1 Introduction

The advent of computer numerical control has enabled adaptation of machine settings for enhanced productivity. An important requirement when adapting machine settings is that the measurements of process and part quality remain within their specified limits so that part integrity is ensured. One adaptation approach in machining is Adaptive Control with Constraints (ACC), which regulates power or cutting force at a specified level (Daneshmend and Pak, 1986; Lauderbaugh and Ulsoy, 1988; Mason and Koren, 1985; Tomizuka and Zhang, 1988). Although ACC can avoid interruptions in the cut due to tool breakages in machining, or safeguard against thermal damage (burn) to the workpiece in grinding, it is not explicitly designed to improve process efficiency in terms of production cost or time.

The adaptation approach which explicitly addresses process efficiency is referred to as Adaptive Control Optimization (ACO) (Koren, 1983). In ACO, the machine settings are adapted so as to minimize production cost or cycle time in response to part and/or process feedback. This interactive approach to process optimization is adopted to enable the ACO systems to maintain constraint satisfaction despite modeling uncertainty, which is the primary factor hindering optimization of machining processes. Modeling uncertainty in machining arises from: (1) the diversity of machining conditions due to variations in material properties, tool/wheel type, and lubrication, (2) the stochastic nature of these processes caused by material inhomogeneity, workpiece misalignment, and measurement noise, and (3) process time-variability due to tool wear.

The first attempt at ACO was the Bendix System (Centner, 1964), where the machining removal rate was continually maximized through changes in the feedrate and spindle speed in response to feedback measurements of cutting torque, tool temperature, and machine vibration. However, the Bendix System was limited in applicability due to the need to estimate tool wear based on an accurate model. A subsequent advancement in ACO was the Optimal Locus Approach (Amitay et al., 1981; Koren, 1989), which made it possible to forego estimation of tool wear. In this approach, the locus of the optimal points associated with various levels of tool wear is computed, and the optimal point is sought where process and part quality constraints become tight. The Optimal Locus Approach can avoid estimation of tool wear by using the tightness of constraints as the measure for optimality. However, it still needs to rely on the accuracy of the process model for computing the optimal locus and determining a priori which constraints are tight at the optimum. Since the success of this approach depends on the premise that modeling uncertainty will have negligible effect on the accuracy of the optimal locus, it will produce sub-optimal results when this premise is invalid.

An ACO method which has been recently developed to overcome the difficulties posed by modeling uncertainty is Recursive Constraint Bounding (RCB) (Ivester and Danai, 1995). Like the Optimal Locus Approach, RCB assumes optimality from the tightness in the constraints using measurements of process and part quality after each workpiece has been finished (cycle). It also uses the model of the process to find the optimal point. However, unlike the Optimal Locus Approach, RCB assumes the model to be uncertain when determining which constraints are to be tight at the optimum and selecting the machine settings for each process cycle. It obtains the machine settings by solving a customized nonlinear programming (NLP) problem, and allows for uncertainty by incorporating conservatism into the NLP problem.

Under deterministic conditions (no modeling uncertainty), the NLP problem would yield the optimal machine settings for the process. In practice, however, the optimal point of the model would differ from that of the process, due to inherent modeling inaccuracies and randomness associated with constraints. As such, there is a strong possibility that the optimal point of the model will violate the process and part quality constraints. In order to avoid constraint violation, a recursive approach to constraint tightening (bounding) is adopted in RCB, where the distance from the constraint measurements of the cycle just completed to the absolute limit of the constraint is defined as the slack in each constraint. The NLP problem is then formulated so
as to minimize the objective function (usually cycle-time or cost) while removing a portion of these slacks, thus yielding more aggressive machine settings for the next cycle. In RCB$_2$, the slack portions removed for each cycle are defined in terms of the confidence levels and noise buffers which account for the inaccuracy and randomness, respectively, of individual modeled constraints. The consideration of separate confidence levels and noise buffers for individual constraints in RCB$_2$ enables the convergence of individual constraints to be tailored according to the severity of modeling uncertainty associated with each constraint. The repeated minimization of the objective function with progressively smaller slacks leads to bound constraints and optimal machine settings. In this paper, the performance of RCB$_2$ is studied in simulation and its effectiveness is demonstrated in cycle-time reduction of cylindrical plunge grinding.

2 RCB$_2$ Method

Optimization of a machining process can be considered as a constrained nonlinear programming (NLP) problem where the machine settings correspond to the control variables, and the process and part quality measurements to the constraints. In general, a constrained NLP problem is defined as (Luenberger, 1989):

$$\text{minimize: } f(x)$$

subject to: $$g(x) \leq 0, \quad h(x) = 0, \quad x_{lb} \leq x \leq x_{ub}$$

where $f(x)$ represents the objective function, $x = [x_1, \ldots, x_n]^T$ denotes the vector of machine settings, $g(x) = [g_1(x), \ldots, g_m(x)]^T$ and $h(x) = [h_1(x), \ldots, h_k(x)]^T$ constitute the vectors of inequality and equality constraints, respectively, and $x_{lb}$ and $x_{ub}$ represent the lower and upper bounds of the machine settings, respectively. For machining processes, the objective function $f(x)$ usually represents cycle-time or cost, and the constraints are associated with part quality and/or machine limitations.

2.1 Methodology. RCB$_2$ relies on the premise that analytical models of machining processes are of the correct form, although they may be imprecise. As such, RCB$_2$ is designed to take advantage of the form of the relationships provided by these models, but compensates for their inaccuracies using measurements of process behavior and part quality as feedback. The basic role of RCB$_2$ is to assess the optimality of the process after each cycle from the measurements of process and part quality as the basis for changing the machine settings for the next cycle (see Fig. 1). RCB$_2$ obtains these machine settings by solving a NLP problem that has been customized for each cycle. The customized NLP problems are obtained by redefining the inequality constraints (Inequality (2)) as

$$\hat{g}(x(j)) = \hat{g}(x(j - 1)) - c[g(x(j - 1)) + n]$$

(5)

to lead to a more aggressive set of machine settings when used as the basis of nonlinear optimization. Inequality (5) redefines the upper limit of the inequality constraints for the next cycle $\hat{g}(x(j))$ in terms of the modeled constraint values $\hat{g}(x(j - 1))$ for the cycle just completed, the measured constraint values $g(x(j - 1))$, the confidence levels $c$ and the noise buffers $n$, representing the allowable changes for individual modeled constraints. Assuming that the process is initiated with conservative machine settings that satisfy the process and part quality constraints, the confidence levels and noise buffers control how much the nonlinear program should tighten the constraints from one iteration to the next.

Constraint redefinition in RCB$_2$ is developed to account for the fact that the value of constraints cannot be accurately determined from the model due to modeling inaccuracies and process randomness (i.e., $g(x) \neq \hat{g}(x)$). Therefore, machine settings that would minimize the objective function while satisfying $g(x) = 0$ do not necessarily ensure $g(x) \leq 0$. In machining, it is generally possible to select conservative settings that satisfy the constraints. After the process is initiated with such settings, RCB$_2$ selects the machine settings such that the objective function will be reduced without violating the constraints. In order to ensure constraint satisfaction, the machine settings for the next cycle $x(j)$ need to be selected such that $g(x(j)) = 0$. However, the only information available to RCB$_2$ is in the form of the model and constraint measurements from the cycle just completed. Therefore, the redefined constraints that replace Inequality (2) need to be formulated in terms of $\hat{g}(x(j))$ as

$$\hat{g}(x(j)) \leq U$$

(6)

The main contribution of RCB$_2$ is its definition of this upper bound such that it is robust to modeling inaccuracy and randomness in the constraint values. As was stated earlier, RCB$_2$ relies on the premise that the model of the process correctly represents its form. Based on this premise, the assumption is made here that this model approximately represents the changes in the constraints due to changes in the machine settings, as

$$g(x(j)) - g(x(j - 1)) = \hat{g}(x(j)) - \hat{g}(x(j - 1))$$

(7)

Although modeling inaccuracy and randomness prevent RCB$_2$ from directly using the above equation for redefining the constraints, it provides the basis for relating $g(x(j))$ to $g(x(j - 1))$, as well as for $\hat{g}(x(j))$ and $\hat{g}(x(j - 1))$ which are available to RCB$_2$ from the model.

In RCB$_2$, allowance for randomness is provided by noise buffers, $n = [n_1, \ldots, n_m]$, which define the width of the noise distributions of $g(x)$. If adequate constraint measurements are available, the noise buffer $n$ can be obtained as $n_i = k_i s_i$, where $s_i$ represents the standard deviation of the $i^{th}$ constraint measurements and $k_i$ denotes a constant typically between 6 and 12. The noise buffer $n$ can alternatively be estimated based on experience if adequate constraint measurements are unavailable. In order to explain how the noise buffers are utilized to establish upper bounds on the constraints, let us consider a case where the machine settings for the next cycle are very close to the settings for the cycle just completed, that is $x(j) = x(j - 1) + \varepsilon = x(j - 1)$. For this case, the upper bounds on the actual constraint values can be defined as

$$g(x(j - 1) + \varepsilon) - g(x(j - 1))$$

$$\leq [\hat{g}(x(j - 1) + \varepsilon) - \hat{g}(x(j - 1))] + n$$

(9)

This inequality provides an upper bound on the change in the constraint measurements, but it is limited to infinitesimal changes in the machine settings. In cases where $x(j) \neq x(j - 1) + \varepsilon$, modeling inaccuracy could result in changes in the constraint measurements that are larger than $[\hat{g}(x(j)) - \hat{g}(x(j - 1))] + n$. In order to extend Inequality (9) so that larger changes in the machine settings can be accommodated, confi-
dence levels $c \in [0, 1]$ are introduced on the right hand side
of inequality (9) as

$$g(x(j)) - g(x(j - 1))$$

$$\leq \frac{1}{c} \hat{g}(x(j)) - \hat{g}(x(j - 1)) + n \quad (10)$$

to account for the inaccuracy of individual modeled constraints.

With the inclusion of the confidence levels, the upper bounds
established in terms of the modeled values of constraints (right
hand side of Inequality (10)) can be made sufficiently large so
as to account for modeling inaccuracy associated with individ-
ual constraints. Accordingly, smaller confidence levels can be
selected for constraints that are less accurately represented
by the model so that a larger upper bound will be placed on the
changes in the constraints.

While Inequality (10) defines the upper bound on the actual
constraint changes, it does not provide the upper bound on
$\hat{g}(x(j))$ (U in Inequality (6)) that is needed for the reformu-
lization of the NLP problem. In order to develop this upper bound,
we note that the absolute requirement in the NLP problem is
$g(x(j)) = 0$. This is equivalent to

$$g(x(j)) - g(x(j - 1)) \leq 0 - g(x(j - 1)) \quad (11)$$

which defines the absolute limit on changes in the actual con-
straints. Satisfaction of this absolute limit in light of Inequality
(10) is ensured when

$$\frac{1}{c} \hat{g}(x(j)) - \hat{g}(x(j - 1)) + n = 0 - g(x(j - 1)) \quad (12)$$

which states that the upper bound for Inequality (10) must be
less than or equal to the upper bound for Inequality (11). In-
equality (12) provides the basis for defining the upper limit on
$\hat{g}(x(j))$ (U in Inequality (6)) so that constraint satisfaction
is guaranteed. Rearranging inequality (12) yields

$$\hat{g}(x(j)) = \hat{g}(x(j - 1)) - c[g(x(j - 1)) + n] \quad (13)$$

which defines the upper bound for $\hat{g}(x(j))$ in terms of the
modeled constraints and their measured values from the cycle
just completed. Inequality (13), which is identical to Inequality
(5), represents the redefined constraints to be used in the cus-
tomized NLP problem in place of Inequality (2). Note that under
deterministic conditions (accurate model, without noise), the
modeled constraint values $\hat{g}(x)$ and their measured values $g(x)$
would be identical. The confidence levels would be assigned the
value of 1 (accurate model) and the noise buffers would have
the value of 0 (noise-free conditions). Under these conditions,
the right hand side of Inequality (13) would be reduced to zero,
and Inequality (13) would be equivalent to Inequality (2).

The salient feature of $RCB_2$ is its robustness to modeling
inaccuracy and noise. The conceptual basis of $RCB_2$'s design
is illustrated in Fig. 2. The dark and light data points in this
figure represent measured and modeled values of a constraint
for successive cycles, respectively, and the dotted arrows point
to the upper limit of the constraint in successively reformulat-
ed NLP problems. The top of the gray area represents the allow-
able limit of a constraint, and the bottom of this area denotes the
limit when noise is taken into consideration. (Note that the
width of the gray area is the value of the noise buffer.) When
the distance from a particular measurement to its limit is less
than its noise buffer (data point within the gray area) the con-
straint cannot be safely tightened. In such cases, the value of
$c[g(x(j - 1)) + n]$ is set to zero (e.g., Cycle 6) signifying
that the modeled constraint value should not be changed. When
the distance from a particular measurement to its limit is greater
than its noise buffer (data point outside the gray area) the con-
straint is tightened using Inequality (13). In such cases, the
distance from each constraint measurement (dark data point)
to its upper limit represents the slack in the constraint ($0 -
g(x(j - 1))$ in Inequality (12)), and the dotted arrows repre-
sent the portion of the slack ($-c[g(x(j - 1)) + n]$) that $RCB_2$
tries to remove by reformulating the NLP problem. If the confidence
level were assigned the value of 1 the actual constraint may fall
above the gray area and result in constraint violation. Assigning
a value less than one to the confidence level provides a safety
margin to improve the likelihood of constraint satisfaction. This
improvement, however, is provided at the cost of reducing the rate
of convergence to the optimum, as will be discussed later in the
simulation study.

As the NLP problem is repeatedly reformulated and solved,
the machine settings approach their optimal values and the pro-
cess and part quality measurements approach their respective
limits. At the steady state, some slack may remain in the con-
straints due to the conservative estimates of the noise buffers,
n. After all of the constraint measurements have converged
within these conservative noise buffers, the process can be re-
peated to obtain more constraint measurements for improving
the estimates of the noise buffers using Eq. (8). In cases where
the new noise buffer estimates are smaller than their original
values, the NLP problem can be reformed with the new
noise buffers so as to further tighten the constraints and reduce
the objective function.

2.2 Analysis in Simulation. The effectiveness of $RCB_2$
is first illustrated in simulation for single-pass turning, where
two machine settings are adjusted so that the state-space can
be depicted graphically. The cutting speed and feed were the
machine settings for this problem, and power and surface
roughness the constrained variables. The constraint measure-
ments were simulated using a turning model (Ivester and Danai,
1995). In order to simulate noise, the values of power and
surface roughness obtained for each cycle were multiplied by
random numbers uniformly distributed between 0.9 and 1.1.

Modeling inaccuracy was simulated by perturbing the coeffi-
cients and exponents of the simulation model within ten percent
of their nominal values before each sequence of cycles. Only
the nominal values of coefficients and exponents were used by
$RCB_2$.

The performance of $RCB_2$ was examined under various con-
iditions. The first study was for a large depth of cut, where tool
wear progressed so rapidly that it was necessary to change the
tool after each cycle. As such, the relationships between the
machine settings and constraints were not affected by accumu-
lated tool wear. The first cycle of each test was begun with
conservative values for the machine settings, so that the con-
straints would be satisfied. Using simulated constraint values

![Fig. 2 Constraint tightening in $RCB_2$](image-url)
and confidence levels of 0.5 for both the power and surface roughness constraints, the redefined constraints for the subsequent cycles were obtained by \( RCB_2 \) using Inequality (13). These redefined constraints were then used to determine the next set of machine settings using the Sequential Quadratic Programming (SQP) algorithm (Powell, 1978). A series of machine settings selected by \( RCB_1 \) is shown in Fig. 3 within the state-space of this problem. The results indicate that the machine settings move towards the optimal point from cycle to cycle without violating the constraints despite the approximate model used by \( RCB_1 \) and the presence of noise in the simulated values of the constraint measurements.

Next, the effect of confidence levels on the speed of convergence of \( RCB_1 \) was studied. For this, runs were performed with various confidence levels for the power and surface roughness constraints. The cycle-times from three runs with confidence levels 0.1, 0.5, and 0.9 are shown in Fig. 4. The results indicate that although the convergence rate of \( RCB_2 \) was improved by increasing the magnitude of the confidence levels, the differences in cycle times were practically insignificant after the first few cycles. However, the rapid convergence rate associated with higher confidence levels is not obtained without a price, since larger confidence levels correspond to greater risks for constraint violation. One case of constraint violation is shown in Fig. 5, where a confidence level of 0.9 was selected inappropriately given the degree of inaccuracy of the model.

Another important feature of \( RCB_2 \) is its adaptability to changing process conditions caused by progression of tool wear. In order to evaluate the performance of \( RCB_2 \) in such circumstances, a smaller depth-of-cut was used to eliminate the need for changing tools at the beginning of each cycle. This made it necessary to cope with process time-variability due to tool wear. The conditions for the first few cycles of this run are very similar to those of the original run. However, as tool wear progresses the constraints become more difficult to satisfy (see Fig. 6). Therefore, in order to continue satisfying the constraints, it becomes necessary to select more conservative machine settings which is reflected in larger values of cycle-times (see Fig. 7). After the cycle-time reaches a certain threshold, it is more economical to change the tool than to continue with a worn tool.

3 Experimental Verification

The \( RCB_2 \) method was validated experimentally for internal cylindrical plunge grinding. In cylindrical grinding, material is removed from the internal cylindrical surface by feeding a grinding wheel that is rotating at a high speed into the workpiece.
which rotates at a much lower speed (see Fig. 8). The infeed control cycle \( u = [u_1, u_2, u_3] \) is typically characterized by three successive stages as illustrated in Fig. 9: (1) roughing with a relatively fast infeed velocity \( u_1 \), (2) finishing with a slower infeed velocity \( u_2 \), and (3) spark-out at zero infeed velocity \( u_3 = 0 \). This is followed by rapid retraction to disengage the wheel from the workpiece.

In response to the controlled infeed, the radial size reduction of the workpiece follows the actual infeed curve as shown in Fig. 9. The transient in the actual infeed at the beginning of each stage is attributed mainly to the elastic deflection of the system and to the radial wear of the grinding wheel. This transient behavior can be approximated by a first order system characterized by a time constant (Malkin and Koren, 1984).

The nonlinear programming problem for this grinding operation can be defined as (Xiao et al., 1993)

Minimize cycle-time: \( T = t_1 + t_2 + t_3 \)

with respect to: \( u_1, u_2, t_1, t_2, t_3, s_d \) \hspace{1cm} (14)

subject to:

\[ g_1 = z_1 - q_2 \leq 0 \] \hspace{1cm} (burning constraint) \hspace{1cm} (15)

\[ g_2 = R_m - R_{\text{max}} \leq 0 \] \hspace{1cm} (surface finish constraint) \hspace{1cm} (16)

\[ g_3 = r - r_{\text{max}} \leq 0 \] \hspace{1cm} (out-of-roundness constraint) \hspace{1cm} (17)

\[ h_1 = u_1 t_1 + u_2 t_2 - \Delta r = 0 \] \hspace{1cm} (size constraint) \hspace{1cm} (18)

The objective function for this problem is the total cycle-time \( T \), which is defined as the sum of the times for the three successive infeed stages, \( [t_1, t_2, t_3] \). The machine settings are the stage times \( [t_1, t_2, t_3] \), the programmed infeed rates for the first two stages \( [u_1, u_2] \), and the dressing lead \( s_d \). For these experiments, the wheel was dressed after each cycle using a single point diamond dresser. The dressing lead \( s_d \), which specifies the crossfeed per revolution of the wheel, determines the initial sharpness of the wheel. Minimization of the total cycle-time requires that tradeoffs among the three stage times be balanced through an examination of their relationships with the various constraints using Eqs. (15)–(18). The burning constraint in Eq. (15) requires that the thermally damaged (burned) layer on the workpiece due to excessive grinding temperatures during the roughing stage be completely removed during the subsequent finishing stage. As such, a deeper layer of thermally damaged material caused by a more aggressive roughing infeed rate can be balanced by a longer time for the finishing stage. An alternative to this burning constraint is to completely avoid thermal damage during the roughing stage, which is more restrictive but may be desirable for grinding of critical components (Xiao et al., 1993). The inequality \( g_2 \) defines the surface finish constraint, where \( R_m \) denotes the measured surface roughness and \( R_{\text{max}} \) its maximum allowable value. The inequality \( g_3 \) defines the out-of-roundness constraint, where \( r \) represents the out-of-roundness value and \( r_{\text{max}} \) the maximum allowable out-of-roundness. The equality \( h_1 \) defines the size requirement, where \( \Delta r \) denotes the radial depth of material to be removed. The relationships among the constraints and machine settings are given in (Xiao et al., 1993).

It has been suggested that the only in-process sensors which can be reliably utilized in the harsh environment of grinding are a power monitor which measures the wheel spindle power and a size gage which measures the workpiece diameter (Rao and Malkin, 1990). The output from the power monitor together with a thermal analysis is used to estimate the depth of the damaged (burned) layer \( z_1 \) on the workpiece. The output from the size gage, which directly indicates the remaining radial depth to be removed, is used to estimate the elastic deflection of the system and the radial wheel wear (or grinding ratio \( G \)). The information derived from these two in-process sensors, together with post-process measurement (inspection) of part quality (e.g., surface roughness and roundness), were used to satisfy the constraints while reducing the cycle-time.

The internal cylindrical plunge grinding system is shown in Fig. 10. The system consists of a Bryant Model 1116 internal grinder modified by the addition of a stepper motor infeed drive, an electrical workpiece drive for computer control in place of the original hydraulic motor, a wheel spindle power monitor (A. F. Green TT2), a diametral size gage (Marposs Micromar 5 and E9 amplifier), a Taylor-Hobson Surtronic 3P surface roughness gage, and a personal computer for data acquisition and control (Rao and Malkin, 1990). Out-of-roundness measurements were not available, so values for the out-of-roundness constraint were simulated based on the model equation. The arithmetic average surface roughness constraint and out-of-roundness constraint were selected as 0.7 \( \mu \text{m} \) and 0.6 \( \mu \text{m} \), respectively. In these experiments, AISI 52100 hardened steel bearing workpieces with an internal diameter \( d_i \) of 70 mm and width \( b \) of 9 mm were machined using a 32A80UMVBE grinding wheel with an external diameter \( d_e \) of 50 mm. The peripheral
Fig. 10  Implementation of RCB3 for cycle-time reduction of internal cylindrical plunge grinding

speeds of the wheel \( v_w \) and the workpiece \( v_w \) were 37 m/s and 0.55 m/s, respectively.

Of the four constraint relationships (Eqs. (15)–(18)) considered in this problem, the first three are inequalities, and the fourth is an equality. The first constraint, depth-of-burn, is not readily measurable, so it was estimated based on the measured value of power. Accordingly, the confidence level and noise buffer for the first constraint were set at 1.0 and 0, respectively. The confidence level and noise buffer for the second constraint (surface roughness) were set at 0.25 and 0.03, respectively. For the third constraint (out-of-roundness), the confidence level and noise buffer were considered as 0.25 and 0.04. Since the fourth constraint is an equality, no confidence level or noise buffer was associated with it.

4 Results

The initial machine settings for these experiments were selected as

\[ [t_1, t_2, t_3, u_1, u_2, s_2] = [13.9, 6.7, 6.6, 16, 4, 110]. \]

with units as given in Table 1. The constraints were then evaluated according to the measurements resulting from these settings as

\[ [g_1, g_2, g_3] = [-0.038, -0.06, -0.45]. \]

The negative constraint values obtained for the first cycle indicate that the constraints are not violated, and that they contain significant slack which can be removed to lead to reduced cycle-times. Using these machine settings in the constraint relationships yielded

\[ [\delta_1, \delta_2, \delta_3] = [-0.038, -0.08, -0.31]. \]

The large negative value for the third constraint indicates that it can be tightened significantly, while the first two constraints have less slack.

<table>
<thead>
<tr>
<th>Iteration Number</th>
<th>Cycle Stage Times (sec)</th>
<th>Infeed Rates (( \mu \text{m/s} ))</th>
<th>Dressing Lead, ( s_2 (\mu \text{m}) )</th>
<th>Cycle Time, ( T ) (sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>13.9 6.7 6.6 16 4</td>
<td>110</td>
<td>27.2</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>9.6 1.1 4.6 25 4</td>
<td>72</td>
<td>19.5</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>9.5 4.5 4.5 24 4</td>
<td>75</td>
<td>18.6</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>9.5 3.7 4.6 25 3.3</td>
<td>80</td>
<td>17.8</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>9.6 3.7 4.4 25 3.2</td>
<td>82</td>
<td>17.6</td>
<td></td>
</tr>
</tbody>
</table>

Table 2  Constraint values obtained after each grinding cycle

<table>
<thead>
<tr>
<th>Iteration Number</th>
<th>Constraints ( g_i )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( g_1 )</td>
</tr>
<tr>
<td>1</td>
<td>-0.038</td>
</tr>
<tr>
<td>2</td>
<td>-0.002</td>
</tr>
<tr>
<td>3</td>
<td>-0.003</td>
</tr>
<tr>
<td>4</td>
<td>-0.001</td>
</tr>
<tr>
<td>5</td>
<td>-0.003</td>
</tr>
</tbody>
</table>

For the second cycle, RCB3 calculated the limits of the redefined constraints based on these constraint values (see Eq. (13)) which were then used to redefine the corresponding constraints for the next cycle as:

\[ \delta_1(x(2)) \leq \delta_1(x(1)) - c_1(g_1(x(1)) + n_1) \]

\[ = -0.038 + 0.038 \]

\[ \delta_2(x(2)) \leq \delta_2(x(1)) - c_2(g_2(x(1)) + n_2) \]

\[ = -0.08 + 0.0075 \]

\[ \delta_3(x(2)) \leq \delta_3(x(1)) + c_3(g_3(x(1)) + n_3) \]

\[ = -0.31 + 0.1025 \]

Using a nonlinear program (Sequential Quadratic Programming (SQP) (Powell, 1978)), the machine settings for the second cycle were obtained as

\[ [t_1, t_2, t_3, u_1, u_2, s_2] = [9.3, 4.1, 6.1, 25, 4, 72]. \]

which represented the optimal point for the redefined optimization problem, such that \([\delta_1, \delta_2, \delta_3] = [0, 0, 0] \). However, when the experiment was run with the above machine settings the constraint values were obtained as

\[ [g_1, g_2, g_3] = [-0.002, -0.03, -0.49] \]

which were again less than zero, with considerable slack in the third constraint.

The above procedure was continued for three more cycles until the cycle-time was approximately minimized. The machine settings and cycle-times for the five cycles are listed in Table 1, with the corresponding measured constraint values listed in Table 2. The cycle-times are plotted in Fig. 11, and the surface roughness and out-of-roundness values are plotted in Fig. 12 against their allowable limits. The results in these figures indicate that the machine settings selected by RCB3 corresponded to progressively tightened constraints (see Fig. 12). For the fifth cycle, both the surface roughness and out-of-roundness

Fig. 11  Gradually reduced cycle-times by RCB3 for grinding
Acknowledgment

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Constraints were within their respective noise buffers, so the cycle-time could not be reduced further.

5 Conclusion

It has been demonstrated in this paper that RCB2 can be used to adjust the machine settings from cycle to cycle in order to reduce cycle-time. For this, a model of the process is required that adequately represents the general form of the relationships between the machine inputs and part quality attributes. RCB2 is designed to cope with modeling uncertainty and process time-variability due to tool wear. It can be used either as a selection guide to the machine operator, or as the basis of a supervisory module for production control. Since RCB2 uses separate confidence levels and noise buffers for each constraint, incorporating additional machine inputs or constraints does not result in a combinatorial increase in computation time or convergence iterations.

Fig. 12 Tightened constraints by RCB2 for grinding