Universal Distributed Sensing via Random Projections

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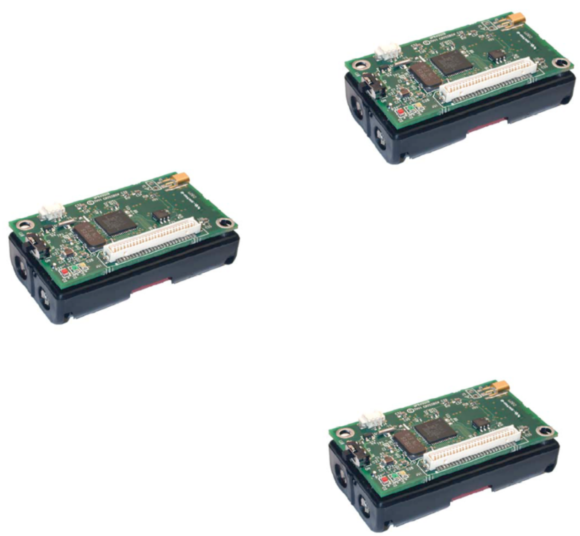

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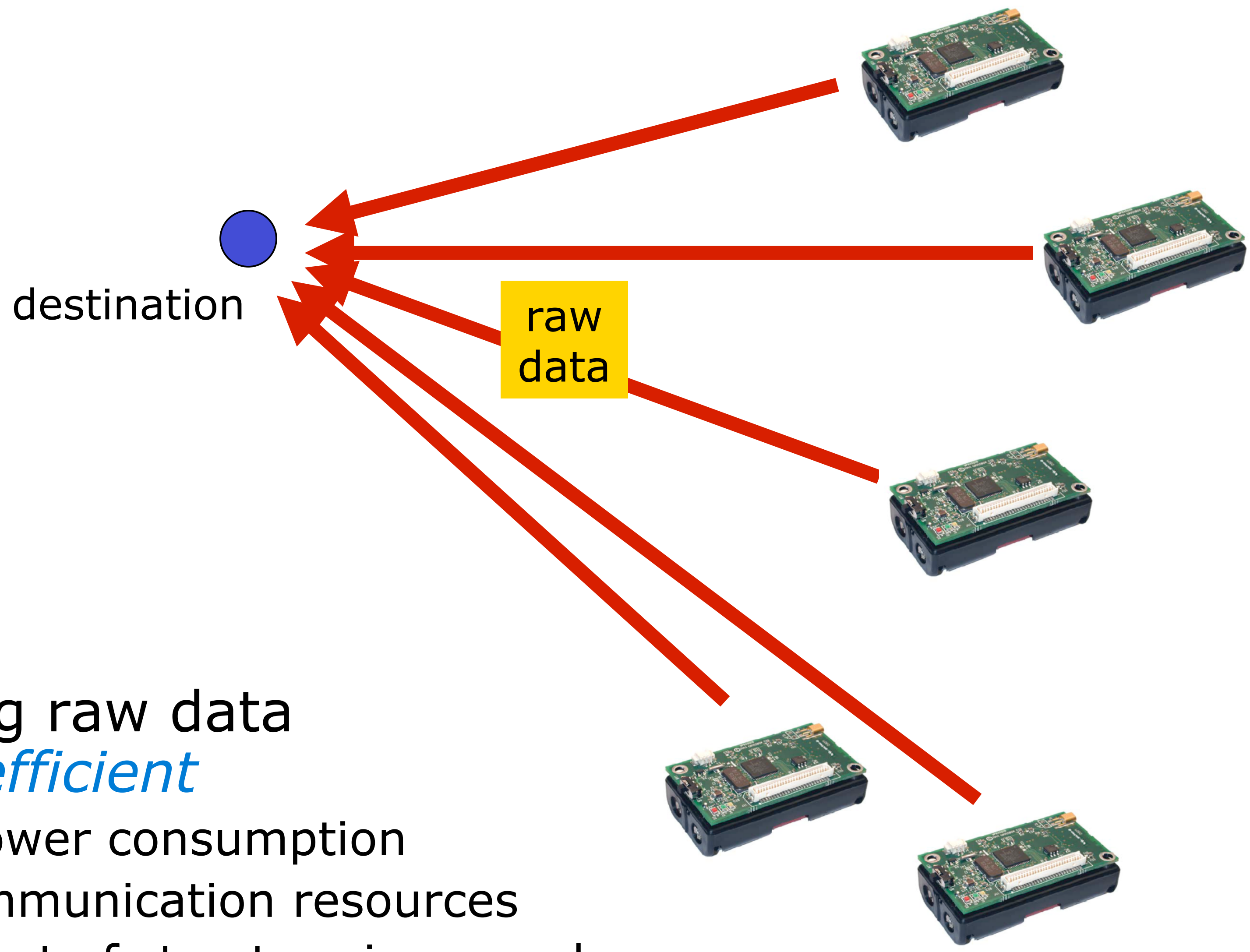
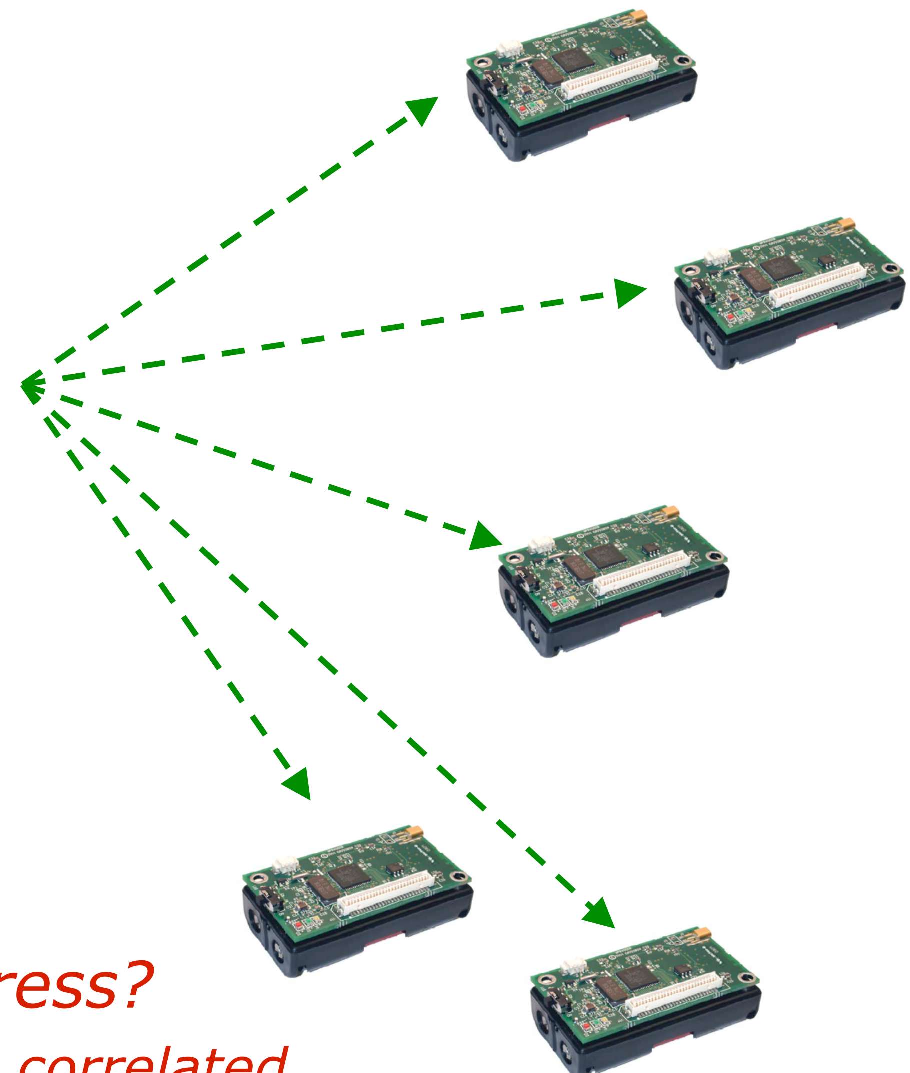
The Need for Compression



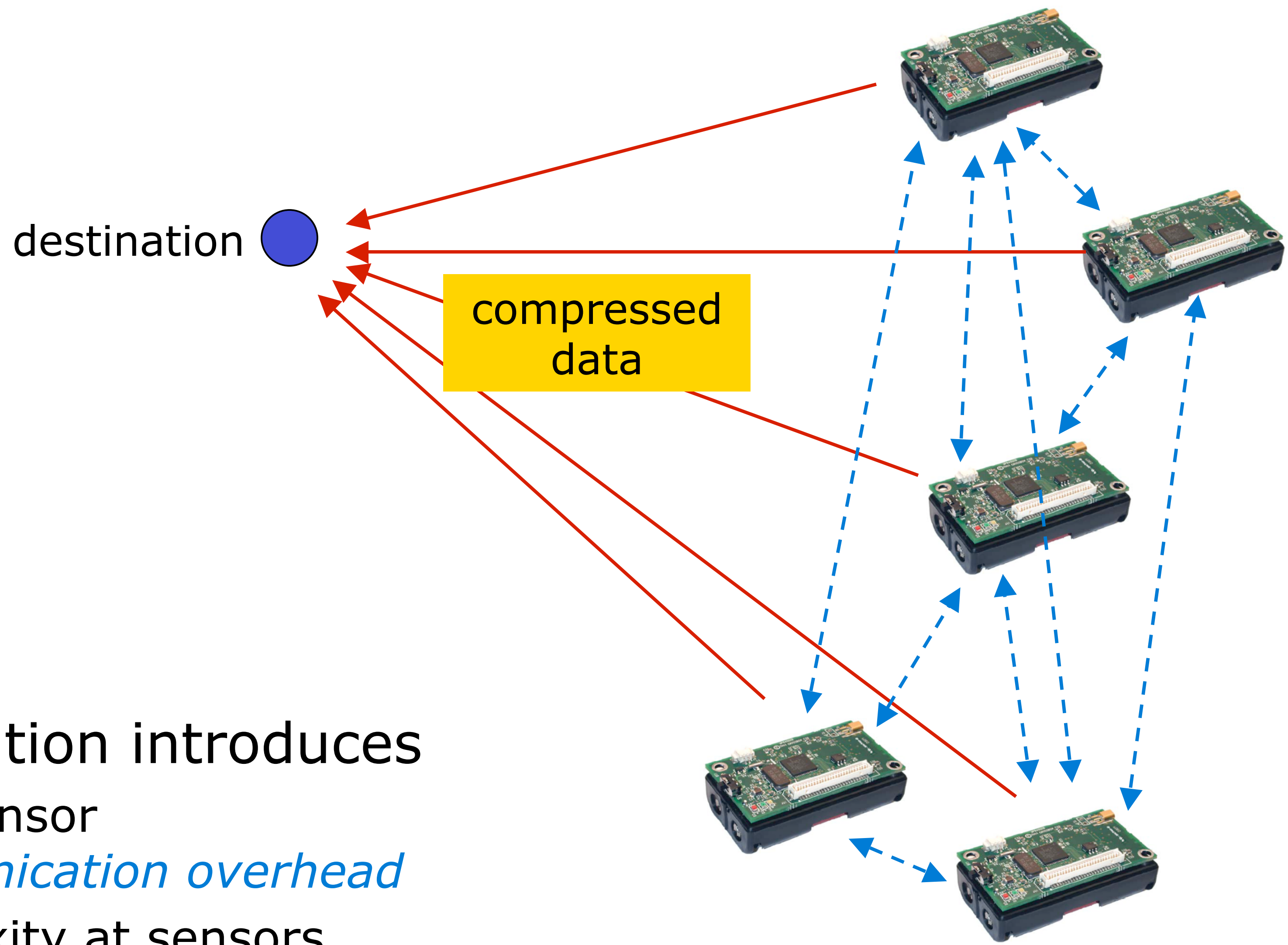
- Transmitting raw data typically *inefficient*
  - reduced power consumption
  - limited communication resources
  - large amount of structure in sensed signals

Correlation



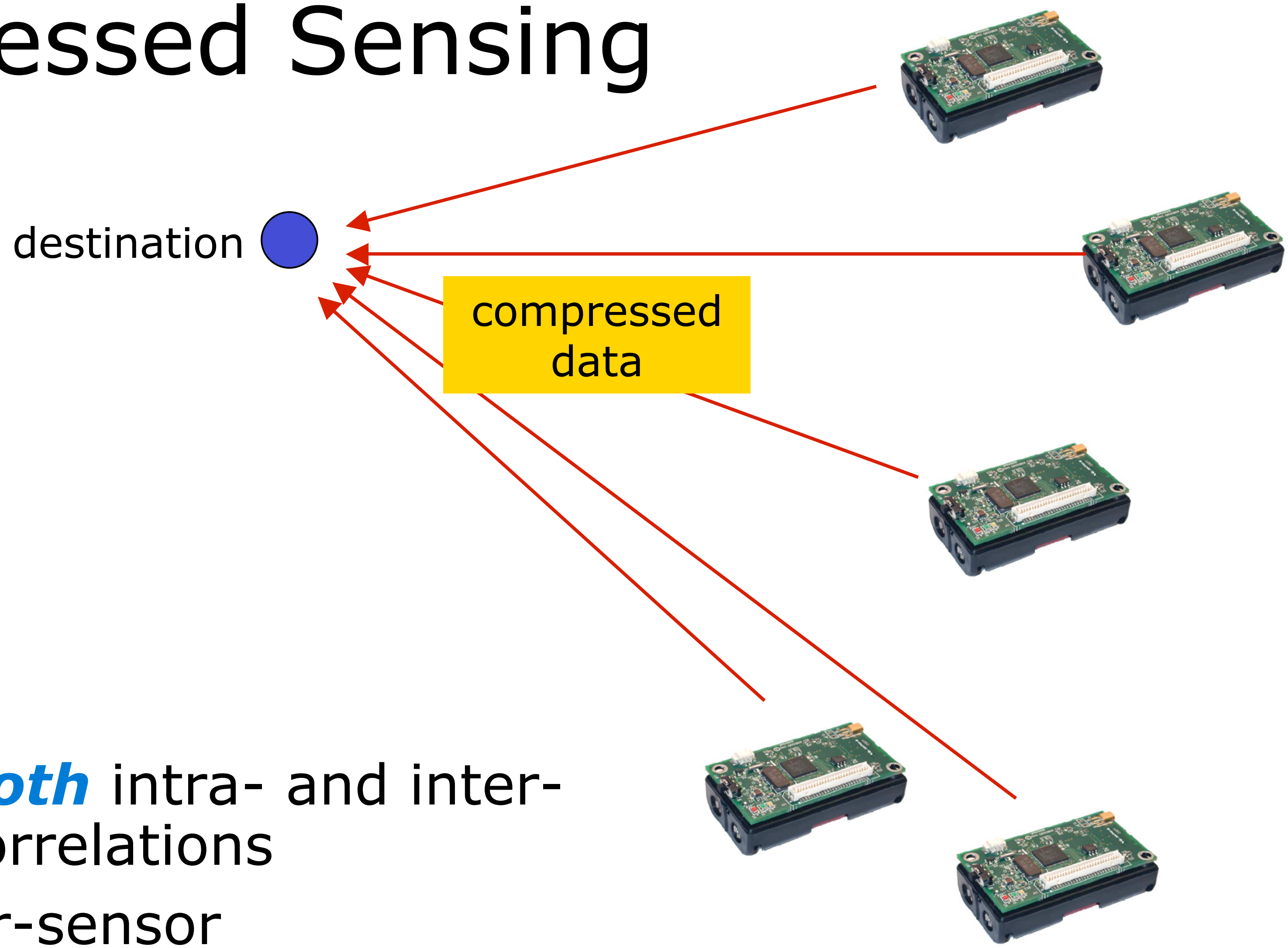
- Can we exploit
  intra-sensor **and**
  inter-sensor
  correlation to *jointly compress?*
  - signals are *compressible* and *correlated*
- *Distributed source coding* problem

Collaborative Compression



- Collaboration introduces
  - inter-sensor *communication overhead*
  - complexity at sensors

Distributed Compressed Sensing (DCS)



Benefits:

- exploit ***both*** intra- and inter-sensor correlations
- zero inter-sensor communication overhead

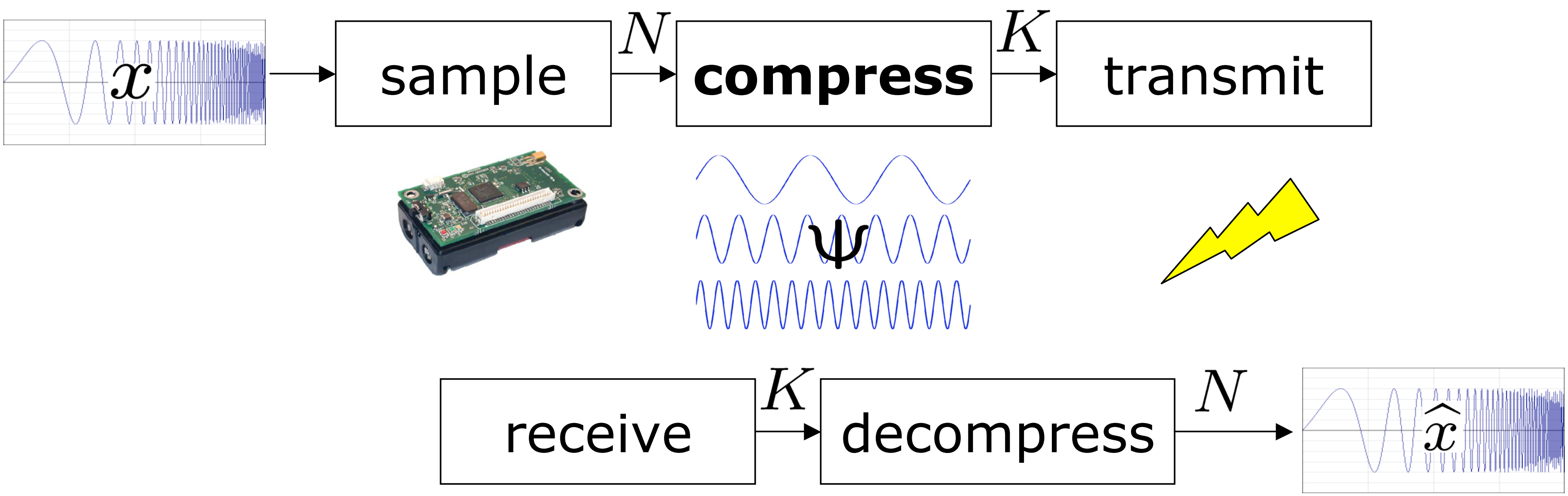
Compressed Sensing

Sensing by Sampling

- Sparse/compressible signals:

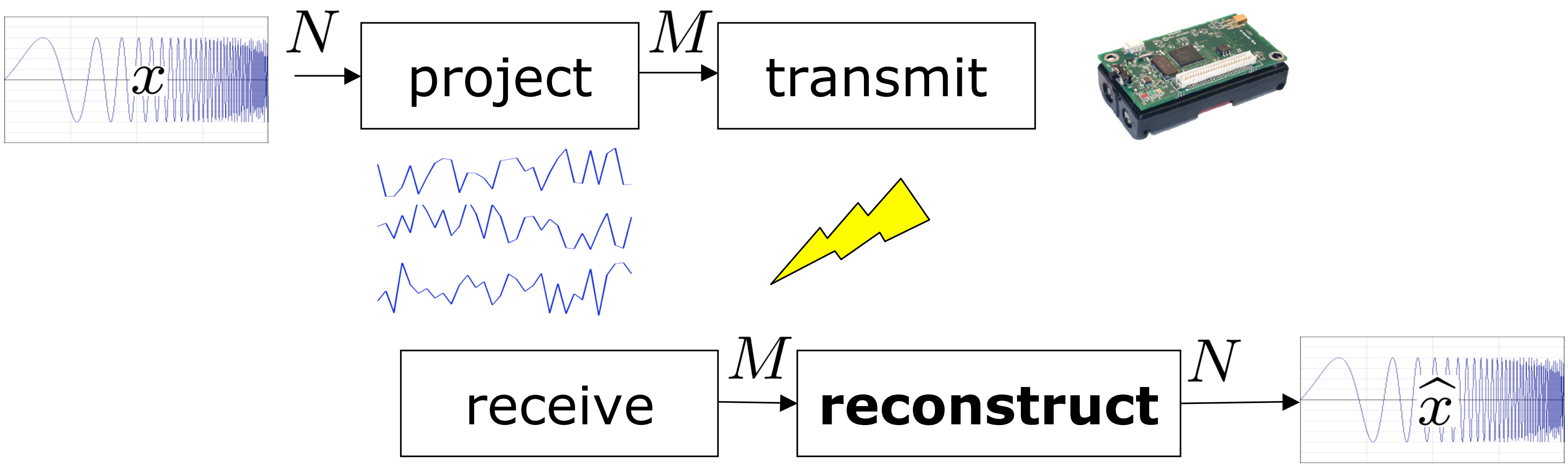
$$x = \Psi \alpha$$

  - Ψ: compression basis (Fourier, wavelets…)
  - α: coefficient vector (few large, many small)
- Compress = transform, sort coefficients, encode largest
- Most computation at *sensor*
- *Lots of work* to throw away >80% of the coefficients



Compressed Sensing (CS)

- Measure linear projections onto *incoherent* basis Φ where data is *not sparse*
  - random sequences are *universally incoherent*
  - mild over-measuring $M \approx 4K \ll N$



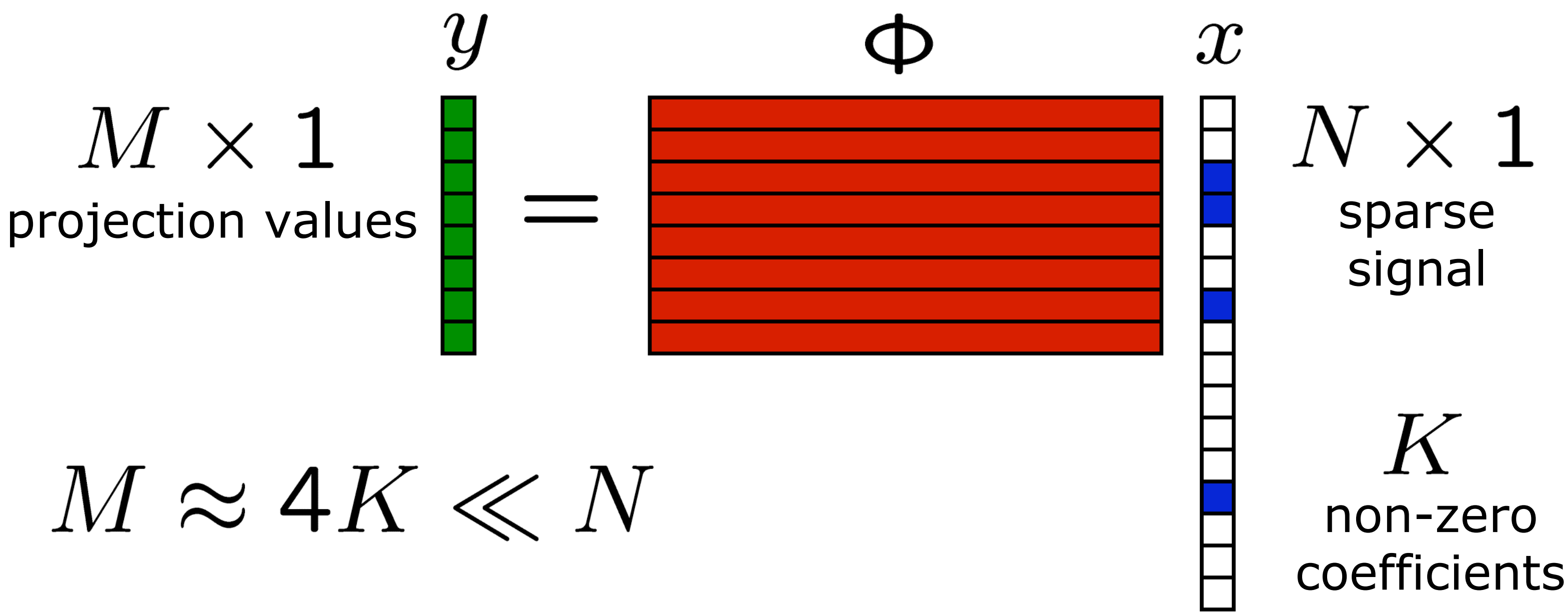
- Computational complexity *shifted* from sensor to receiver

See also Rabbat, Haupt, Singh and Nowak; Bajwa, Haupt, Sayeed and Nowak.

From Samples to *Measurements*

- Replace **samples** by more general **encoder** based on a few *linear projections* (inner products)
  - assume WLOG that x itself is sparse
  - extendable to compressible signals

$$y = \Phi x$$

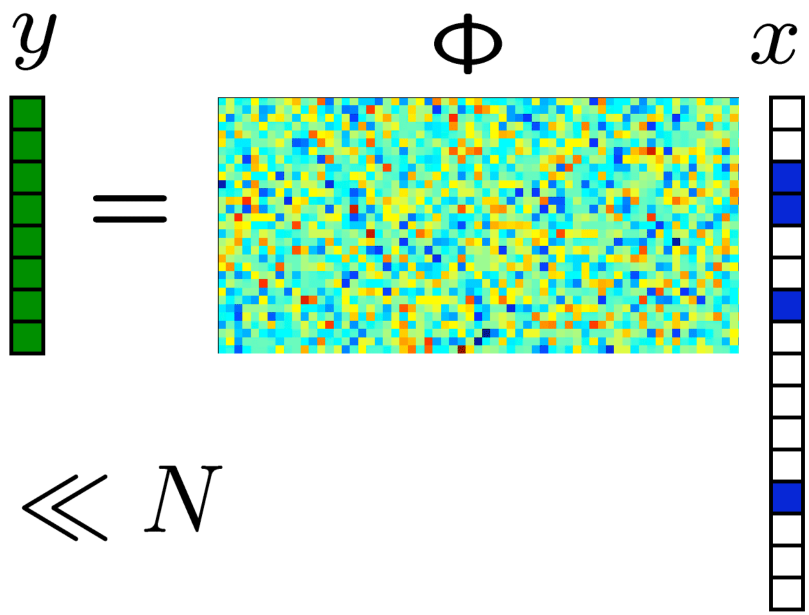


$$M \approx 4K \ll N$$

From Samples to *Measurements*

- **Random** projections

$$y = \Phi x$$



$M \times 1$
projection values

$N \times 1$
sparse
signal

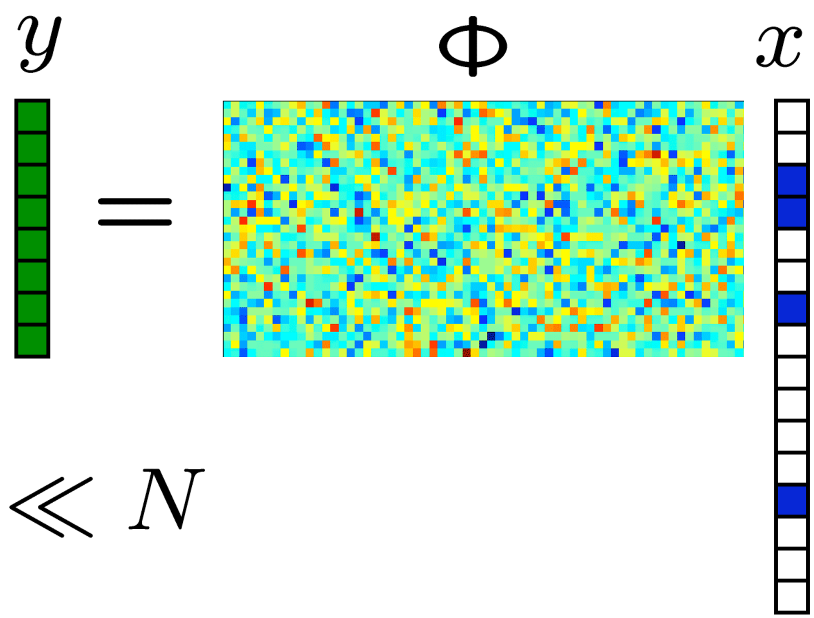
K
non-zero
coefficients

$$M \approx 4K \ll N$$

CS Signal Recovery

- Reconstruction/decoding: given $y = \Phi x$
  (ill-posed inverse problem) find x

$$y = \Phi x$$



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non-zero coefficients

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CS Signal Recovery

- Reconstruction/decoding: given $y = \Phi x$
  (ill-posed inverse problem) find x

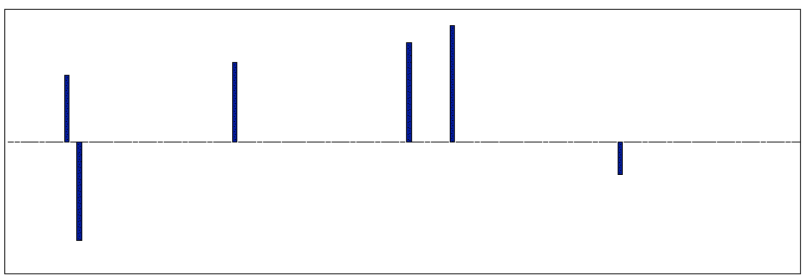
- ℓ_2 **fast** $\widehat{x} = \arg\min_{y=\Phi x} \|x\|_2$

$$\widehat{x} = (\Phi^T\Phi)^{-1}\Phi^T y$$

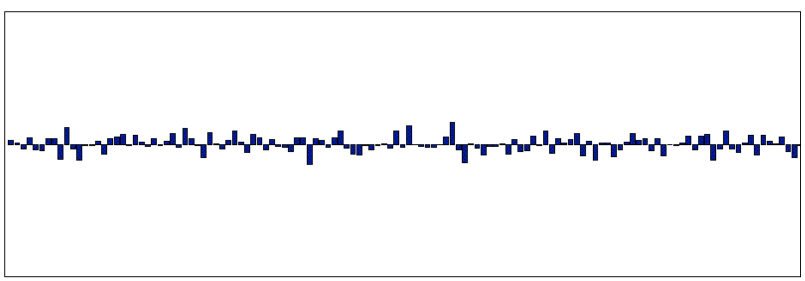
CS Signal Recovery

- Reconstruction/decoding: given $y = \Phi x$
  (ill-posed inverse problem) find x

- ℓ_2 fast, **wrong** $\widehat{x} = \arg\min_{y=\Phi x} \|x\|_2$



x



$$\widehat{x} = (\Phi^T \Phi)^{-1} \Phi^T y$$

CS Signal Recovery

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- ℓ_0 **correct, slow** $\widehat{x} = \arg\min_{y=\Phi x} \|x\|_0$

CS Signal Recovery

- Reconstruction/decoding: given $y = \Phi x$
  (ill-posed inverse problem) find x

- ℓ_2 fast, wrong $\widehat{x} = \arg\min_{y=\Phi x} \|x\|_2$

- ℓ_0 correct, slow $\widehat{x} = \arg\min_{y=\Phi x} \|x\|_0$

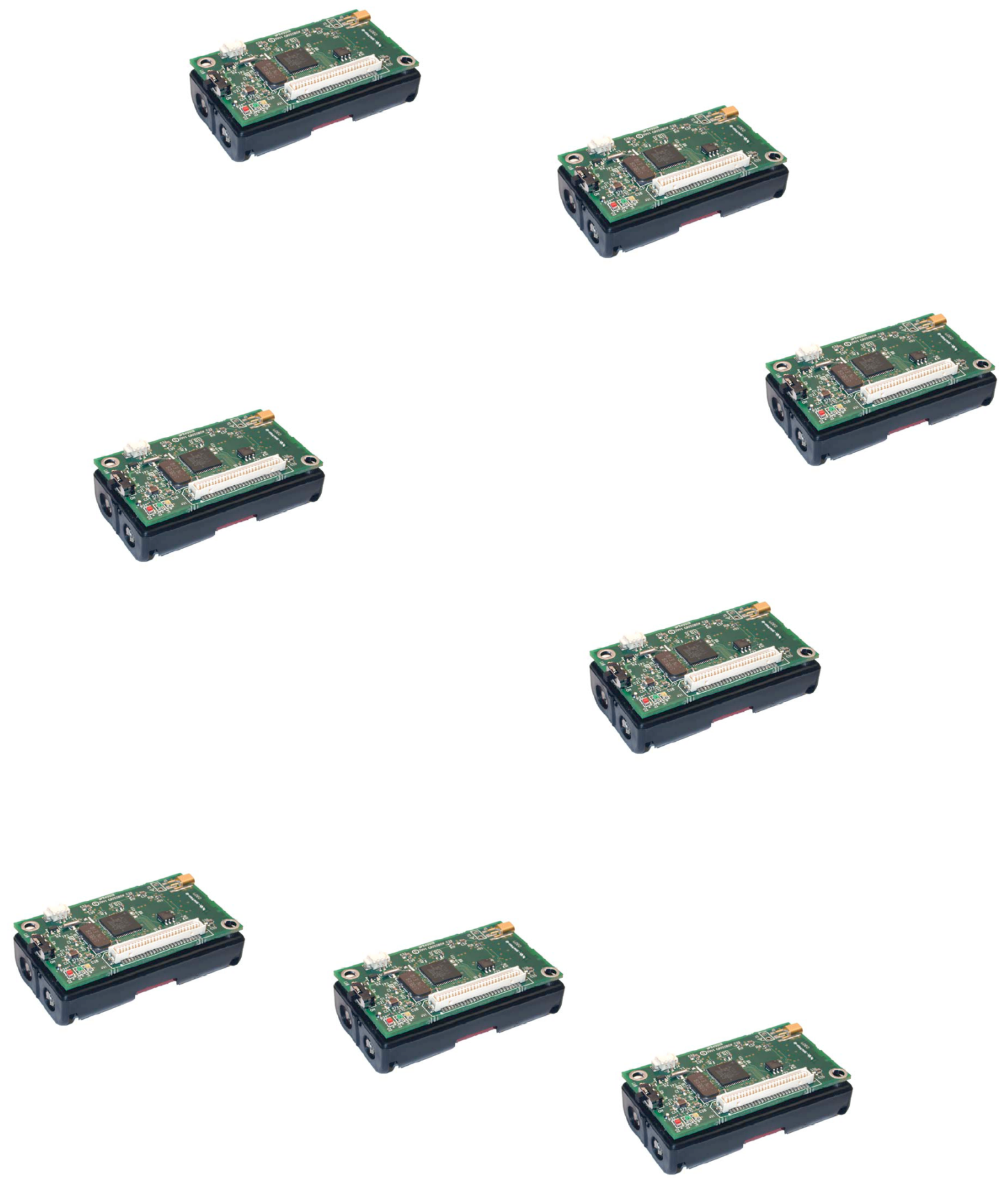
- ℓ_1 **correct,**
  mild oversampling
  [Candes et al, Donoho] $\widehat{x} = \arg\min_{y=\Phi x} \|x\|_1$

  linear program

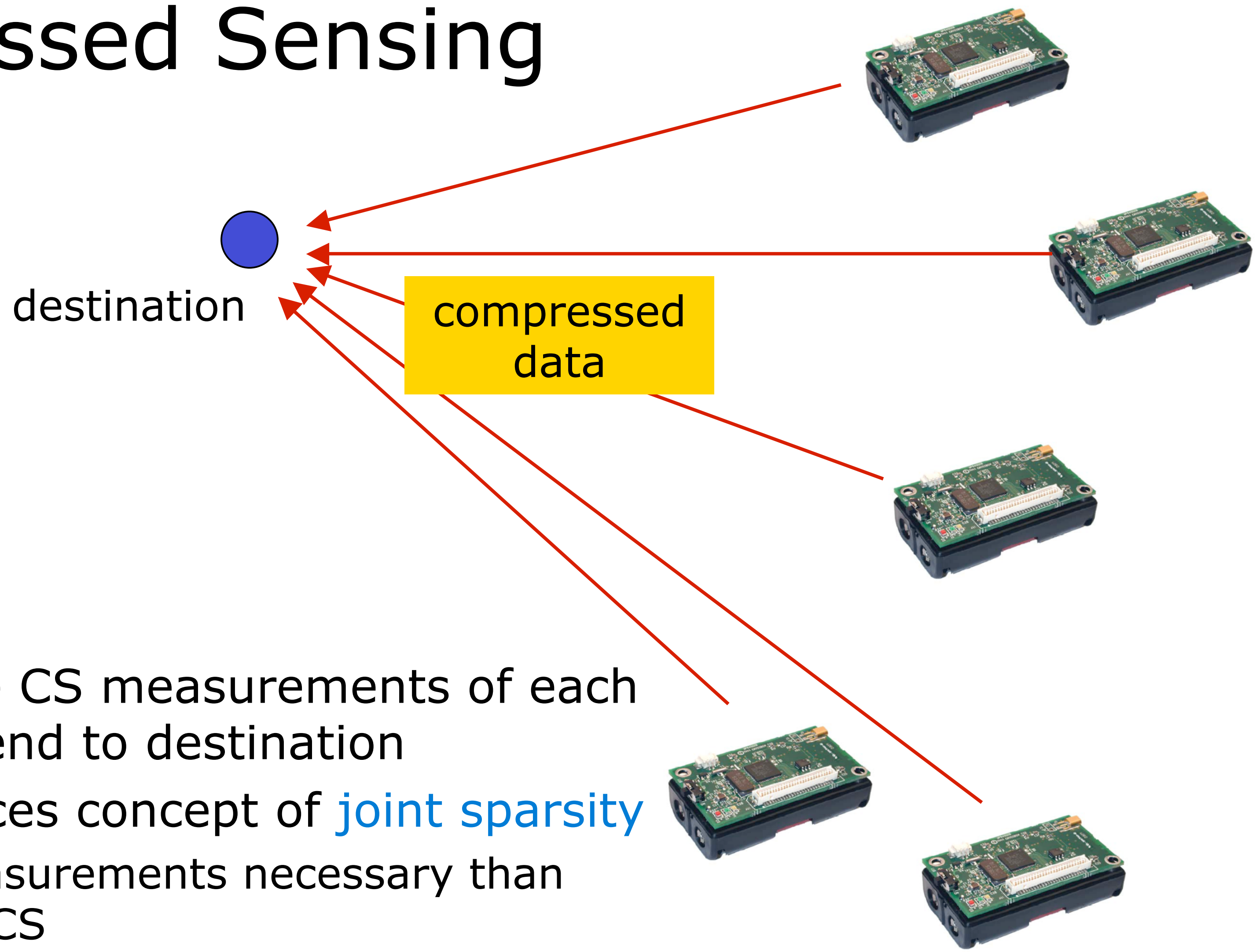
- Greedy [Tropp, Gilbert, Strauss; Rice]

- Complexity-regularization [Haupt and Nowak]

Distributed Compressed Sensing

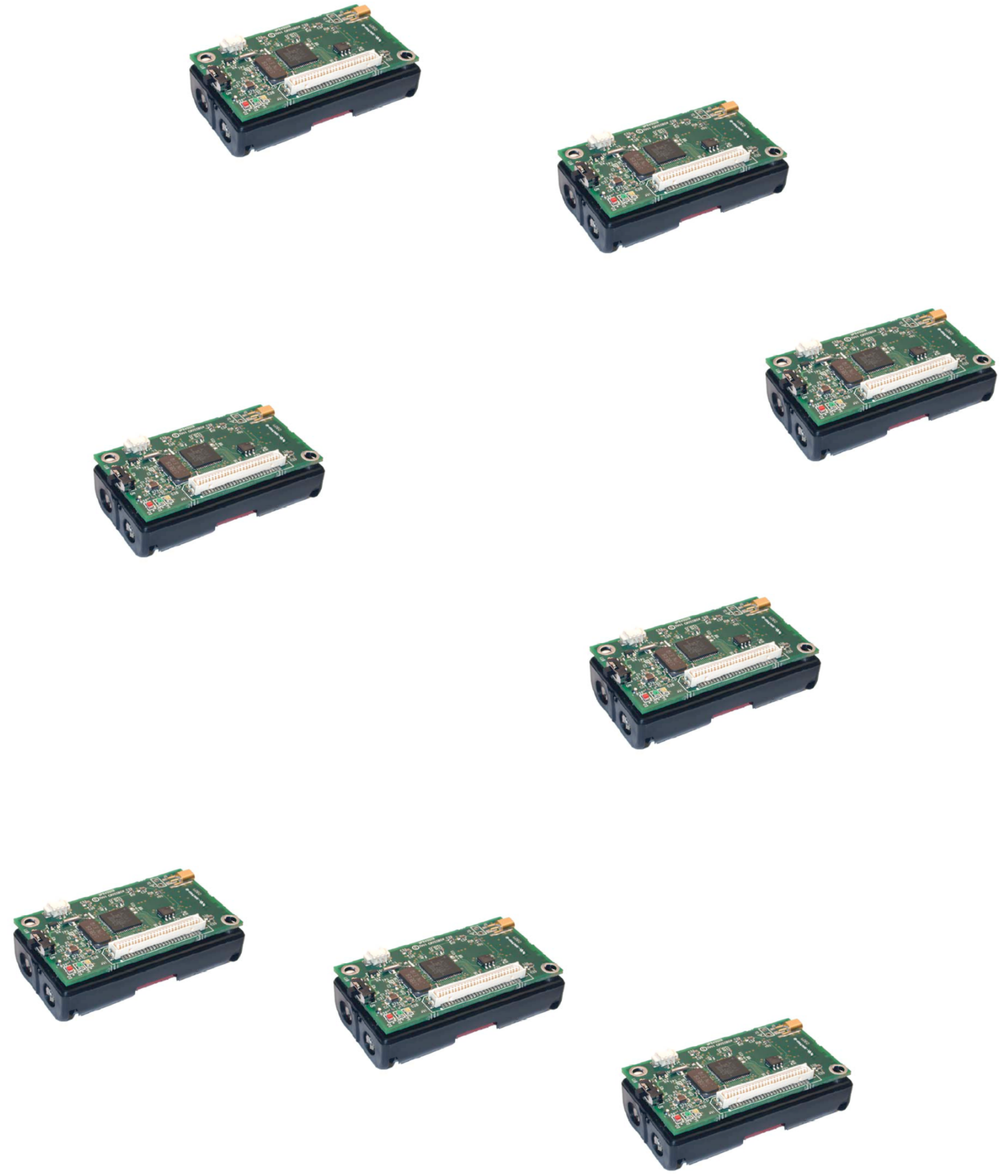


Distributed Compressed Sensing (DCS)



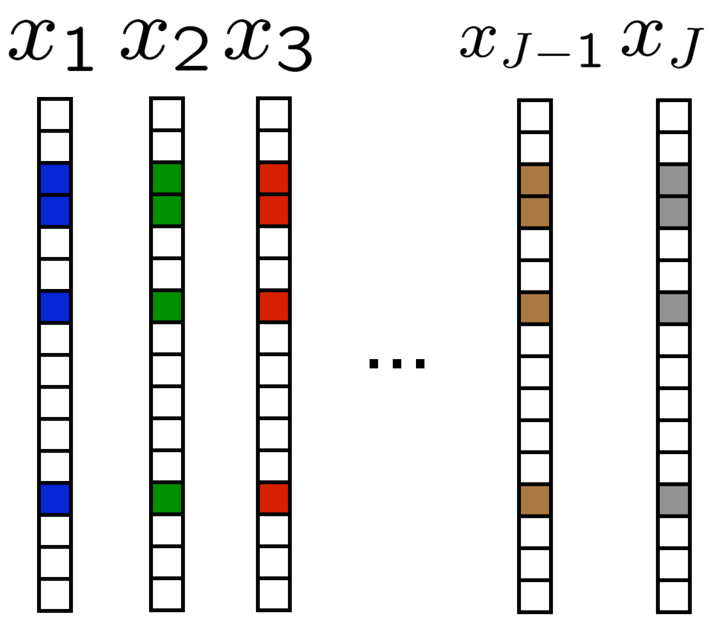
- Sensors take CS measurements of each signal and send to destination
- DCS introduces concept of joint sparsity
  - $\Rightarrow$ Fewer measurements necessary than individual CS
- Different models for different scenarios

Model 1: Common Sparse Supports



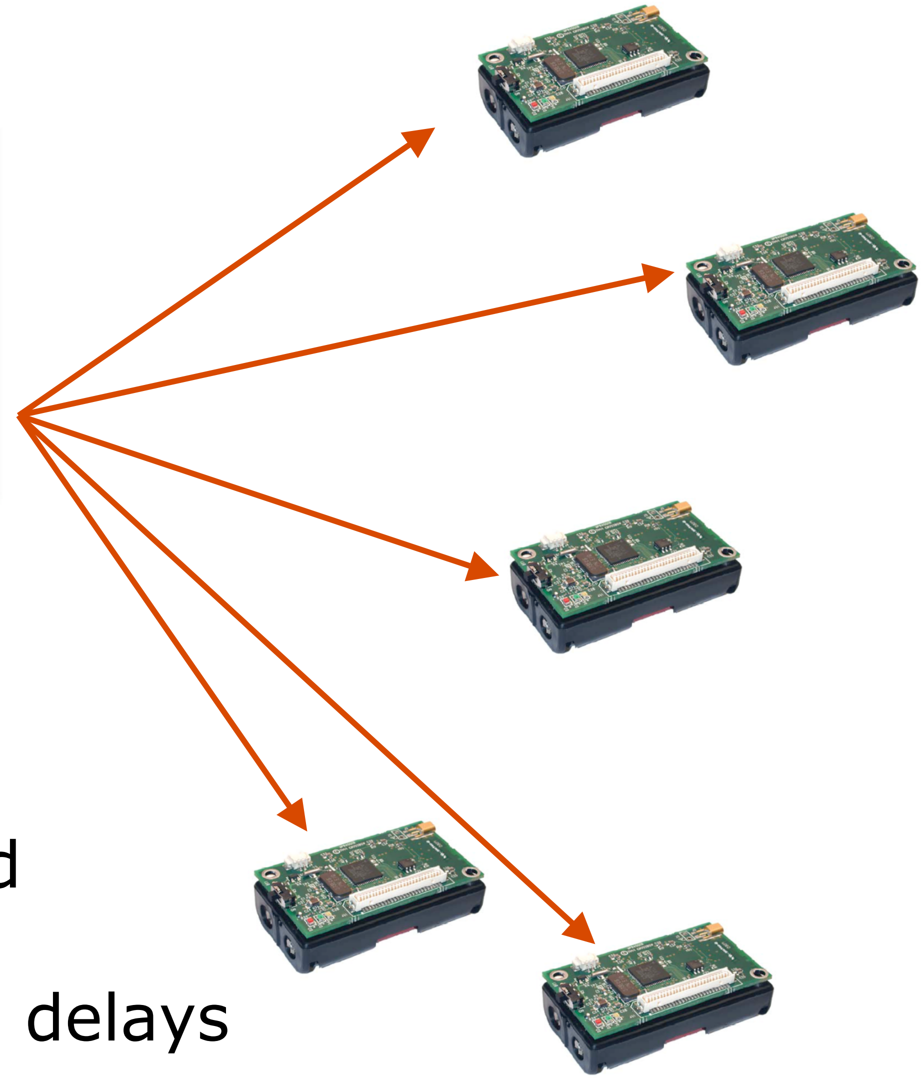
Common Sparse Supports Model

- *Joint sparsity model*:
  - measure J signals, each K-sparse
  - *signals share sparse components, different coefficients*



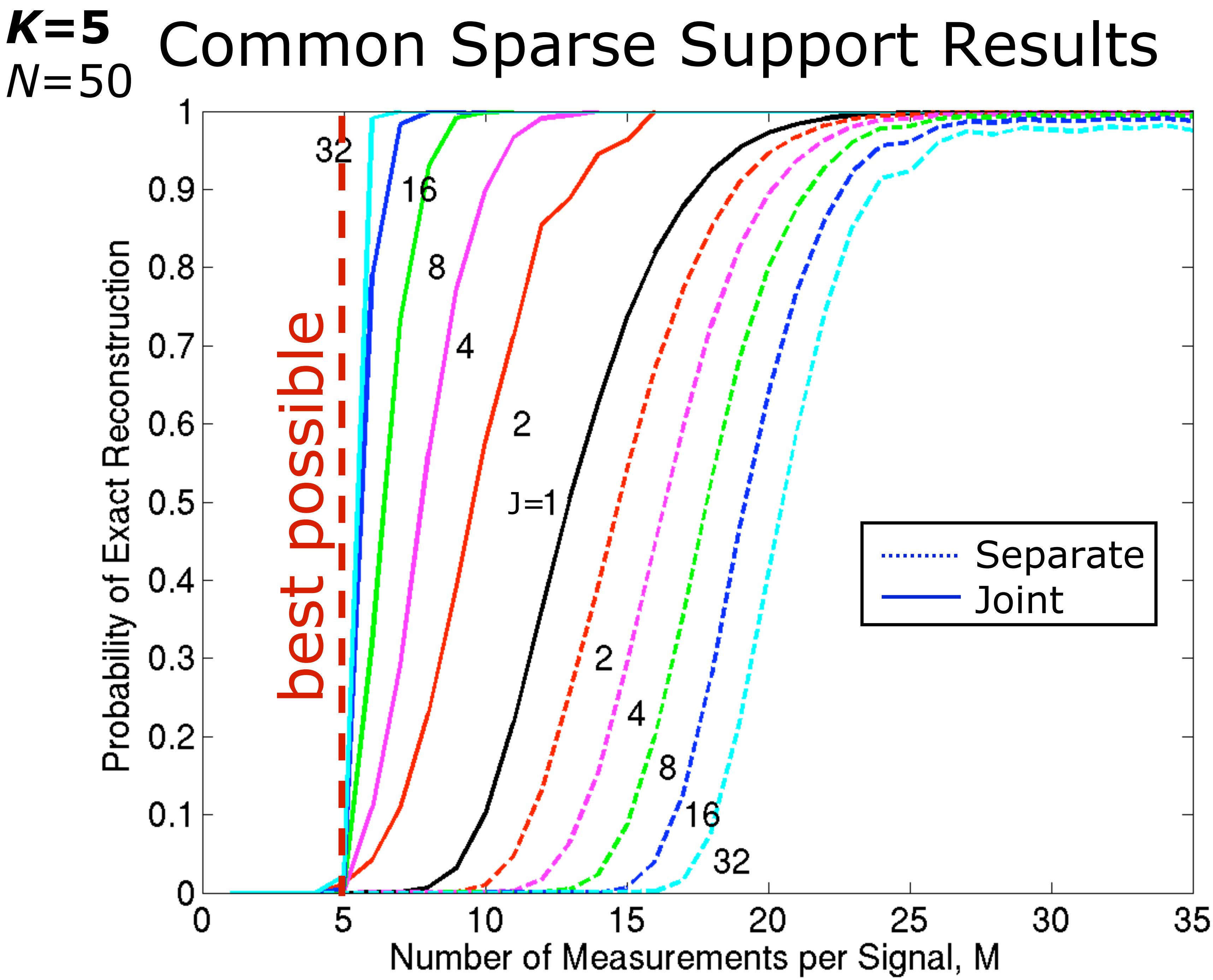
$$
\begin{aligned}
x_j &= \sum_{\omega \in \Omega} x_{j,\omega} \delta_\omega, \\
|\Omega| &= K, \\
y_1 &= \Phi_1 x_1, \\
&\vdots \\
y_J &= \Phi_J x_J
\end{aligned}
$$

Common Sparse Supports Model



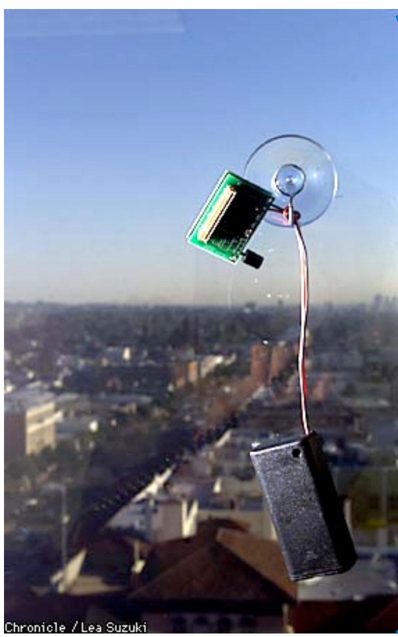
Ex: Audio Signals

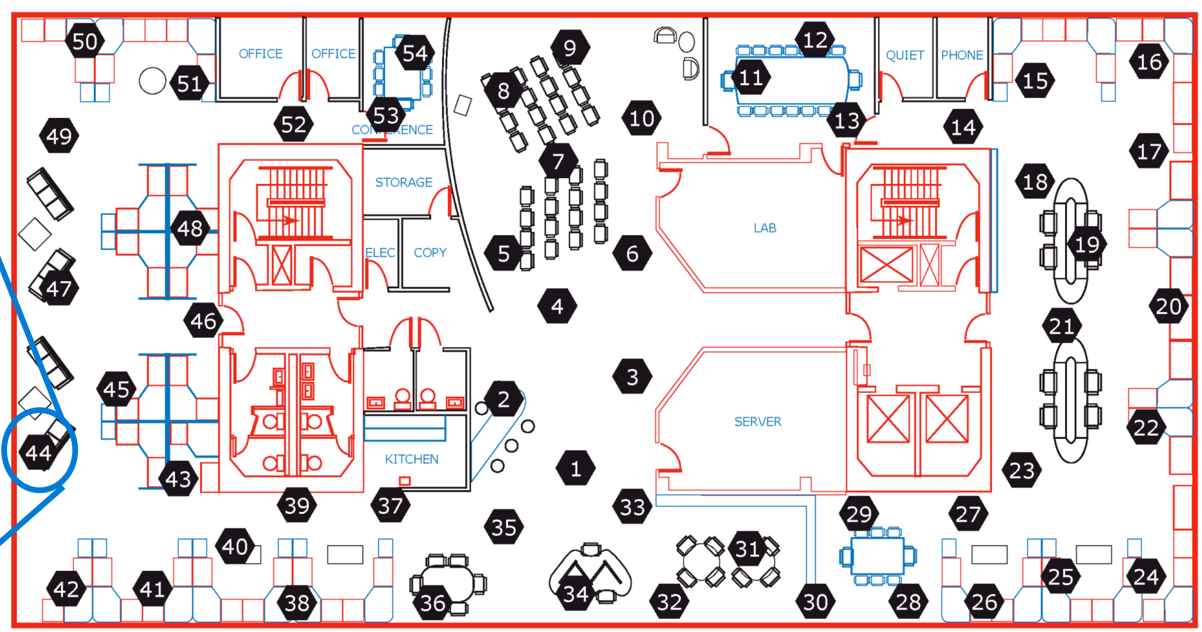
- sparse in Fourier Domain
- same frequencies received by each node
- different attenuations and delays (magnitudes and phases)


K=5
N=50
Common Sparse Support Results
Probability of Exact Reconstruction
Number of Measurements per Signal, M
best possible
32
16
8
4
2
J=1
2
4
8
16
32
Separate
Joint
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
0
5
10
15
20
25
30
35

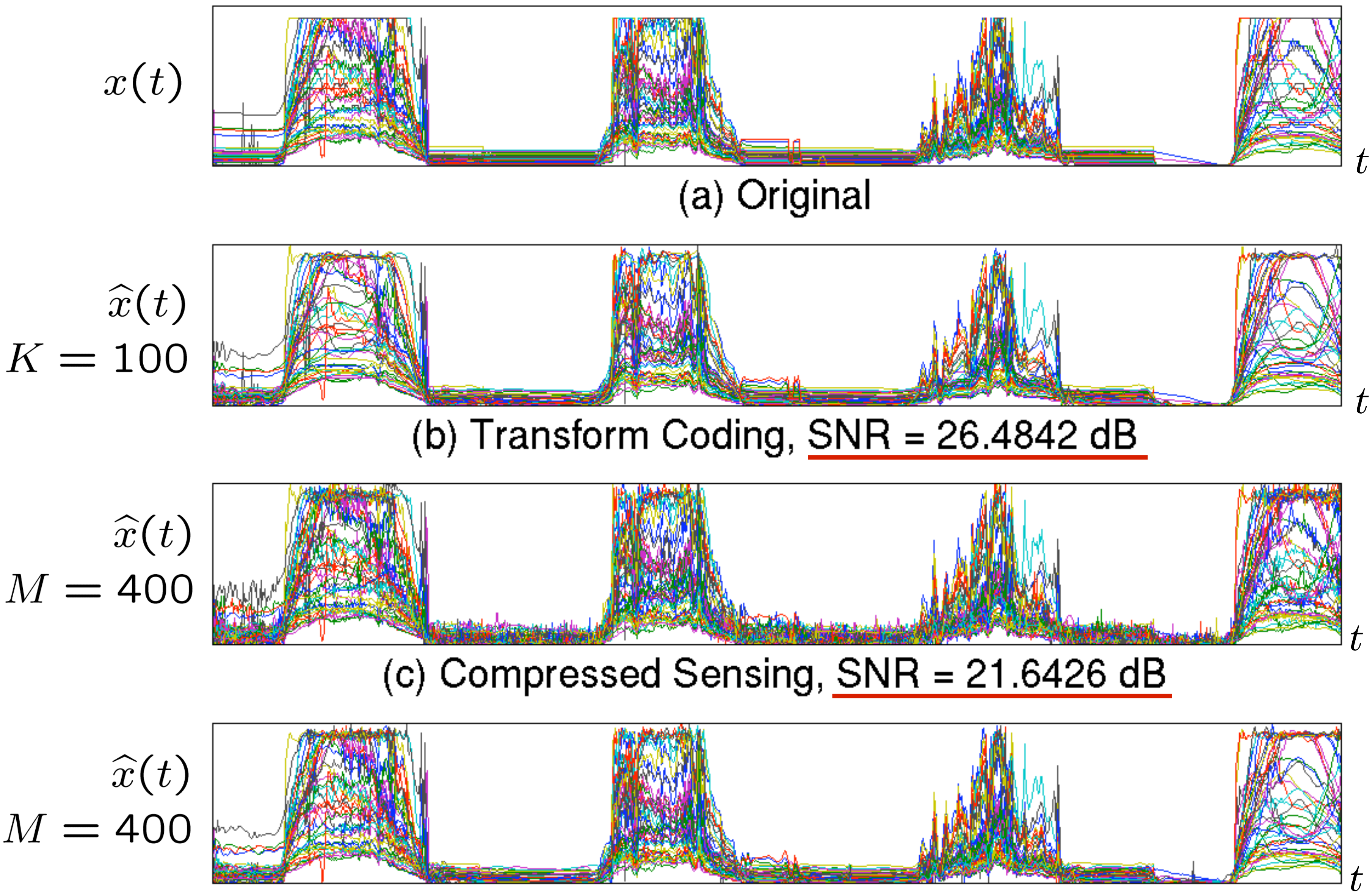
Real Data Example

- Dataset: Indoor Environmental Sensing
- $J = 49$ sensors, $N = 1024$ samples each
- Compare compression using:
  - transform coding approx — K largest terms per sensor
  - independent CS — $4K$ measurements per sensor
  - DCS: common sparse supports — $4K$ measurements per sensor





Light Intensity - Wavelets



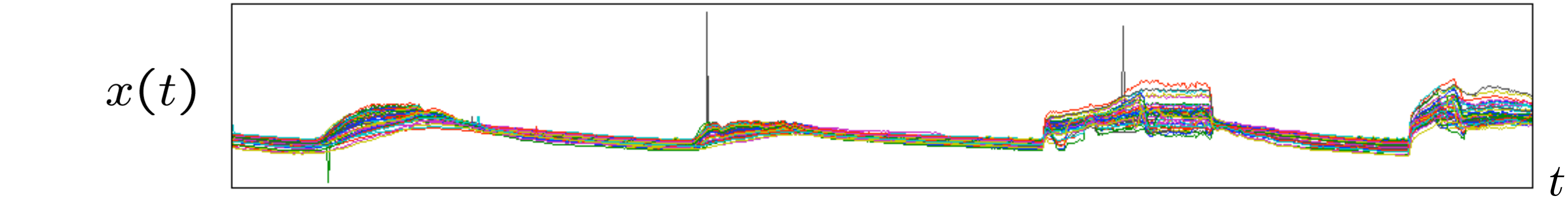
(a) Original

(b) Transform Coding, SNR = 26.4842 dB

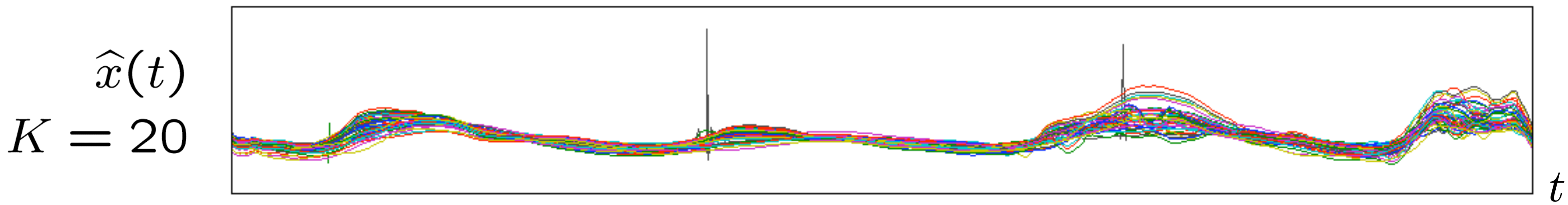
(c) Compressed Sensing, SNR = 21.6426 dB

(d) Distributed Compressed Sensing, SNR = 27.1906 dB

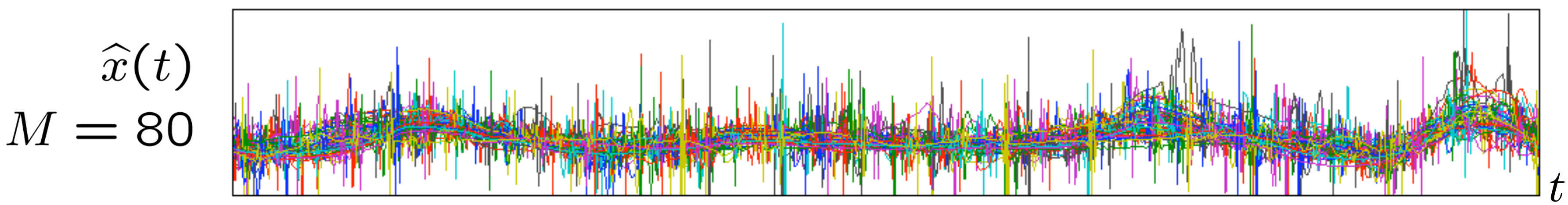
Temperature - Wavelets



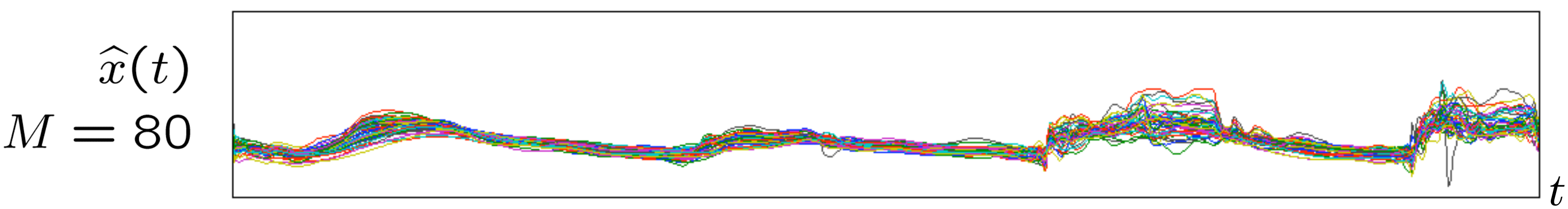
(a) Original



(b) Transform Coding, SNR = 28.589 dB

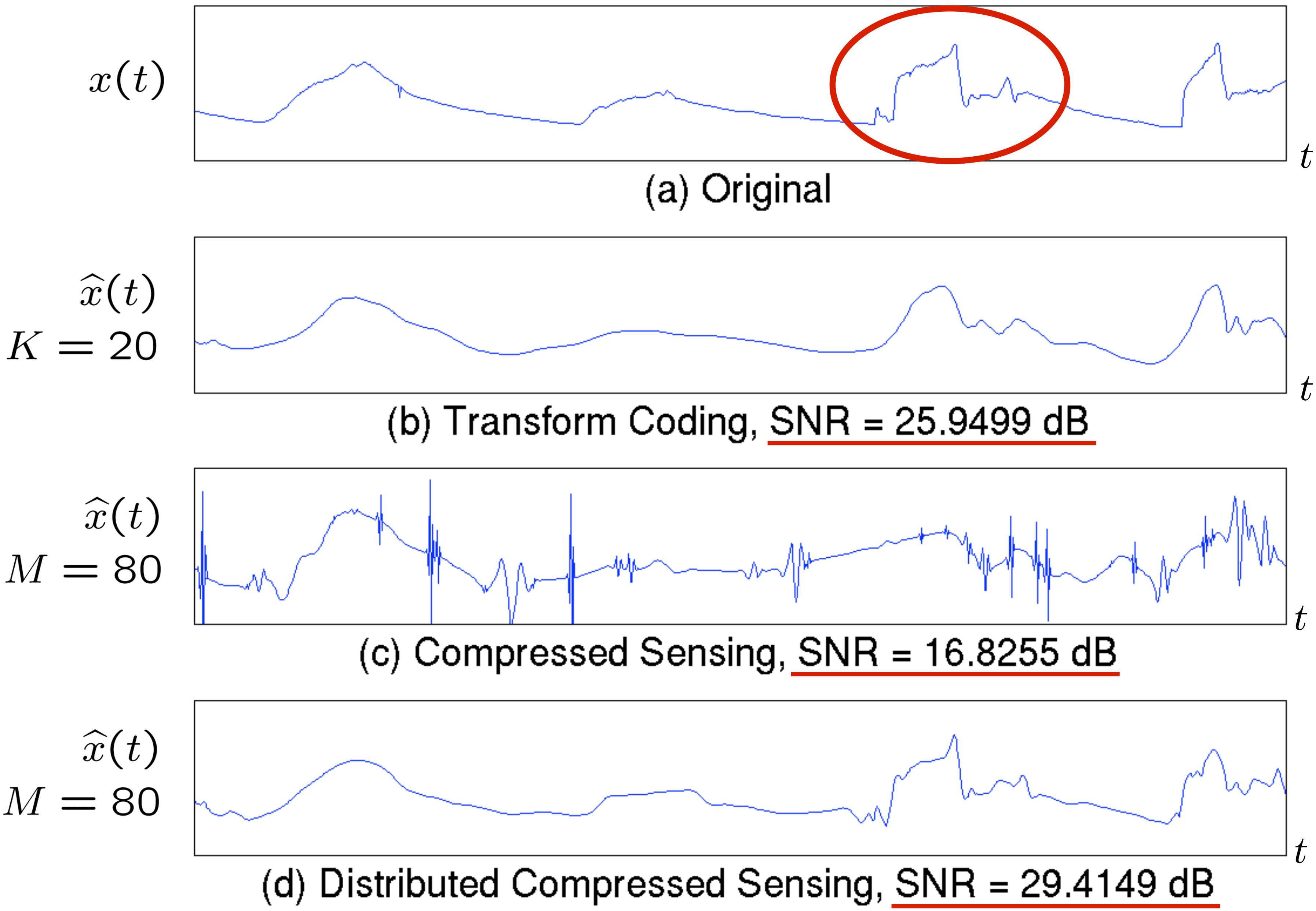


(c) Compressed Sensing, SNR = 18.7817 dB



(d) Distributed Compressed Sensing, SNR = 29.9518 dB

Temperature - Wavelets



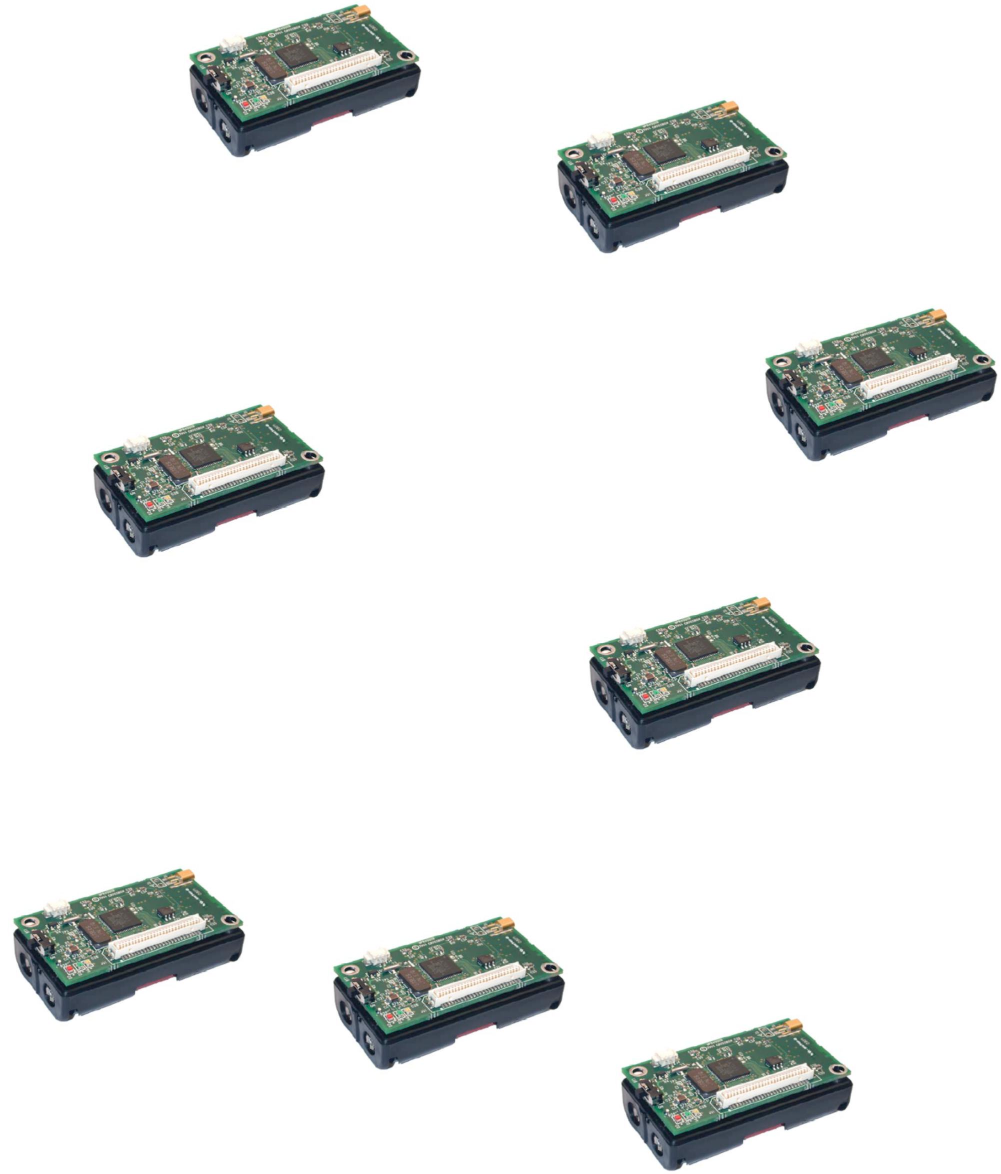
(a) Original

(b) Transform Coding, SNR = 25.9499 dB

(c) Compressed Sensing, SNR = 16.8255 dB

(d) Distributed Compressed Sensing, SNR = 29.4149 dB

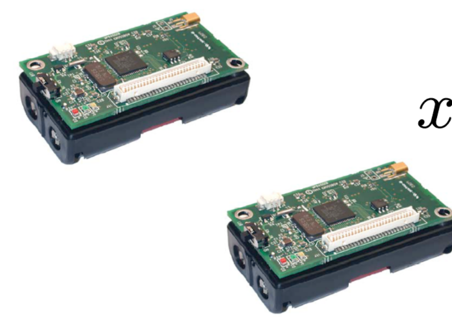
Model 2:
Common + Innovations



Common + Innovations Model

- Motivation: sampling signals in a smooth field

x_1



x_2

- *Joint sparsity model*:
  - length-N sequences x_1 and x_2

$$x_1 = z + z_1$$
$$x_2 = z + z_2$$

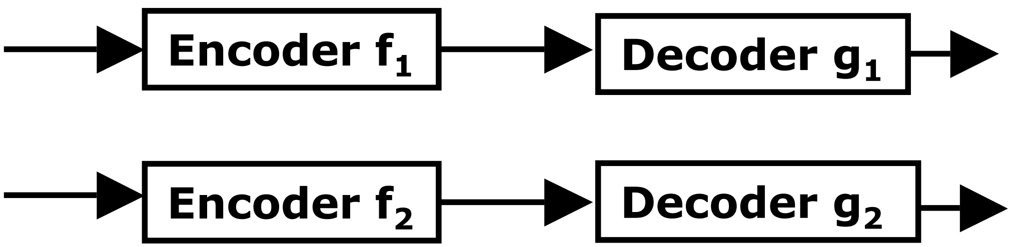
common component sparsity K

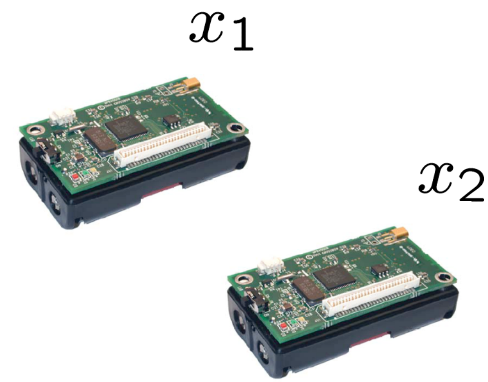
innovation components sparsities K_1, K_2.

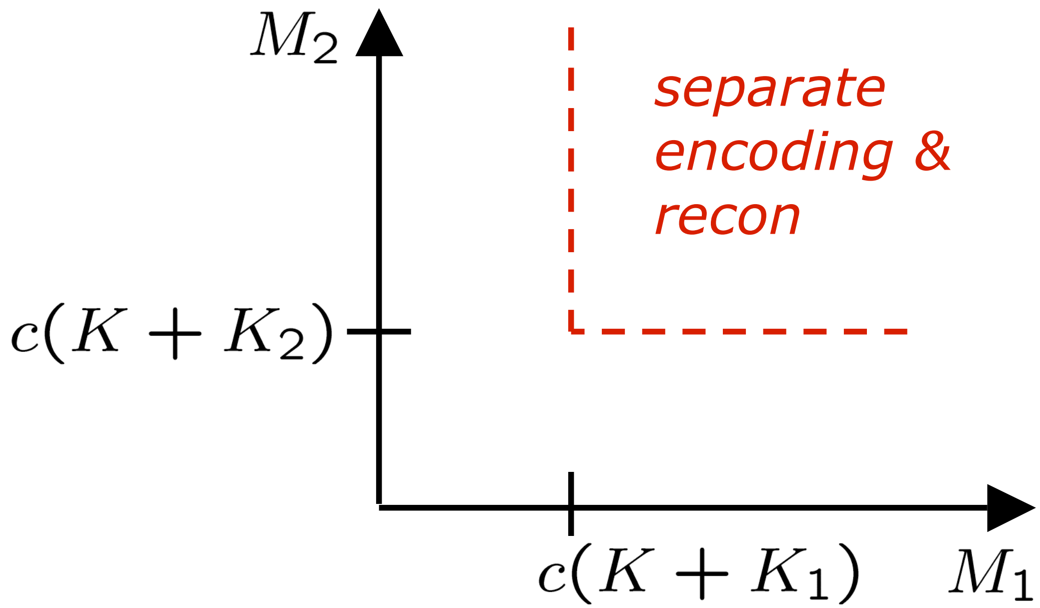
- Measurements

$$\begin{aligned} y_1 &= \Phi_1 x_1 \\ y_2 &= \Phi_2 x_2 \end{aligned}$$

Measurement Rate Region with *Separate* Reconstruction

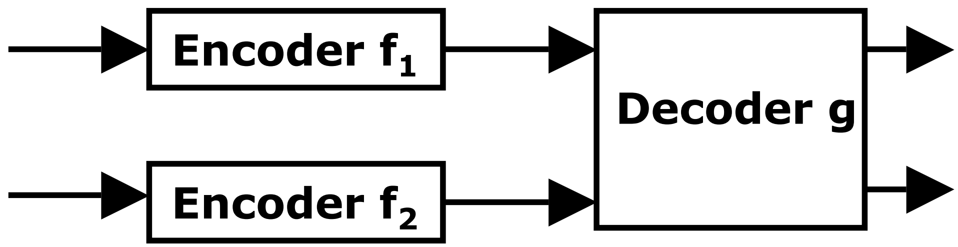


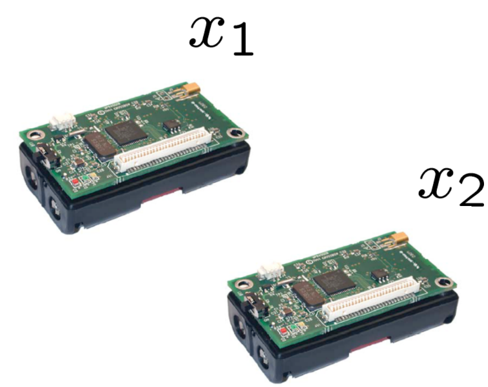


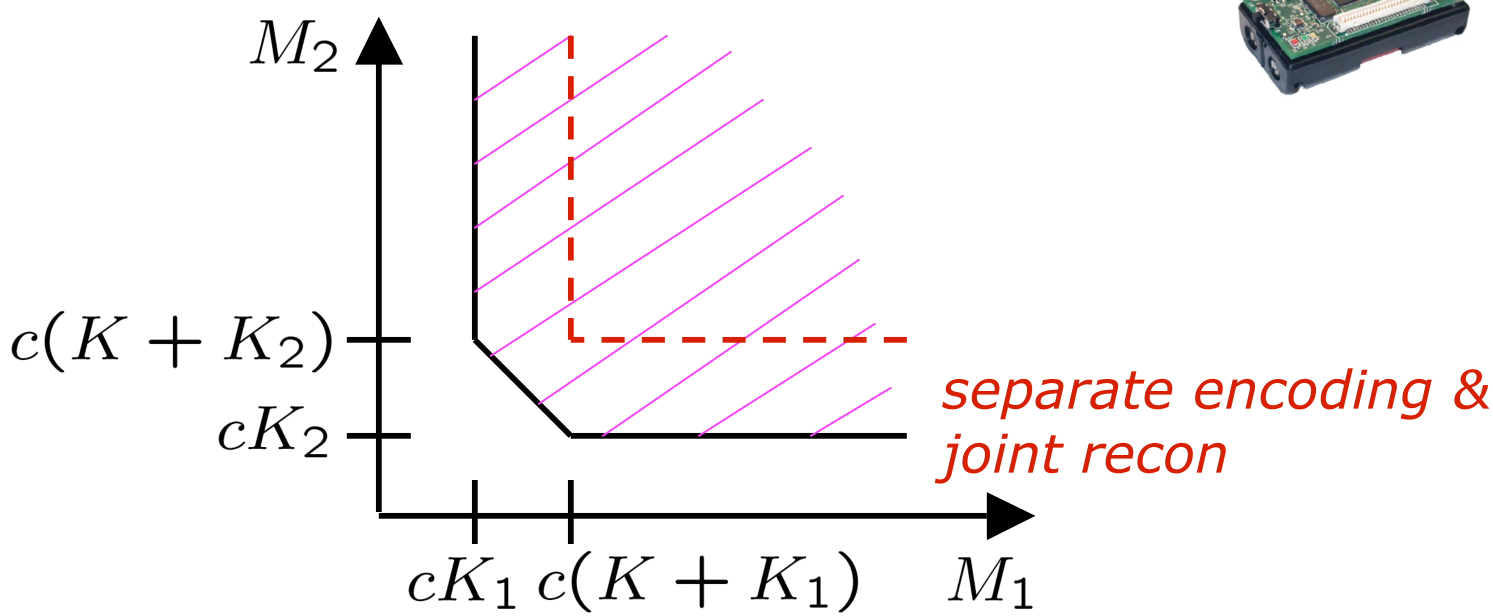


Measurement pair region

Measurement Rate Region with *Joint* Reconstruction







Measurement pair region

DCS Benefits for Sensor Networks

- **Hardware:** ***Universality***
  - same random projections / hardware can be used for *any* signal class with a sparse representation
  - simplifies hardware and algorithm design (generic)
  - random projections automatically encrypted
  - very simple encoding
  - robust to noise, quantization and measurement loss

- **Processing:** **Information scalability**
  - random projections ~ sufficient statistics
  - same random projections / hardware can be used for a range of different signal processing tasks
    - reconstruction, estimation, detection, recognition, …
  - many fewer measurements are required to detect/classify/recognize than to reconstruct
    - implications for power management

Conclusions

- Theme: Compressed Sensing for multiple signals

- Distributed Compressed Sensing
  - exploits *both* intra- and inter-sensor correlation
  - new models for *joint sparsity*
  - many attractive features for sensor network applications

- More
  - additional joint sparsity *models*
  - *theoretical bounds* for compressible signals
  - *statistical signal processing* from random projections
  - *analog* Compressed Sensing
  - *faster* reconstruction algorithms

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