Optimal Energy R&D Portfolio Investments in Response to a Carbon Tax

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***Abstract*—In this paper, we deal with a very timely issue—R&D strategies needed for compliance with a climate policy in an economically optimal way. We provide interesting insights into the composition of R&D portfolios across the main mitigation options for decision makers and policy makers. We address the optimal R&D investment response of a decision maker or an engineering manager—at the firm level with a portfolio of alternative technologies—to a rising carbon tax. Understanding the optimal allocation of investments in these technologies is crucial because like most economic resources, there is a limitation on the investment capabilities of a firm to undertake these innovative efforts. In addition, environmental R&D spending is irreversible and investment decisions made today have multiperiod consequences on the energy technologies landscape. Thus, we explore the reaction of a firm's optimal investment in an energy R&D portfolio comprising four different technologies to increases in a future carbon tax. We find that investment allocation depends on the elasticity of substitution between fossil and nonfossil energy inputs, and the relative costs and efficacy of the R&D programs; and that overall investment tends to decrease in risk depending on firm flexibility and specifications.**

***Index Terms*—Carbon tax, decision-making under uncertainty, elasticity, energy R&D, investment.**

I. Introduction

THE NEGATIVE impact on the climate of greenhouse gas (GHG) emissions confronts decision makers at the firm level, as well as policy makers, with the question of what steps should be taken to ameliorate this growing concern. On one hand, policy makers and regulators are wrestling with determining the optimal policy to spur technological change to improve carbon-intensive technologies or develop noncarbon technologies. On the other hand, decision makers at the firm level are grappling with how to allocate their R&D efforts in the face of several alternatives and under a future policy that is uncertain, but expected to increase in stringency.

The central theme of this paper is to address the optimal R&D investment response of a decision maker—at the firm level with a portfolio of alternative technologies—to a rising carbon tax. Understanding the optimal allocation of investment in these technologies is crucial for four reasons.

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1) There are many new alternative technologies and potential improvements to currently economic technologies with different approaches to reduce GHG emissions. While some of these technologies have zero emissions, others improve on the current methods by reducing their emissions level. Thus, knowing which technology is optimally worthy of investment is important.
2) Like most economic resources, there is a limitation on the investment capabilities of a firm to undertake the research efforts on these improvements and innovative efforts. In addition to this, environmental R&D spending is irreversible.
3) Investment decisions made today have multiperiod consequences on the future shape of energy technologies.
4) Policy makers need a yardstick for evaluating the incentive effects, on firms, of a carbon tax regulation and the overall portfolio of technological change.

Regardless of the domain—product development, environment, or climate change—portfolio investment decisions do not come easy. For example, Loch and Kavadias [1], in their analysis of the dynamic selection of new product development, underscore the combinatorial complexity of allocating a scarce budget over multiple periods. This is not just because decisions have multiperiod consequences, but it is also due to the different return functions on the new product lines that are competing for a common pool of resources and are often interdependent [2]. This phenomenon of interdependence and having different return functions in new product development has similarities with the different ways the energy technologies influence the level of emissions, their effects on the demand for alternatives, and ultimately their interactions through complementary or substitution effects. On the climate-change front, having a portfolio of R&D projects is important because of the several positive ways that each project impacts climate change. The climate-change literature shows that a project portfolio has two advantages: 1) it diversifies uncertainty about the outcome of the technologies and 2) it hedges against the uncertainty about how high the future carbon tax will be.

Understanding the interaction and interdependence properties between the technologies is one part of the discussion. The other part is the cost perspective of reducing GHG emissions by these technologies. For example, in the examination of several technologies in the context of a global integrated assessment model of energy, agriculture, land-use, economics, and carbon cycle processes, Edmonds *et al.* [3] discuss the significance of the development of an expanded suite of technologies, including carbon capture and disposal, hydrogen systems, and biotechnology, because they hold the potential to dramatically reduce the cost of stabilizing GHG concentrations. Similarly, Baker *et al.* [4] derive marginal abatement cost curves under different solar technologies using the MiniCAM model.[1] Using an array of elicited expert definitions of technical success, they show that different technologies, if they achieve success as defined, have different impacts on the marginal cost of abatement. Fishelson and Kroetch [6] show that the possibility that the marginal and total costs are changing at different rates for different innovations justifies the use of more than one type of the technologies available. Although their work focuses on R&D into energy-storage devices, this result still holds in the general climate-change arena.

This paper has two objectives: to determine 1) how an increase in a carbon tax influences a firm's optimal energy R&D spending, in terms of overall investment level and the type of R&D in the portfolio, and 2) how parameters such as substitution elasticity and cost of technical change impact the optimal portfolio both in terms of overall investment size and technology-specific investment. In addition, we explore the impact of riskiness in the carbon tax on the optimal portfolio in order to get insights into the effects of carbon tax uncertainty on investment. These objectives constitute a part of an important list of criteria guiding firms on decision making regarding their investments in response to an increasing carbon tax and uncertainty about climate policy in the presence of different available energy R&D technologies. The differences between these technologies have been widely ignored in the theoretical literature. Therefore, an important motivation is to find out whether the response of a given R&D program to an increasing carbon tax is independent of the consortium of options in the energy R&D portfolio. In other words, do the other programs in the portfolio exert any influence in defining an optimal investment allocation to that technology? For example, how is the optimal investment in nonfossil fuel technologies impacted by the presence of carbon capture and sequestration (CCS) technologies?

Closely related to this, Baker and Shittu [7] examine a firm's profit-maximizing R&D response to an uncertain carbon tax for two R&D programs: cost reduction of nonfossil energy technologies and emissions reductions of currently economic technologies. They consider these different technologies independently, and conclude that the optimal investment in R&D does not always increase monotonically in a carbon tax. This paper extends that analysis by considering four different energy R&D technologies in a portfolio setting. A two-period theoretical model is developed to explore these issues on the four-project energy R&D portfolio: cost reduction of nonfossil energy technologies, emissions reduction of currently economic fossil-based technologies, fossil energy efficiency improvement, and total energy-use efficiency program. Our model focuses on a firm that invests in technological improvements, for example, the American Electric Power (AEP), which both produces and uses R&D.

The contribution of this paper, over and above previous and related work on this topic, comes from an explicit determination of the optimal allocation of R&D resources between different energy technologies in the reduction of GHGs under an uncertain policy. This portfolio approach, as enunciated by Blanford and Clarke [8], is distinct from previous methods because first, it utilizes decreasing returns to scale in R&D such that the marginal productivity of investment is a decreasing function of investment. This allows for an optimal mix of projects under increasing policy stringency because the reduction in marginal returns accompanying increasing funding in one technology allows focus to shift to programs with higher marginal returns on the initial investments. Second, the optimal response to uncertainty in the policy requires that different technologies are included in the mix to address different levels of risk that arise with the uncertainty. In other words, diversification may act as a hedge against any single technology not progressing as far as expected—although in this context, there is no uncertainty in technological change outcomes. Finally, heterogeneity due to variations in technologies and their applications is in line with the expected future deployment of different technologies. This is even more relevant as several carbon-free technologies are expected to cloud the future landscape of meeting demand under a stringent energy policy.

We proceed in Section II with a review of related literature. In Section III, we provide the theoretical framework for the representation of a set of broadly defined R&D programs. Since the demand for energy inputs is central to the relevance of these technologies, Section III-C sets the framework for the overall optimal demand for fossil and nonfossil energy in this four-technology portfolio. Section III-D introduces the computational model. Section IV delves further into the analysis of the developed framework with emphasis on the impact of increasing carbon tax on the levels of investments in each of the technologies in the portfolio and the overall investment. In this section, sensitivity analysis is carried out on the effect of R&D cost coefficient and substitution elasticity between fossil and nonfossil energy on investment. This section also discusses the impact of risk on investment. Section V concludes.

II. Background

The portfolio investment allocation problem has received significant research attention in the past because of its importance to managers and decision makers, and we observe that this problem exists in two relevant literatures—climate change and product development. We review both, but with more emphasis on the climate-change literature. In the climate-change literature, the energy portfolio investment allocation problem is triggered by an exogenous factor—regulatory policy. In this analysis, we explore the influence of an increase in a carbon tax to spur investment in energy technologies. The role of policy uncertainty in inducing technological change in climate control has also attracted considerable research attention.[2]

[1] Brenkert *et al.* [5] and Edmonds *et al.* [3] give a complete description of the model.

[2] The expectation of a tax policy shapes firms' investment decisions on energy R&D. Several earlier studies in this literature, including [9] and [10], show that emissions taxes and emissions permits generally provide more incentives for technological innovation than policies based on standards.

A. Overview of Portfolio Selection and the Role of Uncertainty

In the resource and portfolio allocation literature, Roussel *et al.* [11] discuss the importance of portfolio selection for top management in organizations. They view general managers and R&D managers working as partners to pool their insights into deciding what to do, and why and when to do it, by realistically assessing costs, benefits, and risk/reward, and they balance these variables within a portfolio of R&D activity that best fulfills the purposes of the corporation. This emphasizes several aspects in R&D portfolio management, but a number of papers focus on particular issues.[3] Two case studies, [15] and [16], describe the application of models to R&D project selection at BMW and GM, respectively. Kavadias and Loch [17] cover a wider range of issues concerning portfolio R&D, and more recent advances appear in [18].

In economics, the study of searching for the best alternative traces back to Weitzman [19] in his focus on sequential investment. This research thrust extends into the climate-change literature and energy technology R&D portfolios. For example, Gritsevskyi and Nakicenovic [20] observe that an important policy implication is that future research, development, and demonstration efforts and investments in new technologies should be distributed across "related" technologies rather than directed at only one technology from the cluster. Closely related to this is the impact of R&D efforts on the cost of reducing GHG emissions, as this is important in determining the optimal portfolio. For example, Baker *et al.* [21] point out that a research program to improve the efficiency of coal-fired electricity generation will create a different abatement cost profile from an R&D program into photovoltaic cells. While the first lowers the cost for moderate reductions in GHG emissions, the second will lower the costs of severely reducing emissions. It is clear that having an energy R&D portfolio is the best strategy, but the question that arises is the following: in the face of these different technologies, what is the optimal level of spending on these technologies under an increasing emissions tax?

Pizer [22] shows that uncertainty (without learning) is crucial to investment decisions because it raises the optimal level of emission reductions and leads to a preference for taxes over rate controls. This suggests that analysis that disregards the impact of uncertainty is likely to result in inefficient policy recommendations. For example, Grubler and Gritsevskyi [23] consider the effects of uncertainties such as demand, technology costs, and the size of a carbon tax on technology choice. They find that the entry of an additional source of uncertainty makes the technology portfolio more diversified. While one group of previous efforts—[24] and [25]—look at investment decisions by considering how optimal investment is impacted by uncertainty in prices or demand, another, [26], analyze the same question under technology uncertainty. Hasset and Metcalf [27] argue that random changes in tax policy provide opportunities for firms to wait out high-tax regimes and invest more heavily in low-tax regimes. Other papers show how the optimal R&D investment changes with the risk profile of the technologies and with uncertainty about climate damages.[4]

III. The Model

We model a firm's profit-maximizing choice of energy R&D in the presence of a tax on carbon emissions. We use a two-period theoretical model. Investments in R&D are made in the first period in anticipation of a future carbon tax. For simplicity, we ignore production in the first period. Optimal production is chosen in the second period after the firm learns about the carbon tax and technical change has been achieved, leading to a second-period profit function. We take the market structure for output to be exogenous—the firm faces a known downward sloping demand curve. In the following sections, we define the details of the model used to derive the optimal demand for the energy inputs, and then show how the marginal profit is influenced by investments in energy technology. We then present our computational model—with model parameters—using the defined representations of technical change.

A. Second-Period Profits and Optimal Energy Demand

The firm uses three inputs—nonenergy inputs x, fossil energy inputs ε_c, and nonfossil energy inputs ε_{nc}. Let ε_c be normalized so that when the current technology is used, one unit of fossil energy produces one unit of emissions. Then the total firm-specific price paid for fossil energy is the cost of the fuel P_c plus the price of the carbon emitted, equal to the carbon tax t. Assume that under the current technology, nonfossil energy is more expensive than fossil energy: the firm-specific price of nonfossil energy equals $P_c + \eta$. The price of nonenergy inputs is w. We consider a firm with a nested constant elasticity of substitution production function to produce output y. Thus, in the absence of technical change, the firm chooses the profit-maximizing inputs by solving

$$\pi = \max_{\varepsilon_c, \varepsilon_{nc}, x} yp(y) - ((P_c + t)\varepsilon_c + (P_c + \eta)\varepsilon_{nc} + xw)$$

$$\text{such that } y = (x^\rho + (\varepsilon_c^\gamma + \varepsilon_{nc}^\gamma)^{\rho/\gamma})^{1/\rho} \quad (1)$$

where $p(y)$ is the output price, $\varsigma \equiv 1/1 - \rho$ is the elasticity of substitution between energy and nonenergy inputs, and $\sigma \equiv 1/1 - \gamma$ is the elasticity of substitution between fossil and nonfossil energy inputs. We assume that the firm is facing a constant elasticity demand with inverse demand curve, $p(y) = Ay^{-1/b}$, where A is a constant and b is the price elasticity of demand. The solution to this problem (see the Appendix for details) gives the unconditional demand for fossil energy input and nonfossil energy input, ε_c^* and ε_{nc}^*, respectively, as

$$\begin{aligned} \varepsilon_c^* = {} & P_c^{1/(\gamma-1)} \left(P_c^{\gamma/(\gamma-1)} + P_{nc}^{\gamma/(\gamma-1)}\right)^{(\gamma-\rho)/[\gamma(\rho-1)]} \\ & \times \Big[w^{\rho/(\rho-1)} + \Big(P_c^{\gamma/(\gamma-1)} \\ & + P_{nc}^{\gamma/(\gamma-1)}\Big)^{[\rho(\gamma-1)]/[\gamma(\rho-1)]}\Big]^{[b(1-\rho)-1]/\rho} \left(\frac{b}{b-1}\frac{1}{A}\right)^{-b} \end{aligned} \quad (2)$$

[3]For example, appropriate project sequencing [12], simulating different portfolios to assess the value of information [13], and optimal investment decisions when the return on investment is random [14].

[4]For a detailed review of uncertainty and investment in the context of climate change, see [29].

$$\begin{aligned}\varepsilon_{nc}^{*} =& P_{nc}^{1/(\gamma-1)} \left(P_c^{\gamma/(\gamma-1)} + P_{nc}^{\gamma/(\gamma-1)}\right)^{(\gamma-\rho)/[\gamma(\rho-1)]} \\ &\times \Bigg[w^{\rho/(\rho-1)} + \Big(P_c^{\gamma/(\gamma-1)} \\ &+ P_{nc}^{\gamma/(\gamma-1)}\Big)^{[\rho(\gamma-1)]/[\gamma(\rho-1)]}\Bigg]^{[b(1-\rho)-1]/\rho} \left(\frac{b}{b-1}\frac{1}{A}\right)^{-b} \end{aligned} \tag{3}$$

B. Portfolio Profit Function

Now, we consider the firm's investment in technical change in the first period. We represent technical change as having an impact on the firm-specific cost of energy inputs such that the effective cost is reduced after technical change. Let technical change $\overleftarrow{\boldsymbol{\alpha}}$ represent a vector of cost improvements in the technologies in the portfolio. For each technology, we have $0 \leq \alpha < 1$, with cost reduction in the inputs maximized as α tends to 1. Then the firm's second-period profit function, assuming that the carbon tax t is known, is $\pi(w, P_c + t, P_c + \eta; \overleftarrow{\boldsymbol{\alpha}})$. Let $g(\overleftarrow{\boldsymbol{\alpha}})$ represent the cost vector of the R&D programs aimed at achieving technical change of $\overleftarrow{\boldsymbol{\alpha}}$. We assume that the cost of technical change goes to infinity as $\overleftarrow{\boldsymbol{\alpha}}$ approaches 1: $\lim_{\alpha\to 1} g(\overleftarrow{\boldsymbol{\alpha}}) = \infty$. We assume that $g(\overleftarrow{\boldsymbol{\alpha}})$ is increasing and convex in each argument. Thus, in the first period, the firm chooses $\overleftarrow{\boldsymbol{\alpha}}$, the level of technical change, when the carbon tax is still unknown by solving

$$\max_{\overleftarrow{\boldsymbol{\alpha}}} -g(\overleftarrow{\boldsymbol{\alpha}}) + E_t \pi(w, P_c + t, P_c + \eta; \overleftarrow{\boldsymbol{\alpha}})$$
$$\overleftarrow{\boldsymbol{\alpha}} = \alpha_A, \alpha_C, \alpha_E, \alpha_F \tag{4}$$

where the subscripts $A, C, E,$ and F represent nonfossil, CCS, energy efficiency, and fossil energy efficiency programs, respectively; and E_t refers to the expectation over the uncertain tax. The first-order conditions (FOCs) for each $\boldsymbol{\alpha}_i$ are

$$g'(\alpha_i) = E_t\left[\frac{\partial \pi}{\partial \alpha_i}\right] \qquad \forall i = A, C, E, F. \tag{5}$$

It is clear that the optimal level of R&D spending increases if the right-hand side of (5) increases; this, in turn, depends on the probability distribution of t. Thus, we will focus, computationally, on how a change in the probability distribution over the carbon tax t impacts the optimal investment in each technology in the portfolio. In particular, we will focus on how an increase in t impacts $\partial\pi/\partial\alpha_i$.

C. Representations of Technical Change

In this section, we show how we represent each type of technical change in the portfolio through their effects on the cost of inputs, and thus, on the profit function of the firm in the second period. We use these definitions in a framework that captures the entire portfolio of technical change in the profit-maximization problem. We define energy efficiency improvement as technical change that leads to a higher level of output for the same level of energy input. This, in turn, leads to a reduction in the effective price of both fossil and nonfossil energy inputs per unit output. Examples of energy efficiency improving R&D are general improvements in electricity generation, transmission, and distribution efficiencies. We model this technical change such that both the cost of fossil and nonfossil energy inputs are effectively reduced by $(1 - \alpha_E)$. After technical change is parameterized by α_E, the second-period profit function becomes $\pi[w, (1 - \alpha_E)(P_c + t), (1 - \alpha_E)(P_c + \eta)]$.

Fossil fuel R&D reduces the price of fossil energy by $(1 - \alpha_F)$ by increasing the per-unit efficiency of fossil fuel use, for example, an increase in efficiency of a coal-fired generator such that more output is produced per unit input. The reduction in the price of fossil energy as captured by this technology gives a second-period profit given by $\pi[w, (1 - \alpha_F)(P_c + t), P_c + \eta]$. The representations of R&D into nonfossil fuel technology and CCS technologies follow directly from the work by Baker and Shittu [8]. They model nonfossil fuel R&D as reducing the premium on nonfossil energy from η to $(1 - \alpha_A)\eta$. This program could represent, for example, a firm's research into minimizing the cost of their wind turbines or the development of less expensive solar power, which are nonfossil energy alternatives. They model CCS as reducing the carbon intensity of a unit of fossil energy from 1 to $(1 - \alpha_C)$. Thus, the price of fossil energy is effectively reduced from $P_c + t$ to $P_c + (1 - \alpha_C)t$. This program represents an investment into technology that will capture a fraction α_C of the firm's fossil emissions. Under nonfossil fuel R&D and CCS, the second-period profit functions are $\pi[w, (P_c + t), P_c + (1 - \alpha_A)\eta]$ and $\pi(w, P_c + (1 - \alpha_C)t, P_c + \eta)$, respectively.

Thus, the firm's overall portfolio problem is

$$\begin{aligned}\max_{\overleftarrow{\boldsymbol{\alpha}}} -g(\overleftarrow{\boldsymbol{\alpha}}) + E_t \pi[w, (1 - \alpha_E)(1 - \alpha_F)(P_c + (1 - \alpha_C)t), \\ (1 - \alpha_E(P_c + (1 - \alpha_A)\eta)].\end{aligned} \tag{6}$$

D. Computational Model

In this section, we describe the computational method applied. For a firm facing a constant demand elasticity with inverse demand curve defined in Section III-A, the profit function follows

$$\begin{aligned}\pi = Ay^{*((b-1)/b)} - (1 - \alpha_E)(1 - \alpha_F)(P_c + (1 - \alpha_C)t)\varepsilon_c^* \\ - (1 - \alpha_E)(P_c + (1 - \alpha_A)\eta)\varepsilon_{nc}^* - wx^*\end{aligned} \tag{7}$$

where A is a constant and b is the price elasticity of demand, and asterisks denote that the quantity is optimal. We maximize the firm's profits less overall cost of technical change, $g(\overleftarrow{\boldsymbol{\alpha}})$. In order to get computational results, we need to make an assumption about the cost of R&D. We represent the total cost of technical change as follows:

$$g(\overleftarrow{\boldsymbol{\alpha}}) = \sum_{i\in\{A,C,E,F\}} \frac{\kappa_i \alpha_i^2}{1 - \alpha_i}. \tag{8}$$

We have made the simplifying assumption that all the programs have the same functional form. This functional form has the advantage of simplicity, exhibits decreasing returns to scale in R&D, and ensures that R&D will not bring about zero-cost

TABLE I
MODEL PARAMETERS

Parameter	Symbol	Value
Price of carbon inputs	P_c	1
Premium on non-carbon inputs	η	1
Price of non-carbon inputs	$P_{nc}=P_c+\eta$	2
Price of non-energy inputs	w	1
Output price coefficient	A	1
Demand price elasticity	b	1.1
Elasticity of substitution between energy and non-energy inputs	ζ	0.75
Elasticity of substitution between carbon and non-carbon energy inputs	σ	1.5; 6
Coefficient of investment cost	κ	1

full abatement.[5] It also assumes that the technologies are not complements in terms of R&D programs. The R&D cost coefficients κ_i are subject to sensitivity analyses to determine their influence on the optimal levels of R&D. These programs are effectively different in the way they affect the abatement cost curve. Substituting (7) and (8) into (6), we solve the deterministic maximization problem to determine the maximizing levels of α_i in each technology under different tax levels.

E. Model Parameters

In this section, we state our key assumptions and present the base-case values for model parameters. We start by assigning equal weights to the cost of technical change such that technology *i* has a cost coefficient $\kappa_i = 1$. This assumption presents an unbiased measure of the response to increasing tax of the demand for each technology in the portfolio. We also assume that the prices of the inputs are fixed.

Table I summarizes the baseline values assigned to the parameters. The assumptions about the prices of carbon and noncarbon inputs in this table make fossil and nonfossil technologies become economically equivalent when the tax is 1.

The elasticity values are converted to the parameters in the functions using $\varsigma \equiv 1/(1-\rho)$ and $\sigma \equiv 1/(1-\gamma)$.

Popp [29], in his climate economy model, including endogenous technological change, has calibrated the short-term elasticity of substitution between fossil and nonfossil energy as 1.6, implying that they are substitutes, but not very close substitutes. If we consider electricity generators, then in the long run, fossil and nonfossil energy are perfect substitutes. Goulder and Schneider [30], in their analytical and numerical general equilibrium models in which technological change results from profit-maximizing investments in R&D, used 0.9. For nonelectricity sector firms, we argue that substitution elasticities are even higher because an alternative to their energy supply is electricity that is readily obtainable. We present the following observations under two measures of elasticity of substitution between fossil and nonfossil energy inputs—a high value of 6 to represent the long-term high substitutability between different sources of electricity generation and a low value of 1.5, in the range of Popp's [30] estimate.

[5]In each case, if $\alpha_i = 1$, then emissions would be zero at the profit-maximizing point.

IV. RESULTS OF ILLUSTRATIVE SCENARIOS

In this section, we explore how a firm's optimal investments in these technologies respond to an increasing carbon tax under specific parameter values in order to illustrate general ideas. We discuss the experimental design steps required to understand the behavior of investments in these technologies, and we present results through the lens of how optimal investment in individual technologies respond to changes in parameters. We start by exploring the optimal portfolio reaction to an increasing tax under two specific estimates of the elasticity of substitution between fossil and nonfossil energy inputs—high and low. For each level of flexibility, we assume two types of distributions on the cost coefficients of the technologies. The first is uniform cost coefficients across the technologies and the second is giving higher costs to the efficiency improving technologies.

A. High Elasticity of Substitution

Using the parameters described before, the left-hand side of Fig. 1 shows the profit-maximizing response of total investment in the technologies to different levels of a carbon tax. The right-hand panel shows the breakdown of individual investments across the four technologies in the portfolio. In these figures, the price elasticity of demand is 1.1, the elasticity of substitution between energy and nonenergy input is 0.75, and between carbon and noncarbon energy is 6.

The total investment graph shows that the total optimal R&D investment first increases, then decreases, and finally flattens as

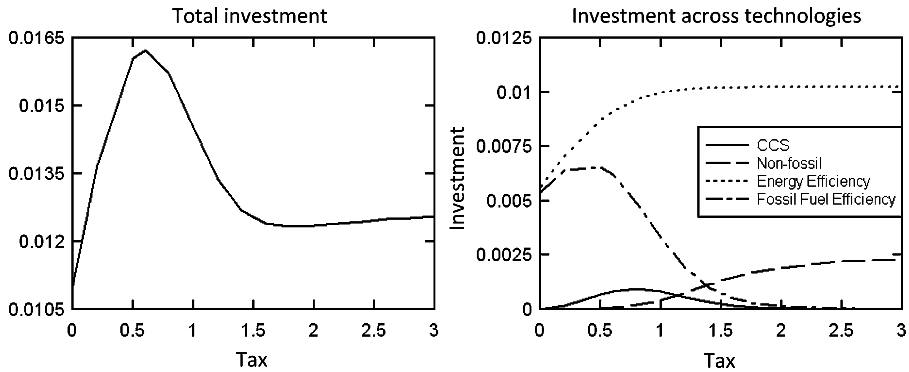

Fig. 1. (Left) Response of total investment in the technologies as carbon tax increases. (Right) Breakdown of individual investments across the four technologies in the portfolio. The price demand elasticity is 1.1, and the elasticity of substitution between energy and nonenergy input is 0.75, and between carbon and noncarbon energy is 6.

the tax increases. Fig. 1 (right) shows how this breaks down technology by technology. We see that investment in all the programs increase in the tax when the tax is low. However, two of the programs—fossil fuel efficiency and CCS—decrease after the tax hits about 0.5 and 0.75, respectively. Energy efficiency and nonfossil fuel investments remain relatively high and unchanging at the high tax levels. To understand these patterns, consider how a carbon tax will effect the firm's demand for fossil and nonfossil inputs. As a carbon tax increases, the demand for fossil inputs will unambiguously decrease. The demand for nonfossil inputs, however, are more ambiguous. The substitution effect will lead to an increase in demand for nonfossil as the carbon tax increases. The output effect will lead to a decrease. It is clear that the demand for energy inputs changes with a carbon tax and drives the optimal level of R&D.

R&D spending in energy efficiency is higher than for other programs because investment in technology that improves on overall efficiency has a double-edged effect, as it affects the prices of both energy inputs simultaneously. Optimal investment in this technology is stable at high levels of the tax. This is because at high levels of the tax, the firm will substitute away from fossil fuel energy inputs and rely mainly on nonfossil inputs. Once a firm has substituted completely away from fossil inputs, the carbon tax is no longer relevant. Thus, one policy implication of this is that a firm that anticipates a high future tax must focus on technologies that use nonfossil inputs. Like energy efficiency, investments in nonfossil fuel technologies monotonically increase as the carbon tax increases. These results reflect the fact that firms substitute toward nonfossil technology as the tax increases, and therefore, invest more in improving nonfossil technologies. This will be true as long as the substitution effect is stronger than the output effect.

Investment in fossil fuel efficiency and CCS improvement programs first increase, and then decrease as the tax increases. The investment increases in the tax when the tax is small, because as long as the tax is small, fossil inputs are less expensive than nonfossil, and the firm will tend to use very large amounts of fossil fuel energy. The R&D takes the hedge off the carbon tax in reducing the carbon emissions per unit output. However, as the total price of fossil fuel rises further, and the firm substitutes away from fossil fuel, the benefits of fossil technology programs start to get small. It is interesting to note that the tax level, $tax = 0.5$, where investments in fossil fuel efficiency program begin to decrease, is equivalent to the point when nonfossil fuel technology comes on stream.

A comparison of CCS and fossil fuel efficiency programs with nonfossil fuel R&D shows that initial investments are higher in fossil technologies—as the tax gets high, however, firms substitute more toward nonfossil energy; thus, investment in technologies that improve nonfossil energy becomes more attractive. Therefore, the economic interpretation of overall investment first increasing and then decreasing in the carbon tax is that investment appears to be highest when the carbon tax is high enough to provide incentives for using CCS, but not so high that firms start to substitute away from fossil energy significantly. The value of technologies that improve nonfossil increases as the carbon tax increases. For technologies that improve fossil fuel technology (CCS and fossil fuel efficiency), the value of the technology follows a Laffer curve—first increases and then decreases. For CCS in particular, the firm has no incentive to invest when the tax is zero, or when the tax is extremely high.

So far, we have assumed that the R&D cost coefficient κ is equal across the technologies. In Fig. 2, we explore the impact of having a high cost of investment in the efficiency programs. Here, we assume that the cost coefficients for both efficiency programs is hundred times that of the other two programs. We see that in this case, the overall spending in the portfolio increases monotonically in a carbon tax. This is because the increase in the nonfossil program dominates the decrease in CCS and fossil fuel efficiency programs. It is evident here that as the carbon tax increases, the substitution effect driving the demand for fossil and nonfossil inputs mentioned earlier is transferred to CCS and nonfossil programs to act as substitutes. The interplay between CCS and nonfossil technologies acting as substitutes under increasing policy stringency increases the level of total investment, and the optimal allocation of investments depends significantly on this interaction. Furthermore, the relatively high costs of R&D into the efficiency programs implies these programs have no observable influence on the substitution effect

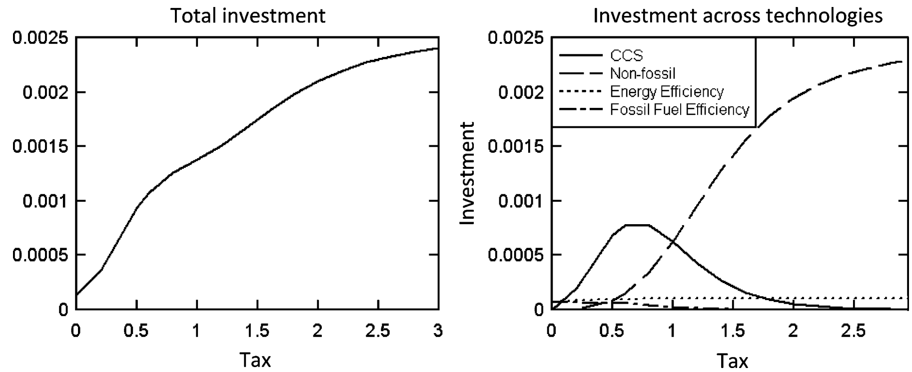

Fig. 2. High-cost efficiency programs/high elasticity of substitution: cost coefficient κ of efficiency programs is higher than for the non-efficiency-improving programs. The elasticity of substitution is 6. The other parameters remain as presented in Table I.

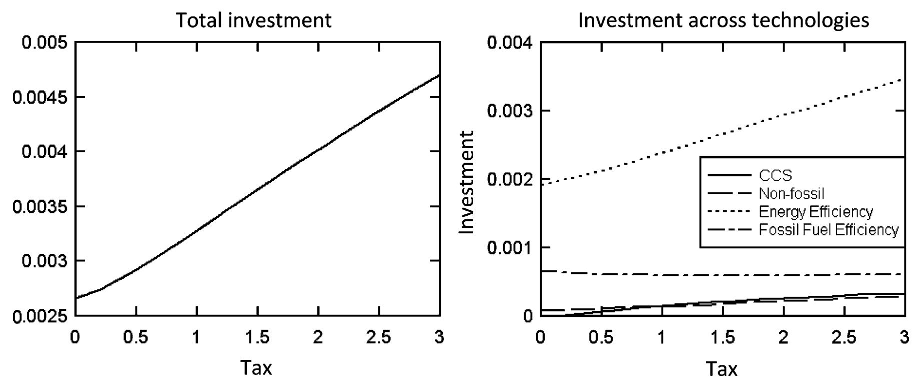

Fig. 3. Elasticity of substitution between fossil and nonfossil energy is 1.5 and the relative cost coefficients are equal.

between CCS and nonfossil technologies because this interaction did not change.

Comparing the right-hand side graphs—*investment across technologies*—of Figs. 1 and 2, we note that the increase in both CCS and nonfossil is not commensurate with the vacuum created by the absence of energy efficiency program. The intuition for this behavior is that energy efficiency R&D dominates, and its exclusion (by means of decreasing its productivity) implies significantly lower R&D investments. This has strong policy implications, suggesting that the quantification of improved energy efficiency R&D spending, especially when measured against the other programs, is crucial to significant mitigation efforts.[6]

Another implication of these analyses is that the crafting of an optimal policy on emissions mitigation strategies cannot hold without giving considerations to the effective carbon price. This is because, on one hand, a policy prescription that makes the carbon price too high discourages firm-level investments in intermediate carbon-based technologies. On the other hand, a policy without *sufficient* impact on the price of carbon will not only have no influence on emissions mitigation, it will also not spur any *appreciable* investments to achieve improved technologies.

B. Low Elasticity of Substitution

In this section, we present the results using a low value for the elasticity of substitution, $\sigma = 1.5$, with all other parameters as in Table I to gain further insights into the interactions between these technologies. The left-hand panel of Fig. 3 shows that in this case, the overall investment increases in a carbon tax; this is in contrast to the results in Fig. 1. Using the right-hand panel of Fig. 3, we focus on the drivers of this result. First, the optimal investment in nonfossil programs is relatively flat. This is because under the assumption of low substitutability between the energy inputs, the substitution effect is smaller and the output effect is larger than what we saw before. The firm does not increase the demand for nonfossil inputs to a great degree under an increasing carbon tax; therefore, the optimal investment in nonfossil technology is also not very responsive. Second, investments in overall efficiency improvement program follow a pattern very similar to above, increasing steadily with a carbon tax. This is because it effects both technologies, and given low substitutability, this is of great benefit. Third, although the demand for CCS technology shows that increasing investment for fossil fuel efficiency improvement is relatively stable and fairly higher. These three factors add up to cause the overall increase in total investment. Note that, eventually, output will decrease with the tax, leading to overall lower investment in technology. This effect will not be seen, however, until a very high carbon tax.

Now we use the same low elasticity, but different R&D cost coefficients to show the influence of the cost coefficients on investment decisions. Fig. 4 illustrates an example of this with the total investment increasing in tax. In this figure, the cost

[6]We gratefully acknowledge a reviewer for pointing out this observation.

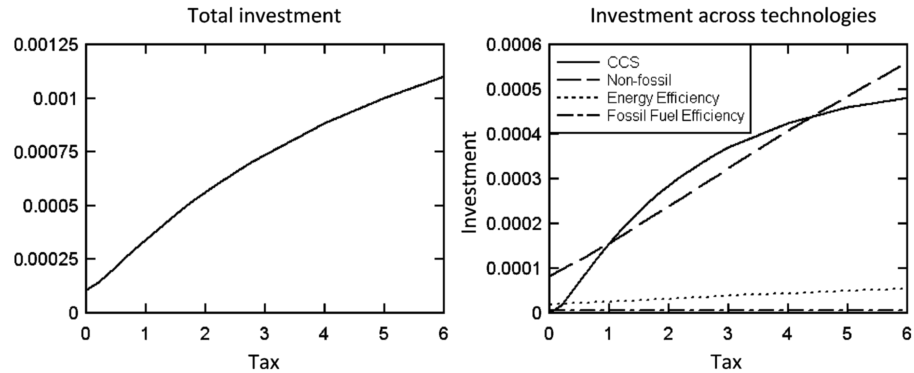

Fig. 4. Elasticity of substitution between fossil and nonfossil energy is 1.5 and the cost coefficients of the efficiency and fossil improvement technologies are 100 times the other programs.

coefficients for both efficiency programs ($\kappa = 100$) are higher than for the other two programs ($\kappa = 1$). We see that overall investment is increasing in the tax, similar to the results in Figs. 2 and 3. A comparison of Figs. 2 and 4 with Figs. 1 and 3 shows that overall investment is lower when efficiency programs are costlier. In a similar manner, a comparison between Figs. 1 and 2 with Figs. 3 and 4 shows that the overall level of spending is significantly higher when elasticity is higher. Firms that are flexible (i.e., have a high elasticity) can substitute toward low carbon inputs as the carbon tax increases. But firms that are less flexible do not have this option; therefore, an increasing carbon tax leads to reduced output. This, in turn, means that firms optimally invest less in any technology.

C. Observations

These specific examples imply two findings. First, the qualitative response of the optimal investment in a portfolio of technologies to an increasing carbon tax depends on the relative costs of the individual programs and the elasticity of substitution between fossil and nonfossil energy inputs. Second, nonfossil and CCS programs act as substitutes, with the investments in CCS first increasing in a carbon tax until the investment benefit is outweighed by the increasing tax. Beyond this threshold, it decreases in tax, and nonfossil increases at a more rapid rate, as the firm substitutes away from the use of fossil technology toward one that uses nonfossil input.

We capture investment behavior in these programs with the following propositions. In the first, we show that as long as the cost of R&D is equivalent for all programs, the investment in energy efficiency programs will always be higher than fossil efficiency, which in turn will be higher than CCS.

Proposition 1: Assume that a portfolio of technologies with equal cost coefficients κ includes efficiency programs, fossil fuel improvement, and CCS technologies (as defined in Section III-C), and that the marginal cost of investment is finite at maximum abatement $g'(1) < \infty$; then the following relation holds between the optimal levels of R&D in energy efficiency, fossil fuel efficiency, and CCS technologies, i.e., $\alpha_E^* \geq \alpha_F^* \geq \alpha_C^*$ for all t.

Proof: Note that the following identities hold at the optimal investment levels, assuming there is no corner point solution. From the FOCs in (5):

$$g'(\alpha_E^*) = E_t[(P_c + t)\varepsilon_c^* + P_{nc}\varepsilon_{nc}^*] \quad (9)$$

$$g'(\alpha_F^*) = E_t[(P_c + t)\varepsilon_c^*]; \quad g'(\alpha_C^*) = E_t[t\varepsilon_c^*] \quad (10)$$

$$g'(\alpha_F^*) - g'(\alpha_C^*) = E_t[P_c\varepsilon_c^*] \geq 0 \text{ and}$$

$$g'(\alpha_E^*) - g'(\alpha_F^*) = E_t[P_{nc}\varepsilon_{nc}^*] > 0 \quad (11)$$

which implies that $g'(\alpha_E^*) \geq g'(\alpha_F^*) \geq g'(\alpha_C^*) \forall t$. This implies $\alpha_E^* \geq \alpha_F^* \geq \alpha_C^*$. If there is a corner point solution, i.e., $g'(\alpha) <$ RHS when $\alpha = 1$, the same result still holds. ■

This result is consistent with our earlier observation that under equivalent cost structures for the technologies, it is optimal to target investments at efficiency programs. This is even more relevant if there is a limitation on investment budget. In the next proposition, we show that optimal investments in nonfossil and CCS technologies are equal when the carbon tax is equal to the premium on nonfossil inputs.

Proposition 2: Assume that the cost coefficient is the same across the technologies and the tax level t is known. Then, the optimal R&D investment in nonfossil and CCS technologies are equal when η, the premium on nonfossil energy, is equal to t.

Proof: From the FOCs in (5) and with known t:

$$g'(\alpha_C^*) = t\varepsilon_c^* \quad \text{and} \quad g'(\alpha_A^*) = \eta\varepsilon_{nc}^*. \quad (12)$$

If $t = \eta$, then $P_c = P_{nc}$, and from (A2) and (A3) in the Appendix, we see that $\varepsilon_c^* = \varepsilon_{nc}^*$ and the right-hand sides of both equations in (12) are equal, implying that $\alpha_C^* = \alpha_A^*$. ■

D. Design of Experiments

The earlier analysis suggests that the qualitative impacts of an increase in the carbon tax on the overall portfolio depend precisely on the values of the parameters. Thus, in this section, we present an in-depth analysis of how the parameters influence the results. We vary five parameters, using a design of experiments approach: the R&D cost coefficients k_i of the four technologies, and the elasticity of substitution between fossil and nonfossil energy inputs. Each parameter has two levels, and thus, we have $2^5 = 32$ experiments. We create a design of ex-

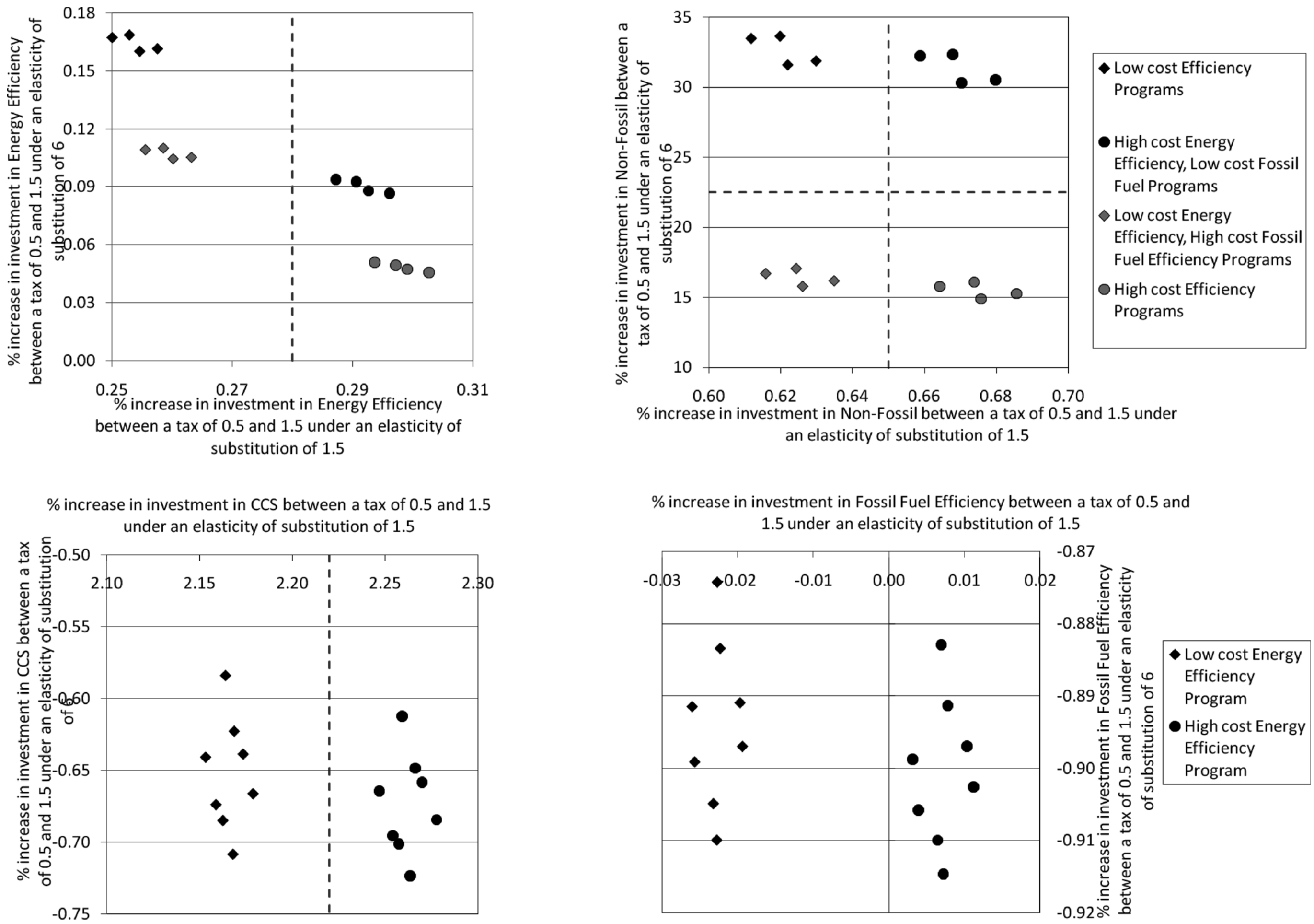

Fig. 5. Percent increase in optimal investments. We use a high cost coefficient of 100 and a low of 1. For the two levels of elasticity, we use 1.5 for low and 6 for high.

periments matrix for these factors. For the two levels of the cost coefficient factors, we use a high coefficient of 100 and a low coefficient of 1. For the two levels of elasticity, we use 1.5 for low and 6 for high.

Fig. 5 shows the results for the four programs in response to the experimental trials. For each experimental run, we find the percent change in optimal R&D investment given a change in the known tax from 0.5 to 1.5: $(\alpha^*(1.5) - \alpha^*(0.5))/\alpha^*(0.5)$, where $\alpha^*(t)$ is the optimal level of R&D, given tax t. In the figure, we show this quantity under a high elasticity on the vertical axis and the quantity under a low elasticity on the horizontal axis. Each point represents a particular combination of cost coefficients. There are 16 possible combinations of high and low cost coefficients. Each of the panels shows these results for a different R&D program. These can be interpreted as follows. A point in the upper left-hand corner of one of the graphs implies that the optimal investment increases significantly in a carbon tax when the elasticity is high, but has a small increase when the elasticity is low; and a point in the lower right-hand corner implies that the optimal investment increases more in a carbon tax under low elasticity, but has a small increase when elasticity is high. Note that some of the scales are in the negative range, implying that in some cases, the optimal investment decreases in the tax. Moreover, the range of the scales are quite different, providing information on which technologies are most strongly effected by changes in parameters. In particular, note that the highest percentage increases are seen for nonfossil R&D when the elasticity is high followed by CCS R&D when the elasticity is low. The biggest decreases are seen for fossil efficiency R&D when the elasticity is high followed by CCS R&D when the elasticity is low.

The axes in the energy efficiency graph in Fig. 5 indicate that the percentage increase is always higher when the elasticity is low than when it is high. The intuition for this is that energy efficiency program is valuable when the tax is high and the flexibility is low since it reduces the cost of both inputs. This figure shows four distinct clusters. The two clusters on the left side represent cases where the increase in R&D investment is relatively higher under a high elasticity; the clusters on the right represent cases where the increase in R&D investment is relatively higher under a low elasticity. All the points in the two clusters on the left have a low R&D cost for energy efficiency technologies. An increase in the R&D cost of energy efficiency technologies leads energy efficiency R&D to be more responsive to an increase in the tax when elasticity is low, but less responsive when elasticity is high. Optimal investments in energy efficiency R&D increase most strongly with an increase in the tax when the cost of the R&D program is high, and firm flexibility is low.

As mentioned earlier, the nonfossil fuel graph on the top right of Fig. 5 shows a significant increase in optimal investment in this technology when the firm is highly flexible; this is most pronounced, interestingly, when the R&D costs of fossil efficiency programs are low (the two clusters of black points). When R&D cost for fossil fuel efficiency is low, the investment in nonfossil is crowded out when the tax is low; when the tax increases, investment in fossil fuel efficiency investment drops off quickly, and is replaced by large investments in nonfossil. The R&D costs of energy efficiency influence the investment in nonfossil fuel program when substitution elasticity is low—leading to a larger increase when the cost of energy efficiency improvement is high—but not when elasticity is high.

The lower left panel shows that CCS has the greatest sensitivity to the elasticity, with significant *increases* in the tax when the elasticity is low, and large *decreases* in the tax when the elasticity is high. This can be explained as follows. When the firm is not flexible, it implies that they have a very hard time substituting away from fossil energy, even as the carbon tax gets very high. This makes CCS a very nice alternative as the tax gets high, allowing the firm to continue to use large amounts of fossil energy without paying the tax. On the other hand, when the firm is flexible, it will optimally substitute away from fossil toward nonfossil as the tax gets high. Thus, R&D into CCS gets less appealing with a very high tax. The two clusters that can be observed in the CCS graph of Fig. 5 are differentiated by the R&D cost coefficient for energy efficiency technology. When energy efficiency R&D is more expensive (and flexibility is low), CCS is an even more attractive investment as the tax gets high. Comparing the CCS graph with the nonfossil graph, we see that an increasing tax is an incentive for investment in nonfossil fuel technology when input substitution is high, whereas it favors improvements in CCS technology when flexibility is low.

The fossil fuel efficiency graph in the bottom right of Fig. 5 shows that a high elasticity generally reduces investment as the tax increases for this technology. On the other hand, optimal investment response is ambiguous when the elasticity is low: when the cost of the energy efficiency program is high, optimal investment in fossil fuel efficiency is increasing in tax; otherwise, it decreases. Thus, fossil fuel R&D increases in a tax only when the firm is less flexible and the R&D cost of general energy efficiency is high.

The foregoing show that the key drivers of investment behavior are the elasticity of substitution and the R&D cost of the energy efficiency program, followed by the R&D cost of the fossil fuel efficiency program. The relative costs of nonfossil and CCS R&D programs have very little effect on the impact of an increasing carbon tax. When the cost of an energy efficiency program is low, then we are always on the left side of the graphs, meaning that when elasticity is low, all of the programs are more responsive to an increase in the carbon tax than if the cost of an energy efficiency program is high. In the case of the energy efficiency program, this can be explained by the relatively small investment into energy efficiency when the cost coefficient is high and the tax is low. Therefore, there is a large percentage increase when the tax goes up. For the other technologies, it is a substitution effect: when the R&D cost of energy efficiency is low, it makes up the bulk of R&D spending; when it is high, the other technologies can take up some of the slack.

All programs increase in a carbon tax when elasticity is low and the cost of energy efficiency is high, with CCS having, by far, the largest percentage increase. When elasticity is high, then CCS and fossil fuel efficiency decrease in a carbon tax, regardless of the relative cost coefficients. Both energy efficiency improvement and nonfossil fuel technologies increase when fossil efficiency is expensive. However, when elasticity is low, investments in both CCS and fossil fuel efficiency programs increase when energy efficiency program becomes costly.

Overall, policy makers saddled with the task of determining the strategies for an optimal energy economy must incorporate the following three factors in their decision making.

1) The makeup of the future landscape of energy technologies is dependent on the price of carbon.
2) The mix of energy technologies meeting demand and total emissions stabilization levels must consider the relative ease of transitions between the technologies.
3) The stringency of policy prescriptions must take into consideration the costs of R&D into these technologies.

E. R&D Investments and Increasing Risk

In this section, we briefly discuss the effect of increasing risk on optimal investment. We define an increase in risk to be a mean-preserving spread (MPS). In Fig. 6, we compare the optimal investment levels in each technology under a certain tax of 1 with the optimal investment levels given a probability of 0.25, 0.5, and 0.25 over taxes of 0, 1, and 2, respectively, and given a 50–50 chance of a carbon tax of 0 or 2. The first situation is least risky, while the last is the most risky. Rothschild and Stiglitz [31] show that any increase in risk can be obtained by a sequence of such MPSs.

In the figure, the slices show the percentage of the total investment that is in each technology, and the decimal values refer to the absolute amount invested in that technology. For these particular MPSs, total optimal investment decreases with increasing risk under the set of parameters in Table I, with elasticity equal to 6. The investment in the nonfossil fuel program, however, increases in risk, in terms of both the proportion of the total investment and the absolute amount of the investment. All the other technologies decrease in absolute value in investment, with CCS decreasing the most. Across this range of increasing risk, the total proportion between CCS and nonfossil is relatively constant, but with nonfossil dominating over CCS.

The effect of uncertainty depends on the elasticity of substitution. In Fig. 7, we show the effect of risk on optimal portfolio investment for low elasticity, and observe that the optimal investment level in energy efficiency program is relatively flat in riskiness, in contrast to higher proportional increases under a high elasticity in Fig. 6. Similarly, in Fig. 7, optimal investment in fossil fuel efficiency improvement increases slightly with

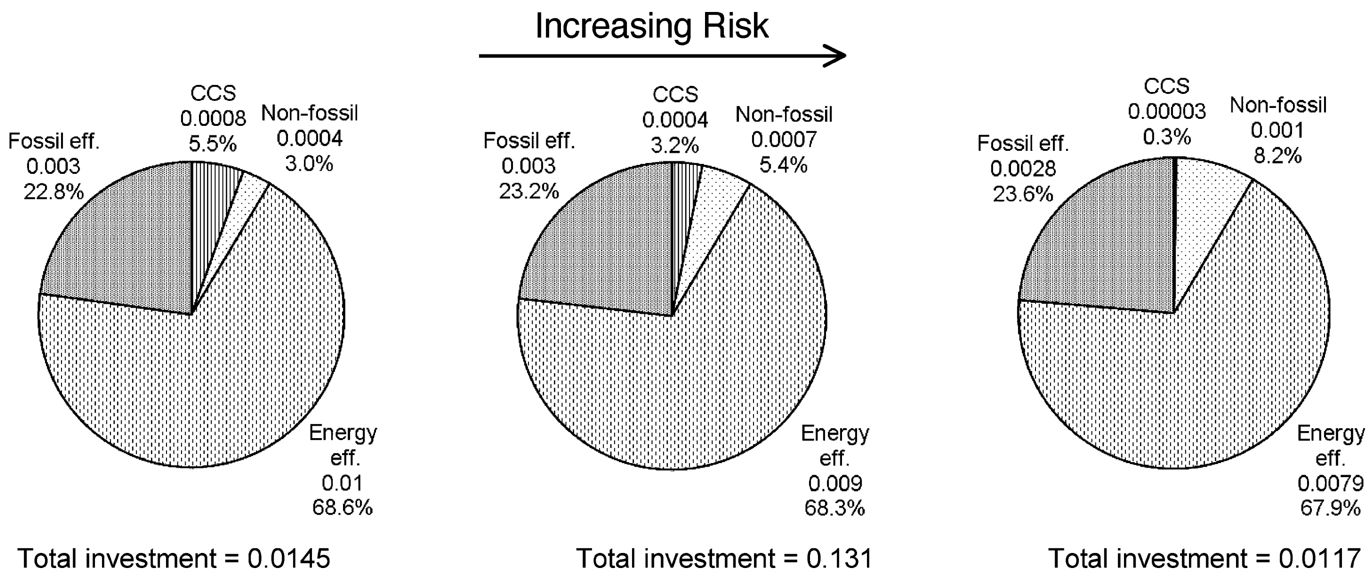

Fig. 6. (Left) Optimal investment distribution in the technologies at a tax of 1. (Middle) MPS of 0.25, 0.5, and 0.25 over taxes of 0, 1, and 2, respectively. (Right) MPS of 50–50 chance of $tax = 0$ and $tax = 2$. The elasticity of substitution between fossil and nonfossil energy is 6, while between energy and nonenergy inputs is 0.75. The price demand elasticity is 1.1.

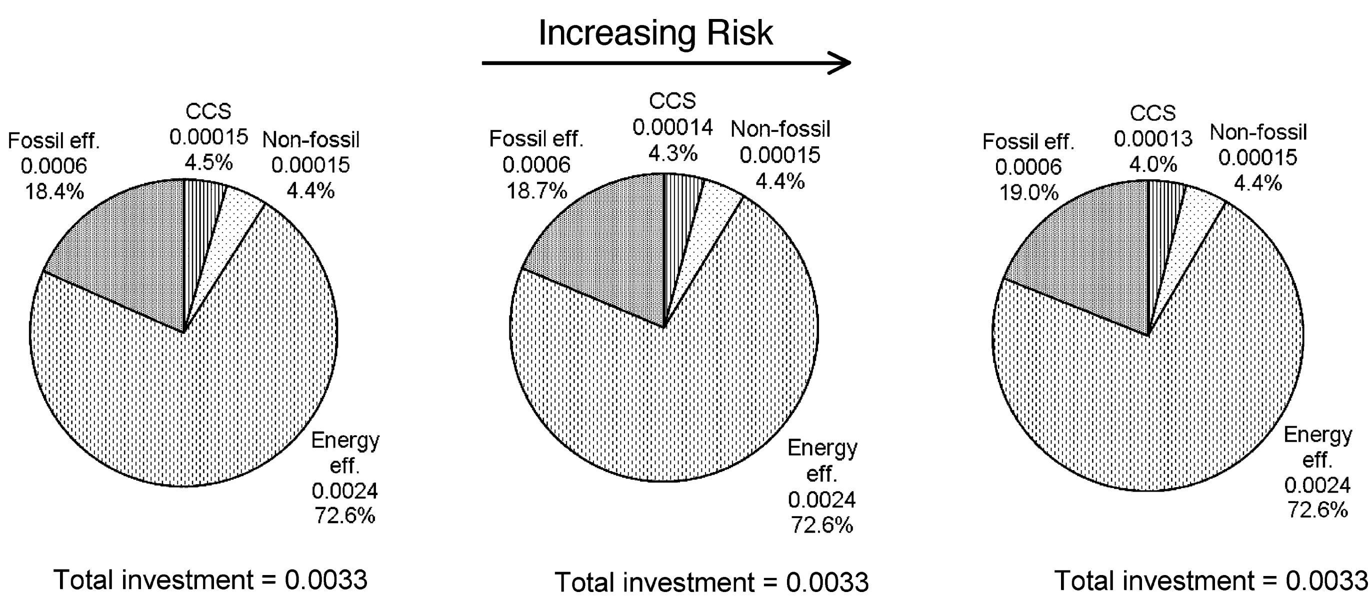

Fig. 7. Effect of increasing risk on optimal portfolio investment. All parameters have same values as in Fig. 6 except substitution elasticity between fossil and nonfossil fuel is low here, i.e., 1.5.

investment in CCS reducing in risk under low flexibility, but it is less than that under high flexibility. It is also evident from these examples that risk has a much smaller overall effect on the optimal investment when the elasticity of substitution is low than when it is high. This is because a firm that is less flexible is already confined to whatever technologies exist in its portfolio, and thus, there is a weak influence of tax uncertainty on the investment levels in the portfolio. However, this result assumes equal cost coefficients for the technologies.

In order to understand the impacts of risk, consider the shapes of the curves in Figs. 1–4. Recall that the expected value of a concave function decreases in risk, while the expected value of a convex function increases; if a function is neither concave nor convex, then the expected value will increase for some increases in risk and decrease for other decreases in risk [32]. Note that in Figs. 1, 2, and 4, overall investment is mostly concave in the tax; thus, we would expect most MPS to lead to a decrease in investment. Similarly, all the programs with the exception of nonfossil fuel program are mostly concave. Specifically, the fossil-based technologies are valuable only when the tax has an intermediate value—neither too low nor too high; thus, investment in these technologies is particularly low when the tax is most risky, i.e., when it will either be quite low or quite high. Nonfossil fuel technology, however, is S-shaped, with a convex region between 0 and 1. This is what leads to the higher investment under uncertainty. Thus, an MPS around a tax of 0.75 leads to an overall increase in investment, since the overall investment is convex in that region.

V. Conclusion

We consider a portfolio of R&D options in terms of reducing the effective cost of inputs. We distinguish between the R&D programs based on their influence on the demand for inputs, which, in turn, drives the effective price of inputs, and thus, the optimal investment level in each of them. We find that R&D investment behavior is influenced by the relative cost of R&D into efficiency programs and the firm's flexibility in substituting between fossil and nonfossil energy inputs.

One of the key drivers of investments in these technologies is the costs of energy efficiency and fossil fuel efficiency programs. For example, when the cost of an energy efficiency program is low and elasticity is low, the impact of an increasing tax is small. Moreover, we show that an increasing tax is an incentive for investment in nonfossil fuel technology when firms are flexible, whereas it favors improvements in CCS technology when firms are less flexible. Overall investment appears to be highest when the carbon tax is high enough to provide incentives for using CCS, but not so high that firms start to substitute away from fossil fuel energy significantly.

The elasticity of substitution between energy inputs is crucial in determining the optimal investment profile. For a firm with a high substitution elasticity between fossil and nonfossil energy, the optimal investment in nonfossil and CCS programs exhibit ambiguous traits—initially, the investments in CCS exceeds that of nonfossil at low tax levels, but at higher levels, the converse is true. This ambiguous response in investment can be attributed to a number of factors. For example, investment in the CCS program increases in a carbon tax to offset the influence of the tax, but at high tax levels, these investments reduce since it is more economical to focus solely on nonfossil programs that are not influenced by the carbon tax.

On the other hand, when short-term elasticity of substitution between fossil and nonfossil energy inputs is low, the carbon tax does not significantly influence the investment level in nonfossil energy programs. Overall portfolio investment reduces in the tax when the firm's ability to substitute away from fossil-related inputs is limited. Uncertainty in the tax decreases the overall optimal investment in the portfolio for a firm with enough flexibility in its use of fossil and nonfossil energy—as observed with nonfossil technology substituting for CCS at considerably high tax levels. Surprisingly, an increase in risk has a much smaller effect on firms that are less flexible than highly flexible firms.

In summary, the contribution of this paper is twofold. First, it provides some insight to firms in terms of R&D investment in energy technologies, as follows:

1) Investments in efficiency that are independent of fuel type are the best.
2) In the case where efficiency is very expensive to attain, overall energy R&D investments should be fairly low when the expected tax is low, and become very large only when the expected tax gets large (in terms of making nonfossil fuel competitive with fossil energy).
3) Given the current uncertainty about a future tax, it looks like optimal R&D investments should be relatively small. However, it appears that a nonfossil fuel program can be a hedge against uncertainty, and thus, more attention should be spent on this technology than would be under uncertainty.

Second, this analysis provides insights to policy makers concerned about setting a carbon tax and crafting R&D policy. It appears that reducing uncertainty will increase investment. Also, it suggests that understanding the distribution of elasticities across energy firms may be very useful for predicting endogenous technical change in response to a carbon tax.

Appendix

Optimal Demand for Carbon and Noncarbon Energy

First, solve the energy subproblem (let ε represent the total energy demand) by considering the cost minimization problem $\min P_c\varepsilon_c + P_{nc}\varepsilon_{nc}$, s.t. $\varepsilon_c^\gamma + \varepsilon_{nc}^\gamma = \varepsilon^\gamma$. Taking FOCs, we get

$$\varepsilon_c = P_c^{1/(\gamma-1)}\left[P_c^{\gamma/(\gamma-1)} + P_{nc}^{\gamma/(\gamma-1)}\right]^{-1/\gamma}\varepsilon$$

and

$$\varepsilon_{nc} = P_{nc}^{1/(\gamma-1)}\left[P_c^{\gamma/(\gamma-1)} + P_{nc}^{\gamma/(\gamma-1)}\right]^{-1/\gamma}\varepsilon. \quad \text{(A1)}$$

Now consider the original problem; rewriting: $\min wx + P_c\varepsilon_c + P_{nc}\varepsilon_{nc}$ by substituting both equations in (A1), we have $\min wx + [P_c^{\gamma/(\gamma-1)} + P_{nc}^{\gamma/(\gamma-1)}]^{(\gamma-1)/\gamma}\varepsilon$, subject to $y = [x^\rho + (\varepsilon_c^\gamma + \varepsilon_{nc}^\gamma)^{\rho/\gamma}]^{1/\rho}$. Substituting and taking FOCs, we have

$$\varepsilon_c = P_c^{1/(\gamma-1)}\left(P_c^{\gamma/(\gamma-1)} + P_{nc}^{\gamma/(\gamma-1)}\right)^{(\gamma-\rho)/[\gamma(\rho-1)]} \times \left[w^{\rho/(\rho-1)} + \left(P_c^{\gamma/(\gamma-1)} + P_{nc}^{\gamma/(\gamma-1)}\right)^{[\rho(\gamma-1)]/[\gamma(\rho-1)]}\right]^{-1/\rho} y \quad \text{(A2)}$$

$$\varepsilon_{nc} = P_{nc}^{1/(\gamma-1)}\left(P_c^{\gamma/(\gamma-1)} + P_{nc}^{\gamma/(\gamma-1)}\right)^{(\gamma-\rho)/[\gamma(\rho-1)]} \times \left[w^{\rho/(\rho-1)} + \left(P_c^{\gamma/(\gamma-1)} + P_{nc}^{\gamma/(\gamma-1)}\right)^{[\rho(\gamma-1)]/[\gamma(\rho-1)]}\right]^{-1/\rho} y. \quad \text{(A3)}$$

A cost function is given as follows:

$$c(y) = y\left[w^{\rho/(\rho-1)} + \left(P_c^{\gamma/(\gamma-1)} + P_{nc}^{\gamma/(\gamma-1)}\right)^{[\rho(\gamma-1)]/[\gamma(\rho-1)]}\right]^{(\rho-1)/\rho}.$$

Now, consider the monopolist's profit-maximization problem: $\max yP(y) - c(y)$, where $P(y) = Ay^{-1/b}$. Hence, substituting, taking FOC, and solving for y gives

$$y = \left(\frac{b}{b-1}\frac{1}{A}\right)^{-b} \left[w^{\rho/(\rho-1)} + \left(P_c^{\gamma/(\gamma-1)} + P_{nc}^{\gamma/(\gamma-1)}\right)^{[\rho(\gamma-1)]/[\gamma(\rho-1)]}\right]^{[b(1-\rho)]/\rho}.$$

Substituting this y into (A2) and (A3), we have the expressions in (2) and (3).

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