

# Non-Linearities in the cost of reducing emissions

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## 1 Introduction

One of the pressing issues in climate change is estimating the cost of reducing emissions. While there is a great deal of uncertainty and disagreement about the cost of abatement in general, there is relatively better knowledge about the cost of small reductions in emissions. As larger reductions in emissions are considered, larger deviations from the status quo must be imagined, thus leading to greater uncertainty. Uncertainty about the cost of small reductions can be interpreted as uncertainty about the level of costs, while uncertainty about large reductions reflect uncertainty about the non-linearity of costs.

While there has been some discussion of the impact of the convexity of the abatement cost curve on the costs of meeting the Kyoto protocol [4] and on optimal instrument choice, there has been little discussion on the impact of nonlinearities on optimal emissions trajectories. This paper considers the impact of the convexity of the abatement cost curve on optimal emissions trajectories, considering both deterministic and stochastic climate damages.

In a multi-period model with discounting, it is optimal to gradually increase abatement

through time: the discounted present value of abatement costs in the future are less than in the present. Additional factors leading to an increasing time profile for abatement are technological change (making it less expensive to reduce emissions in the future), turnover of the capital stock (implying high short term costs of abatement) and uncertainty with learning (it is optimal to delay investment when learning is expected to take place). Counterbalancing these factors, however, is the convexity of the cost curve. If marginal cost is rising very steeply, it is optimal to remain on a lower part of the curve.

Non-linearity of the cost curve may be even more important when considering optimal abatement under uncertainty. Intuitively, if one is facing a highly convex cost curve, then one may prefer to reduce emissions more in the short run, to insure against excessively high abatement costs in the future, should damages turn out to be worse than expected.

We examine these ideas using the integrated assessment model DICE. We provide an example where the level of the abatement cost is less important than the convexity of the cost curve (see Peck and Teisberg [7] for a similar argument related to the damage function). We replace the abatement cost curve in DICE with curves that are both everywhere higher and more convex, and find that the optimal abatement path flattens out. To explore the interactions between convexity and uncertainty, we use a stochastic version of DICE. We find that the impact of learning is tempered in the presence of a convex cost curve.

In the next section we provide an overview of the underlying assumptions impacting the abatement cost curves in a number of integrated assessment models. In section 3 we provide the theoretical underpinnings of why the level of convexity is important to the optimal abatement path, considering both certain and uncertain climate damages. In section 4 we provide a brief

description of our adjustments to the dice model and our empirical results. Section 5 contains a summary and directions for future research.

## 2 Integrated Assessment Models and Abatement Cost Curves

In most Integrated Assessment (IA) models the shape of the abatement cost curve is a result, rather than an assumption. The steepness and convexity of the abatement cost curve is determined by a number of underlying assumptions that vary considerably from model to model. Figure 1 (copied from Weyant and Hill [13]) illustrates the derived marginal cost of abatement for the U.S. for 11 different IA models (See appendix for a list of the models). The curves plot the carbon tax needed to achieve a given percentage reduction in emissions against the given percentage reduction. It is apparent that there is a great deal of disagreement about the marginal cost of reducing emissions. The slope of the marginal cost curve, which is proportional to the convexity of the cost curve, is important for the determination of optimal emissions under certainty. Both the slope and the convexity of the marginal cost curve are important for optimal emissions under uncertainty.

These curves are derived from underlying assumptions in the models. One of the most important drivers of the steepness of the marginal cost curves is assumptions about baseline emissions. In general, the higher the baseline emissions, the more costly it is to reduce emissions by a given percentage. But this is not the whole story. Another key component into the shape of the abatement cost curves is what Jacoby and Wing [4] term the "malleability" of the energy

system. Carbon can be reduced through substitution – between fuels, technologies, capital, labor, and energy services – and through reductions in output. A more malleable or elastic system will reduce emissions mainly through substitution. If the demand for energy is highly elastic, then the marginal cost of reducing emissions will tend to be low. Similarly, if the elasticity of substitution between alternative technologies is high, then costs will be relatively low. In the short term, much depends on the ease of capital turnover. Thus, the structure of the models and assumptions about elasticities and capital turnover are key.

Differences in the convexity of the marginal cost curves depend on the relative contribution of energy intensity reductions and carbon intensity reductions in achieving carbon emission reductions and the implied differences in fuel share adjustments (see Weyant and Hill [13] for a discussion).

Jacoby and Wing [4] discuss some of these issues explicitly, exploring the impacts of adjustment time and capital malleability on the costs of the Kyoto protocol. Similarly, Cooper et al. [3] briefly discuss the impacts of the non-linearity in the abatement cost curve on the costs of the Kyoto protocol. Neither paper, however, addresses the impact of the non-linearity of the cost curve on *optimal* emissions paths.

## **3 Theory**

### **3.1 Deterministic Damages**

We illustrate the impacts of the convexity of the cost curve, and draw a parallel with the elasticity of substitution across time. Consider the following simple model, where we minimize the cost of

abatement plus the damages from climate change.

$$\min_{\mu_1, \mu_2} c(\mu_1) + \delta c(\mu_2) + \delta D(S^* - \mu_1 - \mu_2) \quad (1)$$

$c(\cdot)$  is the abatement cost,  $D(\cdot)$  represents the damages as a function the stock of emissions,  $\mu_1$ ,  $\mu_2$  are the level of abatement in period 1 and 2,  $\delta < 1$  is the discount factor, and  $S^*$  is the business as usual stock of emissions. The cost of abatement is constant over time, thus the driving force that causes optimal abatement to increase over time is the discount factor. On the other hand, the *convexity* of the cost function drives optimal abatement to be spread more evenly over time. For example, if abatement cost were linear, it would be optimal to delay all abatement until the last period.

Note that (1) is nearly identical to an intertemporal utility maximization problem. It is well known that the concavity of the utility function (as measured by the Arrow-Pratt [8] risk aversion coefficient  $-\frac{u''}{u'}$ ) determines the intertemporal elasticity of substitution, with a more concave utility function implying lower intertemporal elasticity of substitution, in turn implying a flatter consumption path. Since our problem is a minimization, it is the convexity of the cost function which implies a lower intertemporal elasticity of substitution. Thus, a more convex cost function will lead to a flatter abatement curve. For example, combining the first order conditions for  $\mu_1$  and  $\mu_2$  derived from (1) above we get

$$\frac{c'(\mu_1)}{c'(\mu_2)} = \delta$$

which means that  $\mu_2 > \mu_1$  if  $c(\cdot)$  is increasing and convex. This illustrates the impact of the

discount factor – we would rather pay later than sooner. If we let  $c(\mu) = \mu^b$  then this simplifies to

$$\left(\frac{\mu_1}{\mu_2}\right)^{b-1} = \delta$$

or

$$\frac{\mu_1}{\mu_2} = \delta^{\frac{1}{b-1}} \quad (2)$$

The right hand side of (2) is increasing in  $b$  (since  $\delta$  is assumed to be less than 1), and therefore the ratio  $\frac{\mu_1}{\mu_2}$  is increasing in  $b$ . Since  $\mu_2$  is higher than  $\mu_1$  this implies that the abatement path is getting flatter as the cost function gets more convex.

### 3.2 Stochastic Damages

Convexity is perhaps even more important when determining the impact of uncertainty and learning on optimal decisions. Intuitively, if costs are very convex, then we may want to reduce emissions now to avoid the high cost of severe abatement if damages turn out to be catastrophic. Mathematically, it has been well known since Rothschild-Stiglitz [10] that the comparative statics of risk depend on the convexity of marginal indirect utility.

A number of papers have shown that in the presence of learning, if emissions are not constrained to be non-negative, and if the cost function is quadratic, optimal abatement decreases in risk (see [1], [5], [11]). We relax the quadratic assumption to investigate the impact of a more convex cost function.

We alter the model (1) above by adding uncertainty (and dropping the discount factor for

simplicity).

$$\min_{\mu_1} c(\mu_1) + \mathbb{E}_Z \min_{\mu_2} [c(\mu_2) + ZD(S^* - \mu_1 - \mu_2)]$$

where  $Z$  represents uncertainty about climate damages. It is assumed that the true value of  $Z$  is learned before choosing second period emissions. Thus, this is a problem of sequential decision making under uncertainty. Let  $S \equiv S^* - \mu_1 - \mu_2$ . Define

$$V(\mu_1, Z) \equiv \min_{\mu_2} c(\mu_2) + ZD(S^* - \mu_1 - \mu_2) \quad (3)$$

We define an increase in risk to be a mean-preserving spread [9]. First period optimal abatement,  $\mu_1$ , is increasing (decreasing) in risk if and only if  $V_{\mu_1}$  is concave (convex) in  $Z$  (subscripts indicate partial derivatives). Thus, assuming differentiability, we need to check the sign of  $V_{\mu_1 ZZ}$  in order to determine how optimal abatement changes with an increase in risk. Assume that damages are quadratic in  $S$ . Let  $\mu_2^* = \mu_2^*(Z, \mu_1)$  be optimal second period abatement. Using the envelope theorem to calculate the partials of  $\mu_2^*$  with respect to  $Z$  we get

$$V_{\mu_1 ZZ} = \frac{D'' D'(S)}{(c''(\mu_2) + ZD'')^3} [2c''(\mu_2^*)^2 + Z[2c''(\mu_2^*)D'' - D'(S)c'''(\mu_2^*)]] \quad (4)$$

Note that  $V_{\mu_1 ZZ}$  is strictly positive when  $Z = 0$ , implying that  $V_{\mu_1}$  is either everywhere convex, or neither convex nor concave. Thus, the first result is that there is no set of assumptions under which abatement will always increase in uncertainty<sup>1</sup>.

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<sup>1</sup>The expected value of a convex function increases in risk, while the expected value of a concave function decreases. Jensen's inequality is a special case of this phenomenon. If a function is neither convex nor concave then the expected value will increase for some increases in risk and decrease for other decreases in risk (see Rothschild and Stiglitz [10] p.240)

**Proposition 1** *Let  $c', c'', D', D'' \geq 0$ . Then there exists random variables  $Z, Z'$  such that  $Z$  is riskier than  $Z'$  and  $\mu_1(Z) \leq \mu_1(Z')$ .*

In fact, (4) will be everywhere positive as long as

$$[2c''(\mu_2^*)D'' - D'(S)c'''(\mu_2^*)] \geq 0 \quad (5)$$

Notice that this is true unambiguously when  $c''' \leq 0$ , a special case of this being when abatement costs are quadratic. On the other hand, if  $c''' > 0$ , then (4) may be negative for some  $Z$ , implying that optimal abatement will increase for some increases in risk, even without a non-negativity constraint on second period abatement. A necessary condition for (4) to be negative is that the quantity in (5) be negative. Under the assumptions of Proposition 4, (5) is negative only if

$$2\frac{D''}{D'} < \frac{c'''}{c''} \quad (6)$$

The value on the right hand side of (6) is a measure of the convexity of  $c'$ , the marginal cost of abatement. Thus, the more convex the marginal costs, the more likely that the right hand side of (6) will be greater than the left hand side for some values of  $\mu_2$ . This suggests that the computational result that optimal abatement decreases in risk may be largely driven by the choice of the convexity of  $c'$ : namely, cost functions that are very close to quadratic, implying marginal costs that are close to linear. This suggests that if marginal costs are assumed to be more convex, then optimal abatement may increase in some increases in risk.

## 4 The Model and Results

We use an adjusted version of William Nordhaus' DICE model in order to test the impact of nonlinearities in the abatement cost curve. The benefit of using DICE is that it uses an explicit abatement cost function. This allows us to determine the impacts of the convexity of the cost curve separately from other assumptions. For the base model see Nordhaus & Boyer [6]. In the original model, reducing emissions by a fraction  $\mu$  requires an investment that reduces output as follows

$$Q_t = (1 - b_1(t) \mu_t^{b_2}) \tilde{Q}_t \quad (7)$$

where  $Q_t$  is available output at time  $t$ , and  $\tilde{Q}_t$  is output without the emissions reduction. Thus, the abatement cost curve is

$$b_1(t) \mu_t^{b_2} \quad (8)$$

with parameters defined as follows:

$$b_1(t) = \frac{b_1(t-1)}{1 + g(t)}$$

$$b_1(0) = .03$$

$$g(t) = g(0) e^{-\delta t}$$

$$g(0) = -.08$$

$$\delta = .05$$

$$b_2 = 2.15$$

Notice that the cost function is not very convex. It is nearly quadratic, and thus the marginal cost curve is nearly linear. In particular, it does not capture our beliefs when it comes to high levels of abatement, particularly in the short run. For example, using this cost function, the reduction in total output resulting from a reduction in emissions of 80% is only 2%. Figure 2 compares the growth of the world economy under assumptions of optimal abatement versus 100% abatement. It shows that, in the DICE model, moving to a no-carbon world immediately would have a fairly mild impact on growth. The DICE model has been calibrated to the more detailed RICE model, matching a base case of no reductions and an optimal case, where abatement runs to about 10%. So the assumption of nearly quadratic abatement costs together with calibration of fairly mild scenarios implies low costs for high abatement. Also, note in Figure 1 above that the RICE model has a linear marginal abatement curve (as do most of the models).

In order to explore the impacts of convexity we investigate two alternative cost functions, each of the form:

$$b_0(t) \mu_t + b_1(t) \mu_t^{b_2}$$

with the parameter values given in Table 1. We assume that  $b_0$  and  $b_1$  increase at the same rate

	<b>Convex Cost 1</b>	<b>Convex Cost 2</b>
$b_0(0)$	.001	.001995
$b_1(0)$	1	1.083
$b_2$	4.15	4.15

Table 1: Abatement Cost Parameters

as in the DICE model. See Figure 3 for a comparison of the cost and marginal cost functions for each of the three parameterizations in the year 2005. In each case, the "convex" cost functions are very close to the Dice cost function for abatement levels between 0 and 12%. Thus the cost does not differ greatly between the two in the range of optimal abatement. Note, however, that



Figure 1: See last page

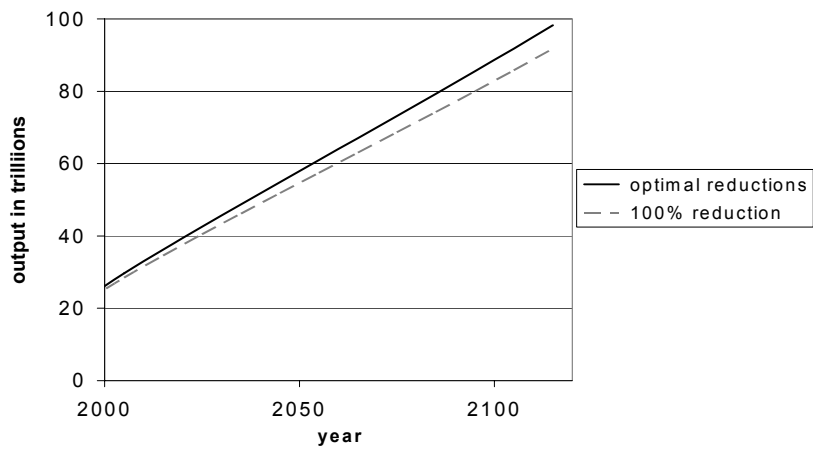


Figure 2: The difference in total output under optimal emissions and under full abatement is small.

both "convex" cost functions are consistently higher than the DICE cost function. This is true until at least 2100. Furthermore, Convex Cost 2 has a higher marginal cost everywhere than DICE. A higher *level* of cost implies lower optimal abatement. On the other hand, as we argue above, a more *convex* cost function leads to a flatter emissions path. Below, we determine which effect is larger.

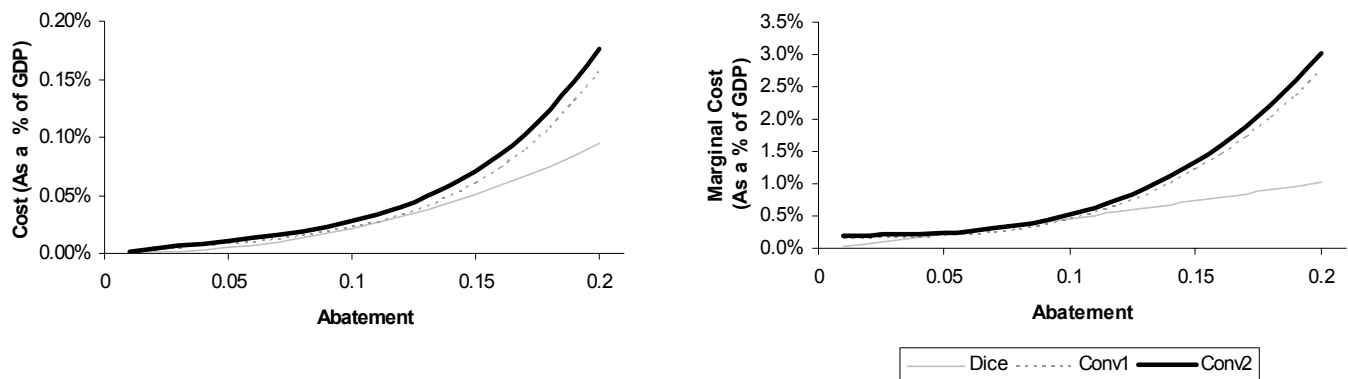


Figure 3: Abatement cost and marginal cost curves under different assumptions about convexity

## 4.1 Deterministic Results

We ran DICE using the regular cost function and our higher, but more convex cost functions. The results are presented in figure 4. In the very short run the two "convex" cost curves actually show lower abatement than the DICE cost curve. This is the result of the linear term  $b_0$  in our "convex" cost functions. When marginal damages are low, the linear term dominates and drives abatement to very low levels. What is happening is that our "convex" cost functions are in fact not convex at all at low levels of abatement. This points to an essential problem of comparing different levels of non-linearity. In order for the cost and marginal cost curves to be close, a

linear term is required. This skews the early results. Once the effect of the linear term wears off, however, we see that abatement is indeed flatter for the more convex cost curves, with higher emissions in the short run and lower emissions in the long run. The convex cost leads to a flatter optimal emissions path, reducing more now, and less later.

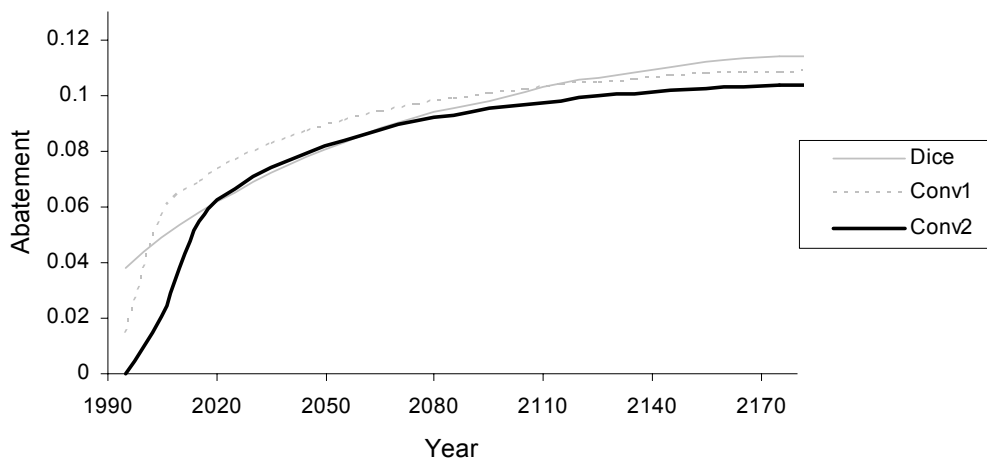


Figure 4: Optimal abatement through time under different convexity assumptions

In order to avoid the problem of the linear term, we use the base DICE cost function (8) with  $b_2 = 4.15$ . Again, comparisons are difficult, because a higher exponent implies significantly lower costs (since  $\mu$  is always less than 1). In Figure 5 we compare the ratio of the optimal abatement level over time to the 1995 level. Clearly, a higher exponent produces a flatter curve, percentage-wise. Similarly, in figure 6 we show the ratio of 2015 abatement to 2115 abatement as the exponent  $b_2$  ranges from 2.15 to 40. It shows that the optimal emissions path gets flatter monotonically as the abatement cost curve gets more convex.

Higher costs imply lower levels of abatement, but more convex costs imply higher levels of near term abatement. The convexity of the cost curve depends on the cost of higher levels

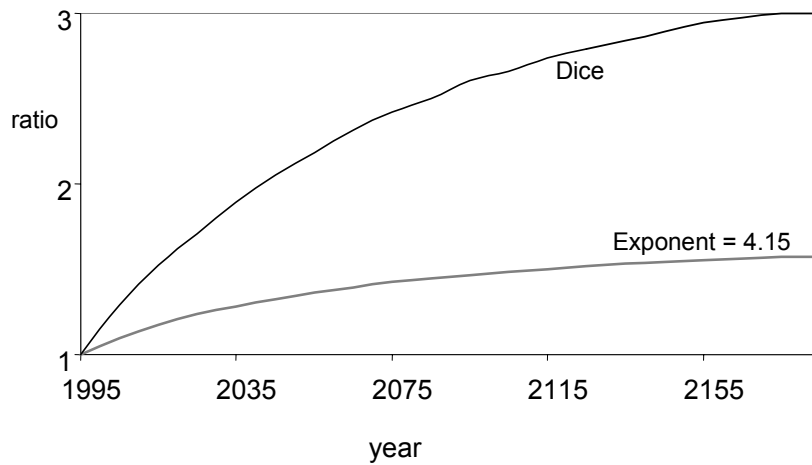


Figure 5: The ratio of annual abatement levels to 1995 abatement.

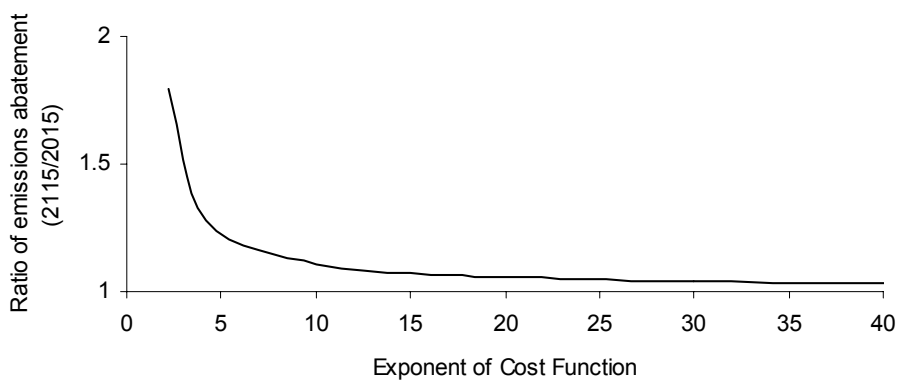


Figure 6: Ratio of optimal abatement for increasing convexities

of abatement, and these are the costs we are least confident about. This implies that careful attention should be given to assumptions that impact the slope of the marginal cost curve, and sensitivity should be done on the impact of these assumptions on optimal emissions. Further, it would be useful to model explicitly the uncertainty about the convexity of the marginal costs and analyze the impact on near term emissions reductions.

## 4.2 Uncertainty

The impact of a changed climate on human welfare is a critical uncertainty for climate change planning. Nordhaus & Boyer [6] devote a full chapter to climate damages and emphasize the potential for catastrophic damages. Thus our model considers low-probability, high-damage outcomes.

In the DICE model, the impact on human welfare is captured as a translation of temperature increase  $T$  into reduced production. Output is multiplied by the fraction  $\Omega = \frac{1}{1+\theta_1 T + \theta_2 T^2}$ . The key parameters in this relationship are  $\theta_1$  and  $\theta_2$ . In our modified version of DICE, we capture uncertainty in climate damages by making  $\theta_2$  a random variable. In order to incorporate learning, we use a stochastic programming approach, solving the model with GAMS/MINOS. The variables are not indexed by the state of the world for the first 5 periods (50 years). Starting in the sixth period each variable is indexed by  $s = h, l$ , representing the state of world being either  $\theta_2 = \theta^H$  or  $\theta_2 = \theta^L$ . For the mean of  $\theta_2$  we use the base value from DICE ( $=.0035$ ). In each case, we choose  $\theta_H > \theta_L$  with probability  $p_H$  that  $\theta = \theta_H$ , keeping the mean constant at  $.0035$ . Using the adjusted dice model we test the prediction that with highly convex costs abatement will increase in risk. The intuition is that if the costs are very convex, then large emissions reductions will be

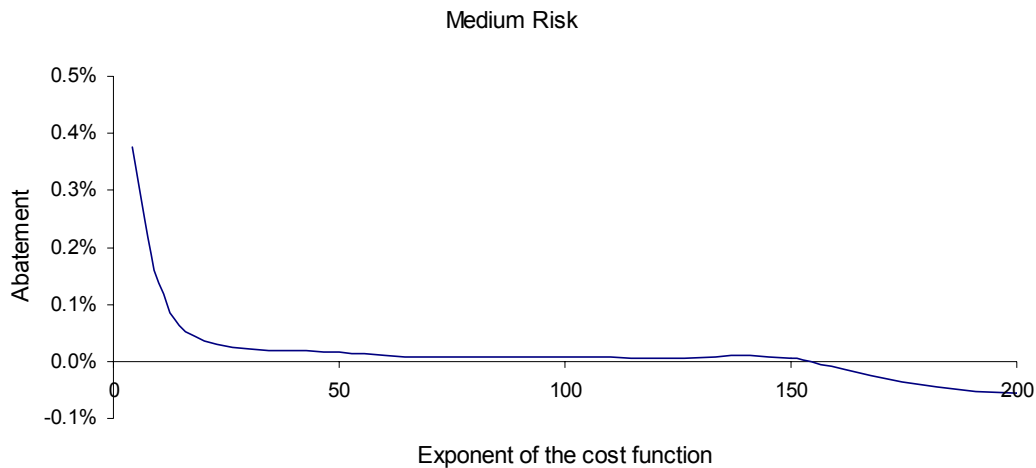


Figure 7: Difference in abatement under certain and stochastic damages

too expensive, thus it may be optimal to protect ourselves by abating more now.

We examine two representations of risk. In each representation the value of  $\theta^L$  is .003. In the first, "high-risk" representation, we let  $\theta^H = .3$  and probability  $p_H = .001684$ . In the "medium-risk" representation,  $\theta^H = .03$  and probability  $p_H = .01852$ . We compare the near-term optimal abatement path under certain damages, where  $\theta_2 = .0035$ , with each of the other two representations. In the high-risk representation, we find that short term emissions under uncertainty are always higher than emissions under certainty, regardless of the convexity of the cost curve. This echoes the results in the literature so far.

In the medium-risk representation, however, this is not true. Figure 7 shows the difference between optimal abatement under certainty and optimal abatement under uncertainty in the year 2015, i.e.  $\mu_{certain}^* - \mu_{uncertain}^*$ , as a function of the exponent  $b_2$ . We find that early emissions decrease in uncertainty if the exponent is high enough. Admittedly, it takes an exponent of nearly 200 to cause this response, which gives us a cost function which is practically a threshold

function – almost zero cost up until full abatement, at which we jump to full cost. Nevertheless, it does point out that the "go slow" policy depends on the level of convexity of the cost function. A more convex cost function indicates higher optimal abatement.

## 5 Conclusion

This paper analyzes the impact of nonlinearities in the cost of abatement. We illustrate theoretically and empirically how a steeper marginal cost curve leads to a flatter optimal abatement path when damages are deterministic. We indicate the importance of the convexity of the marginal cost curve when considering optimal abatement policy under uncertainty with learning, and produce an empirical example where the standard result that optimal abatement decreases in the presence of learning is reversed.

The models and scenarios used in this paper are quite simple, yet some general insights can be inferred. The first is that the *convexity* of the abatement cost curve is as important as the *level* of the costs. Thus, careful attention should be paid to the parameters and model assumptions that determine the level of convexity in abatement cost curves in IA models. This is true for deterministic and stochastic models. Second, since our knowledge about the cost (especially the short term cost) of large amounts of abatement is not firm, it would be helpful to model uncertainty about non-convexities in the abatement cost curve explicitly. In particular, it is of interest how uncertainty about the cost curve interacts with uncertainty about damages.

Future work is required to more fully examine this phenomenon. First, there is a need to systematically investigate which parameters impact the steepness and the convexity of marginal cost curves in IA models. The discussion started in EMF 16 (see Weyant [12] for the papers that

resulted from that exercise) is a starting point for exploring the quantitative impact that different parameters and modeling choices have on the convexity of abatement cost curves. Second, once the most important parameters have been determined, it will be useful to explore the impacts of the uncertainty on these parameters on optimal abatement, both with and without damage uncertainty. Finally, it has been shown that the convexity of the cost curve has important implications for technology policy (see Baker, Clarke, and Weyant [2]). The role of R&D into climate technologies can essentially be seen as shifting the abatement cost curve. If we are not capturing the true level of the convexity of the abatement cost curve, we may under-invest in technologies designed to reduce costs in the future.

## A List of Models used in Figure 1

AIM	Asian-Pacific Integrated Model, National Institute for Environmental Studies, Kyoto, Japan.
CETA	Carbon Emissions Trajectory Assessment, Electric Power Research Institute and Teisberg Associates
MIT-EPPA	Emissions Projection and Policy Analysis Model, MIT
G-Cubed	Global General Equilibrium Model, Australian National University, University of Texas, and U.S. EPA
ABARE-GTEM	Global Trade and Environment Model, Australian Bureau of Agriculture and Resource Economics
MERGE3	Model for Evaluating Regional and Global Effects of GHG Reductions Policies, Stanford University and Electric Power Research Institute
MS-MRT	Multi-sector – Multi-Region Trade Model, Charles River Associates and University of Colorado
Oxford	Oxford Economic Forecasting
RICE	Regional Integrated Climate and Economy Model, Yale University
SGM:	Second Generation Model, Batelle Pacific Northwest National Lab
WorldScan	Central Planning Bureau/RIVM, Netherlands

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Figure 1

Figure 10(a). Marginal Cost of Carbon Emission Reductions in the United States

