelihood values were estimated by approximating the integral $\int_\Gamma p(V(k))dV(k)$ with a summation. The vibration components resulting from each set of blade adjustments were simulated 100 to 200 times and the probability of success was approximated by the average number of times the simulated vibration satisfied the specifications. The set of blade adjustments rendering the maximum probability of success was then selected as the recommended adjustment set.

Performance Evaluation

The proposed method was tested with actual data from 39 UH-60 helicopters. These datasets were generated from test flights performed by Sikorsky during the production acceptance process. For each helicopter, the method was applied iteratively until the simulated vibrations were within their specifications, or an upper limit of 5 process iterations had been reached.

Because of the stochastic nature of the vibration measurements, the track and balance process cannot be evaluated by deterministic measures. Several performance measures have therefore been devised to account for the uncertainty of the process. One such measure that assesses tuning efficiency is the *Average Tuning Iteration Number* (ATIN), which represents the average number of adjustment iterations taken for each helicopter to bring vibration within the acceptable level of 0.2 ips. The number of flights indicated by the method for the 39 helicopters is included in Table 3, along with those actually performed during the production acceptance process. The results reveal that the proposed method indicates a smaller ATIN relative to that actually performed. This result, although not conclusive because of the nonuniformity of the evaluation formats (simulation vs. real aircraft), lends credence to the efficiency of the proposed method.

Table 3. The number of tuning iterations indicated by the proposed method and those actually applied during production acceptance (Actual versus Probability-based)

Helicopter # (39)	Tuning iteration number	
	Actual	Probability-based
176	1	1
179	3	2
180	1	1
184	2	1
⋮	⋮	⋮
260	4	2
⋮	⋮	⋮
861	1	1
Total	71	46
ATIN	1.82	1.18

A fundamental issue to be studied is the impact of the probability component of the model on tuning efficiency, as represented by ATIN. The significance of the probability component in the performance of the method can be analyzed by removing it from the model and using only the deterministic component for blade selection. For this, the blade adjustments can be found by minimizing the distance of the predicted vibration to the center of the specification region (see Fig. 4). The results from this deterministic component, obtained by minimizing the mean square value of the predicted vibration, are listed in Table 4 along with the results from the probability-based model in Table 3. The results indicate that the deterministic component alone is not as effective in tuning and that accounting for the non-uniformity of the probability distributions of the vibration components is important in blade selection.

A preferred aspect of a system of track and balance is its ability to tune the aircraft in one adjustment iteration. This aspect of the method was evaluated by checking the number of helicopters tuned in one iteration.

Table 4. The number of tuning iterations indicated by the proposed method and those actually applied during production acceptance (Probability-based versus Deterministic)

Helicopter # (39)	Tuning iteration number	
	Probability-based	Deterministic
176	1	1
179	2	2
180	1	2
184	1	1
⋮	⋮	⋮
260	2	1
⋮	⋮	⋮
861	1	1
Total	46	57
ATIN	1.18	1.46

For these results, in order to eliminate the difference between the simulation model and the helicopter, only the vibration estimates from simulation were used to evaluate the suitability of the adjustments. The results of this evaluation are shown in Table 5 where the helicopters tuned in one iteration are shown by a $\surd$ and those requiring more than one iteration are denoted by $\times$. The results indicate that the proposed method satisfies this more stringent criterion better than the actual adjustments. This further validates the claim that the proposed method benefits from a more effective search engine.

Because of the randomness of the vibration measurements, repeated application of an adjustment set may lead to slightly different vibration measurements. This, in turn, may cause a variance in the number of iterations produced by adjustments when the resulting vibration is close to the specified threshold. It would be beneficial, therefore, to devise a measure for the probability of success of the adjustments. The empirical measure, Acceptability Index (AI), is defined as

$$AI = \frac{1}{M}\sum_{l=1}^{M} s_l \tag{28}$$

to denote the percentage of times an adjustment set will result in the vibration satisfying the specification. In the above equation, M represents the total number of flights simulated to represent the repeated applications of the same adjustment set, and

$$s_l = \begin{cases} 1 & \text{if vibration of the } l\text{th simulation flight is acceptable} \\ 0 & \text{if vibration of the } l\text{th simulation flight is unacceptable} \end{cases}$$

The Acceptability Index (AI) computed for both the actual and selected adjustments at the first iteration are included in Table 6. The results indicate that the proposed method provides adjustments with a higher probability of success as judged by the acceptability of vibration estimates from the simulation model. These results, which indicate that the selected adjustments from the proposed method can more consistently tune the helicopter in one iteration, imply the better positioning of the adjustments within the feasible region.

Another evaluation basis for the adjustments can be established by comparing them to the actual cumulative adjustments performed during production acceptance. The cumulative adjustment set, Σx, can be defined as

$$\Sigma x = \sum_{k=1}^{N} \Delta x_k \tag{29}$$

where N represents the total number of tuning iterations performed during production acceptance for each helicopter and Δx_k denotes the adjustments applied at the kth iteration. A sample of actual first iteration adjustments, actual cumulative adjustments, and first iteration adjustments from the proposed method is shown in Table 7. The results indicate that the adjustments from the proposed method are closer to the actual cumulative adjustments than are the actual first iteration adjustments. Although the cumulative adjustments may not be the most desirable ones for the aircraft, they represent an acceptable set that have been proven during the production acceptance process. The closeness of the proposed method's solutions to the actual cumulative adjustments provides further evidence of the effectiveness of the search strategy.

Table 5. Tally of helicopters tuned in one iteration

Helicopter # (39)	Vibration of the second flight: Actual	Vibration of the second flight: Proposed method
176	√	√
178	×	√
179	√	√
⋮	⋮	⋮
822	×	√
858	×	√
859	√	√
861	√	√
Total	19	33

Table 6. The value of acceptability index (AI) computed for both the actual and proposed adjustments at the first flight

Helicopter # (39)	Acceptability index: Actual	Acceptability index: Proposed method
176	0.92	0.94
179	0.61	0.80
⋮	⋮	⋮
260	0.40	0.74
261	0.09	0.86
263	0.18	0.13
⋮	⋮	⋮
822	0.0	0.28
857	0.62	0.62
858	0.64	0.94
859	0.94	0.96
861	0.96	0.98
Average	0.581	0.754

Conclusions

A method has been introduced for helicopter track and balance that takes advantage of the probability distribution of vibration measurements

Table 7. Comparison of the first iteration solutions of the proposed method and actual solutions from Sikorsky's production line with the cumulative acceptable adjustments

Helicopter #	Proposed method first iteration solution				Actual cumulative adjustment set ($\sum x$)				Actual first iteration solution			
	PCR_{13}	PCR_{24}	TAB_{13}	TAB_{24}	PCR_{13}	PCR_{24}	TAB_{13}	TAB_{24}	PCR_{13}	PCR_{24}	TAB_{13}	TAB_{24}
801	5	−4	−8	11	2	−4	−4	14	6	−4	−10	11
802	7	−1	−3	5	9	0	−10	10	5	2	0	0
822	6	−3	−21	5	10	−4	−23	13	6	0	−20	0
858	8	0	−15	3	9	−2	−10	3	7	0	−14	0

to cope with their stochastics. In the proposed method, the underlying model comprises two components: a deterministic component as well as a probabilistic one. The method relies on the probability model to estimate the likelihood of the measured vibration satisfying the specifications, and to select the set of blade adjustments with the maximum probability of producing acceptable vibration. The likelihood measures used for blade selection are computed according to the probability distribution of vibration derived from historical track and balance data. Several performance measures were also proposed to account for process uncertainty. The proposed method has been shown to improve the number of iterations used for track and balance based on these performance measures.

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