Complexity-Adaptive Universal Signal Estimation for Compressed Sensing

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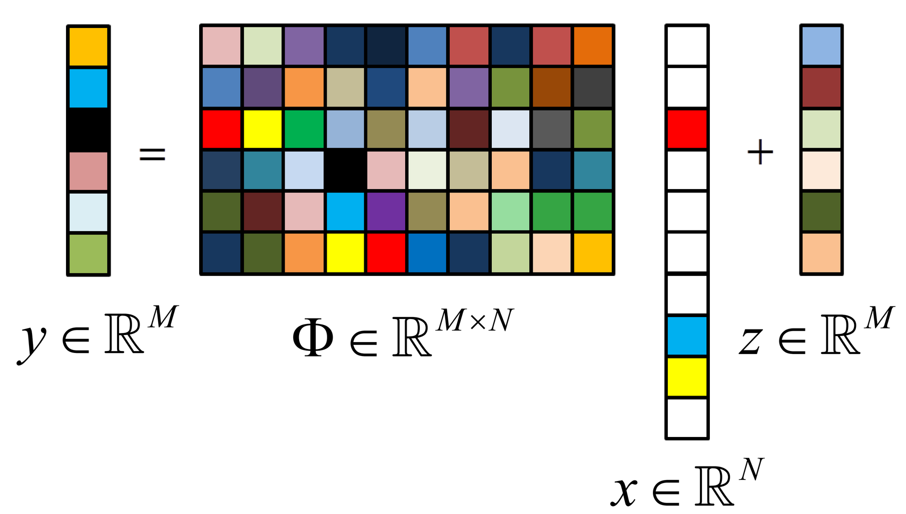
Motivation

Noisy compressed sensing (CS) [1]



x: input signal
Φ: measurement matrix
z: Gaussian noise
y: measurements

- Goal: reconstruct x given y and Φ

Conventional CS algorithms

- Assume sparsity or compressibility
- Need prior knowledge about signal structure

What if prior knowledge not available?

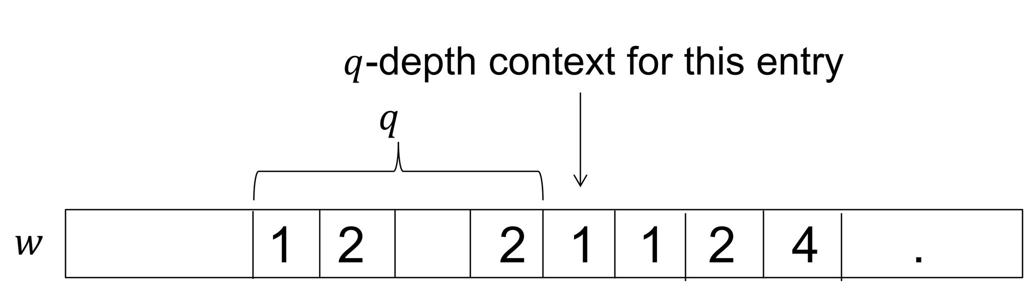
Universal CS algorithms

- Focus on recovery of stationary ergodic non-i.i.d. signals with unknown statistics

Background

Universal MAP estimation [2]

- $x_{MAP} = \arg\max_v f_X(v) f_{Y|X}(y|v) = \arg\min_v \Psi^X(v)$
- $\Psi^X(v) = -\ln(f_X(v)) + \frac{\|y-\Phi v\|_2^2}{2\sigma_z^2}$; optimal risk: $\Psi^X(x_{\text{MAP}})$
- Work on discretized space to reduce complexity
  1. Map indices $j \in \{1,\dots,Z\}$ to $\mathbb{R}$ via discretizer $Q(j)$
  2. Estimate $v = Q(w)$, $w = [w_1,\dots,w_N] \in \{1,\dots,Z\}^N$
- Universal prior [3] $P_U(w) = 2^{-H_q(w)}$
- q-depth conditional empirical entropy $H_q(w)$ [4]



- Objective function $\Psi^{H_q}(w) = N H_q(w) + \frac{\|y-\Phi Q(w)\|_2^2}{2\sigma_z^2 \times \ln(2)}$

MCMC and enhancements

Markov chain Monte Carlo (MCMC) [2,4]

- Boltzmann PMF $p_s(w) = \frac{1}{\zeta_s}\exp(-s\Psi^{H_q}(w))$
- Generate random samples with Gibbs sampler
- Iteration: Process one entry of w with Gibbs sampler
- Super-iteration: Process all entries of w

Does it reconstruct signals well?

- No, fixed quantizer slows down convergence

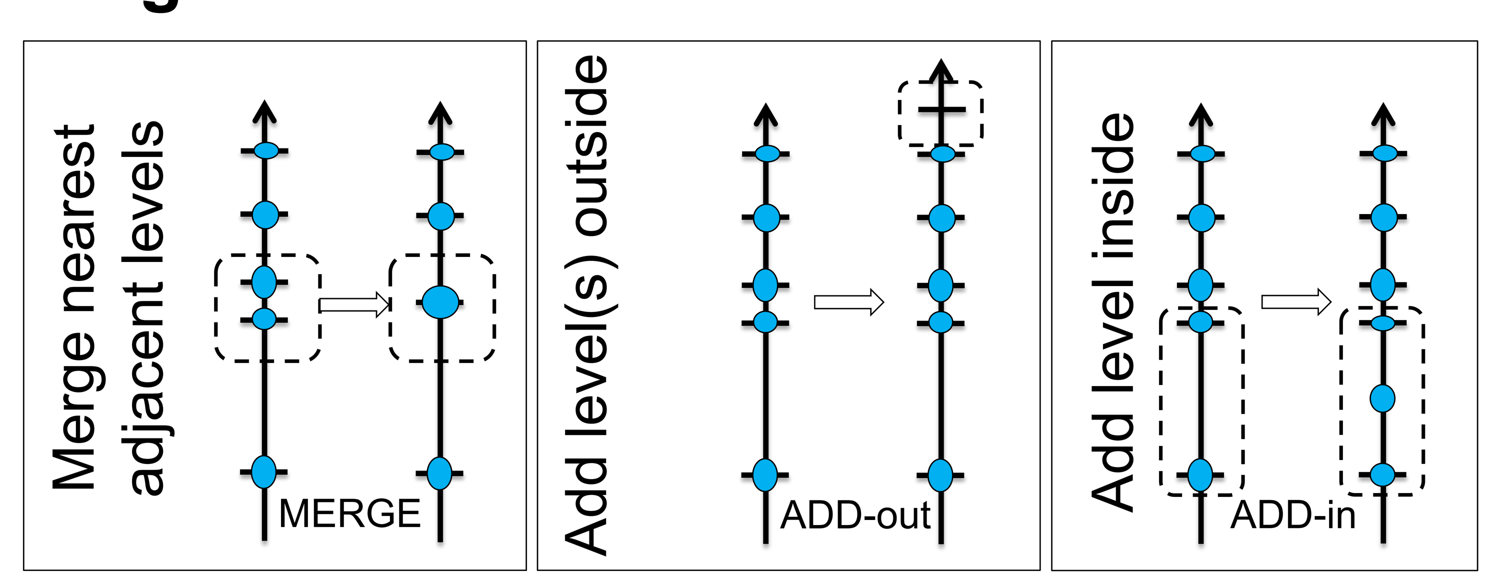
How to minimize l_2 error with fixed number of levels?

Level-adaptive (LA-MCMC) [2]

- $Q_{opt} = \arg\min_{\bar{Q}} \|y - \Phi\bar{Q}(w)\|_2^2$

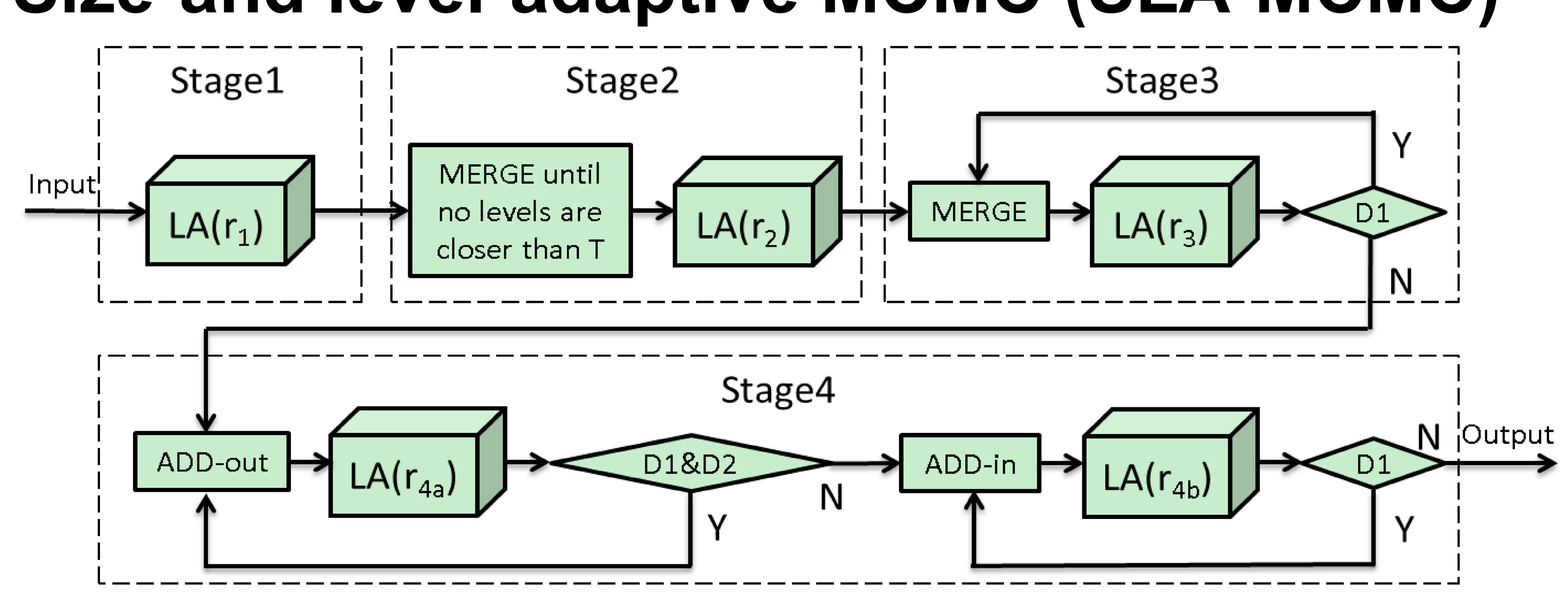
How many levels to use?

Merge / Add levels



Horizontal lines are levels; circle radius reflects population of level

Size-and level-adaptive MCMC (SLA-MCMC)



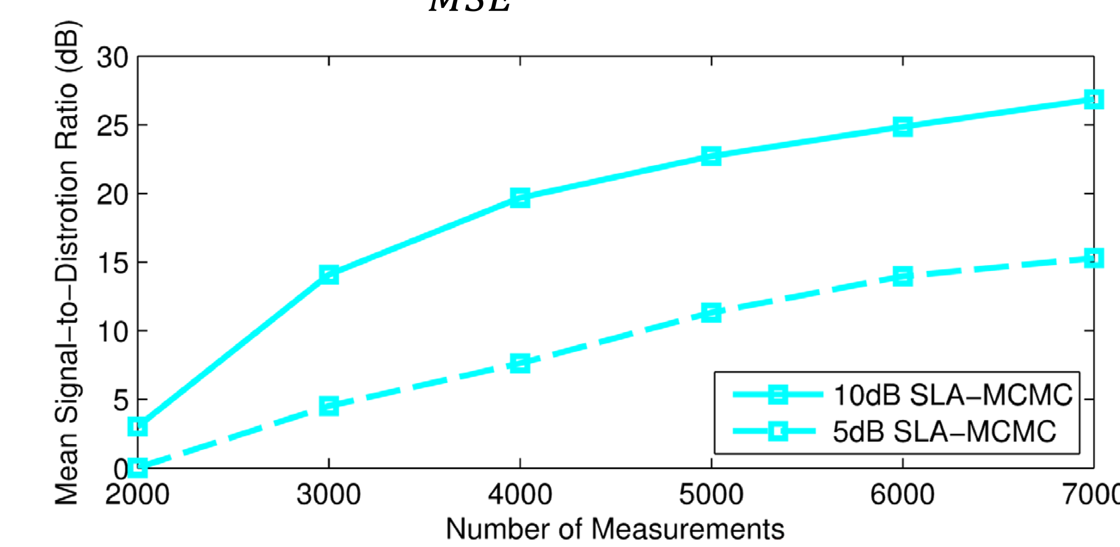
LA(r): run LA-MCMC for r super-iterations; D1 holds if current objective function better than before; D2 holds if newly added level(s) is (are) populated

Numerical results

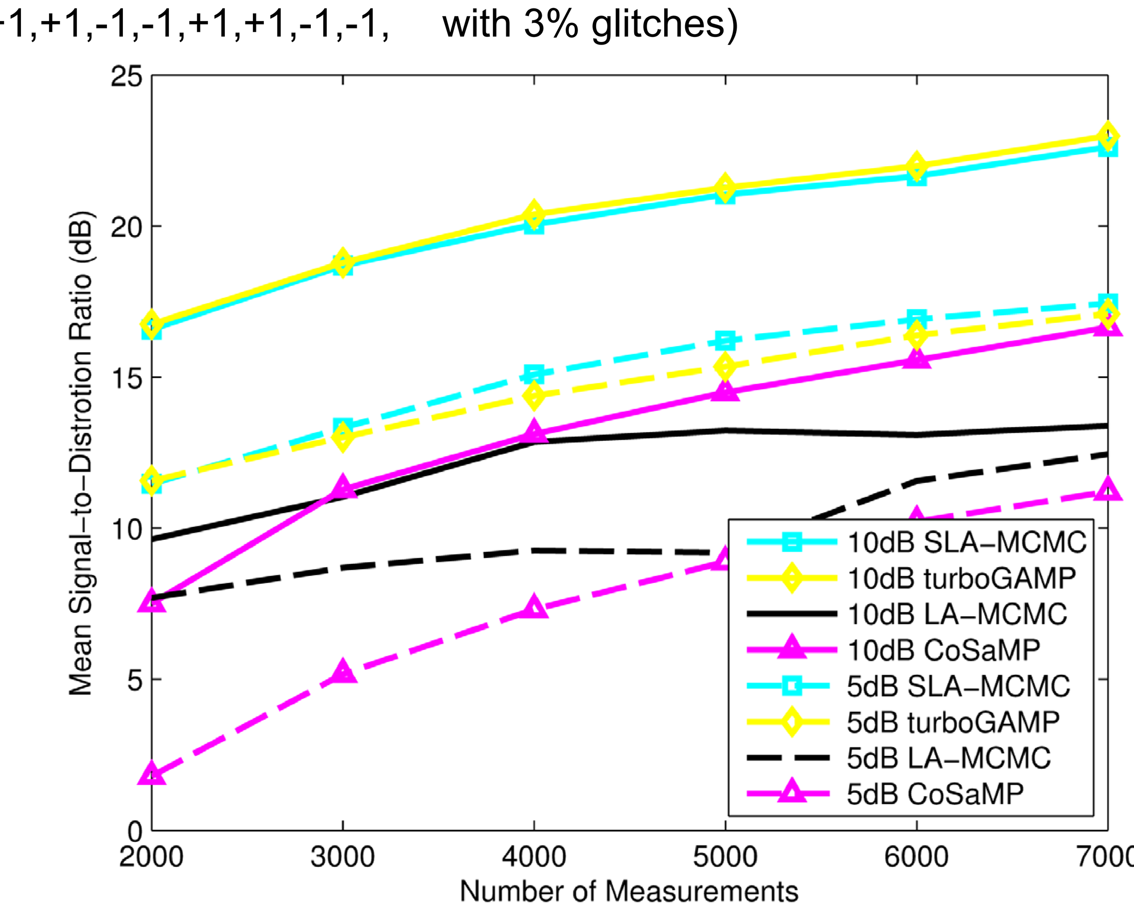
Experimental settings:

- Compare SLA-MCMC with LA-MCMC, CoSaMP, and turboGAMP
- Signal length N=10000, M=2000-7000, AWGN
- SNR=5 and 10 dB
- Error metric: Mean signal-to-distortion ratio

$$MSDR = 10\log_{10}\frac{Var(x)}{MSE}$$



SLA-MCMC estimation results for a four-state Markov switching source (generates pattern +1,+1,-1,-1,+1,+1,-1,-1, with 3% glitches)



SLA-MCMC, LA-MCMC, turboGAMP, and CoSaMP estimation results for a two-state Markov source with non-zero entries drawn from a uniform distribution $U[0,1]$

References and acknowledgements

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[1] D. Donoho, "Compressed sensing," *IEEE Trans. Inf. Theory*, vol. 52, no. 4, pp. 1289–1306, Apr. 2006.

[2] D. Baron and M. F. Duarte, "Universal MAP estimation in compressed sensing," in *Proc. 49th Annual Allerton Conf. Comm., Control, Computing*, Sep. 2011.

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[4] S. Jalali and T. Weissman, "Block and sliding-block lossy compression via MCMC," *IEEE Trans. Comm.*, vol. 60, no. 8, pp. 2187–2198, Aug. 2012.