

# Optimal R&D in Response to a Random Carbon Tax\*

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## **Abstract**

This paper determines a firm's optimal R&D response to an uncertain carbon tax, for two different R&D programs: cost reduction of low carbon energy technologies and emissions reductions of currently economic technologies. We find that optimal R&D does not increase monotonically in a carbon tax. Cost reduction R&D increases only if the firm is flexible enough; emissions reduction R&D first increases then decreases in a carbon tax. Firms that are very flexible may increase cost reduction R&D when the uncertainty surrounding a carbon tax is increased; otherwise firms will generally decrease R&D investment in uncertainty.

# 1 Introduction

A major topic in the discussion surrounding climate change policy is the role of induced technical change – technological innovation induced by climate policies. Understanding induced technical change is important both for choosing an emissions abatement policy and for crafting technology policy. In fact, assumptions about technological progress appear to be the most important determinant of the costs of global warming policy. Another central issue when considering optimal policies for global climate change is the presence of uncertainty and learning about climate damages. There is much written about the impact of uncertainty on optimal abatement. The results from this literature indicate that the optimal response is to go slow, and policy-makers in the U.S. seem to have taken this to heart, and are going slow indeed. Thus, there are no carbon-related policies to induce technical change at this time. Nevertheless, we see at least some firm-supported R&D: some firms are responding to the possibility of future policies. Understanding firm response to uncertain future policies is critical in climate change for crafting climate policies, especially technology policies.

In this paper we examine a firm’s optimal R&D investment in response to a future uncertain carbon tax. A common assumption in the induced technical change literature ([11] [1] [6] [14]) is that a carbon tax will induce environmental R&D. Nevertheless, Sue Wing [17] has shown that this may not be true in a general equilibrium setting, and Farzin and Kort [3] show that the optimal level of environmental R&D can be non-monotonic in an emissions tax. We confirm and expand upon these findings in a general micro-economic

framework. Moreover, a rational, forward-looking firm considering long-term R&D projects should base its decisions on beliefs about future policies, rather than simply react to current policies. Thus, understanding the role of uncertainty and learning is key to understanding firms' R&D investments. Understanding and modeling privately provided R&D is important for both abatement policy and for technology policy.

We consider how changes in the probability distribution of a carbon tax will impact optimal R&D investment. We compare two different R&D programs – the first to lower the cost of low-carbon energy, the second to reduce emissions of the currently economic alternative.<sup>1</sup> We find that these two programs have qualitatively different responses to an increase in a carbon tax. We find, for both programs, that the optimal investment into R&D can decrease with increases in a carbon tax. Investment into cost reduction R&D will increase with a carbon tax only if the firm is flexible enough. Unless zero-emissions technology is optimal, investment into emissions reduction will first increase and then decrease in a carbon tax. Additionally, we analyze how an increase the riskiness (in the Rothschild & Stiglitz sense of a mean-preserving spread [15]) of the carbon tax impacts optimal firm-level R&D spending. We find that, usually, R&D into reducing emissions will decrease in uncertainty. When considering R&D into reducing the cost of low-carbon technology, our analysis suggests that the results depend on the elasticity of substitution between carbon

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<sup>1</sup>Examples of the first program are to lower the cost of solar PVs or wind generation. The second program represents investments into carbon capture and sequestration, but the results are qualitatively the same for programs that aim to increase the efficiency of carbon based energy.

and non-carbon energy. For less flexible firms, R&D decreases unambiguously in uncertainty about a carbon tax. For very flexible firms, R&D will increase as the probability that the carbon tax is "high" increases, but will decrease in the magnitude of the high carbon tax.

We differ from the previous literature in a number of ways. Most of the literature on technical change uses an abstract abatement cost function, and makes assumptions about how technical change will impact this cost function ([2],[4],[6],[7],[9],[10],[12]). Most prominently, the general assumption is that environmental technical change will monotonically decrease the marginal cost of abatement. It is not difficult, however, to find examples where this is not true. For example, consider R&D into improving the efficiency of coal-powered electric generators. If successful, this R&D will decrease the marginal cost of abatement for low levels of abatement. But, since this R&D has no impact on the cost of non-carbon alternatives, it will have no effect on the cost of full abatement, and thus will increase the *marginal cost* of abatement over some range. Since climate change has the potential to be catastrophic it is relevant to consider the whole range of abatement, not just a subset.

We use a profit function approach, with an underlying production function that includes carbon energy, non-carbon energy, and non-energy inputs. The first program we consider lowers the cost of non-carbon energy; the second program lowers the carbon intensity of carbon energy. Both programs lead to increases in the marginal cost of abatement under at least some assumptions and over some ranges. We argue that these programs are more realistic than an abstract technology that lowers the marginal cost of abatement. Additionally, the

profit function approach allows for both input substitution and output reductions. Previous papers based on a production-function approach [3] have allowed for output reduction, but not substitution, while other papers based on a cost function approach [4] have allowed for substitution, but not output reduction. Evidence suggests that the majority of abatement results from input substitution [5]. Yet, at very high levels of abatement, it is likely that output reduction will play a role. When considering uncertainty, there is a positive probability of a very high tax (if damages turn out to be catastrophic) and thus it is important not to assume the possibility of reduced output away.

Finally, the focus in this paper is how R&D responds to the stringency of a policy, rather than on comparing the impacts of alternate policy instruments ([2],[4],[7],[9],[10],[12]) or on the bias that a policy induces (e.g. Magat [8]). Our framework can be used to compare policy instruments, but this paper is primarily intended to improve our understanding of induced technical change.

The two papers that are most closely related to this one are Montero [10] and Farzin and Kort [3]. Montero considers a 3-stage game in which Cournot competitors choose an R&D investment into cost reduction, then choose an emissions level, and finally choose output – thus allowing for the possibility of both substitution and output reduction. He uses a profit function approach to compare the firms’ incentives to invest in R&D across policy instruments including standards and permits. Our study differs from his by allowing for technical change that does not monotonically decrease marginal cost, and considering the

qualitative response of R&D to an increase in the stringency of a policy as well as to an increase in risk. Farzin and Kort use a fully dynamic, optimal control model to consider the impact of both the timing and magnitude of an uncertain tax on the level of investment into abatement technology. They consider emissions-reduction technology and conclude that investment will increase up to a threshold tax, and then decrease; and that the presence of uncertainty discourages investment into abatement technology. While our model is restricted to two periods, we generalize their results by allowing for ex-ante substitution and corner point solutions, by considering different types of R&D, and by considering the impacts of general increases in risk.

In the next section we present the model and the logic of our analysis. We take a decision-theoretic approach: we set up the firm's decision problem and analyze how the optimal investment into R&D changes with changes in the distribution of a carbon tax. We then consider separately two different R&D programs: Cost Reduction of non-carbon energy is discussed in Section 3 and Emissions Reduction is discussed in Section 4. Each of these sections is broken into 3 subsections. We begin each section by introducing our assumptions about technical change and briefly discuss how these programs will lead to increased marginal abatement costs under some conditions, in opposition to the assumptions made in much of the literature. In Subsections 3.1 and 4.1 we analyze the optimal response of each R&D program to an expected increase in the carbon tax. In Subsections 3.2 and 4.2 we consider the impact of an increase in risk on each of the programs. In Section 5 we extend the analysis

to efficiency programs. Section 6 concludes.

## 2 Firm-level decisions about R&D

We consider a hypothetical firm that both produces and uses R&D. The firm also produces carbon emissions as a joint product with output and thus is responsible for the carbon tax. An example of such a firm is American Energy Producers. Based out of Columbus, Ohio they are the largest electricity generator in the United States, and they have a significant R&D budget, including environmental R&D. We do not consider diffusion of R&D, thus this may be considered a model of R&D on firm-specific techniques.

We consider policies that impact the price of carbon such as a carbon tax or a cap-and-trade policy. We assume that there is no price on carbon currently, but that – depending on both scientific and policy variables – there may be a carbon tax in the future. For this study we assume that the probability distribution of the carbon tax is independent of the investment in R&D – the firm is a "policy taker." We assume that decisions about R&D must be made now, so that the changes in the production function can be implemented when the actual carbon tax is realized. We assume that the returns to R&D are deterministic.

We use a profit function approach within a two-period theoretical model. Investments in R&D are made in the first period under uncertainty about a carbon tax. For simplicity, we ignore production in the first period. Optimal production is chosen in the second period after learning has taken place and technical change has been achieved, leading to a second period

profit function. We take the market structure for output to be exogenous – the firm either faces a competitive price for its output or faces a known downward sloping demand curve. Furthermore, while some of the analysis is valid for a price-taking firm, we do not consider equilibrium issues, so most of the results should be considered in the light of a monopolist with a downward sloping demand curve.

Consider a firm that uses three inputs – non-energy inputs  $x$ , carbon energy inputs,  $e_c$  and non-carbon energy inputs,  $e_{nc}$ . Let  $e_c$  be normalized so that, using the current technology, one unit of carbon energy produces one unit of emissions. Then the total firm-specific price paid for carbon energy is the levelized cost of the fuel<sup>2</sup>,  $p_c$ , plus the price of the carbon emitted,  $t$ . Assume that, under the current technology, non-carbon energy is more expensive than carbon energy: the levelized firm-specific price of non-carbon energy equals  $p_c + \eta$ . The price of non-energy inputs is  $w$ . Finally, let  $\alpha$  represent the amount of technical change. Thus, the firm’s second period profit function, assuming that the carbon tax,  $t$ , is known, is  $\pi(w, p_c + t, p_c + \eta; \alpha)$ .

Let  $g(\alpha)$  represent the cost of technical change, and assume that it is increasing and convex. The firm chooses the level of  $\alpha$  in the first period, when the carbon tax is still unknown. In the second period the firm learns the value of the carbon tax  $t$  and maximizes

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<sup>2</sup>Including operating and maintenance cost of the production machinery.

profit accordingly. The firm's two period problem is:

$$\max_{\alpha} -g(\alpha) + \mathbb{E}_t \pi(w, p_c + t, p_c + \eta; \alpha) \quad (1)$$

The first order condition for  $\alpha$  is

$$g'(\alpha) = \mathbb{E}_t \left[ \frac{\partial \pi}{\partial \alpha} \right] \quad (2)$$

The question we ask is, how does the optimal amount of R&D spending change with changes in the distribution of  $t$ ? If the right-hand side of (2) increases, then the optimal level of R&D spending increases. Thus we will focus on how a change in the probability distribution of  $t$  impacts the expected marginal increase in profits due to an increase in R&D. In Subsections 3.1 and 4.1 we consider increases in the expected value of  $t$ , while in Subsections 3.2 and 4.2 we consider a mean-preserving spread of  $t$ .

### 3 Cost reduction of non-carbon alternatives

In this section we consider R&D into reducing the cost of non-carbon alternatives (denoted CR for *cost reduction*). We model CR as reducing the premium  $\eta$  for non-carbon energy by a fraction  $\alpha$ . Thus the second period profit function is

$$\pi(w, p_c + t, p_c + (1 - \alpha)\eta) \quad (3)$$

This program could represent, for example, a firm's research into improving the efficiency of their wind turbines. Note that even this program does not necessarily reduce the marginal cost of abatement. If, for example, the firm had a low elasticity of substitution between carbon inputs and non-carbon inputs, then abatement would be achieved primarily through output reduction. Reducing the cost of non-carbon energy would then increase the marginal cost of abatement.

Under CR the right-hand side of (2) simplifies to:

$$\mathbb{E}_t \left[ \frac{\partial \pi}{\partial \alpha} \right] = -\eta \mathbb{E}_t \left[ \frac{\partial \pi}{\partial p_{nc}} \right] \quad (4)$$

where  $p_{nc}$  represents the total price of non-carbon energy. Applying Hotelling's lemma, the first order condition (2) simplifies to

$$g'(\alpha) = \eta \mathbb{E}_t [e_{nc}^*] \quad (5)$$

where  $e_{nc}^*$  represents the unconditional demand for non-carbon energy. The marginal benefit from CR is perfectly proportional to the demand for non-carbon energy. If demand for non-carbon energy increases in the carbon tax, this implies that optimal investment in CR increases in the expected value of the carbon tax.<sup>3</sup> Similarly, if the expected unconditional demand for non-carbon energy increases in uncertainty, then optimal investment in CR will

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<sup>3</sup>Assuming an interior solution. We show below, however, that if the interior solution is monotone increasing, then the overall optimal solution is also increasing.

increase in uncertainty.

### 3.1 Increase in the expected value of a carbon tax: CR

How will the demand for non-carbon energy respond to an increase in the price of carbon energy? This depends on two, opposite, effects. On the one hand, *conditional* demand (i.e. holding output constant) for non-carbon energy increases in the price of carbon energy; this is the substitution effect. On the other hand, output decreases in the price of an input. Which of these effects dominates depends on the relative values of the elasticity of substitution between carbon and non-carbon energy and the price elasticity of demand the firm is facing. The higher the elasticity of substitution, the more likely that total demand for non-carbon energy will rise in the carbon tax. On the other hand, a higher price elasticity of demand for the firm's output makes it more likely that non-carbon energy will decrease in the carbon tax.

In fact, we show below, that for a monopolist with a nested CES production function facing a CES demand curve, R&D will unambiguously rise in the carbon tax if the elasticity of substitution between carbon and non-carbon energy is higher than both the elasticity of demand and the elasticity of substitution between energy and non-energy.

### 3.1.1 Constant elasticity production and demand

Consider a monopoly with a nested CES production function and constant elasticity demand.

$$f(x, e_c, e_{nc}) = \left[ x^\rho + (e_c^\gamma + e_{nc}^\gamma)^{\frac{\rho}{\gamma}} \right]^{\frac{1}{\rho}} \quad (6)$$

Define  $\zeta \equiv \frac{1}{1-\rho}$  to be the elasticity of substitution between energy and non-energy inputs and  $\sigma \equiv \frac{1}{1-\gamma}$  as the elasticity of substitution between carbon and non-carbon energy inputs. Let inverse demand be  $Ay^{-\frac{1}{b}}$ . To simplify the notation let the total price of carbon inputs be represented by  $\tilde{p}_c$  and the total price of non-carbon inputs be represented by  $p_{nc}$ . Then the unconditional demand for non-carbon inputs is

$$e_{nc}^* = p_{nc}^{\frac{1}{\gamma-1}} \left[ \tilde{p}_c^{\frac{\gamma}{\gamma-1}} + p_{nc}^{\frac{\gamma}{\gamma-1}} \right]^{\frac{\gamma-\rho}{(\rho-1)\gamma}} \left( w^{\frac{\rho}{\rho-1}} + \left[ \tilde{p}_c^{\frac{\gamma}{\gamma-1}} + p_{nc}^{\frac{\gamma}{\gamma-1}} \right]^{\frac{\rho}{\rho-1} \frac{\gamma-1}{\gamma}} \right)^{\frac{-1-\rho b+b}{\rho}} \left( \frac{b}{b-1} \frac{1}{A} \right)^{-b} \quad (7)$$

**Proposition 1** *If the elasticity of substitution between carbon and non-carbon energy is higher (lower) than both the elasticity of substitution between energy and non-energy and the price elasticity of demand, then R&D into reducing the cost of non-carbon energy increases (decreases) in the carbon tax.*

**Proof.** In the appendix we show that the derivative of (7) with respect to  $\tilde{p}_c$  (and therefore with respect to  $t$ ) is positive for all values of  $w, p_c$ , and  $p_{nc}$  if  $\gamma > \bar{\gamma}$  where

$$\bar{\gamma} = \max \left[ \rho, \frac{b-1}{b} \right] \quad (8)$$

and is negative if  $\gamma < \underline{\gamma}$  where

$$\underline{\gamma} = \min \left[ \rho, \frac{b-1}{b} \right] \quad (9)$$

If  $\underline{\gamma} < \gamma < \bar{\gamma}$  then the sign of the derivative depends on the relative values of the prices. In terms of the elasticities of substitution, this says that  $\sigma \geq \max[\zeta, b]$  is a sufficient condition for unconditional demand for non-carbon energy to increase in  $t$ . See the appendix for the extension to corner point solutions. ■

It is interesting to note what happens if neither condition (8) nor (9) holds. If  $\zeta < \sigma < b$ , then if  $p_c$  is high enough<sup>4</sup>, R&D will decrease monotonically in the carbon tax; if  $p_c$  is low, R&D will first increase and then decrease. On the other hand, if  $b < \sigma < \zeta$ , then if  $p_c$  is high enough, R&D will increase monotonically; if  $p_c$  is low then R&D will first decrease and then increase.

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<sup>4</sup>Specifically, if  $p_c$  satisfies  $\left| \frac{-b\gamma+b-1}{\gamma} \right| \left[ p_c^{\frac{\gamma}{\gamma-1}} + p_{nc}^{\frac{\gamma}{\gamma-1}} \right]^{\frac{\rho}{\rho-1} \frac{\gamma-1}{\gamma}} > \left| \frac{\gamma-\rho}{(\rho-1)\gamma} \right| w^{\frac{\rho}{\rho-1}}$

### 3.2 Mean-preserving spread in carbon tax: CR

In this section we determine the impact of an increase in risk on the optimal level of CR. Recall that any change in the distribution of  $t$  that increases the right-hand side of (2) will increase the optimal level of R&D. The expected value of a function increases in risk if the function is convex and decreases if the function is concave.<sup>5</sup> If the function is neither convex nor concave, then it will increase with some increases in risk and decrease with other increases. We use this framework to analyze how the demand for R&D will respond to an increase in the risk of the carbon tax.

In order to determine how the optimal level of R&D changes with an increase in risk, we need to determine if  $e_{nc}^*$  is convex or concave in  $t$ . If  $e_{nc}^*$  is concave in  $t$  then expected demand,  $E_t[e_{nc}^*]$ , decreases in risk, thus  $\alpha^*$ , the optimal amount of R&D, decreases in risk. If  $e_{nc}^*$  is neither concave nor convex it means that the optimal level of R&D decreases with some increases in risk, and increases with other increases in risk.

In this section we will focus on firms that increase investment in R&D with an increase in the carbon tax, i.e. firms who have a high enough elasticity of substitution between carbon and non-carbon. The rationale for this focus is that firms that are inelastic between carbon and non-carbon, and who therefore will decrease R&D in a carbon tax are not very likely to be doing a significant amount of R&D in the absence of a carbon tax, and therefore are not very important.

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<sup>5</sup>Jensen's inequality is a well-known special case of this fact. See Rothschild and Stiglitz [15] and [16] for more details.

Intuitively, we would expect  $e_{nc}^*$  to be concave in  $t$  for large  $t$ . This is because optimal output will decrease in  $t$ , thus tempering the substitution effect. Proposition 2 makes this precise.

**Proposition 2** *Assume that optimal R&D into cost reduction increases in the carbon tax. Then optimal R&D into cost reduction will decrease with some increases in risk.*

**Proof.** Optimal R&D will increase unambiguously in risk if and only if  $e_{nc}^*$  is everywhere convex. The assumption that R&D increases in the carbon tax implies that  $e_{nc}^*$  increases in  $t$ . On the other hand,  $e_{nc}^*$  must be bounded from above, and thus cannot be everywhere convex. ■

Is  $e_{nc}^*$  always concave in  $t$ ? No, it depends again on the elasticity of substitution. Consider a production function that is linear in carbon and non-carbon inputs. As long as  $t \leq \eta$  the firm will use no non-carbon inputs; when  $t > \eta$  the firm will jump to using all non-carbon inputs, and the demand will be constant in  $t$  thereafter. Thus, for some elasticities of substitution,  $e_{nc}^*$  is neither convex nor concave, implying that optimal R&D is ambiguous in risk. To illustrate this further consider a continuation of the example from Section 3.1.1.

### 3.2.1 Constant elasticity production and demand

Consider firms who will do R&D in response to a carbon tax, i.e. assume  $\sigma \geq \max[\zeta, b]$ . We can show that in two special cases there exists  $\bar{\sigma}$  such that if  $\max[\zeta, b] \leq \sigma \leq \bar{\sigma}$  then  $e_{nc}^*$  is concave in  $t$ , and if  $\sigma \geq \bar{\sigma}$  then  $e_{nc}^*$  is neither convex nor concave. See Appendix for proof.

**Proposition 3** *If either  $\zeta = b$  or  $\zeta = \sigma$ , then R&D into lowering the cost of the non-carbon alternative will unambiguously decrease in risk if  $b \leq \sigma \leq b + 1$ .*

We further explore this numerically. Figure 1 illustrates the demand for non-carbon energy as a function of  $t$  for three different elasticities of substitution. We assume that the price elasticity of demand is close to 1 and the elasticity of substitution between energy and non-energy is .75. The qualitative results illustrated in the figure are not sensitive to either of these assumptions. The elasticity of substitution between carbon and non-carbon energy, however, has a significant qualitative impact, as can be seen from the figure. Thus, estimating the value of this elasticity is important. In his climate economy model including endogenous technological change, Popp [13] has calibrated the short-term elasticity of substitution between carbon and non-carbon energy as 1.6. On the other hand, if we consider electricity generators, then in the long run, carbon and non-carbon energy are perfect substitutes. The rigidities inherent in the long lifetimes of generation plants determine the medium-term elasticity. Surprisingly, there appears to be little evidence on this topic.

The figure shows that when the elasticity of substitution between carbon and non-carbon energy is relatively low, then demand for  $e_{nc}^*$  is everywhere concave, and thus R&D decreases unambiguously in risk. When, however, the firm is more flexible, the demand for  $e_{nc}^*$  is convex and then concave. Note that the inflection point,  $t = 1$  on this figure, is the point where non-carbon energy becomes economically competitive with carbon energy, i.e. where  $p_c + t = p_c + (1 - \alpha) \eta$ .

For firms that are somewhat flexible (represented by  $\sigma = 2$  in this graph), an expected carbon tax will induce innovation, but this incentive will be reduced in risk. For firms that are very flexible, an expected carbon tax will induce innovation, and increases in risk may increase or decrease innovation.

For intuition, consider a firm that has perfect substitution between carbon and non-carbon energy. This firm will use only carbon energy until the carbon tax increases to the point that the cost of carbon and non-carbon energy are equal. At that point, the firm will switch to all non-carbon energy, and the carbon tax will have no additional impact. Thus, if the firm expected a small carbon tax, they would do no R&D; if they expected a large carbon tax they would do the maximal amount of R&D. A change in the probability distribution of the carbon tax will induce an increase in R&D if and only if it increases the probability that the price of non-carbon energy will be less than the price of carbon energy. This is made precise in the following proposition.

**Proposition 4** *Assume a firm has perfect substitution between carbon and non-carbon energy. Let  $\alpha^*$  and  $\tilde{\alpha}^*$  denote the optimal investment into R&D when the carbon tax has cumulative distribution function  $G$  and  $\tilde{G}$ , respectively. Then  $\alpha^* \leq \tilde{\alpha}^*$  if and only if  $G((1 - \alpha^*)\eta) \geq \tilde{G}((1 - \alpha^*)\eta)$ .*

**Proof.** Under the assumption of perfect substitution,  $e_{nc}^* = 0$  if  $t < (1 - \alpha)\eta$  and is equal to a constant, say  $M_\alpha$ , if  $t \geq (1 - \alpha)\eta$  for any given level of  $\alpha$ . If the CDF of  $t$  is  $G$  then  $E_t[e_{nc}^*] = \int_{(1-\alpha)\eta}^{\infty} M_\alpha dG = M_\alpha [1 - G((1 - \alpha)\eta)]$ . If  $G((1 - \alpha^*)\eta) \geq \tilde{G}((1 - \alpha^*)\eta)$  then

$g'(\alpha^*) \leq M_{\alpha^*} \left[ 1 - \tilde{G}((1 - \alpha^*)\eta) \right]$ , thus  $\tilde{\alpha}^*$  must be larger than  $\alpha^*$  to preserve optimally. Likewise, if  $G((1 - \alpha^*)\eta) < \tilde{G}((1 - \alpha^*)\eta)$  then  $g'(\alpha^*) > M_{\alpha^*} \left[ 1 - \tilde{G}((1 - \alpha^*)\eta) \right]$  and  $\tilde{\alpha}^*$  must be smaller than  $\alpha^*$ . ■

In general, if  $e_{nc}^*$  is convex before the inflection point (that is the point where non-carbon energy becomes competitive with carbon energy) and concave after, then any mean-preserving spread (MPS) to the right of the inflection point decreases the incentive to invest in R&D, while any MPS to the left increases it.

## 4 Emissions Reduction

In this section we consider R&D into reducing emissions of the currently economic technology (denoted ER for *emissions reduction*). We model ER as reducing the carbon intensity of a unit of carbon energy from 1 to  $(1 - \alpha)$ . Thus, effectively, the price of carbon energy is reduced from  $p_c + t$  to  $p_c + (1 - \alpha)t$ . This program represents an investment into technology that will capture a percentage  $\alpha$  of the firm's carbon emissions. In Section 5 we show that an R&D program into increasing the efficiency of carbon energy has a qualitatively similar response to a carbon tax as ER. Thus, the results of this section can be applied more broadly.<sup>6</sup>

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<sup>6</sup>Additionally we can include the possibility of fuel switching. Our model has only two energy inputs, carbon and non-carbon. We can generalize by allowing for a third energy input, say cleaner carbon energy  $e_{cc}$ . If we assume that  $e_c$  and  $e_{cc}$  are perfect substitutes, then all of the results are the same. If they are imperfect substitutes, then we would need to consider the elasticity of substitution between carbon and cleaner carbon in a similar manner as the other elasticities.

Under ER the second period profit function is

$$\pi(w, p_c + (1 - \alpha)t, p_c + \eta) \quad (10)$$

This program will lower the cost of abatement for low levels of abatement, but it will not have an impact on full abatement, thus the marginal cost of abatement will be higher over some regions. The right-hand side of (2) simplifies to:

$$\mathbb{E}_t \left[ \frac{\partial \pi}{\partial \alpha} \right] = - \mathbb{E}_t \left[ t \frac{\partial \pi}{\partial p_c} \right] \quad (11)$$

Applying Hotelling's lemma, the first order condition (2) simplifies to

$$g'(\alpha) = \mathbb{E}_t [te_c^*] \quad (12)$$

where  $e_c^*$  represents the unconditional demand for carbon energy. The marginal benefit from ER is equal to the total expected carbon tax expenditures of the firm. Assuming an interior solution, if total tax expenditures increase in the carbon tax, then ER increases in the expected value of the carbon tax. Similarly, if the expected tax expenditures increase in uncertainty, then optimal investment in ER will increase in uncertainty. However, as we show in the next section, this problem is more complicated than CR for two reasons: the marginal benefits to R&D are non-monotonic in the carbon tax, and corner-point solutions

play a prominent role.

#### 4.1 Increase in the expected value of a carbon tax: ER

ER may decrease if the tax gets too high. Why? Because as the tax gets higher firms will substitute away from carbon energy, thus dampening the benefit from ER. First consider the marginal benefits to ER, which, as shown above, are proportional to the total carbon tax expenditures. For a given level of ER,  $\alpha < 1$ , tax expenditures will generally follow a Laffer curve – they first increase and then decrease in the carbon tax. In fact, we can show that for a monopolist with a CES production function and constant elasticity demand, there exists a  $t^*$  that maximizes carbon tax expenditures.<sup>7</sup> This effect in turn depends on the elasticity of substitution between carbon and other inputs and the elasticity of demand. The more elastic are substitution and demand, the sooner tax expenditures fall in the carbon tax. Thus, we would expect the interior solution for optimal ER to generally increase, and then decrease in a carbon tax.

We illustrate this using the same CES monopolist from Sections 3.1.1 and 3.2.1. Figure 2 illustrates carbon tax expenditures at  $\alpha = 0$ , as a function of the carbon tax for three different values of  $\sigma$ . The point where tax equals 1 again represents the point where carbon and non-carbon energy become economically competitive. Again, the elasticity of substitution between energy and non-energy has no qualitative impact. The price elasticity of demand,

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<sup>7</sup>See Appendix for proof Proposition 8.

however, has a significant impact, as can be seen from the right-hand panel where price elasticity is assumed to be 1.1. For the higher price elasticity and for higher elasticity of substitution, tax expenditures and R&D are maximized at a point that is before carbon and non-carbon energy become competitive.

Since the marginal benefits to ER are non-monotonic, however, corner point solutions are important. First, if zero-emissions technology (i.e.  $\alpha^* = 1$ ) is optimal for some  $\hat{t} < \infty$ , then it is optimal for every  $t \geq \hat{t}$ , even though the interior solution may fall. Second, if zero-emissions technology is not optimal for some finite  $t$ , then as  $t$  gets large and the firm substitutes away from carbon energy, it is likely that the corner solution  $\alpha^* = 0$  becomes optimal.

First we assume that emissions from carbon energy cannot be reduced to zero (i.e.  $g(1) = \infty$ ), and show that optimal ER will decrease as  $t$  gets high. Next we show that if  $g, g'$  are finite at  $\alpha = 1$ , then if zero-emissions technology is not optimal for a finite  $t$  then optimal ER will decrease as  $t$  gets high. Both of these results depend on the corner point solution where  $\alpha^* = 0$ . The logic is this: as long as  $\alpha$  is strictly less than one, the cost of carbon energy will strictly increase in the carbon tax; for a high enough tax the firm will substitute almost entirely away from carbon energy (or stop producing altogether if they are not flexible enough to substitute). If a firm is not using carbon energy, then ER has no value to them; thus the optimal amount of R&D at that point is zero.

**Proposition 5** *Assume that  $g$  is continuous,  $g(0) = 0$  and  $g(1) = \infty$ , and that the total*

*demand for carbon energy goes to zero as its price goes to infinity. Then optimal ER will not monotonically increase in  $t$ .*

**Proof.** Define  $\alpha(t)$  as a local maximum ( i.e. it satisfies the first and second order conditions) of the decision problem (1) assuming ER, and assume that it is monotonically increasing in  $t$ . If these assumptions do not hold then Proposition 5 is true by default. Since  $\alpha$  is monotonically increasing in  $t$  and is bounded above by 1, the limit of  $\alpha$  exists. First, assume that  $\lim_{t \rightarrow \infty} \alpha(t) = 1$ , then consider

$$\lim_{t \rightarrow \infty} -g(\alpha(t)) + \pi((1 - \alpha(t))t) \quad (13)$$

where we are suppressing the prices  $p_c, p_{nc}$ , and  $w$  in the profit function. Note that  $0 \leq \pi((1 - \alpha(t))t) \leq \pi(0)$ : profits are bounded below by zero and above by business-as-usual profits. Since  $g$  is continuous,  $\lim_{t \rightarrow \infty} -g(\alpha(t)) = -\infty$ , so (13) goes to negative infinity as  $t$  gets large; since  $\pi(t) \geq 0$ , the corner point solution  $\alpha^* = 0$  is optimal for large enough  $t$ .

Now assume that  $\lim_{t \rightarrow \infty} \alpha(t) = c < 1$ . The profit function is continuous [18], thus

$$\lim_{t \rightarrow \infty} -g(\alpha(t)) + \pi((1 - \alpha(t))t) \quad (14)$$

$$= -g(c) + \pi((1 - c)\infty) \quad (15)$$

$$= -g(c) + \pi(\infty) \quad (16)$$

$$< -g(0) + \pi(\infty) = \pi(\infty) \quad (17)$$

Note that (17) is the value of the maximand at the corner point solution  $\alpha^* = 0$ . ■

**Proposition 6** *Assume that  $g' < \infty$ . Then either zero-emissions technology is optimal for a finite tax, or optimal R&D falls for a high enough carbon tax.*

**Proof.** Assume that  $g(1) < \infty$ . Proof by contradiction: assume that zero-emissions technology is never optimal for finite  $t$  but that  $\alpha$  increases monotonically in  $t$ ; according to the Karush-Kuhn-Tucker Theorem (KKT), that implies that for  $\forall t$   $g'(1) > te_c^*(t=0)$ , but this is impossible as long as  $g' < \infty$ . ■

The logic of Proposition 5 is this: if the interior solution is getting arbitrarily close to one, then at some point the cost of R&D will outweigh the benefits, and the interior solution will not be optimal. If, on the other hand, the interior solution is bounded by some constant strictly less than one, then as the tax gets very large, the cost of the carbon input is getting very large, even after R&D. For a large enough tax, the difference between profits with and without R&D becomes small, and so the cost of producing R&D is no longer worth it.<sup>8</sup> The reason we don't get this result in CR is that the limit of profits as  $t$  goes infinity is strictly different with and without R&D; for ER profits converge to the same amount with or without R&D, as long as R&D is less than 1.

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<sup>8</sup>Note that in the special case that  $g(1) < \infty$  but  $g'(1) = \infty$ , one can construct an example in which the optimal  $\alpha$  monotonically increases in  $t$ , but is strictly less than 1.

## 4.2 Mean-preserving spread in carbon tax: ER

Since optimal ER is low when the tax is low, and low when the tax is high, ER will generally decrease in risk. The following proposition shows that there always exists some increase in risk that causes ER to fall.

**Proposition 7** *Optimal R&D into emissions reduction will decrease with some increases in risk regardless of the characteristics of the firm.*

**Proof.** In the uninteresting cases where  $\alpha^* = 0$  or 1 for all  $t$ , then every increase in risk is weakly decreasing  $\alpha^*$ . Assume that  $\alpha^* = 0$  when  $t = 0$  and there exists a  $\hat{t}$  such that optimal ER  $\hat{\alpha} > 0$ . For every  $t^H$  there exists a  $p = \frac{\hat{t}}{t^H}$  such that  $\hat{t} = p * t^H + (1 - p) * 0$ . Consider

$$\alpha^* \equiv \arg \max_{\alpha} -g(\alpha) + p\pi((1 - \alpha)t^H) + (1 - p)\pi(0) \quad (18)$$

$$= \arg \max_{\alpha} -g(\alpha) + p\pi((1 - \alpha)t^H) \quad (19)$$

$$= \arg \max_{\alpha} -g(\alpha) + \frac{\hat{t}}{t^H}\pi((1 - \alpha)t^H) \quad (20)$$

Since profits are strictly bounded,

$$\lim_{t \rightarrow \infty} -g(\alpha) + \frac{\hat{t}}{t^H}\pi((1 - \alpha)t^H) = -g(\alpha) \quad (21)$$

which is strictly negative for  $\alpha > \hat{\alpha}$ . Thus the optimal  $\alpha^*$  must decrease for  $t^H$  high enough, i.e. we have shown that there exists a MPS around  $\hat{t}$  that induces a decrease in  $\alpha^*$ . ■

Furthermore, as we mention above, the marginal benefits of ER first increase and then decrease in the carbon tax. Thus, even assuming an interior solution, the optimal amount of R&D will tend to drop sharply in a MPS of the tax. For example, consider the right-hand panel of Figure 2, when the elasticity of substitution between carbon and non-carbon is 7 or above. The marginal benefits of R&D under a certain tax  $t = 1$  is about .3; the expected marginal benefits of R&D given a 50-50 chance of a carbon tax of 0 or 2 is an order of magnitude smaller, about .025.

## 5 Efficiency

In this section we briefly indicate how R&D into increased efficiency will fare under this framework. First we consider an improvement in efficiency of carbon-energy, say an improvement in combined cycle gasification which will allow the firm to generate more energy per unit of carbon input. This is equivalent to reducing the effective price of  $e_c$ . The second period profit function would be  $\pi(w, (1 - \alpha)(p_c + t), p_{nc})$  and the first order condition (2) simplifies to

$$g'(\alpha) = \text{Et} [(p_c + t) e_c^*] \quad (22)$$

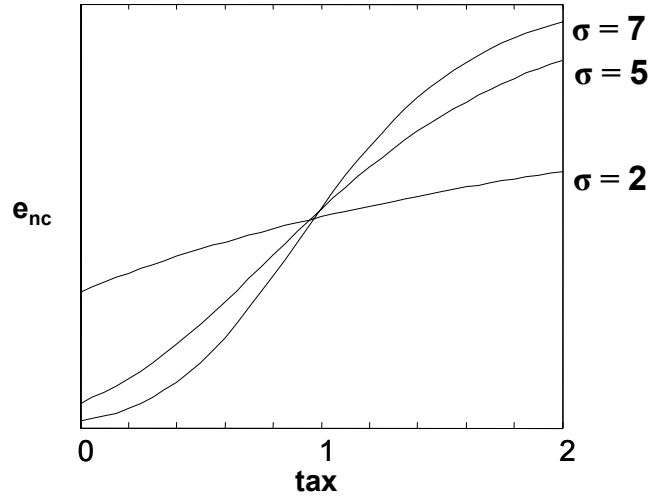


Figure 1: Unconditional demand for non-carbon energy as a function of the carbon tax. Price elasticity of demand is 1.1. Elasticity of substitution between energy and non-energy is .75.

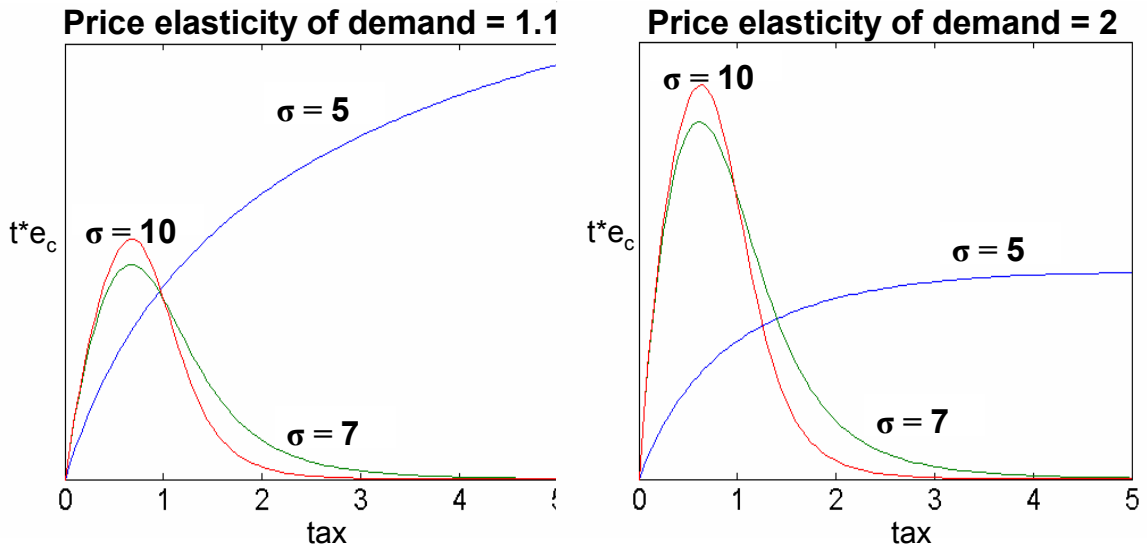


Figure 2: Total carbon tax expenditures as a function of the carbon tax. The elasticity of substitution between energy and non energy is .75.

The optimal response of this kind of R&D will be qualitatively similar to the optimal response of ER: it will be non-monotonic in a carbon tax, and generally decrease in uncertainty. Note, however, that the threshold carbon tax, beyond which R&D decreases, will tend to be lower in this case, and strictly decreases in  $p_c$ .

Second we consider a general improvement in energy efficiency. If we assume that the production function for energy is homogenous of degree one, then energy efficiency would have the effect of reducing the effective price of both carbon and non-carbon energy. The second period profit function would be  $\pi(w, (1 - \alpha)(p_c + t), (1 - \alpha)p_{nc})$  and the first order condition (2) simplifies to

$$g'(\alpha) = E_t [(p_c + t) e_c^*] + p_{nc} E_t [e_{nc}^*] \quad (23)$$

Thus, the marginal benefit of R&D into energy efficiency is equal to the total expected cost of energy. The impact on energy efficiency R&D of an expected increase in a carbon tax will be qualitatively the same as the impact of an increase in the carbon tax on the overall cost of energy, which in turn depends on the characteristics of the firm in a similar manner as in the above analysis.

## 6 Conclusion

The first result is that if we consider R&D in terms of reducing the effective cost of inputs, then we can analyze the optimal amount of R&D in terms of how a change in price will change the unconditional demand for the inputs. An expected carbon tax will cause firms to invest in R&D to reduce the cost of low-carbon energy to the degree that a carbon tax will increase unconditional demand for non-carbon inputs. On the other hand, R&D into emissions-reduction will increase to the degree that a carbon tax raises total carbon tax expenditures. Similarly, an expected tax will increase R&D into efficiency to the degree that a carbon tax will increase the overall total cost of energy.

Taking this approach we have been able to show that environmental R&D will not monotonically increase in an expected carbon tax. If the elasticity of substitution between carbon and non-carbon energy is not high enough, then investment into R&D aimed at reducing the cost of non-carbon alternatives will drop in a carbon tax. If, on the other hand, the elasticity of substitution is high enough, then this kind of R&D will increase monotonically in a carbon tax. If zero-emissions technology is not optimal, then R&D into reducing emissions responds non-monotonically to a carbon tax, first increasing, then decreasing as the tax gets too high: there is a threshold tax above which optimal R&D decreases. The level of this threshold tax decreases as both demand and production get more elastic.

We go on to analyze the firm's optimal response to an increase in risk in the carbon tax. When considering R&D into cost reduction of low-carbon alternatives we find that the results

depend on the firm characteristics. For less flexible firms, R&D decreases unambiguously in risk. For more flexible firms, R&D will increase with some increases in risk and decrease with others. In particular, if the elasticity of substitution between carbon and non-carbon energy is very high, then the optimal amount of R&D depends on the probability that the carbon tax will be high enough to induce a switch to non-carbon energy. When considering R&D into emissions reduction, the optimal level of R&D will generally decrease in risk.

Our results emphasize the fact that different R&D programs have different optimal responses to policy. Thus, when modeling induced technical change, care must be taken. In particular, the assumption that environmental R&D necessarily decreases marginal abatement cost may need to be revisited.

We have made a number of simplifying assumptions. We consider uncertainty about the magnitude of a tax, but not the timing. This can be partially justified by the idea that R&D projects can take a long time; but future work on the timing of R&D investments is needed. We assume that the returns from R&D are deterministic. It will be important to include both uncertainty in policy and uncertainty in returns to R&D to determine how they interact. We have assumed risk neutrality. Initial theoretical work indicates that firms will be more likely to increase R&D spending in response to an increase in risk as they get more prudent; however, our numerical examples showed no impact of risk aversion even for very high levels of prudence. Another key issue that must be addressed is that the providers of R&D may be different from those who implement the new technology. If the producers of

R&D hold significant market power, it may impact the results. We have used a price-based mechanism to represent policy, and considered a firm's response to a MPS in this price. An alternative is to consider an MPS in an emissions standard. We note, however, that if ex-post policy is efficient, and both the tax and the standard are based on underlying scientific uncertainty, the results are qualitatively the same.

## A Appendix

### A.1 Proof of Proposition 1

**Proof.** We take the derivative of (7) with respect to  $\tilde{p}_c$ , dropping terms that are constant in  $\tilde{p}_c$ :

$$\begin{aligned}
\frac{\partial e_{nc}^*}{\partial p_c} &= \frac{\gamma - \rho}{(\rho - 1)\gamma} \left[ \tilde{p}_c^{\frac{\gamma}{\gamma-1}} + p_{nc}^{\frac{\gamma}{\gamma-1}} \right]^{\frac{\gamma-\rho}{(\rho-1)\gamma}-1} \frac{\gamma}{\gamma-1} \tilde{p}_c^{\frac{\gamma}{\gamma-1}-1} \left( w^{\frac{\rho}{\rho-1}} + \left[ \tilde{p}_c^{\frac{\gamma}{\gamma-1}} + p_{nc}^{\frac{\gamma}{\gamma-1}} \right]^{\frac{\rho}{\rho-1} \frac{\gamma-1}{\gamma}} \right)^{\frac{-1-\rho b+b}{\rho}} \\
&\quad + \left[ \tilde{p}_c^{\frac{\gamma}{\gamma-1}} + p_{nc}^{\frac{\gamma}{\gamma-1}} \right]^{\frac{\gamma-\rho}{(\rho-1)\gamma}} \frac{-1-\rho b+b}{\rho} \left( w^{\frac{\rho}{\rho-1}} + \left[ \tilde{p}_c^{\frac{\gamma}{\gamma-1}} + p_{nc}^{\frac{\gamma}{\gamma-1}} \right]^{\frac{\rho}{\rho-1} \frac{\gamma-1}{\gamma}} \right)^{\frac{-1-\rho b+b}{\rho}-1} \times \\
&\quad \frac{\rho}{\rho-1} \frac{\gamma-1}{\gamma} \left[ \tilde{p}_c^{\frac{\gamma}{\gamma-1}} + p_{nc}^{\frac{\gamma}{\gamma-1}} \right]^{\frac{\rho}{\rho-1} \frac{\gamma-1}{\gamma}-1} \frac{\gamma}{\gamma-1} \tilde{p}_c^{\frac{\gamma}{\gamma-1}-1} \tag{24} \\
&= \frac{\gamma}{\gamma-1} \tilde{p}_c^{\frac{\gamma}{\gamma-1}-1} \left[ p_c^{\frac{\gamma}{\gamma-1}} + p_{nc}^{\frac{\gamma}{\gamma-1}} \right]^{\frac{\gamma-\rho}{(\rho-1)\gamma}-1} \left( w^{\frac{\rho}{\rho-1}} + \left[ \tilde{p}_c^{\frac{\gamma}{\gamma-1}} + p_{nc}^{\frac{\gamma}{\gamma-1}} \right]^{\frac{\rho}{\rho-1} \frac{\gamma-1}{\gamma}} \right)^{\frac{-1-\rho b+b}{\rho}-1} \times \\
&\quad \left\{ \frac{\gamma-\rho}{(\rho-1)\gamma} w^{\frac{\rho}{\rho-1}} + \left( \frac{\gamma-\rho}{(\rho-1)\gamma} + \frac{-1-\rho b+b}{\rho} \frac{\rho}{\rho-1} \frac{\gamma-1}{\gamma} \right) \left[ \tilde{p}_c^{\frac{\gamma}{\gamma-1}} + p_{nc}^{\frac{\gamma}{\gamma-1}} \right]^{\frac{\rho}{\rho-1} \frac{\gamma-1}{\gamma}} \right\}
\end{aligned}$$

Dropping constants whose sign is positive, the sign of (24) is equal to the sign of:

$$\frac{\gamma}{\gamma-1} \left\{ \frac{\gamma-\rho}{(\rho-1)\gamma} w^{\frac{\rho}{\rho-1}} + \left( \frac{\gamma-\rho}{(\rho-1)\gamma} + \frac{-1-\rho b+b}{\rho} \frac{\rho}{\rho-1} \frac{\gamma-1}{\gamma} \right) \left[ \tilde{p}_c^{\frac{\gamma}{\gamma-1}} + p_{nc}^{\frac{\gamma}{\gamma-1}} \right]^{\frac{\rho}{\rho-1} \frac{\gamma-1}{\gamma}} \right\} \quad (25)$$

This quantity can be unambiguously signed for all values of  $w, p_c$  and  $p_{nc}$  only when the coefficients of both terms inside the curly brackets have the same sign. Consider  $\gamma > 0$ .

Then  $\frac{\gamma}{\gamma-1} < 0$  (since  $\gamma < 1$ ).  $\frac{\gamma-\rho}{(\rho-1)\gamma} < 0$  if  $\gamma > \rho$ . The second coefficient inside the curly brackets simplifies to

$$\frac{-\rho - \rho b \gamma + b \gamma + 1 + \rho b - b}{(\rho-1)\gamma} \quad (26)$$

$$= \frac{-b\gamma + b - 1}{\gamma} \quad (27)$$

The quantity in (27) is negative if

$$-b\gamma + b - 1 < 0 \quad (28)$$

which is true if

$$\gamma > \frac{b-1}{b} \quad (29)$$

This calculation can be repeated for the other 3 cases.

Consider possible corner point solutions. Define  $\alpha(t)$  as the interior solution to (1). If

$\alpha^* = 0$  is the global maximum at tax  $t$ , KKT implies that

$$-g(\alpha(t)) + \pi(w, p_c + t, p_c + (1 - \alpha(t))\eta) < \pi(w, p_c + t, p_c + \eta) \quad (30)$$

But, under the assumptions of the proposition, the right-hand side of (30) is decreasing in  $t$  faster than the left-hand side<sup>9</sup>, thus if the interior solution  $\alpha(\hat{t})$  is the global maximum for  $t = \hat{t}$ , then the interior solution  $\alpha(t)$  is the global maximum for  $t \geq \hat{t}$ . ■

## A.2 Proof of proposition 3

**Proof.** If  $\zeta = \sigma$  then  $\rho = \gamma$ , and (7) simplifies to the following

$$e_{nc}^* = K_1 \left[ K_2 + \tilde{p}_c^{\frac{\gamma}{\gamma-1}} \right]^{\frac{-1-\gamma b+b}{\gamma}} \quad (31)$$

where  $K_1 = p_{nc}^{\frac{1}{\gamma-1}} \left( \frac{b}{b-1} \frac{1}{A} \right)^{-b}$  and  $K_2 = w^{\frac{\gamma}{\gamma-1}} + p_{nc}^{\frac{\gamma}{\gamma-1}}$  are constants in terms of  $p_c$ . If, alternatively,  $\zeta = b$ , then (7) also simplifies to (31), with  $K_1$  defined the same and  $K_2 = p_{nc}^{\frac{\gamma}{\gamma-1}}$ .

Dropping the constant  $K_1$  and taking the derivative of (31) twice with respect to  $p_c$  we

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<sup>9</sup>The derivative of both sides with respect to  $t$  is  $-e_c^*$ . Under the assumptions of the proposition,  $e_c^*$  is increasing in  $p_{nc}$ .  $p_{nc}$  is strictly higher when  $\alpha = 0$ , so the slope of the rhs is more negative.

get

$$\frac{\partial^2 e_{nc}^*}{\partial p_c^2} = \frac{-1 - \gamma b + b}{\gamma} \left( \frac{-1 - \gamma b + b}{\gamma} - 1 \right) \left( \frac{\gamma}{\gamma - 1} \right)^2 \left[ K_2 + \tilde{p}_c^{\frac{\gamma}{\gamma-1}} \right]^{\frac{-1-\gamma b+b}{\gamma}-2} \tilde{p}_c^{2\frac{\gamma}{\gamma-1}-2} \quad (32)$$

$$\frac{-1 - \gamma b + b}{\gamma} \frac{\gamma}{\gamma - 1} \left( \frac{\gamma}{\gamma - 1} - 1 \right) \left[ K_2 + \tilde{p}_c^{\frac{\gamma}{\gamma-1}} \right]^{\frac{-1-\gamma b+b}{\gamma}-1} \tilde{p}_c^{\frac{\gamma}{\gamma-1}-2} \quad (33)$$

Collecting terms this simplifies to

$$\frac{-1 - \gamma b + b}{\gamma} \frac{\gamma}{\gamma - 1} \left[ K_2 + \tilde{p}_c^{\frac{\gamma}{\gamma-1}} \right]^{\frac{-1-\gamma b+b}{\gamma}-2} \tilde{p}_c^{\frac{\gamma}{\gamma-1}-2} \times \quad (34)$$

$$\left\{ \frac{\gamma}{\gamma - 1} \left( \frac{-1 - \gamma b + b}{\gamma} - 1 \right) \tilde{p}_c^{\frac{\gamma}{\gamma-1}} + \frac{1}{\gamma - 1} \left[ K_2 + \tilde{p}_c^{\frac{\gamma}{\gamma-1}} \right] \right\} \quad (35)$$

$$= \frac{-1 - \gamma b + b}{\gamma} \frac{\gamma}{\gamma - 1} \left[ K_2 + \tilde{p}_c^{\frac{\gamma}{\gamma-1}} \right]^{\frac{-1-\gamma b+b}{\gamma}-2} \tilde{p}_c^{\frac{\gamma}{\gamma-1}-2} \left\{ \frac{b - b\gamma - \gamma}{\gamma - 1} \tilde{p}_c^{\frac{\gamma}{\gamma-1}} + \frac{1}{\gamma - 1} K_2 \right\} \quad (36)$$

A necessary condition for this to be negative for all  $\tilde{p}_c$  is that the first term inside the curly brackets is negative. This is true if  $\gamma \leq \frac{b}{b+1}$ , which is true if  $\sigma \leq b + 1$ . ■

### A.3 Proposition 8

In this section we consider the worst case – a Leontiff production function. We show that there exists a  $t^*$  that maximizes carbon tax expenditures, and we characterize  $t^*$  for the Leontiff technology. We argue that since there exists such a  $t^*$  for the Leontiff technology, then it follows that the same is true for all other CES production functions, since if the firm is able to substitute away from carbon energy then they will as the tax increases,

meaning that  $e_c^*$  will decrease faster in  $t$ . Consider a simple Leontiff production function with  $y = \min[x, e_c, e_{nc}]$ . The demand for carbon energy  $e_c^*$  is equal to the total optimal output  $y^*$ . Assume the same inverse demand as in Section 3.1.1. Then optimal output is

$$y^* = (w + p_c + (1 - \alpha)t + p_{nc})^{-b} K \quad (37)$$

Where  $K$  is a constant equal to  $\left(\frac{b}{A(b-1)}\right)^{-b}$ . Thus carbon tax expenditures  $te_c^*$  are equal to  $t(w + p_c + (1 - \alpha)t + p_{nc})^{-b} K$  and are maximized at

$$t^* = \frac{w + p_c + p_{nc}}{(b - 1)(1 - \alpha)} \quad (38)$$

This proves the following proposition.

**Proposition 8** *For a monopolist with CES production and constant elasticity demand, and for  $\alpha < 1$ , carbon tax expenditures first increase and then decrease in the carbon tax.*

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