

# Uncertainty and Learning in a Strategic Environment: Global Climate Change\*

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## Abstract

Global climate change is rife with uncertainties. Yet, we can expect to resolve much of this uncertainty in the next 100 years or so. Therefore, current actions should reflect the value of flexibility. Nevertheless, most models of climate change, particularly game-theoretic models, abstract from uncertainty. A model of the impacts of uncertainty and learning in a non-cooperative game shows that the level of correlation of damages across countries is crucial for determining optimal policy.

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# 1 Introduction

How should the uncertainty inherent to global climate change impact optimal policy? Policy makers and advocates seem to fall into two groups regarding the implications of uncertainty. One group argues that the most prudent course is to wait for more knowledge and then act. The other group invokes the precautionary principle, saying the world should act to reduce emissions now, before we are engulfed in catastrophe. One way to find a middle ground between these two groups is to consider the impacts of what – and how much – we expect to learn.

It can reasonably be expected that a great deal – if not all – of the scientific and economic uncertainty will be resolved in the next 100 years or so. As more is learned about climate change, policies can be tailored accordingly. Therefore, current actions should reflect the value of flexibility. The question is, what preserves more flexibility – reducing emissions now or waiting for information?

A selection of papers have considered the effect of learning about the climate on near-term emissions decisions. They have found that the possibility of learning combined with the ability to alter future behavior implies that a hedging strategy is optimal: reduce emissions a small amount now, then wait and see what happens.

Some questions, however, have been left open. First, all but Kolstad's (1996) computational model assume perfect learning. In particular, analytical results show that emissions are higher under perfect learning than under no learning. In this paper, we consider the impact of an incremental increase in learning. This extension is not trivial – it has been well known since Epstein (1980) that considering the difference between no learning and perfect learning is a special case. Second, most of the papers assume a single decision maker, when in fact, climate change is a global

problem involving many different independent players. We analyze the impact of partial learning on equilibrium emissions in a non-cooperative game. Understanding the non-cooperative equilibrium is an important part of crafting a cooperative agreement, since the non-cooperative equilibrium is often the “threat-point” that holds a cooperative agreement together.

Using results from Baker (2004) on the comparative statics of learning, we present more general analytical results on the optimal timing of emissions for both a single decision maker and a non-cooperative game. Furthermore, we analyze the importance of key assumptions made in the literature, such as quadratic functional forms and irreversibility, by exploiting the importance of convexity in the comparative statics of risk and of learning.

This paper builds on the model presented by Ulph and Ulph (1996) and Ulph and Maddison (1997), in which two non-cooperative players attempt to maximize utility by choosing a level of emissions, taking the other’s emissions as given. The damages from climate change increase in the stock of emissions. The exact relation between emissions and damages is assumed to be initially uncertain with information revealed over time. The players know that they will be learning and, therefore, use a framework of sequential decision making under uncertainty. Each player makes decisions in the short run under uncertainty, knowing that the uncertainty will be (partially) resolved in the future and that future decisions will be made under greater certainty (see Figure 1). Additionally, we expand results on a single decision maker from Ulph and Ulph (1997).

The papers above consider how equilibrium emissions change with perfect learning in an open-loop game, and note that the correlation of damages across players impacts the computational results in a closed-loop game. We provide a proposition indicating how the coefficient of correlation of damages across countries impacts equilibrium emissions under partial learning, considering three

cases. If damages are highly positively correlated across countries then first period equilibrium emissions increase in both uncertainty and learning. If damages are independent or negatively correlated across countries, the results are reversed. Since these results depend on strong assumptions about the functional forms of the benefits and damages, we provide an analysis of the importance of the assumptions, and a simple computational model to test the tightness of the restrictions.

The rest of the paper is organized as follows. In the next section this work is put in the context of previous literature. In section 3 we provide a brief discussion of the comparative statics of risk and of learning. We present a theorem which allows us to greatly simplify the analysis of the impacts of partial learning. In section 4 we discuss the case of a single decision maker (SDM). We provide some new results, and emphasize how the comparative statics of risk and of learning is related to the convexity/concavity of marginal damages. In Section 5 we extend the SDM model to the case of multiple decision makers (MDM). We use the same method as in Section 4 – in this case it is the reaction functions of the decision makers that are impacted by the convexity/concavity of marginal damages. Applying results from monotone comparative statics allows us to extend the results to the equilibria of the games. In Section 6 we provide a simple computational model to test the importance of the assumptions. We provide a brief discussion about the linearity of the uncertainty in Section 7, and Section 8 concludes.

## 2 Human-induced Climate Change

### 2.1 Uncertainty and Learning in Climate Change

The most influential climate change models in the policy debate are the large and complex integrated assessment models such as MIT-EPPA (Jacoby and Wing, 1999), RICE (Nordhaus and Yang, 1996), and MERGE (Manne, Mendelsohn and Richels, 1993). They have provided important insights into the costs and benefits of mitigation. While it has been a challenge to include uncertainty in such complex models, two topics related to uncertainty, learning, and climate change have been covered in the literature: the value of information (See Nordhaus and Popp, 1997; Manne and Richels, 1992) and the optimal timing of emissions reductions (See Hammitt *et al.*, 1992 and Scott *et al.*, 1999). Kolstad (1996) looks explicitly at the effect of partial learning about the climate in the face of irreversibilities. He finds that the possibility of learning causes an increase in emissions. Similar results are found in Ulph and Ulph (1997), Manne (1996), and a recent paper by Keller *et al.* (2001). On the other hand, Webster (2002) and Gollier *et al.* (2000) illustrate that these results can be reversed by considering stronger interactions between periods or utility functions with high levels of prudence.

### 2.2 Game Theory and Uncertainty

Since climate change exhibits both uncertainty and multiple non-cooperative players, a game-theoretic model with uncertainty in the state of nature is appropriate. Nevertheless, very few such models exist. In the literature on fisheries, Sandler and Sterbenz (1990) find that harvest uncertainty will reduce exploitation in a tragedy of the commons game if players are risk averse and

if all actions are *ex-ante*. Looking at climate change, Na and Shin (1998) find that since countries are more likely to be facing similar conditions *ex ante* the possibility of coalition formation is enhanced the sooner negotiations take place. Using a numerical model, Hammitt and Adams (1996) find that the expected benefits of a non-cooperative solution are very close to the expected benefits of the cooperative solution in a game with perfect learning and perfect correlation across players.

In the real options literature, there have been some recent attempts to model the effect of strategic interactions on the value of waiting to make irreversible decisions (see Zhu and Weyant, 2003; Kilatilaka and Perotti, 1998; Trigeorgis, 1996). A key assumption in all these papers is that there is a single uncertain variable that effects all players. Reinganum (1989), on the other hand, assumes that the random variables affecting each player are independent in her work combining game theory and uncertainty to analyze R&D races.

In climate change, however, Ulph and Ulph (1996) and Ulph and Maddison (1997) have indicated that the level of correlation between players is an important issue. Using essentially the same model as we use, these papers solve explicitly for the equilibrium levels of emissions for an open loop game, for two cases: no learning and perfect learning. Ulph and Ulph (1996) show that in an open loop game emissions are always higher under perfect learning than under no learning, regardless of the coefficient of correlation. Ulph and Maddison (1997) assume that each of the stochastic shift parameters will take one of two values – 0 or  $\theta$  – and have a coefficient of correlation of  $\rho$ . They then show that the value of perfect information is an increasing function of  $\rho$ . Both papers indicate the key role of the correlation of damages across countries. However, because of the special nature of the assumptions – no learning versus perfect learning and open loop games – there is a need to generalize the results. In Section 5.1 we show how equilibrium emissions change with both an

incremental increase in uncertainty and an incremental increase in learning.

### 3 Comparative Statics of Uncertainty and Learning

Epstein (1980) presents a method for analyzing optimal decisions under partial learning. While innovative, it requires determining if a functional defined on an infinite dimensional space is convex or concave. Because of the difficulty of such work, this method has rarely been applied, and never in a game-theoretic setting. Baker (2004) provides a theorem which collapses the infinite dimensional problem down to one dimension, thus simplifying calculations greatly.

Consider the two following decision problems. The first is more general; the second assumes perfect learning before the second period.

$$\max_{x_1} \mathbb{E}_Y \max_{x_2 \in C(x_1)} \mathbb{E}_{Z|Y} U(x_1, x_2, Z) \tag{1}$$

$x_1, x_2 \in \mathbb{R}$  are the first and second period decision variables,  $U$  is a payoff function,  $C(x_1) \subseteq \mathbb{R}$  represents the choice set for  $x_2$ , which may or may not be constrained by  $x_1$ .  $Y$  and  $Z$  are random variables defined on a given probability space  $(\Omega, \mathcal{A}, P)$ , thus  $Y$  may provide information about  $Z$ .  $\mathbb{E}$  is the expectation operator, where  $\mathbb{E}_Z$  means the expected value over  $Z$ . The related problem with perfect learning is:

$$\max_{x_1} \mathbb{E}_Z \max_{x_2 \in C(x_1)} U(x_1, x_2, Z) \tag{2}$$

Theorem 1 below is built upon a well-known theorem from Blackwell (1951). Define a signal  $Y$  as

being more informative than  $Y'$  if all decision makers are better off with  $Y$ . Define an increase in risk or uncertainty as a mean-preserving spread, in the Rothschild and Stiglitz (1970) sense. Then Blackwell's Theorem says that a signal is more informative if and only if it induces a generalized mean preserving spread of the posterior distributions. Lemma 2 from Baker (2004) applies the logic of this theorem to show that if  $Y$  is more informative than  $Y'$  then  $E[g|Y]$  is riskier than  $E[g|Y']$  for any function  $g(z)$  for which the expectation is defined.

**Theorem 1** *Let  $x_1^*$  solve (1) and  $x_1^{**}$  solve (2). Assume that  $U$  is linear in some function  $g(z)$ . Then  $x_1^*$  is increasing (decreasing) in informativeness if and only if  $x_1^{**}$  is increasing (decreasing) in uncertainty around  $g$ . The effect of increasing informativeness on  $x_1^*$  is ambiguous if and only if the effect of increasing risk on  $x_1^{**}$  is ambiguous.*

**Proof.** Define

$$x_1^*(Y) \equiv \arg \max_{x_1} E_Y \max_{x_2 \in C(x_1)} U(x_1, x_2, E[g|Y]) \quad (3)$$

and

$$x_1^{**}(Z) \equiv \arg \max_{x_1} E_Z \max_{x_2 \in C(x_1)} U(x_1, x_2, g) \quad (4)$$

where  $x_1^*$  and  $x_1^{**}$  are functions of the distributions of  $Y$  and  $Z$  respectively, as opposed to a particular realization of the variables. Let  $Z$  be riskier than  $Z'$  and assume  $x_1^{**}$  is increasing in uncertainty around  $g$ . This means that  $g$  riskier than  $g' \Rightarrow x_1^{**}(Z) \geq x_1^{**}(Z')$ . But since  $E[g|Y]$  plays the same role in (3) as  $g$  plays in (4), the above is equivalent to saying that  $E[g|Y]$  riskier than  $E[g|Y'] \Rightarrow x_1^*(Y) \geq x_1^*(Y')$ . Lemma 2 from Baker (2004) tells us that if  $Y$  is more informative than  $Y'$  then  $E[g|Y]$  is riskier than  $E[g|Y']$ . Therefore if  $Y$  is more informative than  $Y'$  then

$x_1^*(Y) \geq x_1^*(Y')$  :  $x_1^*$  is increasing in informativeness. The proof for the decreasing case uses the same logic with the opposite inequalities. See Baker (2004) for the proof of the converse and for the last statement in the theorem. ■

The broad intuition of this result is as follows: if in problem (1) we expect to have more information before we choose  $x_2$  then we will want to choose  $x_1$  in such a way to leave ourselves more flexibility to react to what is learned. Similarly, the more prior risk we face in problem (2), the more flexibility we would like when choosing  $x_2$ . Hence, we might expect an increase in informativeness and an increase in uncertainty to have similar effects on  $x_1$ .

The logic of Theorem 1 carries over to non-cooperative games (see Baker, 2004 for details). Thus, this theorem allows us to analyze the impacts of increasing risk on first period emissions, using methods from Rothschild and Stiglitz (1971), and then directly apply these results to the impacts of increasing informativeness.

## 4 Single Decision Maker

We consider a two-period decision problem. The uncertainty is assumed to be resolved before the 2nd period, allowing the SDM to adjust behavior. Thus, it is a model of sequential decision making under uncertainty. The two periods can be thought of as *now* when there is uncertainty about the nature of damages caused by global climate change, and *later*, when that uncertainty will be resolved. Emissions  $x_i$  (for periods  $i = 1, 2$ ) are released into the atmosphere and dissipate slowly, reflected by the constant  $\gamma$ . The stock of emissions in the second period is  $s = \gamma x_1 + x_2$ . Damages are assumed to be zero in the first period. The time discount factor from one period to the next is

$\delta$ .

The SDM balances the benefits of emissions<sup>1</sup>  $b(x)$  against the uncertain damages caused by climate change  $\varepsilon D(s)$ .  $b(\cdot)$  represents the net benefits of the energy use that creates an emission level of  $x$ , and is assumed to be strictly concave – implying that the marginal cost of reducing emissions is increasing – and to have a unique maximum point, commonly referred to as the business as usual level.<sup>2</sup>

$D(\cdot)$  is increasing, strictly convex, and deterministic – implying that the *shape* of the damage function is known. The uncertainty is represented by  $\varepsilon$ , a stochastic shift parameter that multiplies the deterministic portion of the damage function. We assume throughout that  $\varepsilon \geq 0$ .

The problem is solved using backward induction. Optimal 2nd period emissions  $x_2(x_1, \varepsilon)$  are characterized by the first order condition for  $x_2$ :

$$b'(x_2) = \varepsilon D'(s) \tag{5}$$

If emissions are irreversible then there is a second constraint  $x_2 \geq C$ . The second period stock  $s(x_1, \varepsilon)$  is a function of first-period emissions and the stochastic shift parameter, since second period emissions are completely determined by these variables. In the first period the SDM maximizes the benefit from first-period emissions *plus* the discounted expected benefit of the second period emissions *minus* the discounted expected damages from the stock. The decision problem is as

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<sup>1</sup>To simplify the presentation, we use the same benefit function for both time periods. All results hold if benefit functions are different for the two periods.

<sup>2</sup>Benefits from emissions can be related to  $c(\mu)$ , the cost of abating a percentage of emissions,  $\mu$ , in the following manner:  $c(\mu) = b(x^*) - b((1 - \mu)x^*)$ , where  $x^*$  is the business-as-usual emissions level. The assumptions on  $b$  imply that abatement costs are increasing and convex in  $\mu$ .

follows:

$$\max_{x_1} b(x_1) + \delta \mathbf{E} \{b(x_2(x_1, \varepsilon)) - \varepsilon D(s(x_1, \varepsilon))\} \quad (6)$$

The first order condition derived from (6) is

$$b'(x_1) = \delta \mathbf{E} \{\varepsilon D'(s(x_1, \varepsilon))\} \quad (7)$$

Marginal benefits equal expected marginal damages. Any change in the distribution of  $\varepsilon$  that increases expected marginal damages will lead to a decrease in optimal first period emissions. We are particularly interested in the impact of an increase in risk. The expected value of a function increases (decreases) in risk if and only if the function is everywhere convex (concave) (Rothschild and Stiglitz, 1961). Thus, we explore when the marginal damages – the quantity inside the brackets on the right hand side of (7) – are convex or concave.

#### 4.1 Impact of Risk and of Learning on Emissions: SDM

In this section we present two propositions about how increasing risk and increasing learning impact optimal emissions for a SDM. These propositions are interesting in their own right, but they also serve to set the stage for the MDM case. We start by considering the simpler case, where there is no constraint on second period emissions. We then go on to present a result in the more complex case, where second period emissions are constrained to be non-negative.

We are interested in focusing on the impact of learning, and in particular, disentangling the impact of learning and of risk aversion. Thus, this model presents a risk neutral decision maker (implied by the linear payoff function). In the absence of learning, therefore, an increase in risk has

no impact on welfare and no impact on first period decisions. When learning takes place, however, the story changes. After the SDM learns the true value of  $\varepsilon$ , he chooses the optimal level of second period emissions,  $x_2$  in response. The higher the value of  $\varepsilon$ , the lower emissions. Thus, in the presence of learning, high damages can be ameliorated somewhat. An increase in risk means that there is a higher probability of both good and bad outcomes, but bad outcomes are ameliorated, so overall expected damages decrease with an increase in risk. Thus, both welfare and first period emissions increase in risk.

Theorem 1 above implies that these results extend to an increase in learning. Intuitively, it is the presence of learning that allows the decision maker to react to bad outcomes by reducing emissions. The more the SDM expects to learn, the more he expects he will be able to tailor emissions to the actual outcome, and in particular, to ameliorate the bad outcomes. Thus, expected damages decrease with an increase in learning – again increasing both welfare and first period emissions in a way commensurate with an increase in risk.

Proposition 1 below formalizes these ideas.<sup>3</sup> We assume that  $b$  and  $D$  are differentiable. All proofs are presented in the appendix.

**Proposition 1** *Assume there is a single decision maker,  $b'''$ ,  $D''' \leq 0$ , and emissions are reversible. Then optimal first period emissions increase in uncertainty and informativeness.*

We illustrate the concept using a graphical analysis. Recall that optimal emissions increase as expected marginal damages decrease; and expected marginal damages decrease in risk if marginal damages  $\varepsilon D'(s(\varepsilon))$  are concave in  $\varepsilon$ . The ray coming out of the origin in Figure 2 illustrates

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<sup>3</sup>Ulph and Ulph (1997) show that emissions under perfect learning are higher than under no learning, assuming that benefits and damages are quadratic.

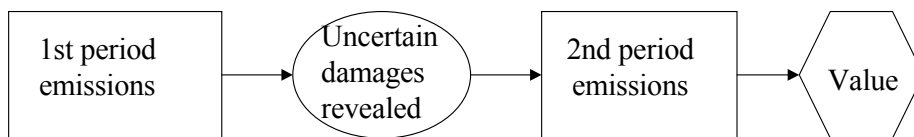


Figure 1: Sequential decision making under uncertainty

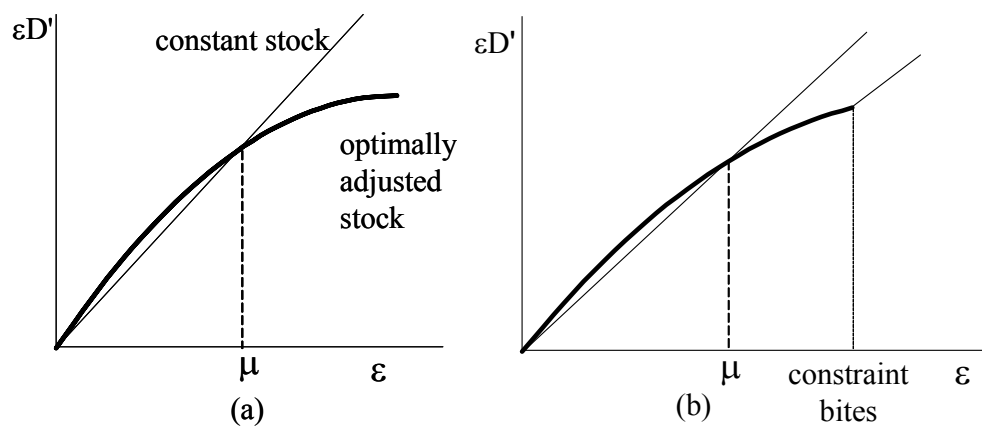


Figure 2: Marginal damages as a function of the random shift parameter  $\varepsilon$ , with and without learning. (a) shows no constraint on emissions. (b) shows constrained emissions.

marginal damages as a function of  $\varepsilon$  when there is no learning. In that case, optimal second period emissions depend only on the expected value of the shift parameter (denoted by  $\mu \equiv \mathbb{E}[\varepsilon]$  in the figure). Therefore, the stock of emissions is constant in the shift parameter and expected marginal damages are proportional to  $\varepsilon$ .<sup>4</sup> When learning takes place, optimal emissions are higher when  $\varepsilon < \mu$  and lower when  $\varepsilon > \mu$ . Thus marginal damages  $\varepsilon D'(s)$  are above the ray when  $\varepsilon < \mu$  and below the ray when  $\varepsilon > \mu$ , crossing the line where  $\varepsilon = \mu$ . The heavy line in panel (a) illustrates marginal damages when  $b$  and  $D$  are quadratic and there is no non-negativity constraint on emissions. The concavity of the marginal damages imply that expected marginal damages decrease in risk, and thus optimal first period emissions increase.

Under what assumptions are marginal damages not everywhere concave? We can start by showing that marginal damages are always concave at  $\varepsilon = 0$ , regardless of the higher derivatives of  $b$  and  $D$ , or reversibility. Hence the next proposition.

**Proposition 2** *Assume there is a single decision maker, and  $x_2$  is constrained to be greater than  $C \geq -\infty$ . Then optimal first period emissions  $x_1^*$  increase with some increases in risk and with some increases in informativeness. Formally, there exist signals  $Y$  and  $Y'$  such that  $Y$  is more informative than  $Y'$ , and  $x_1(Y) > x_1(Y')$ , where  $x_1(Y)$  indicates optimal first period emissions given the distribution of signal  $Y$ .*

**Proof.** The second derivative of  $\varepsilon D'$  when evaluated at  $\varepsilon = 0$  is  $\frac{2D'D''}{b''} < 0$ . Thus, marginal damages are either everywhere concave, or are neither convex nor concave. It follows from Rothschild & Stiglitz (1970, p.240), that expected marginal damages will decrease with some increases

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<sup>4</sup>This is true independent of assumptions about the functional forms of  $b$  or  $D$ . It depends on the assumption that the payoff function is linear in  $\varepsilon$ .

in risk. The extension to informativeness follows from Theorem 1. First period emissions increase if expected marginal damages decrease. ■

Proposition 2 differs from the result in Ulph and Ulph (1997)[UU]. That paper shows that if the irreversibility constraint bites for  $E[\varepsilon] = \mu$ , then optimal emissions will be lower under perfect learning than under no learning. But, results for perfect learning do not always carry through for partial learning. In Appendix C we present an example in which the conditions in UU are met, and yet emissions increase in learning, as predicted by Proposition 2.

Panel (b) of Figure 2 illustrates marginal damages when second period emissions are constrained. Once the constraint bites, the stock is fixed and marginal damages increase linearly with slope  $D'(\bar{s})$ , where  $\bar{s}$  represents the minimum possible stock given the constraint. The slope of the line is always greater than the slope of the marginal damages without constraint<sup>5</sup>, implying that marginal damages are neither convex nor concave, and thus optimal emissions will increase with some increases in risk and will decrease with others.

#### 4.1.1 The Role of the Assumptions

In this section we consider how assumptions on the higher derivatives of  $b$  and  $D$  impact Proposition 1. To visualize the impact of the assumption on  $b'''$ , see Figure 3. The Figure illustrates that if marginal benefits are very convex, then optimal emissions (and therefore the stock of emissions) are also very convex in  $\varepsilon$ . In fact, very convex marginal benefits lead to optimal emissions that are nearly constant in  $\varepsilon$  for higher values of  $\varepsilon$ : the impact of learning is reduced. Since the stock is

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<sup>5</sup>The slope of  $\varepsilon D'$  is  $D'(s) + \varepsilon D'' \frac{\partial s}{\partial \varepsilon}$  evaluated at  $\varepsilon = \bar{\varepsilon}$ , the point where the constraint bites. Since  $\frac{\partial s}{\partial \varepsilon} < 0$ , this slope is less than  $D'(\bar{s})$ .

decreasing very little at high values of  $\varepsilon$ , marginal damages may bend back up again, and thus not be everywhere concave. Irreversibility can be considered a special case of convex marginal costs of abatement: marginal costs jump to infinity at the point of constraint. Next we consider the restriction on the damage function. If  $D''' > 0$ , then  $D'$  is convex in  $s$ , thus  $\varepsilon D'$  will tend to be convex for higher values of  $\varepsilon$ .

We conclude that optimal emissions *may* decrease with some increases in learning if (1) emissions are constrained to be non-negative or (2) either marginal abatement costs or marginal damages are convex.

## 5 Multiple Decision Makers

How does the problem change when considering multiple decision makers? If damages are not perfectly correlated then there may be winners and losers after learning takes place. The losers' bad luck will be compounded by the winners' good luck – the winners will choose a relatively higher level of emissions, thus increasing the stock. It turns out that this possibility of being a loser induces precaution in the first period. To illustrate this we modify the model from Section 4 to include 2 players in a closed-loop dynamic game. We then discuss how the same type of analysis we use above – an analysis of expected marginal damages – can be used to predict how the equilibrium of the non-cooperative game will change with uncertainty and learning. We present our central proposition, and discuss why each of the results holds true and how our assumptions impact the result.

Consider the model in Section 4, but with two players, X and Y, representing individual nation

states, with emissions  $x_i$  and  $y_i$ , respectively. The stock of emissions in the second period is  $s = \gamma(x_1 + y_1) + x_2 + y_2$ .

The players use a hedging strategy in a non-cooperative framework. The solution concept is feedback Nash equilibrium. Emissions in the second period depend on the realized value of the random variables and on first period emissions. In the first period, emissions are chosen recognizing that the second period emissions of both players depend on first period emissions. Thus, the model is “closed-loop”. The game is played with complete information – the distributions of the random variables  $\varepsilon_x$  and  $\varepsilon_y$  are common knowledge.

2nd period equilibrium emissions  $x_2(x_1 + y_1, \varepsilon_x, \varepsilon_y)$  and  $y_2(x_1 + y_1, \varepsilon_x, \varepsilon_y)$  are characterized by first order conditions similar to (5). If emissions are irreversible then there is a second set of constraints  $x_2, y_2 \geq C$ . The second period equilibrium stock  $s(x_1 + y_1, \varepsilon_x, \varepsilon_y)$  is a function of first-period emissions and the two stochastic shift parameters. Each player solves an optimization problem similar to (6), taking the other player’s first period emissions as given. The first order condition for Player X is

$$b'(x_1) = \delta \mathbf{E} \left\{ \varepsilon_x D'(s(x_1 + y_1, \varepsilon_x, \varepsilon_y)) \left[ \gamma + \frac{\partial y_2}{\partial x_1} \right] \right\} \quad (8)$$

Again, we have the familiar condition that marginal benefits equal expected marginal damages. Note that X’s first period emissions are based on X’s damage alone, rather than on the total damage  $\varepsilon_x + \varepsilon_y$ . This is the tragedy of the commons. The marginal effect of X’s first period emissions on X’s second period emissions drops out of the equation by the envelope theorem<sup>6</sup>. This

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<sup>6</sup>  $(b'(x_2) - \varepsilon_x D'(s)) \frac{\partial x_2}{\partial x_1} = 0$  by the first order conditons.

is not true, however, of the effect of X’s first period emissions on Y’s second period emissions. This is the effect of strategic behavior seen in a “closed loop” model. Each player realizes that an increase in first period emissions will cause the other player to reduce second period emissions (i.e.  $\frac{\partial y_2}{\partial x_1} \leq 0$ ; see appendix for proof). Therefore internalized expected marginal damages (the right-hand side of (8)) are *additionally* lower than in a typical one-period or open-loop Tragedy of the Commons. When players are sophisticated there is not only the tactical tragedy of the commons, there is also a strategic increase in emissions reflecting the knowledge that higher emissions now will force their opponents to emit less in the future.

## 5.1 The Impact of Risk and Learning on Emissions: MDM

### 5.1.1 Method

In this section we apply the methods from Section 4 – namely analyzing expected marginal damages – to predict the impact of risk and of learning on equilibrium first period emissions. This extension to equilibria is based on the seminal work of Milgrom and Roberts (1994) on comparing equilibria using monotone comparative statics. The idea is as follows. The equilibria of a game is a set of fixed points, say  $\{X : G(X, \theta) = X\}$ . In our case  $X = (x_1, y_1)$ ,  $\theta$  represents the level of risk or informativeness, and  $G$  is the implicit reaction function defined by the first order conditions for  $x_1$  and  $y_1$ . When we analyze the impacts of risk on expected marginal damages, we are saying something about how the function  $G$  changes with changes in risk,  $\theta$ . What Milgrom and Roberts show is that if the function  $G$  is monotone in all its arguments, then the set of equilibria are monotone as well, in the sense that the highest and lowest equilibria are both monotone in the

same direction. In the game presented in Section 5, the relationships are indeed monotone – each player’s optimal emissions are a decreasing function of the other player’s emissions. Thus, when we show a monotone relationship between risk and emissions in the reaction functions, we can conclude that this relationship holds for the set of equilibria.

### 5.1.2 Results

We find that the results for MDMs are different than for SDM, depending on how damages are correlated across players. In general, learning and information can play very different roles in a game than in an SDM. For example, for an SDM, information always has a non-negative value, but for MDM, information (if it is common knowledge) can have a negative value<sup>7</sup>. We have seen above that for an SDM, learning provides the possibility of reducing emissions and thus ameliorating bad outcomes. In a game, however, learning provides an additional possibility – of the opponent increasing emissions, and thus exacerbating bad outcomes.

We formally present the results in Proposition 3. The proof consists of showing whether the quantity inside the brackets on the right-hand side of (8) is convex or concave, then applying monotone comparative statics to extend the results to the equilibria of the game (see appendix for details of proofs). The three cases we consider are (i) perfect negative correlation:  $(\varepsilon_x, \varepsilon_y) = (\varepsilon, 2\mu - \varepsilon)$  where  $\varepsilon$  is a symmetric, non-negative random variable with mean  $\mu$ , (ii) independence, and (iii) perfect correlation:  $\varepsilon_x = \varepsilon_y$ .

**Proposition 3** *Assume there are two non-cooperative players,  $b(\cdot)$  is quadratic, and emissions are*

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<sup>7</sup>See Hirshleifer (1971) for the seminal example. Zhu and Weyant (2003) present an example in a strategic framework.

reversible. (i) If damages are perfectly negatively correlated then the highest, lowest, and symmetric first period equilibrium emissions  $(x_1^*, y_1^*)$  **decrease** in uncertainty and informativeness. Additionally assume that  $D(\cdot)$  is quadratic. Then (ii) if damages are independent, then highest and lowest first period equilibrium emissions  $x_1^*$  (a) **increase** in own-uncertainty (or in increased learning about  $\varepsilon_x$ ) when  $\varepsilon_y < -\frac{2}{3} \frac{b''}{D''}$  almost surely, and (b) **decrease** in opponent's uncertainty (or in increased learning about  $\varepsilon_y$ ) unconditionally, and (iii) if damages are perfectly correlated then the highest, lowest, and symmetric first period equilibrium emissions  $(x_1^*, y_1^*)$  **increase** in uncertainty and informativeness.

Parts (iia) and (iii) from Proposition 3 reinforce the results in the SDM literature: if the irreversibility of emissions doesn't bite, then the possibility of learning causes emissions to increase. This result is not substantially changed by including a strategic framework.

Parts (i) and (iib) reverse the findings in the SDM literature and expand the results from Ulph and Ulph (1996). In a strategic situation, the possibility of an opponent's learning can be damaging enough to outweigh the benefits of own-learning. In a game, the level of correlation across players fundamentally affects behavior in the face of uncertainty. The implication for climate change policy is that if the coefficient of correlation across nations is low enough, the possibility of learning may indicate a decrease in emissions, rather than the increase found in the SDM literature. This result stands even without strong irreversibility, without assuming risk averse players, and for partial learning. Below we discuss what drives these results, in particular considering the impact of the quadratic assumptions and reversibility of emissions.

**Perfect Correlation** Under perfect correlation, the stock of emissions is impacted by risk in the same manner as under a SDM: as  $\varepsilon$  increases, both players decrease emissions, hence, given the assumptions of Proposition 3, marginal damages are concave in the random variable, as in Figure 2. Similar to the SDM case, these results may be weakened if emissions are constrained to be non-negative or marginal abatement costs or marginal damages are convex.

**Independence** Under independence, emissions increase in own-risk (and own-learning) following the logic for the SDM: Player X decreases emissions as  $\varepsilon_x$  increases. Player Y increases emissions in response, but not enough to dampen the effect, i.e.  $\frac{\partial y_2}{\partial x_2} > -1$ . Thus, again, the stock of emissions decreases in the shift parameter and Figure 2 holds.

On the other hand,  $\varepsilon_y$  only impacts Player X through a change in the overall stock of emissions. If  $\varepsilon_y$  is high, then emissions are low; and vice versa. But, since it gets more and more expensive to reduce emissions, each unit increase in  $\varepsilon_y$  leads to a smaller reduction in emissions. This implies that emissions are convex in  $\varepsilon_y$ , and thus, a mean preserving spread in  $\varepsilon_y$  leads to a mean-increasing spread in emissions. An increase in Y's risk leads to a higher expected stock and therefore higher expected damages for X.

Consider the impact of the assumptions. First, an irreversibility constraint on emissions would cause the stock to be more convex in  $\varepsilon_y$ , and thus reinforce the result. Second, if  $b''' < 0$  – marginal benefits are concave – then emissions may be concave in the shift parameter (See Figure 3), implying that marginal damages may be concave. In that case, expected marginal damages decrease in risk, and emissions increase. Third, if  $D'$  is concave in  $s$  ( $D''' < 0$ ) then marginal damages will generally be neither convex nor concave in  $\varepsilon_y$ . In summary, optimal emissions will decrease in other's risk

(or learning about other's damages) unless either marginal costs or marginal damages are concave.

**Perfect Negative Correlation** When damages are perfectly negatively correlated and benefits are quadratic (and symmetric), then the stock of emissions is constant in  $\varepsilon$ : every decrease by Player X in response to worse damages is countered by an increase by player Y in response to less severe damages. A constant stock implies marginal damages are linear in  $\varepsilon$  (see the straight line in figure 2). Thus, in an open loop game, equilibrium emissions are independent of risk. In the closed loop game, however, marginal damages in equation (7) include a strategic term  $\frac{\partial y_2}{\partial x_1}$ , to reflect that an increase in  $x_1$  causes Player Y to optimally reduce second period emissions. As  $\varepsilon_x$  increases, however, the impact of this effect diminishes. For example when  $\varepsilon_x = 2\mu$ , then  $\varepsilon_y = 0$ , and  $x_1$  has no impact on Y's choice of emissions. Thus X's damages are being compounded (i.e.  $\frac{\partial y_2}{\partial x_1}$  is getting less negative) and X reduces first period emissions in response to greater risk and/or informativeness.

If irreversibility holds, or if marginal benefits are convex, then the stock is no longer constant in  $\varepsilon$ , but rather has a U shape. If marginal benefits are concave, then the stock of emissions has an inverted U shape. In each of these cases equilibrium emissions may be ambiguous in risk.

## 6 Computational Sensitivity Analysis

The discussion in Section 5 provides some indication of how the third derivatives of the benefit and damage functions impact the results, but it is difficult to get unambiguous results, particularly for a closed-loop game. We test the implications from Section 5, and gauge the tightness of the quadratic restrictions through a simple computational model of a non-cooperative game, using non-quadratic

functional forms.

## 6.1 Computational Model Description

The following functional forms are used in the computational model.

$$b(x) = x - \frac{x^{e_1}}{e_1} \quad (9)$$

$$D(s) = \frac{s^{e_2}}{100} \quad (10)$$

The business-as-usual emissions are  $x = 1$  for any level of the exponent,  $e_1$ . Marginal benefits get more concave as  $e_1$  increases; marginal damages get more convex as  $e_2$  increases.

Each stochastic shift parameter can take on two possible values,  $\theta_l^H$  and  $\theta_l^L$ ,  $l = x, y$ . Let  $p(\theta_x, \theta_y)$  represent the probability that  $(\varepsilon_x, \varepsilon_y) = (\theta_x, \theta_y)$ . Then  $p(\theta_x^H, \theta_y^H) = p(\theta_x^L, \theta_y^L) = \frac{1+\rho}{4}$  and  $p(\theta_x^H, \theta_y^L) = p(\theta_x^L, \theta_y^H) = \frac{1-\rho}{4}$  where  $\rho$  is the coefficient of correlation. We compare a certain case  $\theta_x = \theta_y = 10$ , with risky cases where  $\theta^H > 10$  and  $\theta^L < 10$ ,  $\frac{\theta^H + \theta^L}{2} = 10$ . Emissions are constrained to be non-negative.

The first order conditions for second period emissions are modeled as an explicit constraint for each pair of  $(\theta_x, \theta_y)$ :

$$1 - x_2^{(e_1-1)} - \theta_x \frac{e_2 s^{(e_2-1)}}{100} = 0 \quad (11)$$

where  $s = x_1 + y_1 + x_2 + y_2$ . GAMS/MINOS is then run iteratively on the two first period payoff functions, taking the other player's optimal emissions from the previous iteration as given. We consider only symmetric equilibria.

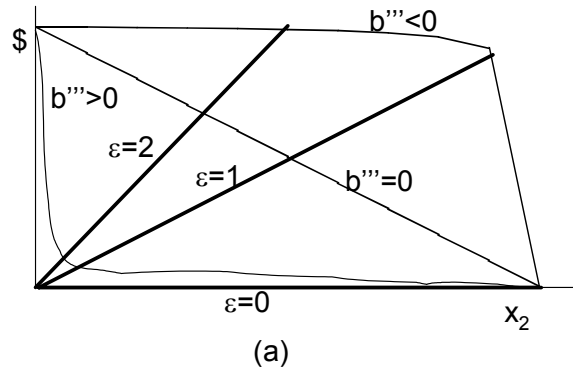
## 6.2 Results

Table 1 indicates the sensitivity of the results from Section 5.1 to the assumption of quadratic benefits and damages. The first column lists the assumption about correlation. The second column indicates whether equilibrium emissions increase or decrease with an increase in informativeness, given the quadratic assumptions. The third and fourth columns show the value of the coefficient that caused our results to be reversed. Figure 4 illustrates the impact of the exponent on the curvature of  $b'$  and  $D'$ .

The result that equilibrium emissions increase in risk or informativeness (the first two cases) appears very sensitive to the curvature of the damage function. If damages are slightly more convex than quadratic, we see a decrease in equilibrium emissions in this simple model. This corresponds to the case when  $D''' > 0$ . This suggests that results in the single decision maker literature may be sensitive to the curvature of the damage function. The analysis in Section 5.1.2 implies that results may be impacted by  $b''' > 0$ , but this restriction does not show up in the computational model, and thus does not appear very tight. On the other hand, the result that equilibrium emissions decrease in uncertainty (the last two cases) appears to be sensitive to the curvature of the benefit function, especially in the case of independence. If the marginal benefits are just slightly concave (i.e.  $b''' < 0$ ), then equilibrium emissions increase in the risk and informativeness of the other's damages. For perfect negative correlation, if marginal benefits are convex, then equilibrium emissions increase in risk. Restrictions on the damage function do not appear to be tight for these two cases.

In summary, the results are most sensitive to damages that are very convex and to marginal

Marginal benefits and marginal damages



Optimal emissions as a function of  $\varepsilon$

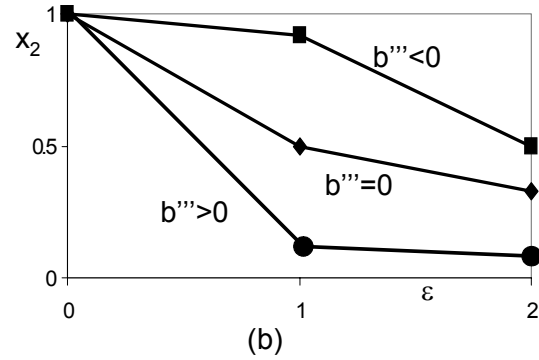


Figure 3: (a) The heavy, upward sloping lines represent marginal damages for three values of  $\varepsilon$ : 0,1,2. The lighter, downward sloping lines represent marginal benefits for three values of  $b'''$ . (b) Optimal emissions as a function of  $\varepsilon$  for each of the three marginal benefit curves shown on the left.

Correlation	Impact	$e_1$	$e_2$
Perfect	↑	-	2.5
Independent - own	↑	-	3
Independent - others	↓	2.5	-
Perfect negative	↓	1.3	-

Table 1: Sensitivity of results to changes in the curvature of the marginal benefit and marginal damage functions.

abatement costs that are either concave, or very convex.

## 7 The Role of Linearity

This paper investigates the role of uncertainty and learning in a special case – linear uncertainty. The benefit of this assumption is that we are able to get unambiguous results both for SDM and MDM, and that the results extend directly to learning. But, given that this is a restrictive assumption, we ask how useful are the results of this paper? First, theoretical results based on linear SDM models appear to be consistent with numerical results from non-linear models (See Ulph and Ulph, 1997, Manne, 1996). For example, in the model with threshold uncertainty presented in Keller et al. (Forthcoming), early emissions increase in risk, consistent with the broad results of Section 4.1.<sup>8</sup> Second, this paper indicates the importance of correlation across players in a game, and makes predictions that can be tested on non-linear game-theoretic models.

Third, the results in this paper for increasing learning are more general than the linear case, encompassing cases where damages are non-linear in the random variable. Table 2 presents four examples of non-linear damage functions that satisfy the constraints of Theorem 1. The qualitative results for increasing informativeness in Propositions 1 - 3 are identical for each of the damage functions shown in Table 2. This is not true for increasing risk. The curvature around the random variable has no impact on the comparative statics of information, while it is very important for the comparative statics of risk. The reason is that if a signal is more informative for  $\varepsilon$ , then it is

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<sup>8</sup>Additionally, we have done some preliminary work testing the impact of skew-increasing and skew-decreasing increases in risk on a stochastic version of the DICE model. The numerical results match the qualitative predictions of the linear model even though the the random variable is quite non-linear in the numerical model.

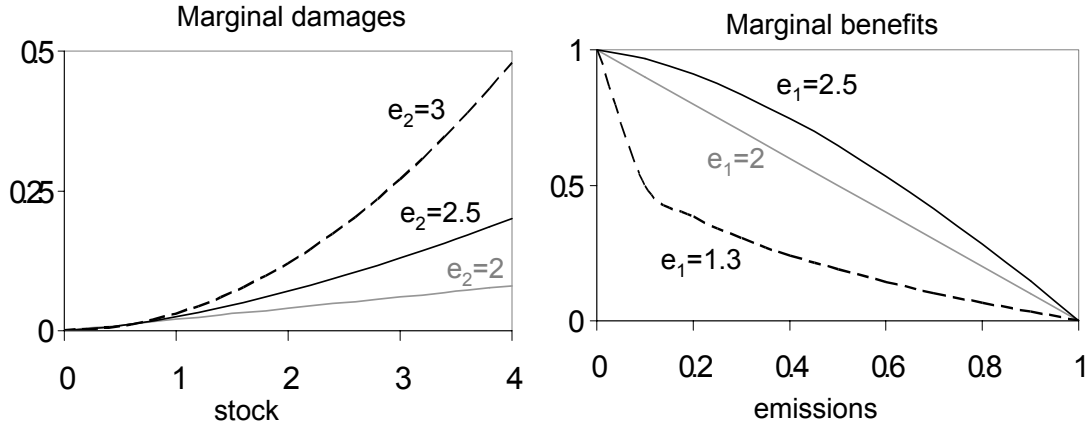


Figure 4: The left panel shows marginal damages for three different exponents:  $e_2 = 2$  is the standard assumption in the paper. The results for perfect correlation and independence are reversed when  $e_2 = 2.5$  and  $3$ , respectively. The right panel shows marginal benefits for three different exponents:  $e_1 = 2$  is the standard assumption. The results for independence -other's risk and for perfect negative correlation are reversed when  $e_1 = 2.5$  and  $1.3$  respectively.

$$\begin{array}{cccc}
 1 & 2 & 3 & 4 \\
 (\varepsilon s)^n & \left(\frac{s}{1-\varepsilon}\right)^n & \left(\frac{\varepsilon s}{1+\varepsilon}\right)^n & \left(\frac{\varepsilon}{1+\varepsilon}\right) s^n
 \end{array}$$

Table 2: Non-linear damage functions. For Proposition 1  $n$  must be greater or equal to 2, for Propositions 2 and 3i,  $n$  must be greater than or equal to 1; for all other propositions  $n=2$ .

equally more informative for any function of  $\varepsilon$ . The analogous statement is not true for increasing risk – a mean-preserving increase in risk will change the mean of any non-linear function of  $\varepsilon$ . For example, even holding the stock constant, an increase in risk will increase expected damages for damage function 1 and decrease them for damage function 4.

It is interesting to compare these results to those in Gollier et al. (2000). They model second period utility as  $u(c_2 - \varepsilon(\gamma c_1 + c_2))$  where  $u$  represents a utility function and  $c_i$ , consumption in period  $i$ , is assumed to be proportional to damages. They show that optimal first period emissions decrease in informativeness if utility  $u$  exhibits a high degree of prudence<sup>9</sup>. The difficulty in interpreting this result, however, is the unavoidable problem (in an expected utility framework) of conflating risk aversion with the elasticity of substitution across time.<sup>10</sup> The results of this paper – indicating that the curvature of the payoff function around the random variable has no impact in the separable case – suggest that the results from Gollier et al. may reflect the impact of a very low elasticity of substitution across time, rather than prudence in the risk-avoiding sense.<sup>11</sup>

## 8 Conclusion

This paper analyzes the impact of risk and of learning on the optimal or equilibrium emissions abatement path. Using a theorem on the comparative statics of risk and of learning, we are able to generalize (and in one case reverse) previous results in the literature. Considering a single decision maker, we show that there is no condition under which emissions unambiguously decrease in risk or

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<sup>9</sup>Prudence is defined as  $-\frac{u'''}{u''}$ . The condition is that  $-\frac{u'''}{u''} \geq -\frac{u''}{u'}$

<sup>10</sup>Increasing concavity in the utility function simultaneously represents increasing risk aversion and decreasing elasticity of substitution across time.

<sup>11</sup>See Baker (2004) for another example and discussion. See Epstein and Zin (1989) for a framework that separates preferences for smoothed consumption from risk aversion.

informativeness. This contrasts with the result for perfect learning found in Ulph and Ulph (1997). Unambiguous results – that emissions increase in risk and informativeness – can only be obtained when a constraint on emissions will never bite.

We confirm the importance of the correlation of damages across countries in determining the equilibrium emissions level in a non-cooperative game. In particular, we show that equilibrium emissions decrease in risk and in learning when damages are perfectly negatively correlated. This result is independent of assumptions about damages and appears to be fairly robust to assumptions about abatement costs.

These results have implications for integrated assessment modelers, particularly as interest grows in incorporating uncertainty and learning into climate policy models. First, it is crucial that a "high damage" case be high enough to cause any constraint on emissions to bite. Otherwise, it is certain that optimal emissions will increase in risk for a single decision maker. Second, the analysis in Section 5 combined with the computational results in Section 6 indicate where sensitivity analysis on functional forms will be most effective. Results that show emissions increasing in risk appear to be sensitive to more convex damages, while results that show emissions decreasing with risk are sensitive to the curvature of the marginal abatement costs.

The results in this paper imply that the correlation of damages across countries is important. Yet, there seems to be little understanding of this parameter. Most analyses, in fact, give the impression that damages are perfectly correlated across countries by focusing on global variables such as global mean temperature. Yet regional analyses and common sense imply otherwise. For example, it is not clear whether the average temperature in Europe will increase or decrease with a moderate increase of global mean temperature. Given the lack of discussion of correlation, it is

quite likely that policy makers are unconsciously assuming that damages are perfectly correlated across countries. To the degree that this assumption is wrong, current emissions policy may be skewed.

In order to test these results more robustly will require empirically-based computational models, combining integrated assessment models, uncertainty and learning, and multiple non-cooperative players. Such a combined model could explore the impacts of multiple, asymmetric players over long time periods. If we consider two asymmetric players that differ mainly in size, then the results will tend toward the single decision maker results as the size differential increases. It may be of more interest to consider a developing country whose current economy is small, but is expected to grow considerably in the future. Finally, considering the impact of the non-cooperative game on cooperative agreements under uncertainty and learning (see Kolstad 2003) may provide new insights into international environmental agreements under uncertainty.

## A The Strategic Effect

We show that  $\frac{\partial y_2}{\partial x_1} \leq 0$

**Proof.** Totally differentiate the first order conditions for  $x_2$  and  $y_2$  to get

$$\begin{aligned} (b'' - \varepsilon_x D'') dx_2 - \varepsilon_x D'' dy_2 - D' d\varepsilon_x - \varepsilon_x D'' \gamma dx_1 - \varepsilon_x D'' \gamma dy_1 &= 0 \\ -\varepsilon_y D'' dx_2 + (b'' - \varepsilon_y D'') dy_2 - D' d\varepsilon_y - \varepsilon_y D'' \gamma dx_1 - \varepsilon_y D'' \gamma dy_1 &= 0 \end{aligned} \tag{12}$$

Applying Cramer's rule

$$\frac{dy_2}{dx_1} = \frac{\varepsilon_y D'' b'' \gamma}{[(b'' - \varepsilon_x D'')(b'' - \varepsilon_y D'') - \varepsilon_x \varepsilon_y D''^2]} \leq 0 \quad (13)$$

■

## B Proof of Proposition 1

**Proof.** The second derivative of  $\varepsilon D'(s)$  is

$$2D'' \frac{\partial s}{\partial \varepsilon} + \varepsilon D''' \left( \frac{\partial s}{\partial \varepsilon} \right)^2 + \varepsilon D'' \frac{\partial^2 s}{\partial \varepsilon^2} \quad (14)$$

(14)  $\leq 0$  if  $D''' b'' - D'' b''' \geq 0$  (see author for details), which is satisfied if  $D''', b''' \leq 0$ . Thus  $E[\varepsilon D'(s)]$  is decreasing in risk, and  $x_1$  is increasing in risk. ■

## C Example of Partial Learning

In order to meet the conditions in Ulph and Ulph (1997), the benefit function must be different in the two periods. Let  $b_1(x_1) = x_1 - \frac{x_1^2}{2}$ ,  $b_2(x_2) = \frac{x_2}{2} - \frac{x_2^2}{4}$ , and  $D(S) = S^2$  where  $S = x_1 + x_2$ . Say that the probability distribution of the shift parameter is

$$\varepsilon = \left\{ \begin{array}{ll} .1 & \text{prob } \frac{1}{11} \\ 1.1 & \text{prob } \frac{10}{11} \end{array} \right\} \quad (15)$$

When there is no learning, optimal emissions in both periods are chosen using the expected value of  $E[\varepsilon] = 1$ . This results in an optimal solution of  $x_1^* = 1/3, x_2^* = 0$ . Thus the irreversibility constraint bites at the mean: Ulph and Ulph conditions are met. Now consider three signals defined in (16), each more informative than the one before.

True value of $\varepsilon$	Signal 1	Signal 2	Signal 3	
$\varepsilon = .1$	$y_1 = 0$	$y_2 = 0$	$y_3 = 0$	(16)
$\varepsilon = 1.1$	$y_1 = 0$	$y_2 = \left\{ \begin{array}{l} 0 \text{ prob } \frac{4}{70} \\ 1 \text{ prob } \frac{66}{70} \end{array} \right\}$		$y_3 = 1$

Signal 3 is perfectly informative; Signal 2 is more informative than Signal 1, which provides no information. The full decision problem is

$$\max_{x_1} x_1 - \frac{x_1^2}{2} + E_y \left\{ \max_{x_2 \geq 0} E_{\varepsilon|y} \left[ \frac{x_2}{2} - \frac{x_2^2}{4} - \varepsilon (x_1 + x_2)^2 \right] \right\} \quad (17)$$

The optimal values of  $x_1$  for each of the three signals is presented below:

	Signal 1	Signal 2	Signal 3	
$x_1^*$	.333	.322	.330	(18)

We see that while emissions with no learning are always higher than emissions with learning, nevertheless, emissions increase with an increase in learning between Signal 2 and Signal 3.

## D Proof of Proposition 3

**Proof.** If

$$f(\varepsilon_x) = \varepsilon_x D'(s) \left[ \gamma + \frac{\partial y_2}{\partial x_1} \right] \quad (19)$$

is concave, it implies that  $x_1$  is increasing in risk, holding  $y_1$  constant. To determine whether (19)

is concave or convex, consider the second derivative:

$$\begin{aligned} f''(\varepsilon) &= 2D'' \left[ \gamma + \frac{\partial y_2}{\partial x_1} \right] \frac{\partial s}{\partial \varepsilon_x} + 2D'(s) \frac{\partial^2 y_2}{\partial x_1 \partial \varepsilon_x} + 2\varepsilon D'' \frac{\partial^2 y_2}{\partial x_1 \partial \varepsilon_x} \frac{\partial s}{\partial \varepsilon_x} + \\ &\varepsilon_x D'' \left[ \gamma + \frac{\partial y_2}{\partial x_1} \right] \frac{\partial^2 s}{\partial \varepsilon_x^2} + \varepsilon_x D'(s) \frac{\partial^3 y_2}{\partial x_1 \partial \varepsilon_x^2} \end{aligned} \quad (20)$$

The proof proceeds by calculating the partial derivatives under each assumption, then determining the sign of (20). The derivatives are calculated by totally differentiating the first order conditions for  $x_2$  and  $y_2$  (see the proof in Appendix A) and then applying Cramer's rule. We first calculate for the independent case, then note what the specific assumptions imply.

$$\frac{\partial s}{\partial \varepsilon_x} = \frac{\partial x_2}{\partial \varepsilon_x} + \frac{\partial y_2}{\partial \varepsilon_x} = \frac{D'b''}{[(b'' - \varepsilon_x D'')(b'' - \varepsilon_y D'') - \varepsilon_x \varepsilon_y D''^2]} \quad (21)$$

$$\frac{\partial^2 s}{\partial \varepsilon_x^2} = \frac{2D'D''b''^2}{[(b'' - \varepsilon_x D'')(b'' - \varepsilon_y D'') - \varepsilon_x \varepsilon_y D''^2]^2} \quad (22)$$

$$\frac{\partial y_2}{\partial x_1} = \frac{\varepsilon_y D'' b'' \gamma}{[(b'' - \varepsilon_x D'')(b'' - \varepsilon_y D'') - \varepsilon_x \varepsilon_y D''^2]} \quad (23)$$

$$\frac{\partial^2 y_2}{\partial x_1 \partial \varepsilon_x} = \frac{\gamma \varepsilon_y b''^2 D''^2}{[(b'' - \varepsilon_x D'')(b'' - \varepsilon_y D'') - \varepsilon_x \varepsilon_y D''^2]^2} \quad (24)$$

$$\frac{\partial^3 y_2}{\partial x_1 \partial \varepsilon_x^2} = \frac{\gamma \varepsilon_y b''^3 D''^3}{[(b'' - \varepsilon_x D'')(b'' - \varepsilon_y D'') - \varepsilon_x \varepsilon_y D''^2]^3} \quad (25)$$

$$\frac{\partial^2 y_2}{\partial x_1 \partial \varepsilon_y} = \frac{\gamma b''^2 D'' (b'' - \varepsilon_x D'')}{[(b'' - \varepsilon_x D'')(b'' - \varepsilon_y D'') - \varepsilon_x \varepsilon_y D''^2]^2} < 0 \quad (26)$$

$$\frac{\partial^3 y_2}{\partial x_1 \partial \varepsilon_y^2} = \frac{\gamma b''^3 D''^2 (b'' - \varepsilon_x D'')}{[(b'' - \varepsilon_x D'')(b'' - \varepsilon_y D'') - \varepsilon_x \varepsilon_y D''^2]^3} > 0 \quad (27)$$

Since the players are symmetric  $\frac{\partial s}{\partial \varepsilon_x} = \frac{\partial s}{\partial \varepsilon_y}$ , thus when damages are perfectly correlated ( $\varepsilon = \varepsilon_x = \varepsilon_y$ ) then  $\frac{\partial s}{\partial \varepsilon} = 2 \frac{\partial s}{\partial \varepsilon_x}$ . Similar observations can be used to calculate the other partials under different assumptions.

(i) When damages are perfectly negatively correlated

$$\frac{\partial s}{\partial \varepsilon} = \frac{D' [b''(y_2) - b''(x_2)]}{[(b'' - \varepsilon D'')^2 - \varepsilon^2 D''^2]} \quad (28)$$

which is equal to zero if  $b$  is quadratic. Since  $D'''$  will always be multiplied by  $\frac{\partial s}{\partial \varepsilon}$ , its sign is irrelevant. Simplifying (23), (24), and (25) shows that  $\frac{\partial y_2}{\partial x_1} \leq 0$ ,  $\frac{\partial^2 y_2}{\partial x_1 \partial \varepsilon} > 0$ , and  $\frac{\partial^3 y_2}{\partial x_1 \partial \varepsilon^2} = 0$ . Thus, (20) is positive, and  $x_1$  is decreasing in uncertainty. Since the players are symmetric the same argument holds for  $y_1$  holding  $x_1$  constant. The relationships are monotone and thus we can apply Theorem 3 from Milgrom and Roberts to conclude that the highest, lowest, and symmetric equilibria are all increasing in risk.

(ia) Substituting the expressions above into (20) shows that

$$f''(\varepsilon_x) = \frac{D'(s) D'' \gamma b''^3}{[(b'' - \varepsilon_x D'')(b'' - \varepsilon_y D'') - \varepsilon_x \varepsilon_y D''^2]^3} [2b''^2 - 2\varepsilon_x D'' b'' - 2\varepsilon_y^2 D''^2 - 3\varepsilon_x \varepsilon_y D''^2] < 0 \text{ if}$$

$$\varepsilon_y < -\frac{2}{3} \frac{b''}{D''} \quad (29)$$

(iib) Consider

$$f''(\varepsilon_y) = 2\varepsilon_x D'' \frac{\partial^2 y_2}{\partial x_1 \partial \varepsilon_y} \frac{\partial s}{\partial \varepsilon_y} + \varepsilon_x D'' \left[ \gamma + \frac{\partial y_2}{\partial x_1} \right] \frac{\partial^2 s}{\partial \varepsilon_y^2} + \varepsilon_x D'(s) \frac{\partial^3 y_2}{\partial x_1 \partial \varepsilon_y^2} \quad (30)$$

Note that  $\left[ \gamma + \frac{\partial y_2}{\partial x_1} \right] = \gamma b'' \frac{b'' - \varepsilon_x D''}{[(b'' - \varepsilon_x D'')(b'' - \varepsilon_y D'') - \varepsilon_x \varepsilon_y D''^2]} > 0$  and that  $\frac{\partial s}{\partial \varepsilon_y} = \frac{\partial s}{\partial \varepsilon_x} < 0$  and  $\frac{\partial^2 s}{\partial \varepsilon_y^2} = \frac{\partial^2 s}{\partial \varepsilon_x^2} > 0$ ,  $\frac{\partial^2 y_2}{\partial x_1 \partial \varepsilon_y} < 0$ , and  $\frac{\partial^3 y_2}{\partial x_1 \partial \varepsilon_y^2} > 0$ : Therefore each term in (30) is positive and the right hand side is convex in  $\varepsilon_y$ .

The reaction functions are increasing in the risk of own-damages and decreasing in the risk of other's-damages. We again apply Theorem 3 from Milgrom and Roberts (1994) to show that first period equilibrium emissions increase for the player whose risk is increasing and decrease for the opponent (whose risk is held constant).

(iii) Simplifying (21)–(25) under assumptions of perfect correlation, plugging into (20) and simplifying the resulting expression gives

$$f''(\varepsilon) = \frac{2D'' \gamma b''^3 D'}{[(b'' - \varepsilon D'')^2 - \varepsilon^2 D''^2]^3} [-4\varepsilon b'' D'' + 2\varepsilon^2 D''^2 + 3b''^2] < 0 \quad (31)$$

Since the players are symmetric the same argument holds for  $y_1$  holding  $x_1$  constant. Thus the highest, lowest, and symmetric<sup>12</sup> equilibria are decreasing in risk. ■

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<sup>12</sup>The symmetric equilibrium is unique. Consider two symmetric equilibria  $(x, y) > (x', y')$ . If  $y > y'$  then  $\delta E \left\{ \varepsilon_x D' (s(x_1 + y, \varepsilon_x, \varepsilon_y)) \left[ \gamma + \frac{\partial y_2}{\partial x_1} \right] \right\} > \delta E \left\{ \varepsilon_x D' (s(x_1 + y', \varepsilon_x, \varepsilon_y)) \left[ \gamma + \frac{\partial y_2}{\partial x_1} \right] \right\}$  implying that  $x' < x$ , which contradicts the assumption.

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