OPTIMAL POLICY UNDER UNCERTAINTY AND LEARNING ABOUT CLIMATE CHANGE: A STOCHASTIC DOMINANCE APPROACH

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Abstract

Global climate change presents a classic problem of decision making under uncertainty with learning. We provide stochastic dominance theorems that provide new insights into when abatement and investment into low carbon technology should increase in risk. We show that R&D into low-carbon technologies and near-term abatement are in some sense opposites in terms of risk. Abatement provides insurance against the possibility of major catastrophes; R&D provides insurance against the possibility that climate change is marginally worse than average. We extend our results to the comparative statics of learning.

1. Introduction

While scientists largely agree that humans are changing the climate, there is a great deal of uncertainty about the degree to which emissions of the greenhouse gasses that cause global warming will cause damages in the future. This is part of what is causing a very large debate about how best to proceed in the fight against global climate change. Possible near term policy responses include both restrictions on emissions (through emissions limits or taxes) and investment in environmentally friendly technologies. In this paper, we consider how the optimal emissions path and the optimal technology investment are simultaneously impacted by increases in uncertainty about climate damages. We show that in many cases abatement and R&D act as "risk-substitutes": changes in risk that induce an increase in one, induce a decrease in the other.

Our approach to this problem is as follows. We develop an optimization model with two decision variables—abatement (i.e., a reduction of emissions) and investment in improving low-emissions technologies. We investigate how the optimal levels of the two variables change with changes in risk and with

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We thank the Associate Editor for helpful insights and comments.
Received December 8, 2006; Accepted February 9, 2008.

Journal of Public Economic Theory, 11 (5), 2009, pp. 721–747.

changes in the amount expected to be learned in the future. It is well known that in models of climate change, the impact of risk (defined as a mean-preserving spread Rothschild and Stiglitz 1970) on the decision variables is ambiguous. Thus, we define and interpret additional conditions on mean-preserving-spreads (MPS) that lead to unambiguous changes in the vector of decision variables; and relate these conditions to the shape of the marginal benefit functions. By focusing on the shape of the marginals, we provide a unifying approach to analyzing the comparative statics of optimal climate policy under uncertainty and learning.

We provide a brief background on the state of the literature on the comparative statics of uncertainty and learning in the rest of this section. In Section 2, we introduce our model of climate change policy and the key assumptions that lead to unambiguous results. In Section 3, we discuss what is known about the shape of the marginal benefits to abatement and to R&D, and re-interpret the current literature in this context. In Section 4, we present some new stochastic dominance theorems that are necessary for comparative statics results in the climate change problem. In Section 5, we apply these theorems to get insights on how the optimal emission policy and the optimal technology investment policy are impacted by the uncertainty around climate change. In Section 6, we extend the results to a partial learning problem. We conclude in Section 7.

1.1. Comparative Statics of Risk and Uncertainty

Our approach is related to the stochastic dominance literature. Papers in this literature consider different subsets of utility functions, and characterize the changes in probability distributions that cause expected utility to increase or decrease (see e.g., Hadar and Russell 1969). The classic literature considers increasing concave utility functions. There is also some recent work, inspired by prospect theory, considering increasing S-shaped and reverse S-shaped functions (Levy and Wiener 1998). We develop stochastic dominance theorems for less standard sets of functions, guided by the shapes of the marginal benefits to abatement and to R&D, and apply them to two-period decision problems to get at the comparative statics of risk and learning.

We start this section by defining an increase in risk, and discussing why we focus on subsets of MPS. In Section 1.1.2, we introduce a generic decision problem to discuss the comparative statics of risk, and discuss how our approach compares to other approaches in the literature.

1.1.1. Definition of Risk

In this paper we focus on increasing risk or MPS, as defined by Rothschild and Stiglitz (1970). Let Z represent a random variable.

DEFINITION 1: *Z is **riskier** (or **more uncertain** or **more variable** or an **MPS**) than Z' iff $E_Z U(Z) \leq E_{Z'} U(Z')$ for all concave U.*

By definition then, risk is exactly what risk-averters do not like.[1] We focus on MPS because they do not conflate the impact of an increase in mean with an increase in risk; and they are relatively easy to characterize and interpret. There is also an added benefit to considering MPS—there is a strong parallel between MPS and an increase in informativeness, in the Blackwell (1951, 1953), sense. Thus, through studying what kinds of MPS increase the optimal value of a decision variable, we can infer what kinds of signals increase the optimal value of a decision variable. Rothschild and Stiglitz (1970) showed that this definition is equivalent to Z having more weight in the tails than Z' (characterized by what are often called the integral conditions). We will add additional conditions to these integral conditions, and thus will characterize subsets of the set of all MPS.

1.1.2. Generic Decision Problem

Consider a generic decision problem, where the costs are known but the benefits are uncertain:

$$\max_x E_z[b(x;Z)] - c(x), \tag{1}$$

where x is a decision variable, Z is a random variable, E_z is the expectation operator over Z, and $b(\cdot;\cdot)$ and $c(\cdot)$ are the benefits and costs of the decision x. We assume that the problem is well-behaved: $\frac{\partial b}{\partial x}$, c', $c'' \geq 0$ and $\frac{\partial^2 b}{\partial x^2} \leq 0$. The first-order condition for this problem is

$$c'(x) = E_z[Mb(x;Z)], \tag{2}$$

where $Mb = \frac{\partial b}{\partial x}$ represents the marginal benefits to x.[2] Any change in the probability distribution of Z that increases (decreases) the expected marginal benefits will cause the optimal value of the decision variable to increase (decrease).[3] Expected marginal benefits will increase (decrease) for all increases in risk if and only if the marginal benefits are convex (concave) in Z.[4] This is a direct result of the definition of increasing risk.

Stochastic Dominance theory has been applied to decision theory for about 50 years (see Levy 1992, for a review). This problem, however, was

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[1] This definition differs from what is often called second-order stochastic dominance (SOSD), in that SOSD is defined only for *increasing*, concave U. The result of this different definition is that "increasing risk" only orders random variables with equal means; SOSD orders a larger set. In general, the more restrictions that are put on the set of functions U, the larger is the set of probability functions that can be ordered; and vice versa. See Athey (2000) and Osborn (2004) for a discussion of this relationship.

[2] Assuming the problem is well behaved, i.e., that $E_z[b(x;Z)]$ exists for every x; and $Mb(x;Z)$ is continuous in x for every Z.

[3] Throughout the paper we will use the term "increasing" to mean nondecreasing, and will say "strictly increasing" when that is what we mean.

[4] To be precise, define $x(F)$ as the optimal value of the decision variable x given the probability distribution F. Then we say x is **increasing (decreasing) in risk** if $x(G) \geq (\leq) x(F)$ whenever G is riskier than F.

prominently analyzed in the economics literature by Rothschild and Stiglitz (1971).[5] They focused on finding the conditions under which various decision problems had *Mb* either everywhere convex or everywhere concave. Unfortunately, in more cases than not, marginal benefits are neither convex nor concave. This is especially the case in two period decision problems with learning, such as follows:

$$\max_{x_1} E_z\left[\max_{x_2} b(x_1, x_2; Z) - c_2(x_2)\right] - c_1(x_1). \tag{3}$$

Unless the benefits are temporally separable, the second period decision, x_2 will generally be a function of the first period decision, x_1. Thus, even with extremely simple and well-behaved primitives, it is rare to find *Mb* that are everywhere convex or concave. When *Mb* are neither convex nor concave it implies that the impact of risk is ambiguous: the optimal action will increase for some increases in risk and decrease for other increases in risk. This problem—the lack of convexity/concavity of the *Mb*—has been noted in an extensive literature, and has been approached in two ways. The first approach creates new definitions of increasing risk such that *Mb* will decrease for all risk averters (Meyer and Ormiston 1985, Gollier 1995). The second approach puts restrictions on the set of payoff functions (or more specifically the utility functions of the decision makers) so that *Mb* will always decrease (Kimball 1990).

In this paper, we first characterize the *Mb* in the climate change problem. These *Mb* do not conform to the general decision problem set up in the literature, which tends to focus on the impact of risk aversion on decisions (e.g., Jewitt 1989, Eeckhoudt and Gollier 1995, Hadar and Seo 1988). Based on the characteristics of *Mb*, we find the subsets of MPS under which the expected *Mb* will always decrease. Thus, we take clues from each of the two approaches.

1.1.3. Learning

The generic problem in (1) was further generalized by Epstein (1980), who considered a two-period problem with partial learning before the second period, as follows:

$$\max_{x_1} E_Y\left[\max_{x_2} E_{Z|Y} b(x_1, x_2; Z) - c_2(x_2)\right] - c_1(x_1), \tag{4}$$

where Y is a "signal," a random variable defined on the same sample space as Z, thus possibly providing information about Z. Let $B(x_1, F_{Z|Y}) \equiv \max_{x_2} E_{Z|Y} b(x_1, x_2; Z) - c_2(x_2)$. Then the marginal benefit to the first period action x_1 is

$$MB = \frac{\partial B}{\partial x_1}.$$

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[5] See also Laffont (1989) for a similar discussion.

Using Blackwell's (1951) definition of a more informative signal, Epstein shows that the first period action x_1 increases (decreases) with a more informative signal if and only if the *MB* are convex (concave) in the conditional probability distribution $F_{Z|Y}$. Baker (2006) exploits the parallel between increasing risk and increasing informativeness, showing that, if the benefit is separable in a function of Z, then the action in (4) increases with an increasingly informative signal if and only if the action in (3) increases in risk. Thus, once we describe the subset of MPS that causes an action to increase or decrease, we can immediately describe the subset of informative signals that will cause an action to increase or decrease.

2. Climate Change Model

In this section, we present a simple climate change decision model. We represent uncertainty in climate change through a single random variable Z that impacts the damage curve. Above we referred to general decisions x_1 and x_2. Here, there are two first period decisions on which we concentrate: abatement, μ, and R&D investment, α. The second period decision that provides the flexibility is second period abatement, μ_2.

$$\min_{\mu \leq 1, \alpha} c_1(\mu) + g(\alpha) + E_Z\Big[\min_{\mu_2 \leq 1} c_2(\mu_2; \alpha) + D(S - \mu - \mu_2; Z)\Big], \tag{5}$$

where μ, μ_2 are first and second period abatement (measured as the fraction of emissions reduced below the business-as-usual level of emissions); α is the impact on the second period abatement cost curve from technological change; S is the current stock of emissions; Z is the random variable (with range $\mathbb{R}^+$) that impacts damages; c_1, c_2 are the abatement cost functions in the first and second period; g is the cost of achieving technical change equal to α, and D is the damage from global climate change. We make the standard assumptions that g, c_1, c_2 are increasing and convex; and that D is increasing and convex in the stock $s = S - \mu - \mu_2$.

Here, we explicitly list three assumptions that relate to the damage function. We assume that the random variable Z is defined so that both damages and marginal damages are increasing and (weakly) convex in z. These assumptions imply that uncertainty is something to be concerned with: it increases both the expected damages and the expected marginal damages. Formally:

ASSUMPTION 1: *Marginal damages are increasing in z: $D(s_H, z) - D(s_L, z)$ is increasing in z for all $s_H > s_L$.*

ASSUMPTION 2: *Marginal damages are convex in z: $D(s_H, z) - D(s_L, z)$ is (weakly) convex in z for all $s_H > s_L$.*

Assumption 1 is sufficient to assure that optimal abatement increases with damages.[6] Note that Assumption 2 holds in the special case that damages are linear in z.

Throughout the paper, we assume that technical change takes a specific form, namely that it will decrease the cost of second period abatement proportionally: $c_2(\mu_2;\alpha) = (1-\alpha)c_2(\mu_2)$.[7] Baker, Clarke, and Weyant (2006) have argued that this assumption is a good representation of technical change aimed at reducing the cost of low-carbon alternatives, and that this kind of technical change can be a hedge against some increases in uncertainty.[8]

Abatement in the second stage is assumed to be optimal, and to depend on climate damages. Formally, the optimal interior value of $\mu_2^*(z)$ (where lower-case z represents a realization of the random variable Z) satisfies the first-order condition

$$\frac{\partial c_2}{\partial \mu_2} = \frac{\partial D}{\partial s}. \tag{6}$$

We may, however, have a corner point solution, where $\mu_2^*(z) = 1$ and $\frac{\partial c_2}{\partial \mu_2} \leq \frac{\partial D}{\partial s}$. We define $\bar{z}$ as the "full abatement point," the level of damage that induces the corner point solution: $\mu_2^*(z) = 1$ for all $z \geq \bar{z}$. This point is central to the results in Section 3.3.1 and for Proposition 1, thus we state the assumption formally.

ASSUMPTION 3: *For every μ and α there exists a $\bar{z}(\mu,\alpha)$ such that $\mu_2^*(z,\mu,\alpha) = 1$ for all $z \geq \bar{z}(\mu,\alpha)$.*

The full abatement point $\bar{z}$ is the level of damage that induces a corner point solution in the second period. Figure 1 illustrates optimal second period abatement when a full abatement point, $\bar{z}$ exists.

The first-order conditions for μ and α are as follows:

$$c_1'(\mu) = E_z\left[\frac{\partial D\big(S-\mu-\mu_2^*;Z\big)}{\partial S}\right] \tag{7}$$

$$g'(\alpha) = E_z\big[c_2\left(\mu_2^*\right)\big], \tag{8}$$

where μ_2^* is second period optimal abatement.

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[6] It is not, however, necessary. For example, the slightly weaker single-crossing property is also sufficient for optimal abatement to increase (Milgrom and Shannon 1994).

[7] See Jung, Krutilla, and Boyd (1996), Montero (2002), Goulder and Mathai (2000), Parry (1998), Fischer, Parry, and Pizer (2003), Goulder and Schneider (1999), Downing and White (1986), and Milliman and Prince (1989) for papers that make similar assumptions about technical change.

[8] We note here that other assumptions on how technical change impacts the cost curve can have fundamentally different results. For example, technical change into efficiency of fossil-fuels is not typically a hedge against risk (see Baker, Clarke, and Weyant 2006, Baker and Shittu 2006, Baker and Adu-Bonnah 2008). We do not mean to imply that all technical change is the same.

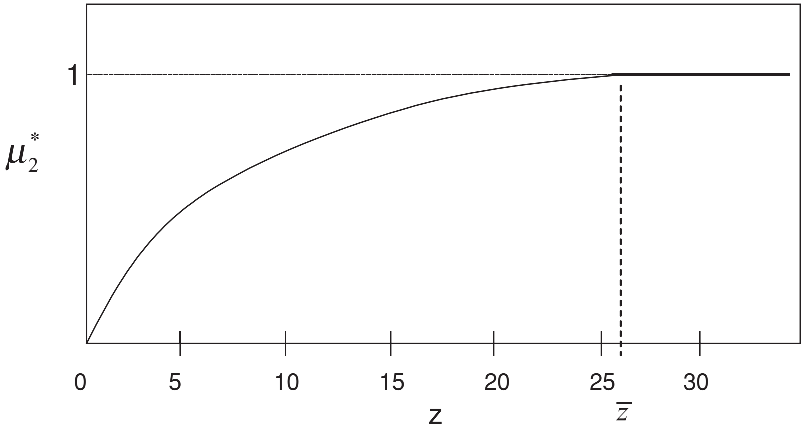

Figure 1: Optimal second period abatement as a function of the random variable *z*. Full abatement is optimal if $z \geq \bar{z}$.

The right-hand sides of (7) and (8) are the marginal benefits to near term abatement and to R&D. The right-hand side of (7) is the marginal change in damages due to a reduction in the stock of emissions, assuming that second period abatement is optimal; we will call this optimal marginal damages from now on (to distinguish from marginal damages, holding second period abatement constant).[9] The marginal benefits to abating in the first period are equal to the expected optimal marginal damages. On the other hand, the marginal benefits to investing in R&D are equal to the expected cost of optimal second period abatement. Thus, we need to investigate the shape of optimal marginal damages and the shape of the cost of optimal second period abatement, as a function of the random variable Z.[10]

3. The Shape of the Marginals

At the heart of this paper (and, we argue, driving all previous results) are the shapes of the marginal functions. We illustrate the concepts using the shapes induced from quadratic assumptions. We then use this context as a unifying device to interpret the state of the current literature. We go on to discuss how different sets of assumptions lead to implications about the shapes of the marginals, which then lead to comparative statics results.

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[9] Formally, marginal damages are defined as $\frac{\partial D(s,z)}{\partial s}$; optimal marginal damages are defined as $\frac{\partial D(S-\mu-\mu_2^*,\, z)}{\partial S}$.

[10] Note that the impact of risk on the net marginal benefits (the benefits minus the costs) are exactly equal to the impact of risk on the gross marginal benefits.

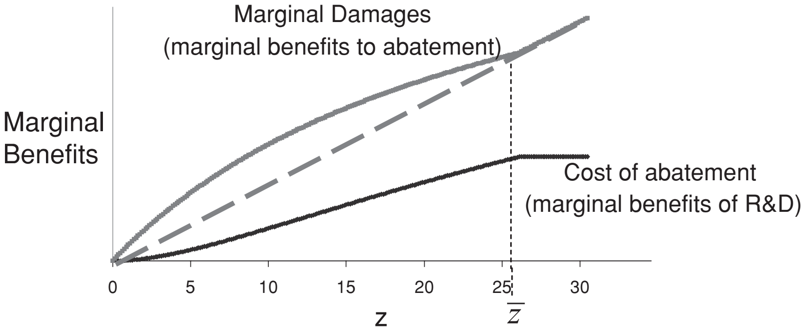

Figure 2: The marginal benefits of abatement and R&D as a function of multiplicative uncertainty on damages, Z. The marginal benefits to abatement are equal to $ZD'(S-\mu-\mu_2^*)$; the marginal benefits to R&D are equal to $c(\mu_2^*)$. The dashed line represents the marginal benefits to abatement, holding second period abatement constant at full abatement $\mu_2 = 1$: $ZD'(S-\mu-1)$.

3.1. An Illustration

Figure 2 illustrates the shape of the marginal benefits to abatement and to R&D, under the assumptions that costs and damages are quadratic, damages are linear in z, and abatement in both periods is restricted to be less than or equal to 1. The marginal benefits to abatement (the upper, solid line) are equal to $zD'(S-\mu-\mu_2^*)$ and here are graphed as a function of z. Note that changes in z have a direct effect on optimal marginal damages (increasing linearly in this case), and an indirect effect through optimal second period abatement μ_2^*. They are concave in z, until the full abatement point, $\bar{z}$. This reflects the fact that before full abatement, second period optimal emissions decrease as damages get worse. Thus, optimal marginal damages are concave (in fact, this is true as long as $D''' \leq 0$). When the full abatement point is reached, however, second period emissions can no longer be reduced and optimal marginal damages become linear in the variable z. The slope of the linear part of the curve is always higher than the slope of the concave part of the curve, thus the curve is reverse S-shaped with inflection at $\bar{z}$ (see Baker 2005, for proofs). The key points here are: (1) marginal damages are initially concave since there is room to respond optimally in the second period; (2) there is an inflection point at full abatement; and (3) the shape of the marginal benefits to abatement after the inflection point depends only on the direct effect of the random variable on the marginal damages.

The marginal benefits to R&D are equal to $c_2(\mu_2^*)$. This is impacted indirectly by z, since z impacts optimal emissions, μ_2^*. This marginal has a shape that is in some sense opposite to the marginal for abatement. The cost of optimal second period abatement is convex at $z = 0$, since $c_2(\cdot)$ is convex and μ is increasing in z. However, optimal second period abatement is concave in z—it becomes increasingly expensive to reduce the next unit of emissions, therefore the marginal reduction in emissions slows down. This leads to an inflection point $\hat{z} \leq \bar{z}$ after which the cost of optimal second period abatement is concave.[11] Finally, the cost of optimal second period abatement is constant for $z \geq \bar{z}$ since second period abatement is constant for that range. This leads the marginal benefits of R&D to be S-shaped. The key points here are: (1) optimal cost is initially convex if cost is convex and second period abatement is increasing rapidly near the origin; (2) optimal cost will generally become concave in high damages, since optimal second period abatement will be strongly concave (as it approaches a maximum level); and (3) if a full abatement point exists, then optimal cost will be constant after this point.

The figure shows that for both control variables the marginal benefits are neither convex nor concave under even these extremely simple assumptions, and thus both optimal first period abatement and optimal R&D will increase with some increases in risk and decrease with other increases in risk.

3.2. Prior Results and the Shape of the Marginals

There has been a substantial amount of work analyzing the impact of uncertainty and learning on optimal abatement. The most common result has been that the impact of uncertainty and learning on abatement is ambiguous. This result is driven by two conflicting irreversibilities. On the one hand, the ability to react in the second period (increasing second period abatement if damages are high) puts a downward pressure on first period abatement. This is because the resources allocated to abatement in the first period cannot be recouped in the second period. On the other hand, if the damages from climate change turn out to be very high, then the ability to react is limited: second period abatement is restricted to full abatement. Thus, the irreversibility of emissions will bite in this case. This puts an upward pressure on abatement, but only in proportion to the probability that damages are high enough to induce full second period abatement. The first irreversibility induces the concavity in marginal damages seen near the origin; the second irreversibility induces the convexity at the full abatement point.

A prominent example of illustrating the ambiguity of results on optimal abatement is found in Ulph and Ulph (1997). They go on to show in Lemma 3(iii) that if the mean damages are high, $E[z] > \bar{z}$, then abatement will be higher under risk than under no risk.[12] To understand how this result is related to the shape of the marginal damages see Figure 3. Note that any simple

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[11] Specifically, if $c_2(\mu) = a\mu^2 + b\mu$ and $D(S-\mu) = (S-\mu)^2$ then $\bar{z} = \frac{a+\frac{b}{2}}{S-1}$ and $\hat{z} = \min[\frac{a(aS-\frac{b}{2})^2+ab}{2aS+b}, \bar{z}]$.

[12] Their result is stated in terms of learning versus no learning. Given their linear model, their result is true if and only if abatement is higher under risk than under no risk. See Baker (2006) for details of the equivalence between increases in risk and increases in learning.

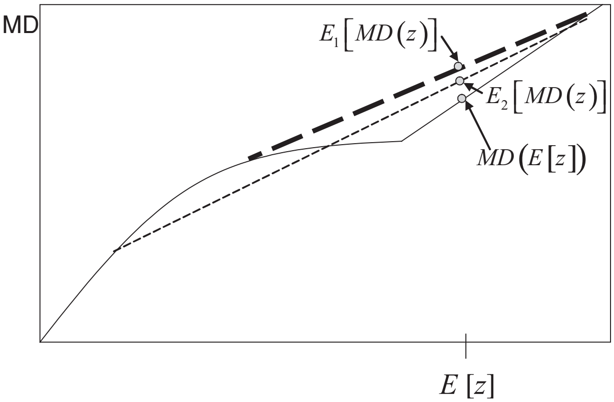

Figure 3: The thick dashed line represents an MPS in the random variable *Z*; the thinner dashed line represents an additional MPS. Expected marginal damages first increase, and then decrease with the successive MPS.

MPS around any point above $\bar{z}$ will lead to (weakly) higher expected marginal damages due to the convexity induced by the full abatement point. This is illustrated in this case by both dotted lines. However, note that the thinner dotted line represents a riskier distribution than the thicker, and yet the expected marginal damages decrease with a move to the riskier distribution. In Proposition 1 below we provide a related, but more general result, removing the restriction on the mean and allowing for successive increases in risk.

Webster (2002) analyzes a similar model and concludes again that the impact of risk is ambiguous, and that what matters is how the probability is redistributed. This paper will address this question explicitly. Kolstad (1996), Gollier, Jullien, and Treich (2000), Karp and Zhang (2006), and Baker (2005) each consider the impact of an increase in learning, and arrive at varying conclusions, finding conditions under which learning causes optimal emissions to increase or decrease. A literature is starting to emerge considering how uncertainty in damages impacts optimal technology R&D (e.g., Baker and Adu-Bonnah 2008, Baker, Clarke, and Weyant 2006, Farzin and Kort 2000. See Baker and Shittu 2008, for a review). The impact of uncertainty on optimal technology investment depends on how the technology is modeled, which impacts the profile seen in Figure 2.

3.3. What can be Inferred about the Shape of the Marginals?

From the simple analysis above we note that optimal marginal damages tend to be convex around the full abatement point, and concave near the origin; optimal costs tend to be concave around the full abatement point and convex near the origin. In this section, we provide sufficient conditions that formalize these observations. In the first subsection below we discuss how

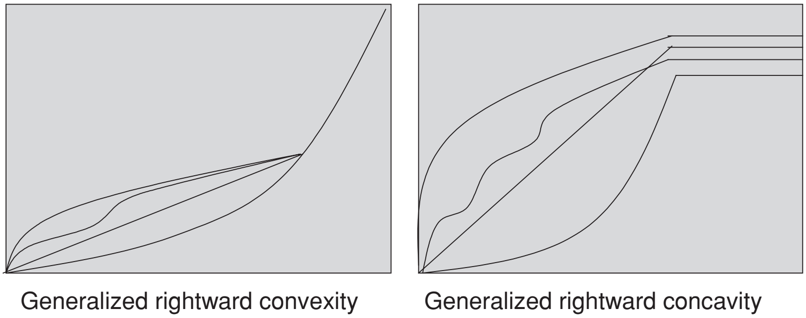

Figure 4: Each panel illustrates four representative functions. Each of the functions in the left-hand panel belongs to the set $\Delta_{\bar{z}}$ of generalized convex functions; each of the functions in the right-hand panel belongs to the set $\Psi_{\bar{z}}$ of generalized concave functions.

weak assumptions about the costs and damages of climate change, coupled with the assumption that a full abatement point exists, lead to something we call generalized convexity/concavity in the marginals. In the following subsection we discuss, both qualitatively and mathematically, what kinds of assumptions lead to marginals that are convex/concave near the origin.

3.3.1. Generalized Convexity/Concavity

Here we focus on the convexity/concavity induced by the full abatement point.

Optimal marginal damages: convex around the full abatement point. In this section, we define the set of "Generalized rightward convex" functions, and show that optimal marginal damages are in this set under very weak assumptions. Formally, we define the set $\Delta_{\bar{z}}$: all increasing functions $d : \mathbb{R} \to \mathbb{R}$, such that there exists a continuous, convex function $\kappa(z)$ with the following properties

1. $d(z) \geq \kappa(z)$ for all $z \leq \bar{z}$;
2. $d(z) = \kappa(z)$ for $z \geq \bar{z}$.

The left-hand panel in Figure 4 illustrates four representative functions in this set, each with the same underlying convex function $\kappa(z)$.[13] These functions are convex around all points to the right of the inflection point $\bar{z}$ (that is for any points x, x' and $0 \leq \gamma \leq 1$ such that $\gamma x + (1 - \gamma) x' \geq \bar{z}$, it is

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[13] Note that if $d(z) \in \Delta_{\bar{z}}$ then, by definition, there will exist at least one $\kappa(z)$ satisfying the conditions; there may, however, be many such $\kappa(z)$. On the other hand, each convex function $\kappa(z)$ forms the basis for a number of Generalized rightward convex functions.

true that $\gamma d(x) + (1-\gamma)d(x') \geq d(\gamma x + (1-\gamma)x'))$. Note that all convex functions are members of $\Delta_{\bar{z}}$.

LEMMA 1: *Under Assumptions 1, 2, and 3 above, optimal marginal damages belong to the set* $\Delta_{\bar{z}}$.

Proof: Assumption 1 assures that optimal marginal damages are increasing in z. Assumptions 2 and 3 imply that a convex function $\kappa(z)$ exists that is equal to optimal marginal damages for $z \geq \bar{z}$, namely $\kappa(z) \equiv \frac{\partial D(S-\mu_2^*(\bar{z});z)}{\partial S}$, marginal damages when second period abatement is full. The standard assumption that damages are convex in the stock of emissions implies that optimal marginal damages are greater than $\kappa(z)$ for $z \leq \bar{z}$: $\frac{\partial D(S-\mu_2^*(z);z)}{\partial S} \geq \frac{\partial D(S-\mu_2^*(\bar{z});z)}{\partial S}$. Thus, under the assumptions above, optimal marginal damages belong to the set of generalized rightward convex functions. ■

Optimal costs: concave around the full abatement point. In this section, we define the set of "Generalized rightward concave" functions, and show that the cost of optimal second period abatement is in this set under very weak assumptions. Formally, we define the set $\Psi_{\bar{z}}$ as all increasing functions that are constant for $z \geq \bar{z}$. The right-hand panel in Figure 4 illustrates four representative functions in this set. These functions are concave around points to the right of the inflection point $\bar{z}$ (that is for any points x, x' and $0 \leq \gamma \leq 1$ such that $\gamma x + (1-\gamma)x' \geq \bar{z}$, it is true that $\gamma f(x) + (1-\gamma)f(x') \leq f(\gamma x + (1-\gamma)x')$.

LEMMA 2: *Under Assumptions 1 and 3 above, the cost of optimal second period abatement,* $c(\mu_2^*)$, *is in the set* $\Psi_{\bar{z}}$.

Proof: If marginal damages are increasing in z, then optimal second period abatement is weakly increasing in z. This implies that the cost of optimal second period abatement is (weakly) increasing in z; and that it is constant in z for $z \geq \bar{z}$. ■

3.3.2. Concavity/Convexity Near the Origin

In this section, we focus on the shapes of the marginals near the origin. In order to derive unambiguous results we make the following strong, but common assumption.

ASSUMPTION 4: *Damages are linear in the random variable Z.*

We stress, however, that this is only a sufficient condition. Below, we discuss qualitatively what drives the results and the likelihood that necessary conditions obtain.

Marginal damages: concave near the origin. Formally, define the set $\delta_{\hat{z}}$ as all increasing functions that are concave for $z \leq \hat{z}$.

LEMMA 3: *Under Assumptions 1 and 4, there exists a* $\hat{z} \geq 0$ *such that optimal marginal damages,* $\frac{\partial z D(S - \mu_2^*(z))}{\partial S}$, *belong to the set* $\delta_{\hat{z}}$.

Proof: Under the assumption that damages are linear in Z, Proposition 2 in Baker (2005) shows that optimal marginal damages are always strictly concave at $z = 0$. ■

Optimal marginal damages are concave in z for a similar reason that the standard economic cost function is concave in prices: the ability to act in response to a higher z moderates the impact of the higher z. In the absence of second period abatement marginal damages would be linear in z; the ability to react to a higher z by abating more induces optimal marginal damages to be concave. From this argument it is clear that the linearity assumption is sufficient, but not necessary. As long as marginal damages are not too convex at $z = 0$, optimal marginal damages will be concave.

Optimal costs: convex near the origin. Define the set $\psi_{\hat{z}}$ as all increasing functions that are convex for $z \leq \hat{z}$. In order to show that optimal costs are in this set we must make one more assumption.

ASSUMPTION 5: *Marginal costs are weakly concave:* $c_2''' \leq 0$; *and costs are sufficiently more convex than damages:* $\frac{c_2''}{c_2'} > 2\frac{\frac{\partial^2 D}{\partial s^2}}{\frac{\partial D}{\partial s}}$.

LEMMA 4: *Under Assumptions 4 and 5, there exists* $\hat{z} > 0$ *such that optimal costs,* $c(\mu_2^*)$, *belong to the set* $\psi_{\hat{z}}$.

See the Appendix for proof.

The reason that we expect optimal costs to be convex near the origin, is that second period abatement tends to increase relatively quickly at the origin, and costs are convex in second period abatement. This will be true as long as the cost of abatement is convex in the abatement level; and optimal second period abatement is not "too concave" in damages. Assumption 5 is sufficient, but not necessary, to ensure that optimal second period abatement is not "too concave."

3.3.3. Summary

In summary, under fairly weak restrictions, the marginal benefits to abatement are convex around the full abatement point, and the marginal benefits to R&D are concave around the full abatement point. Under the assumption that damages are linear in Z and costs are more convex in abatement than damages, the marginal benefits to abatement are concave near the origin, and the marginal benefits to R&D are convex near the origin.

Table 1: Summary of set definitions

Set Notation	Set Definition	Member of Set	Subset of MPS that Increase (Decrease)
$\Delta_{\bar{z}}$	$\left\{\begin{array}{l} f:\mathbb{R}\rightarrow\mathbb{R} \mid \exists\kappa(z) \text{ continuous, convex s.t.} \\ f(z)\geq\kappa(z)\forall z\leq\bar{z};\ f(z)=\kappa(z)\forall z\geq\bar{z} \end{array}\right\}$	Optimal Marginal Damages	RMPS
$\Psi_{\bar{z}}$	$\{f:\mathbb{R}\rightarrow\mathbb{R},\ \text{nondecreasing; constant}\ \forall z\geq\bar{z}\}$	Optimal Costs	(RMPS)
$\delta_{\hat{z}}$	$\{f:\mathbb{R}\rightarrow\mathbb{R},\ \text{nondecreasing; concave}\ \forall z\geq\hat{z}\}$	Optimal MD	$\left(\begin{array}{l} MPS; F=G \\ \text{for } z\geq\bar{z} \end{array}\right)$
$\psi_{\hat{z}}$	$\{f:\mathbb{R}\rightarrow\mathbb{R},\ \text{nondecreasing; convex}\ \forall z\geq\bar{z}\}$	Optimal Costs	$MPS; F=G$ for $z\geq\bar{z}$

4. Stochastic Dominance Theorems

In this section, we present new stochastic dominance theorems based on the shapes we determined for the marginals above. Table 1 summarizes the sets of functions (related to the shapes of the marginals) used in the stochastic dominance theorems. We focus on two subsets of MPS. In the first section, we focus on MPS that stretch the tail of the distribution to the right; in the second, we consider MPS where the risk is mainly increased to the left of an extreme point.

4.1. Generalized Convexity/Concavity

In this section, we present a stochastic dominance theorem for the sets of generalized convex and concave functions, $\Delta_{\bar{z}}$ and $\Psi_{\bar{z}}$. Since the members of each of these sets are convex/concave around points to the right of the inflection point, we consider an MPS that stretches or skews the distribution to the right, while leaving the relative probabilities to the left of the inflection point unchanged. Let F and G be cumulative probability distributions. First, we define the probability distribution to the left and the right of the inflection point. These are the original probability distributions normalized.

$$F_L \equiv \left\{\begin{array}{ll} \dfrac{F(x)}{F(\bar{z})} & x\leq\bar{z} \\ 1 & x\geq\bar{z} \end{array}\right| \tag{9}$$

$$F_R \equiv \left\{\begin{array}{ll} 0 & x\leq\bar{z} \\ \dfrac{F(x)-F(\bar{z})}{1-F(\bar{z})} & x\geq\bar{z} \end{array}\right| . \tag{10}$$

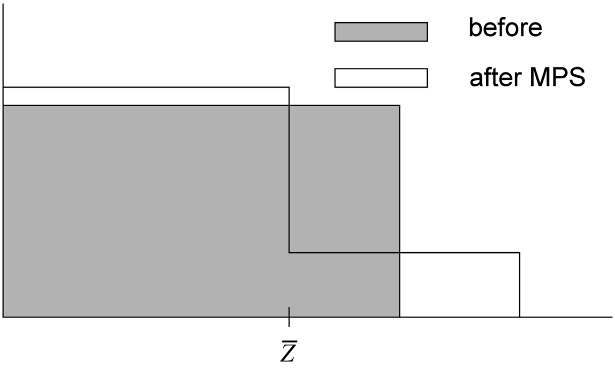

Figure 5: A right-skewed MPS around $\bar{z}$.

Let the means of F_L and F_R be written as η_{FL} and η_{FR}. Now we define an MPS that is right-skewed around a point $\bar{z}$.

DEFINITION 2: *G RMPS*$_{\bar{z}}$ *F if G MPS F and* $G_L = F_L$.

If G RMPS$_{\bar{z}}$ F then it follows that $G(\bar{z}) \geq F(\bar{z})$, $\eta_{GR} \geq \eta_{FR}$, and that G RMPS$_{\hat{z}}$ F for all $\hat{z} \leq \bar{z}$. Figure 5 provides an example of an RMPS. In the riskier distribution, the probability of an extreme event (defined as any $z \geq \bar{z}$) is lower, but the expected damages given an extreme event (i.e., $E[Z \mid Z \geq \bar{z}]$) are higher.

Following the stochastic dominance literature, we say that G dominates F on the set Υ if

$$\int u(x)\, dG \geq \int u(x)\, dF \ \forall u \in \Upsilon. \tag{11}$$

Now we present a stochastic dominance theorem for generalized rightward convexity and concavity. See the Appendix for proof.

THEOREM 1: *If G RMPS*$_{\bar{z}}$ *F then G dominates F on* $\Delta_{\bar{z}}$, *the set of generalized rightward convex functions; and F dominates G on* $\Psi_{\bar{z}}$, *the set of generalized rightward concave functions.*

4.2. Concave/Convex Near the Origin

In this section, we present a stochastic dominance theorem for functions that are convex or concave near the origin. Thus, we consider an MPS near the origin, leaving the tail of the distribution unchanged.

THEOREM 2: *if G MPS F and* $F = G$ *for* $x \geq \hat{z}$ *then G dominates F on* $\delta_{\hat{z}}$ *and F dominates G on* $\psi_{\hat{z}}$.

5. Application of Theorems to Climate Change

5.1. Results

In this section, we combine the discussions in Section 3 on the shape of the marginals with the Theorems from Section 4 to get comparative statics results for climate change. The first proposition says that if we believe that a full abatement point exists, and that uncertainty is something to be concerned about (in the sense that expected damages increase in uncertainty) then increases in risk that stretch the tail of the probability distribution lead to optimally higher abatement and lower R&D investments.

PROPOSITION 1: *Under Assumptions 1 and 3, optimal R&D will decrease with* $RMPS_{\bar{z}}$. *Under the additional Assumption 2, optimal first period abatement will increase with* $RMPS_{\bar{z}}$. *Precisely, let* $\alpha(F)$, $\mu(F)$, *be the optimal values of the decision variables given distribution of damages F, and* $\bar{z}(F)$ *the full abatement point given* $\alpha(F), \mu(F)$. *If G* $RMPS_{\hat{z}}$ *F for any* $\hat{z} \geq \bar{z}(F)$ *then* $\alpha(G) \leq \alpha(F)$ *and* $\mu(G) \geq \mu(F)$.

Proof: Lemma 2 says that $c(\mu_2^*) \in \Psi_{\bar{z}}$, the set of generalized rightward concave functions. Theorem 1 implies that the right-hand side of (7), $E[c(\mu_2^*)]$, is higher under G, and therefore the optimal μ is higher. Lemma 1 says that $\frac{\partial D(S-\mu-\mu_2^*; Z)}{\partial S} \in \Delta_{\bar{z}}$, the set of rightward convex functions. Theorem 1 implies that the right-hand side of (8), $E[\frac{\partial D(S-\mu-\mu_2^*; Z)}{\partial S}]$, is lower under G, and therefore optimal α is lower. ■

An RMPS means that there is a higher likelihood of being in the lower part of the distribution (where the lower part is defined by the inflection point, $z \leq \bar{z}$); but given that damages are in the higher part of the distribution, expected damages are now worse. In terms of R&D this means that the overall probability of full abatement is lower, thus the expected benefits from R&D decrease with the RMPS. For abatement, this means that if the irreversibility constraint does bite, it bites harder. Thus, it provides an incentive for leaving a little more flexibility to reduce emissions.

The next proposition makes slightly stronger assumptions that lead to marginal damages being concave and optimal cost being convex near the origin. Under these assumptions, a change in the distribution of damages that is riskier in the left-hand side of the distribution, but leaves the right part of the distribution unchanged, will decrease abatement and increase optimal R&D.

PROPOSITION 2: *Under Assumptions 1, 4, and 5, for every F there exists* $\hat{z}(F) \geq 0$ *such that optimal abatement is decreasing* $(\mu(G) \leq \mu(F))$ *and optimal R&D is increasing* $(\alpha(G) \geq \alpha(F))$ *whenever G MPS F and* $F = G$ *for* $\forall z > \hat{z}(F)$.

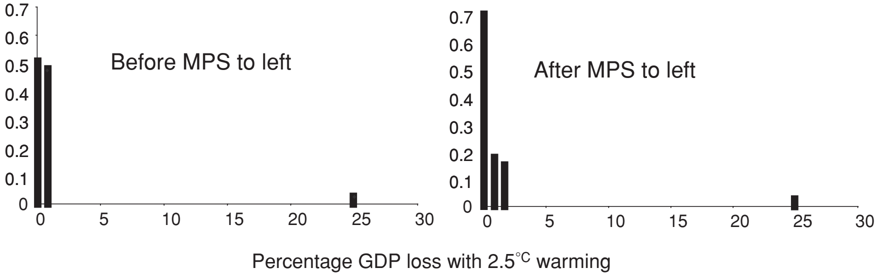

Figure 6: Both panels illustrate probability distribution functions over $z \equiv$ percentage GDP loss given a 2.5 °C warming. The right-hand panel represent an "MPS to the left" as compared to the left-hand panel, where the inflection point could be any point between 5 and 20.

Proof: The assumptions together with Lemmas 3 and 4 imply that there exists a $\hat{z} \geq 0$ such that optimal marginal damages are in the set $\delta_{\hat{z}}$ and optimal costs are in the set $\psi_{\hat{z}}$. Theorem 2 implies that expected optimal marginal damages are decreasing and expected costs are increasing whenever G MPS F and $F = G$ for $\forall z > \hat{z}(F)$. This implies that optimal abatement is decreasing and optimal R&D is increasing. ■

Figure 6 illustrates an MPS near the origin, as in Proposition 2. If, for example, we assume that full abatement is optimal given a 20% loss of GDP, then such an MPS would cause optimal first period abatement to decrease but optimal spending on low-carbon technology R&D to increase. Abatement decreases because expected second period abatement is lower after the MPS, and neither the likelihood nor the magnitude of suffering high damages has changed. R&D increases because the expected benefits from R&D are higher after the MPS.

5.1.1. Summary

Optimal abatement increases with MPS that stretch the distribution of damages to the right; and decreases with MPS near zero (which in this case is near the mean). When we consider optimal technology investment we get almost opposite results. Optimal R&D decreases with MPS that stretch the distribution of damages to the right; and increases with MPS near zero.

5.2. Role of Assumptions

5.2.1. Costs and Damages Not Separable

The decision problem in (5) implies that the decision maker is risk-neutral, has perfect elasticity of substitution across time, and that costs of abatement are separable from damages. All of these assumptions are reversed if we put a utility function around second period costs and damages.[14] Theoretically, this can change the shape the marginal benefits to abatement and to R&D. Specifically, the right-hand sides of the first-order conditions (8) and (7) will be multiplied by the marginal (dis)utility. If utility reflects extreme prudence (that is, if $-\frac{u'''}{u''}$ is very high), then the marginal benefits to both abatement and R&D could be everywhere convex, implying that both abatement and R&D increase unambiguously in risk. For example, Gollier, Jullien, and Treich (2000) have shown, in a slightly different model, that marginal damages can become everywhere convex, implying that optimal abatement increases unambiguously in risk. The level of curvature required, however, was very large, and thus may not be very relevant. Moreover, the results presented here are consistent with Baker, Clarke, and Weyant (2005), who used the DICE model, which has a log utility function, to test how optimal R&D responds to increases in risk.

5.2.2. Full Abatement

In Proposition 1, we assume that full abatement is achievable, but that abatement is capped at 1. It may be argued, on the one hand, that full abatement is not possible, that the cost of abatement goes to infinity as abatement approaches one. It may be argued, on the other hand, that there is no strict limit on abatement, that it is physically possible to remove carbon emissions from the atmosphere. Sufficient conditions for a full abatement point to exist are that the cost of abatement is finite at full abatement, it is impossible to reduce the flow of emissions below zero, and damages increase indefinitely as z increases. Even if a full abatement point does not exist, however, the results for optimal abatement will hold as long as (1) there exists some value k such that $\mu_2^*(z) \leq k\ \forall z$ and (2) $\bar{z}$ is such that $\mu_2^*(z) \geq k - \varepsilon\ \forall z \geq \bar{z}$ where ε is sufficiently small. This is because the key to the abatement results is that the second period ability to react must become severely limited at some point.

On the other hand, in the absence of a full abatement point, it is theoretically possible that the optimal cost of abatement is everywhere convex, in particular, if marginal damages are very convex in the stock and full abatement is not achievable. In this case, the optimal investment in alternative R&D would unambiguously increase in risk. It would be interesting to see if any technologically detailed integrated assessment models could induce such a result, and what kinds of assumptions it would require.

A second question is whether the full abatement point, if it exists, is relevant. To answer that requires more in-depth expert assessments on the possible damages from climate change than have been done to this point.

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[14] First period utility does not impact the comparative statics of risk or learning, so it is ignored.

We note that some models indicate that full abatement may be optimal if our goal is to stabilize the stock of emissions at 350 ppmv (Wigley, Richels, and Edmonds 1996), and that this stabilization goal may be optimal if damages turn out to be severe. Thus, it would appear that there is a positive probability that climate change could induce full abatement.

6. Learning

These results can be extended to partial learning. We can re-write the climate change decision problem as a problem of sequential decision making under partial learning, as follows:

$$\min_{\mu,\alpha} c_1(\mu) + g(\alpha) + E_Y\Big[\min_{\mu_2} E_{Z|Y}\{c_2(\mu_2;\alpha) + D(S-\mu-\mu_2;Z)\}\Big], \quad (12)$$

where Y is a random variable defined on the same space as Z, thus it may give some information about Z. We define a more informative signal in the Blackwell sense: Y is more informative than Y' if every decision maker is (weakly) better off under signal Y than Y'. Baker (2006) shows that if the damages D are linearly separable in a function of Z, then the impact of learning more is the same as the impact of increasing risk. To illustrate the intuition behind this, assume that D is linear in Z. Then (12) simplifies to

$$\min_{\mu,\alpha} c_1(\mu) + g(\alpha) + E_Y \min_{\mu_2}[c_2(\mu_2;\alpha) + E[Z \mid Y]D(S-\mu-\mu_2)]. \quad (13)$$

As Y increases in informativeness, the random variable $E[Z \mid Y]$ increases in risk. For intuition consider the two extreme cases. If Y is independent from Z then $E[Z \mid Y] = E[Z]$ for all possible realizations of Y: the random variable $E[Z \mid Y]$ is constant. On the other hand, if Y provides perfect information about Z, that implies that $E[Z \mid Y] = Z$, i.e., the variability of $E[Z \mid Y]$ will be exactly equal to the variability of Z. Thus, an increase in the informativeness of the signal has the same qualitative impact as an increase in the risk of the random variable.

The broad intuition of this result is as follows: if in problem (13) we expect to have more information before we choose second period abatement μ_2 then we will want to choose first period abatement and R&D in such a way to leave ourselves more flexibility to react to what is learned. Similarly, the more prior risk we face in problem (5), the more flexibility we would like when choosing μ_2. Hence, an increase in informativeness and an increase in risk have similar effects on first period decisions.

Just as current decisions in (5) depend on the probability distribution of Z; in the learning model (13) current decisions depend on the distribution of $E[Z \mid Y]$. Thus, a more informative signal that induces an RMPS in $E[Z \mid Y]$ will have the same impact as an RMPS in Z. Here we present a proposition

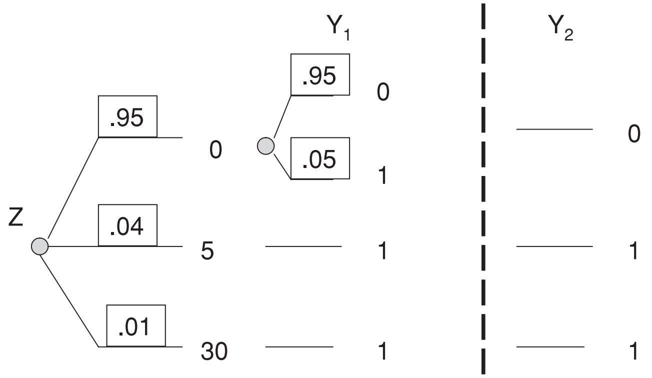

Figure 7: Y_2 is more informative than Y_1. It is isomorphic to an RMPS.

that generalizes Proposition 1. The proofs of Propositions 3 and 4 follow from an application of theorem 1 of Baker (2006).

PROPOSITION 3: *Assume that damages are multiplicatively separable in the random variable:* $D(s, Z) = h(Z)\tilde{D}(s)$. *Then, Under Assumptions 1, 2, and 3, optimal R&D will decrease and optimal first period abatement will increase with any signal that induces a* $RMPS_{\bar{z}}$ *in* $E[Z \mid Y]$.

Say that Y' is more informative than Y, and induces an $\mathrm{RMPS}_{\bar{z}}$ in $E[Z \mid Y]$. This means that the probability distribution over $E[Z \mid Y]$ is the same as the probability distribution over $E[Z \mid Y']$ given that $E[Z \mid Y], E[Z \mid Y'] \leq \bar{z}$. Stated another way, given that the signals indicate that damages are "low" ($E[Z \mid Y] \leq \bar{z}$), we cannot differentiate between the signals. However, the probability that the more informative signal says "low" is higher; and given that the signal says "high" ($E[Z \mid Y] \geq \bar{z}$), expected damages are higher under the more informative signal. Consider signals that have only two outcomes, high or low. Then the more informative signal is an RMPS if it has the same chance of a false negative ($\Pr(Z > \bar{z} \mid Y = low) = \Pr(Z > \bar{z} \mid Y' = low)$) but a lower chance of a false positive $\Pr(Z < \bar{z} \mid Y = high) > \Pr(Z < \bar{z} \mid Y' = high)$.

Figure 7 presents an example of a more informative signal that is isomorphic to an RMPS. Uncertainty about damages is represented by the random variable Z. Assume that full abatement is optimal for all $Z \geq 5$. If true damages are below 5, then the first signal, Y_1 will correctly indicate this 95% of the time; but 5% of the time it will produce a signal associated with high damages. The second signal Y_2 will always indicate whether damages are above or below 5. The probability that $y_1 = 0$ is $.95 * .95 \approx .9$. $E[z \mid y_1 = 0] = 0$; $E[z \mid y_1 = 1] \approx 5$. The probability that $y_2 = 0$ is .95. $E[z \mid y_2 = 0] = 0$; $E[z \mid y_2 = 1] = 10$. So $E[z \mid y_2]$ is a RMPS of $E[z \mid y_1]$. If we expect to get signal 2 rather than signal 1, we would do less R&D, but more abatement, in the short run. We do less R&D because in both cases the value of R&D is the same if we get the high signal, but we now have a lower probability of using R&D. On the other hand, if we are getting signal 2, we want to leave ourselves more flexibility to decrease emissions, since if we get bad news, it will be worse.

We now present a proposition that generalizes Proposition 2.

PROPOSITION 4: *Assume that damages are separable in the random variable:* $D(s, Z) = h(Z)\tilde{D}(s)$. *Then, under Assumptions 1, 4, and 5, optimal abatement is decreasing and optimal R&D is increasing with a more informative signal, when the signal is only more informative about outcomes to the left of* $\hat{z}$.

This implies that if we expect (in the future) to get a better handle on damages near the mean of the distribution, while leaving the tail unexplored, then we should optimally spend more on R&D and less on abatement. Given better information around the mean, we expect to get more value, on average, from R&D. This is because, near mean damages, the cost of abatement is increasing rapidly; thus if we learn that damages are a bit higher than the mean, we get a fair amount of value from R&D. On the other hand, given better information around the mean means that we can more accurately respond in the second period in terms of abatement; when damages are near the mean, we know that we will not hit a constraint, so first period abatement has relatively less value since we will have flexibility to respond in the second period; thus there is an option value to waiting to implementing higher abatement.

Thus, to the degree that climate change research is aimed at analyzing the impact of a given mean temperature change, and mean damages resulting from the temperature change, more should be spent on R&D and less on short term abatement. On the other hand, to the degree that climate change research is aimed at increasing our understanding of the probability of a high damage event, less should be spent on R&D and more should be spent on abatement.

7. Conclusions

In this paper, we provide a new perspective for analyzing the comparative statics of uncertainty and learning in the climate change problem. By focusing on the shape of the marginals we are able to provide new, unambiguous results on how optimal policy changes with changes in risk and changes in what we expect to learn. We have shown that abatement and investment in alternative energy R&D may be risk-substitutes in many cases: changes in risk that optimally increase one, decrease the other. In particular, we have shown that if a "full abatement point" exists, then a mean-preserving-spread that stretches the probability distribution over damages to the right of that point (or analogously, an expectation of learning more about the tail of the damage distribution than about the mean) will increase optimal abatement and decrease optimal alternative energy R&D spending. On the other hand, if we consider multiplicative damage uncertainty, and assume that neither damages nor costs are too convex in abatement, then a mean-preserving-spread near the mean (or analogously, an expectation of learning more around the mean than around the tail) will result in a decrease in optimal abatement and an increase in optimal alternative energy R&D spending.

Abatement is a hedge against *catastrophic* damages—damages that make society want to abate more than the total flow of emissions. If there is a possibility that damages will turn out to be catastrophic and society may find itself up against a wall, there is value in reducing emissions now in order to preserve the flexibility to respond in the future. On the other hand, if damages turn out to be just a little worse than expected, society can simply abate more in response—there is value to waiting to learn. Technology R&D is a hedge against damages that may be a little worse than average. The reason is that if damages are a little worse than expected, but not catastrophic, then an incremental improvement in technology leads to two benefits: the cost of abating a given amount is lower *and* the optimal abatement is higher. The double payoff is what makes the investment more attractive. However, in the event of a catastrophe, improvement in technology will not lead to higher abatement, it will only lead to cost savings. Since the probability of this cost savings is lower under riskier damages, the optimal investment is smaller.

The focus on the shapes of the marginals presents a unifying framework for interpreting the prior work on the comparative statics of uncertainty and learning in the climate change problem. For example, we show how an assumption that mean damages are very high leads to higher optimal abatement under risk than under no risk. Additionally, we point out that the essential role played by the assumption of "prudence" is to induce concavity in a reverse S-shaped marginal.

In order to provide comparative statics results, we have presented and proved new stochastic dominance theorems, in particular for the sets of "generalized" convex/concave functions. These theorems may be applicable in other problems in which an irreversibility constraint induces a bend in the marginals that prevents the application of standard stochastic dominance theorems.

There is much room for future work expanding the results in this paper. In particular, we have provided insights based on very simple models of climate change. One contribution of this work is to provide a structure for running computational experiments on more detailed models of climate change. Without the theoretical results, there is a danger of over-generalizing specific computational results. Thus, an important direction for future work is to run computational experiments on more detailed models to determine to what degree the general results in this paper hold; and to what degree increases in risk are quantitatively important.

Appendix

Proof of Theorem 1:

(1) *Generalized rightward convex functions:* Let $d(z) \in \Delta_{\bar{z}}$; in particular let $\kappa(z)$ be a convex function such that $\kappa(z) \leq d(z)$ for $z \leq \bar{z}$ and $\kappa(z) = d(z)$ for $z \geq \bar{z}$. Since G MPS F it is true for all convex functions κ that $\int_0^\infty \kappa(z)\,dF \leq \int_0^\infty \kappa(z)\,dG$. Expanding this inequality using the definitions of probability distributions to the right and the left given in (9) and (10), we get

$$F(\bar{z}) \int_0^{\bar{z}} \kappa(z)\,dF_L + (1 - F(\bar{z})) \int_{\bar{z}}^\infty \kappa(z)\,dF_R \leq G(\bar{z}) \int_0^{\bar{z}} \kappa(z)\,dG_L + (1 - G(\bar{z})) \int_{\bar{z}}^\infty \kappa(z)\,dG_R. \tag{A1}$$

Rearranging terms

$$(1 - F(\bar{z})) \int_{\bar{z}}^\infty \kappa(z)\,dF_R - (1 - G(\bar{z})) \int_{\bar{z}}^\infty \kappa(z)\,dG_R \leq G(\bar{z}) \int_0^{\bar{z}} \kappa(z)\,dG_L - F(\bar{z}) \int_0^{\bar{z}} \kappa(z)\,dF_L. \tag{A2}$$

Since $F_L = G_L$

$$(1 - F(\bar{z})) \int_{\bar{z}}^\infty \kappa(z)\,dF_R - (1 - G(\bar{z})) \times \int_{\bar{z}}^\infty \kappa(z)\,dG_R \leq (G(\bar{z}) - F(\bar{z})) \int_0^{\bar{z}} \kappa(z)\,dF_L. \tag{A3}$$

The fact that $\kappa(z) \leq d(z)$ for $z \leq \bar{z}$ implies that

$$\int_0^{\bar{z}} \kappa(z)\,dF_L \leq \int_0^{\bar{z}} d(z)\,dF_L. \tag{A4}$$

Combining (A3) and (A4) implies

$$(1 - F(\bar{z})) \int_{\bar{z}}^\infty \kappa(z)\,dF_R - (1 - G(\bar{z})) \times \int_{\bar{z}}^\infty \kappa(z)\,dG_R \leq (G(\bar{z}) - F(\bar{z})) \int_0^{\bar{z}} d(z)\,dF_L. \tag{A5}$$

Rearranging terms and noting that $d(z) = \kappa(z)$ for $z \geq \bar{z}$ we get that

$$F(\bar{z}) \int_0^{\bar{z}} d(z)\,dF_L + (1 - F(\bar{z})) \int_{\bar{z}}^\infty d(z)\,dF_R \leq G(\bar{z}) \int_0^{\bar{z}} d(z)\,dG_L + (1 - G(\bar{z})) \int_{\bar{z}}^\infty d(z)\,dG_R \tag{A6}$$

and thus, for all $d \in \Delta_{\bar{z}}$

$$\int_0^\infty d(z)\,dF \le \int_0^\infty d(z)\,dG. \tag{A7}$$

(2) *Generalized rightward convex functions:* For any $c \in \Psi_{\bar{z}}$

$$\int_0^\infty c(z)\,dF = F(\bar{z}) \int_0^{\bar{z}} c(z)\,dF_L + (1 - F(\bar{z}))\,c(\bar{z}) \tag{A8}$$

$$= F(\bar{z}) \int_0^{\bar{z}} c(z)\,dG_L + (1 - F(\bar{z}))\,c(\bar{z}) \tag{A9}$$

$$\ge G(\bar{z}) \int_0^{\bar{z}} c(z)\,dG_L + (1 - G(\bar{z}))\,c(\bar{z}) = \int_0^\infty c(z)\,dG. \tag{A10}$$

The last inequality holds true since $G(\bar{z}) \ge F(\bar{z})$ and $\int_0^{\bar{z}} c(z)\,dG_L \le c(\bar{z})$. ■

Proof of Theorem 2: Assume G MPS F and $G = F$ for $x \ge \hat{z}$. In particular, note that $G(\hat{z}) = F(\hat{z})$. Recall the "integral conditions" for a MPS (Rothschild and Stiglitz 1970): *G MPS F* if and only if

$$\int_{-\infty}^{x} G(t) - F(t)\,dt \ge 0 \;\forall x \tag{A11}$$

$$\int_{-\infty}^{\infty} G(t) - F(t)\,dt = 0. \tag{A12}$$

The fact that $G(\hat{z}) = F(\hat{z})$ and $G = F$ for $x \ge \hat{z}$ implies that

$$\int_{-\infty}^{x} \frac{G(t)}{G(\hat{z})} - \frac{F(t)}{F(\hat{z})}\,dt \ge 0 \;\forall x \tag{A13}$$

$$\int_{-\infty}^{x} \frac{G(t)}{G(\hat{z})} - \frac{F(t)}{F(\hat{z})}\,dt = 0 \;\forall x \ge \hat{z} \tag{A14}$$

thus G_L MPS F_L. Thus, if h is convex for $z \le \hat{z}$ then

$$\int_0^{\hat{z}} h(x)\,dG_L(x) \ge \int_0^{\hat{z}} h(x)\,dF_L(x). \tag{A15}$$

Since

$$\int_0^\infty h(x)\,dG(x) = G(\hat{z}) \int_0^{\hat{z}} h(z)\,dG_L + (1 - G(\hat{z})) \int_{\hat{z}}^\infty h(z)\,dG_R$$

and $G_R = F_R$ condition (A15) implies that

$$\int_0^{\infty} h(x)dG(x) \geq \int_0^{\infty} h(x)dF(x) \;\; for\ h \in \delta_{\hat{z}}.$$

If, on the other hand, h is concave for $z \leq \hat{z}$ then $\int_0^{\hat{z}} h(x)dG_L(x) \leq \int_0^{\hat{z}} h(x)dF_L(x)$ and it follows that $\int_0^{\infty} h(x)dG(x) \leq \int_0^{\infty} h(x)dF(x)$ for $h \in \psi_{\hat{z}}$. ■

Proof of Lemma 4: We show that the second derivative of $c(\mu^*)$ with respect to z is positive at $z = 0$. The second period optimization problem is

$$\min_{\mu}(1-\alpha)c(\mu) + zD(S-\mu) \tag{A16}$$

$$\text{FOC}: (1-\alpha)c'(\mu) - zD'(S-\mu) = 0 \tag{A17}$$

$$\text{SOC}: (1-\alpha)c''(\mu) + zD''(S-\mu) > 0. \tag{A18}$$

Applying the envelope theorem and differentiating again we get

$$\frac{\partial \mu}{\partial z} = \frac{D'(S-\mu)}{\text{SOC}} \tag{A19}$$

$$\frac{\partial^2 \mu}{\partial z^2} = -2\frac{D'D''}{\text{SOC}^2} - \frac{D'D''}{\text{SOC}^3}[(1-\alpha)c''' - zD'''] \tag{A20}$$

$$\frac{\partial^2}{\partial z^2}c(\mu^*) = c''(\mu^*)\left[\frac{\partial \mu^*}{\partial z}\right]^2 + c'(\mu^*)\frac{\partial^2 \mu^*}{\partial z^2} \tag{A21}$$

$$= c''\frac{D'^2}{\text{SOC}^2} - 2c'\frac{D'D''}{\text{SOC}^2} - c'\frac{D'D''}{\text{SOC}^3}[(1-\alpha)c''' - zD'''] \tag{A22}$$

$$= \frac{D'}{\text{SOC}^2}\left[c''D' - 2c'D''\right] - c'\frac{D'D''}{\text{SOC}^3}[(1-\alpha)c''' - zD''']. \tag{A23}$$

The first term is strictly positive if $\frac{c''}{c'} > 2\frac{D''}{D'}$. The second term is positive if $c''' \leq 0$ and either $D''' \geq 0$ or z is small. Thus, optimal costs are convex for small z. ■

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