Continuous Optimal Infeed Control for Cylindrical Plunge Grinding, Part 1: Methodology

A new methodology is developed for optimal infeed control of cylindrical plunge grinding cycles. Unlike conventional cycles having a few sequential stages with discrete infeed rates, the new methodology allows for continuous variation of the infeed rate to further reduce the cycle time. Distinctive characteristics of optimal grinding cycles with variable infeed rates were investigated by applying dynamic programming to a simulation of the grinding cycle. The simulated optimal cycles were found to consist of distinct segments with predominant constraints. This provided the basis for an optimal control policy whereby the infeed rate is determined according to the active constraint at each segment of the cycle. Accordingly, the controller is designed to identify the state of the cycle at each sampling instant from on-line measurements of power and size, and to then compute the infeed rate according to the optimal policy associated with that state. The optimization policy is described in this paper, and the controller design and its implementation are presented in the following paper [1]. [DOI: 10.1115/1.1751423]

1 Introduction

Grinding is widely used as a final machining operation in the production of components requiring fine surfaces and precise tolerances. The most common type of grinding operation is cylindrical plunge grinding, such as illustrated in Fig. 1 for machining of internal surfaces. Material removal during cylindrical plunge grinding occurs by radially infeeding a rapidly rotating grinding wheel into a slowly rotating workpiece at a command infeed rate \( u(t) \) [2]. A typical grinding cycle as shown in Fig. 2 consists of three stages each having a pre-defined infeed rate: (1) roughing with a fast infeed rate \( u_1 \), (2) finishing with a slower infeed rate \( u_2 \) (typically about 25% of \( u_1 \)), and (3) spark-out with zero infeed rate \( u_3 = 0 \). This is followed by rapid retraction of the wheel from the workpiece. In response to the controlled infeed rate, the actual radial infeed into the workpiece follows the curve \( q(t) \) in Fig. 2 whose slope \( v(t) = q(t) \) is the actual instantaneous infeed rate. The actual infeed lags behind the controlled infeed mainly due to elastic deflection of the system and also due to radial wear of the grinding wheel. The present investigation was undertaken to explore the prospects for more efficient grinding cycles by allowing for continuously variable infeed rates instead of just a few specified values.

Most of the controllers which have been developed for grinding machines have attempted to maintain a predetermined value of a particular parameter, usually normal force or power [3–9]. Other attempts at control of grinding processes have been concerned with reducing the cycle time by shortening or even eliminating the spark-out stage at the end of the grinding cycle. For example, Gao et al. and Allanson et al. [10–13] limited the spark-out time in order to just satisfy size and roundness requirements. A more sophisticated approach to minimizing or even completely eliminating the spark-out stage utilizes the system dynamics in a different way by overshooting the controlled infeed followed by rapid retraction of the wheel to reach the required size and roundness [14–18]. Malkin and Koren [15] derived an accelerated spark-out method according to optimal control theory, and Webster and Zhao [16] incorporated on-line identification of the system time constant from size measurements in order to make the method more robust.

Fewer attempts have dealt with the more challenging problem of optimizing a conventional multi-stage grinding cycle. Peters and Aarens [19] used an oversimplified process model to identify optimal infeed rates for a three stage cycle without intermediate wheel dressing. A somewhat different method developed by Amtay et al. [20] used a detailed model of the grinding process to find the maximum infeed rate and dressing conditions subject to constraints associated with thermal damage to the workpiece and surface roughness. This was the first reported attempt to optimize not only the grinding parameters, but also the dressing parameters. For this purpose, an optimal locus of the plunge grinding cycle was derived containing all the possible optimal operating points with the grinding power for thermal damage and the surface roughness at their allowable limits. On-line measurements of power and off-line measurements of surface roughness were used to identify the optimal operating point on the optimal locus. Probably the most advanced optimization method was reported by Xiao et al. [21,22] who applied a monotonicity analysis to a comprehensive grinding model in order to determine the optimal grinding and dressing conditions for a three stage grinding cycle. One critical aspect of this system which facilitated implementation in industry was the inclusion of a comprehensive set of constraints typical of those encountered in production: included size tolerance, surface roughness, out-of-roundness, wheel wear, and thermal damage. Each of these constraints was found to affect the optimal grinding cycle and dressing parameters in a different way. Artificial intelligence techniques have also been used for optimization of the conventional cycle [23]. For example, Vishnuapad and Shin [24] have used fuzzy logic to determine the optimal setpoints for various grinding processes.

A conventional grinding cycle with a few discrete infeed rates applied sequentially is readily implemented with older conventional machine control methods (e.g., ladder logic), but it can limit the extent of cycle-time reduction by optimization. The present investigation was undertaken to develop a new optimal control methodology that allows for a grinding cycle with a continuously varying infeed rate. An optimal continuously varying infeed rate should provide significant savings in cycle time and be readily implemented with modern open-architecture controllers.

2 Optimization Problem

The objective of the grinding optimization in the present study is to find the command infeed rate \( u(t) \) which minimizes the cycle
Minimize cycle time: 
\[ t_c = \sum_{i=1}^{N} \delta_i \]  
(1)

subject to the constraints:

\[ g_1: \quad z(k) - \sum_{i=1}^{N} v(i) \delta t \leq 0 \]  
(2)

\[ g_2: \quad R_e(N) - R_{e,\max} \leq 0 \]  
(3)

\[ g_3: \quad r(N) - r_{\max} \leq 0 \]  
(4)

\[ g_4: \quad \sum_{i=1}^{N} v(i) \delta t - Q \leq \text{tol} \]  
(5)

\[ g_5: \quad u_i - u(k) \leq 0 \quad \text{for all} \quad k = 1, \cdots, N \]  
(6)

\[ g_6: \quad u(k) - u_0 \leq 0 \quad \text{for all} \quad k = 1, \cdots, N \]  
(7)

In the above formulation, the inequality \( g_1 \) defines the depth of thermal damage limit above a critical temperature referred to as “workpiece burn” where \( z(k) \) represents the depth of burn produced up to step \( k \), and \( \Sigma_{i=1}^{N} v(i) \delta t \) denotes the remaining depth of removal after step \( k \). In the limit with \( z = 0 \), this constraint reverts to the “no-burn constraint” where the infed rate is controlled such that thermal damage does not occur. As mentioned above, one purpose for monitoring the grinding power is to estimate the depth of thermal damage during the cycle. The inequality \( g_2 \) defines the surface roughness requirement where \( R_e(N) \) denotes the final surface roughness and \( R_{e,\max} \) its maximum allowable value. The inequality \( g_3 \) defines the out-of-roundness requirement where \( r(N) \) is the value of out-of-roundness at the end of the cycle, and \( r_{\max} \) is its maximum allowable value. The inequality \( g_4 \) defines the size requirement where \( Q \) denotes the total depth of removal and \( \text{tol} \) represents the size tolerance. The size constraint is enforced by on-line size measurement during the cycle. The last two constraints represent limitations on the machine infed rate: \( u_i \) is the lower bound and \( u_e \) the upper bound. The detailed relationships among the process variables, process parameters, and part constraints are presented in the Appendix.

From the system viewpoint, the input variable to the grinding process is the command infed rate \( u(t) \). However, the solution to the cycle-time minimization problem to be defined at each sampling instant is the actual infed rate \( v(t) \). The dynamics between the command and actual infed rates can be approximated as [2]:

\[ v(t) = \frac{1}{\tau}[u(t) - u(t)] \]  
(8)

where \( \tau \) denotes the characteristic time constant (\( \tau = F_e/k_e \)), \( k_e \) the system stiffness, and \( F_e \) the proportionality constant between the normal force and actual infed rate. For a constant command infed rate, the relationship between the discrete values of actual infed rate \( v_i \) and actual infed \( q_i \) between two consecutive sampling instants can be written for this first order system as:

\[ \delta q_i = u_i \delta t + \tau[(u_i - v_{i-1})e^{-\delta t/\tau} - (u_i - v_{i-1})] \]  
(9)

3 Minimum-Time Cycles

The characteristics of optimal cycles were explored by applying dynamic programming (DP) [26–28] to a simulation program of cylindrical plunge grinding [29]. The performance index \( J \) for minimization of the cycle-time was defined as

\[ J_{k,N}(u(k)) = \delta t_k + J_{k+1,N}(u(k+1)) \]  
(10)

where \( k \) and \( N \) denote, respectively, the current and final steps of the cycle and \( \delta t_k \) represents the time period between two consecutive steps at the constant control variable \( u(k) \). Specifics of the cycle-time minimization procedure are as follows:
Cycle-time minimization is defined as a sequential decision making problem, which entails discretizing the total infed into a finite number of small steps with incremental infed \( \delta q \).

The state of the process is defined at each step. The complete state of the grinding process includes the surface roughness, out-of-roundness, depth of burn, and the actual infed rate. Previous research indicates that the surface roughness is affected mainly by the wheel dressing parameters, and only to a much lesser extent by the actual infed rate [2]. The roughness constraint can be satisfied by selecting the dressing lead according to the empirical roughness model [2]:

\[
S_r = \frac{R_o}{R_0} \frac{v_y}{\omega_d \gamma_d} \sqrt{a_d y}
\]

where \( R_o \), \( x \), \( y \) and \( \gamma \) are empirical constants and the dressing depth \( a_d \) is kept constant. Furthermore, out-of-roundness and depth of burn can be defined as functions of the actual infed rate, so it alone is used to completely define the state of the process. For discretization purposes, the range of the actual infed rate is divided into \( M \) values called grid points, each corresponding to one possible infed rate. The DP then searches through all of the infed rates at each step for the best candidate, except for the first and last steps. The infed rate must be zero at the first step, and it is determined by the roundness specification for the last step. Theoretically the final workpiece out-of-roundness \( r \) should be equal to the infed per revolution when the wheel disengages the workpiece [15]:

\[
r = u / n_w
\]

where \( n_w \) denotes the rotational speed of the workpiece. But experimental results indicate that there is an additional residual out-of-roundness \( r_m \) which is machine tool dependent [22], so the out-of-roundness becomes:

\[
r = r_m + u / n_w
\]

Therefore in order to satisfy the roundness constraint, the actual infed rate at the last step \( u_f \) is defined as

\[
u_f = (r_{max} - r_m) / n_w
\]

From step 2 to step \( N - 1 \), the actual infed rate is constrained by the system dynamics. As such, infed rates not reachable from the previous step without violating the upper limit of the command infed rate are eliminated from consideration.

Dynamic programming starts from the last step of the cycle and continues backward to the first step. The command infed rate associated with each step is selected according to the following procedure:

1. At step \( N - 1 \), the constraints are first evaluated for each reachable infed rate. Since the out-of-roundness constraint needs to be evaluated only at the end of the cycle, the only constraint that is monitored is the depth of burn (see Eq. (2)). Infed rates that violate this constraint are rendered infeasible. For each reachable infed rate \( u_{N-1,j} \), the control (command infed rate) \( u_j \) and time \( t_j \) to reach step \( N \) can be calculated by solving simultaneously the incremental infed and inverse dynamic equations represented, respectively, as:

\[
\delta q = u_j + \int (u_j - u_{N-1,j}) e^{-t_j/r} dt_j - (u_j - u_{N-1,j})
\]

\[
u_j = u_j - u_{N-1,j} e^{-t_j/r} - \frac{t_j e^{-t_j/r}}{1 - e^{-t_j/r}}
\]

If \( u_j \) violates the control bound constraints represented by Eq. (6) and Eq. (7), the infed rate is considered infeasible.

2. At step \( N - 2 \), similar to step \( N - 1 \), the feasible infed rates are first determined and then for each reachable infed rate \( u_{N-2,j} \) the time and control values to reach individual feasible infed rates in step \( N - 1 \) are calculated using the same method in step \( N - 1 \), by replacing \( u_j \) and \( u_{N-1,j} \) with their corresponding counterparts in Eqs. (15) and (16). The total times from step \( N - 2 \) to step \( N \) for all the feasible infed rates at steps \( N - 2 \) and \( N - 1 \) are stored in memory.

3. The procedure in 2 is repeated for each step from \( N - 3 \) to 1.

• The command infed rates that result in the minimum cycle time from step 1 to step \( N \) are determined as optimal.

The DP algorithm was applied to the simulation program for cylindrical plunge grinding [29] in order to explore the effects of the process parameters and constraint values on the optimal cycle. Optimal profiles of the command and actual infed rates for the nominal parameters in Table 1 are shown in Fig. 3. It can be seen that the command infed rate stays at its allowable limit for almost half of the cycle (the ripples are caused by discretization), and it remains at a very low level (near zero) for the last part of the cycle. These two sections are similar to the roughing and spark-out stages of the conventional cycle (see Fig. 2). However, the intermediate section of the cycle is now different. The time saved with this continuous infed cycle was assessed by comparing it with an optimal three-stage cycle obtained for the same case (Table 1) using the strategy proposed by Xiao and Malkin [22]. The optimal time for the resulting three-stage cycle totaled 19.4 seconds (roughing=11.06 s, finishing=4.01 s, and spark-out=4.27 s), about 5 seconds or 35% longer than the optimal time of 13.9 seconds with continuous infed (N=50).

Some insight into how the continuous optimal infed cycle is achieved can be gained by examining the corresponding remaining infed and depth of burn shown in Fig. 4. The results in this

![Fig. 3 Optimal infed rates obtained by dynamic programming for the process conditions in Table 1](image-url)
figure were computed according to Eqs. (21) and (24) in the Appendix. During the initial section of the cycle, the remaining infeed is larger than the depth of burn, but then it becomes equal to the depth of burn during the intermediate section due to a sharp drop in the controlled infeed rate. Following another sharp drop in the command infeed rate at the end of the intermediate section, the grinding power in Fig. 5, computed according to Eq. (22) in the Appendix, becomes smaller than the corresponding burning power. The remaining burned material is then removed, and the infeed rate continues to decrease until the end of the cycle so as to satisfy the final roundness and size tolerance.

The number of steps N in the DP solution constitutes the maximum number of stages to be included in the cycle. While the above solution (Figs. 3–5) with N=50 has enough steps to be considered continuous, the question arises as to how the number of steps affects the cycle time. This point was addressed by obtaining optimal cycles with fewer steps. The infeed rates for two optimal cycles with N=20 and N=10 are shown in Fig. 6. The cycle times are slightly longer for the continuous infeed cycle (N=50), but still significantly shorter than the time of 19.4 s with N=3. Near optimal “continuous” cycles can be achieved with as few as 10 steps.

A significant portion of the time for both discrete and continuous cycles is associated with the transients. The transient time at the end of the cycle can be reduced by accelerated spark-out [14–18] which consists of overshooting the controlled infeed and then retracting the wheel (negative controlled infeed rate) to reach the required size and roundness. To explore the potential for additional time reduction by accelerated spark-out, the optimal cycle was recomputed by DP while also allowing for negative controlled infeed rates. The result in Fig. 7 indicates an optimal cycle time of 11.9 seconds, an additional savings of about 2 seconds.

Another precaution that is often used in practice is to avoid thermal damage during the entire cycle. In order to enforce such a constraint, the infeed rates were computed by DP with the grinding power maintained below the corresponding burning power throughout the cycle. The simulated command and actual infeed rate profiles, shown in Fig. 8 for the no-burn constraint, indicate a cycle similar to the conventional three-stage cycle. The no-burn cycle lacks an intermediate section to balance the thermal damage against the remaining depth of removal in Fig. 9.

Fig. 4 Remaining infeed and depth of burn during the optimal cycle obtained for the process conditions in Table 1

Fig. 6 Optimal infeed rates obtained by dynamic programming with different step numbers

Fig. 5 Grinding power during the optimal cycle along with the burning power for the process conditions in Table 1

Fig. 7 Optimal cycle with the accelerated spark-out provision

Fig. 8 Optimal Infeed rates when thermal damage is avoided

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The optimization study was extended to also observe the effect of system dynamics and wheel dullness. The effect of system dynamics was explored for the nominal conditions in Table 1 but with two additional time constants (τ = 1.2 s and 2.0 s). The optimal command infeed rates with these two time constants are shown in Fig. 10 together with that for the nominal cycle (τ = 1.5 s). As expected, a shorter optimal cycle was obtained with a smaller time constant, but the configuration for all three cycles remained essentially the same.

Wheel dullness in the process model is characterized in terms of the effective wear flat area A_{eff}. Neglecting for now the surface roughness constraint, coarser dressing should result in a sharper wheel with a smaller wear flat area and lower grinding power, thereby enabling a faster removal rate at the thermal constraint. The command infeed control obtained for an optimal grinding cycle with a smaller effective wear flat area (A_{eff}=0.015) is shown in Fig. 11 together with the results for the nominal cycle. The optimal cycle with the sharper wheel (A_{eff}=0.03) now consists of only two sections. There is no need for an intermediate section that was necessary for the duller wheel (A_{eff}=0.03, denoted by the dashed line) to remove thermal damage because the grinding power is below the corresponding burning power throughout the entire cycle (Fig. 12). As a practical matter, the surface roughness requirement limits how coarse the dressing can be and does not usually allow for such an optimal two stage cycle. For practical implementation, the value of the wear flat area can be either estimated from the dressing conditions or computed from the measured power, as discussed in the following paper [1].

The roundness constraint mainly affects the last section of the optimal cycle. A tighter out-of-roundness specification requires a lower final actual infeed rate (Eq. (14)) and a longer spark-out. This effect can be seen in Fig. 13 which shows how changing only the out-of-roundness requirement affects the optimal time for the nominal cycle.

4 Concluding Remarks

Dynamic Programming (DP) was applied to a simulation of the cylindrical grinding process in order to explore the possibilities for optimal grinding cycles with continuously variable infeed control. While the long computation times required by DP preclude its use for on-line machine control, the results from this investigation provide important insight into the features of minimum
time cycles that can be implemented on-line. The following paper [1] is concerned with the design of the controller and implementation of the optimal machine tool control.

Optimal cycles obtained by DP were found to consist of distinct sections each with dominant constraints. If thermal damage is generated within the bounds of the allowable infed rates, three sections are necessary for the cycle, otherwise two sections suffice. In both cases, the first section uses the maximum allowable infed rate similar to the roughing stage in the conventional cycle, and the last section is similar to a spark-out stage to satisfy the roundness constraint. The intermediate section, if necessary, balances the depth of thermal damage against the remaining infed. Although the wheel dullness and system dynamics both affect the optimal cycle, dullness can have a more dramatic influence on the shape of the optimal infed profile.

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Appendix

The following equations obtained from grinding theory are used for calculating and estimating the process parameters (Malkin, [2]):

\[ a = \frac{1}{n_w} \]

\[ n_w = \frac{d_w}{(\pi d_w)} \]

\[ d_z = \left( \frac{d_w d_w}{d_x} \right) \]

\[ v_t = \frac{1}{(u_i - v_i - v_{i-1})} \exp \left( \frac{t_i}{\tau} \right) \]

\[ q_t = \frac{1}{(u_i - v_i - v_{i-1})} \exp \left( \frac{t_i}{\tau} \right) \left( u_i - v_i - v_{i-1} \right) \]

\[ P = 0.0138 \pi d_w v + 9.62 \times 10^{-7} v_z + 0.842 (d_w u)^{0.85} A_{eff} = \frac{1}{(n_w)^{0.85}} \]

\[ P_w = 0.00617 \pi d_w v + 0.0072(\pi d_w d_v v_w)^{0.25} \]

\[ z = -1.449 \left( \frac{v_w t_w}{4 \alpha} \right)^{0.37} \frac{\pi k c \theta_{mb} v_w}{v_w} \]

\[ \times \ln \left( \frac{6.2 \pi d_w v}{4 \alpha} \left( v_w t_w \right)^{0.53} \right) \left( p_w - 0.45 v_c h \right) \]

\[ p_w = P / (\pi d_v w) \]

\[ l_v = (\pi d_w d_v / v_w)^{0.5} \]

\[ v = \frac{1}{n_w + n_m} \]

\[ R_a = R_0 S^{d_w} \]

\[ A_{eff} = -0.008 A_0 n(1.4 \times 10^4 m) \]

\[ a = \text{depth of cut} \]

\[ a_d = \text{dressing depth} \]

\[ A_0 = \text{constant} \]

\[ A_{eff} = \text{wear flat area} \]

\[ d_e = \text{equivalent diameter} \]

\[ d_w = \text{diameter of part} \]

\[ d_x = \text{diameter of wheel} \]

\[ k = \text{thermal conductivity of part} \]

\[ l_z = \text{contact length} \]

\[ m = \text{constant} \]

\[ n_w \] rotational speed of workpiece

\[ P \] grinding power

\[ P_b \] burning power

\[ q_t \] actual infed for the ith stage

\[ r \] actual value of out-of-roundness

\[ r_0 \] constant

\[ R_a \] measured surface finish

\[ R_0 \] constant

\[ s_d \] dressing lead

\[ t_i \] grinding time for the ith stage

\[ p_w \] specific energy for chip formation

\[ u_i \] programmed infed rate

\[ v \] actual infed rate

\[ v_i \] actual infed rate for the ith stage

\[ v_z \] velocity of wheel

\[ v_w \] velocity of workpiece

\[ x \] constant

\[ y \] constant

\[ z \] depth of burn

\[ \alpha \] thermal diffusivity of part

\[ \delta \] equivalent dressing infed angle

\[ \theta_{mb} \] critical temperature for burning

\[ \tau \] time constant

References


