

# Non-Homogeneous Hidden Markov Chain Models for Wavelet-Based Hyperspectral Image Processing

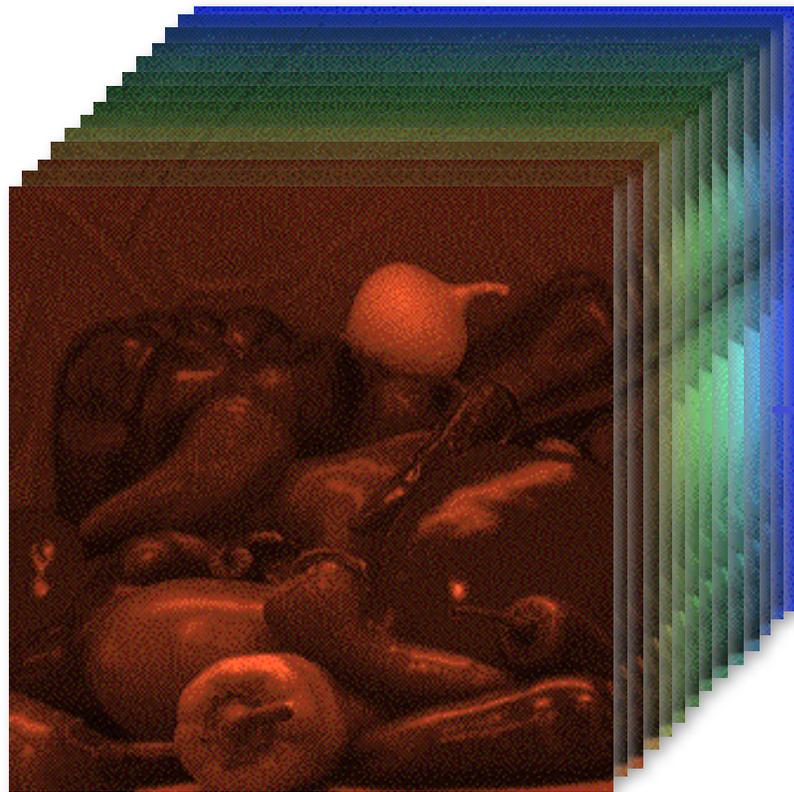
Marco F. Duarte

Mario Parente



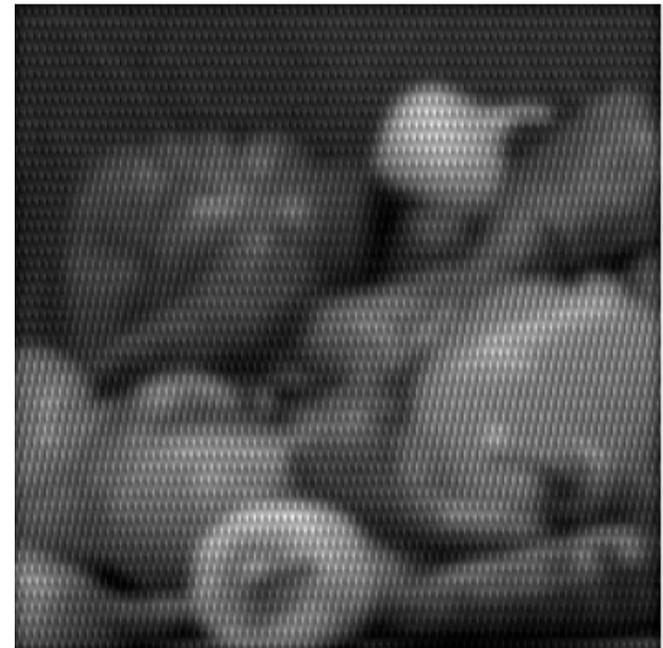
UMASS  
AMHERST

# Hyperspectral Imaging



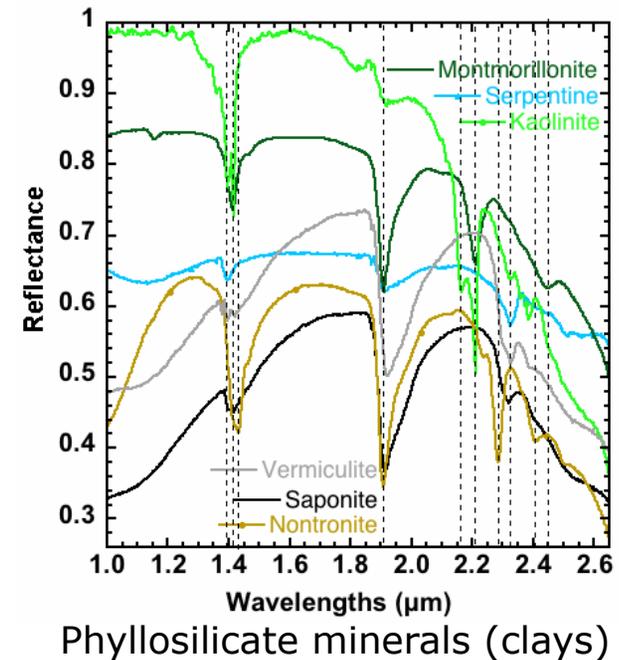
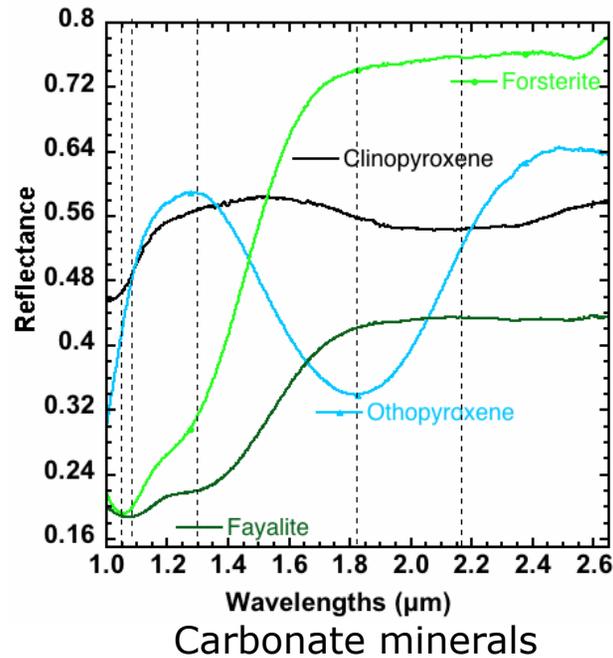
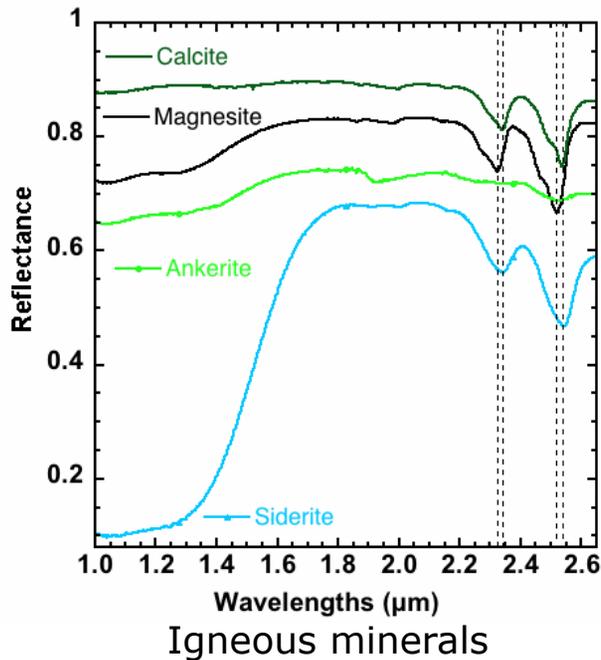
Hyperspectral datacube

One signal/image per band



Spectrum at each pixel represents  
composition/physical state of subject  
(remote sensing, industrial process monitoring, etc.)

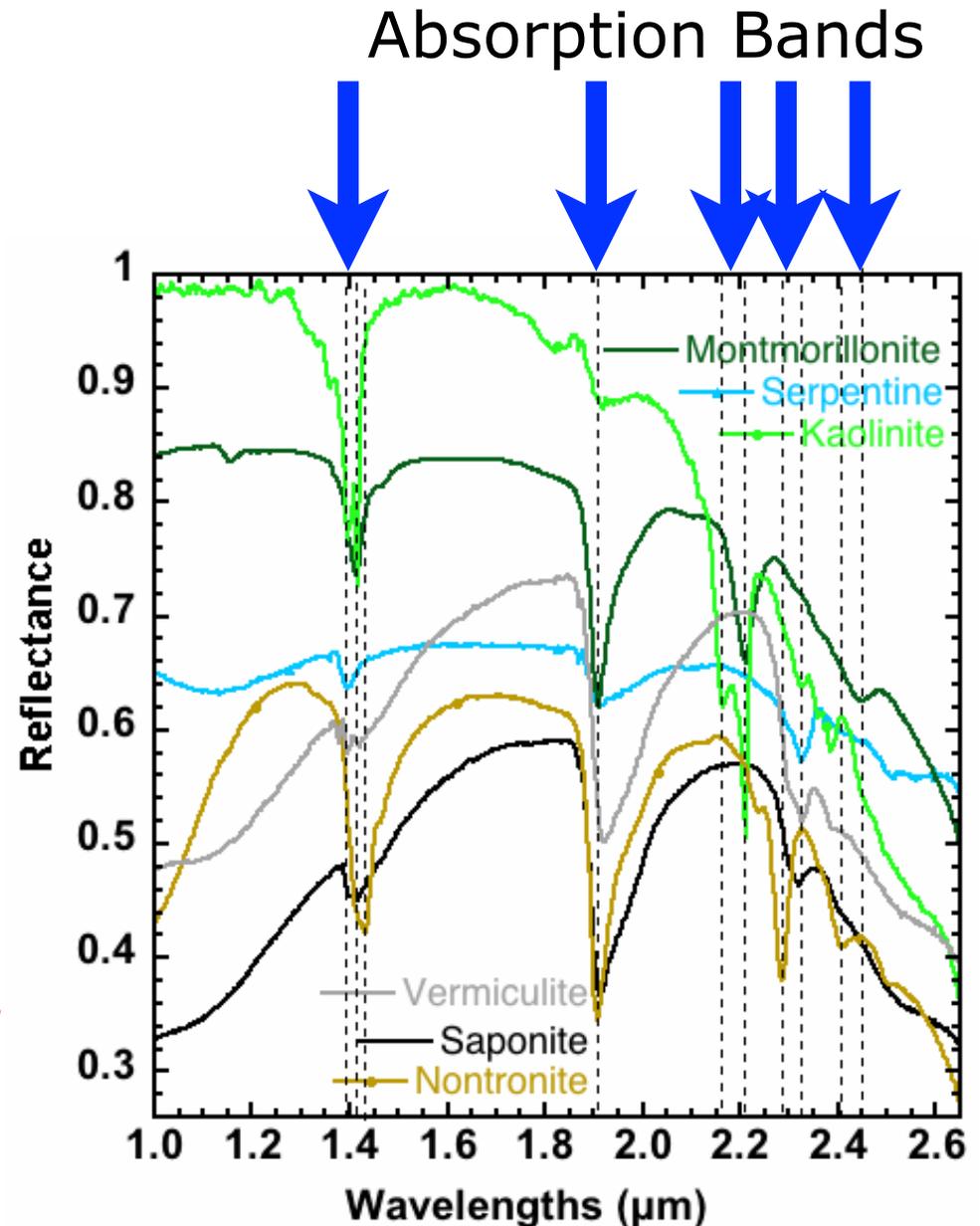
# Hyperspectral Signatures



- Encode **reflectivity of material surface** over a variety of wavelengths of light (100+)
- Differences **evident** between materials/minerals of different classes; more **subtle** within a class
- Signature **fluctuations** used in ad-hoc fashion for material identification
- **Positions** and **shapes** provide identifiability

# Hyperspectral Classification

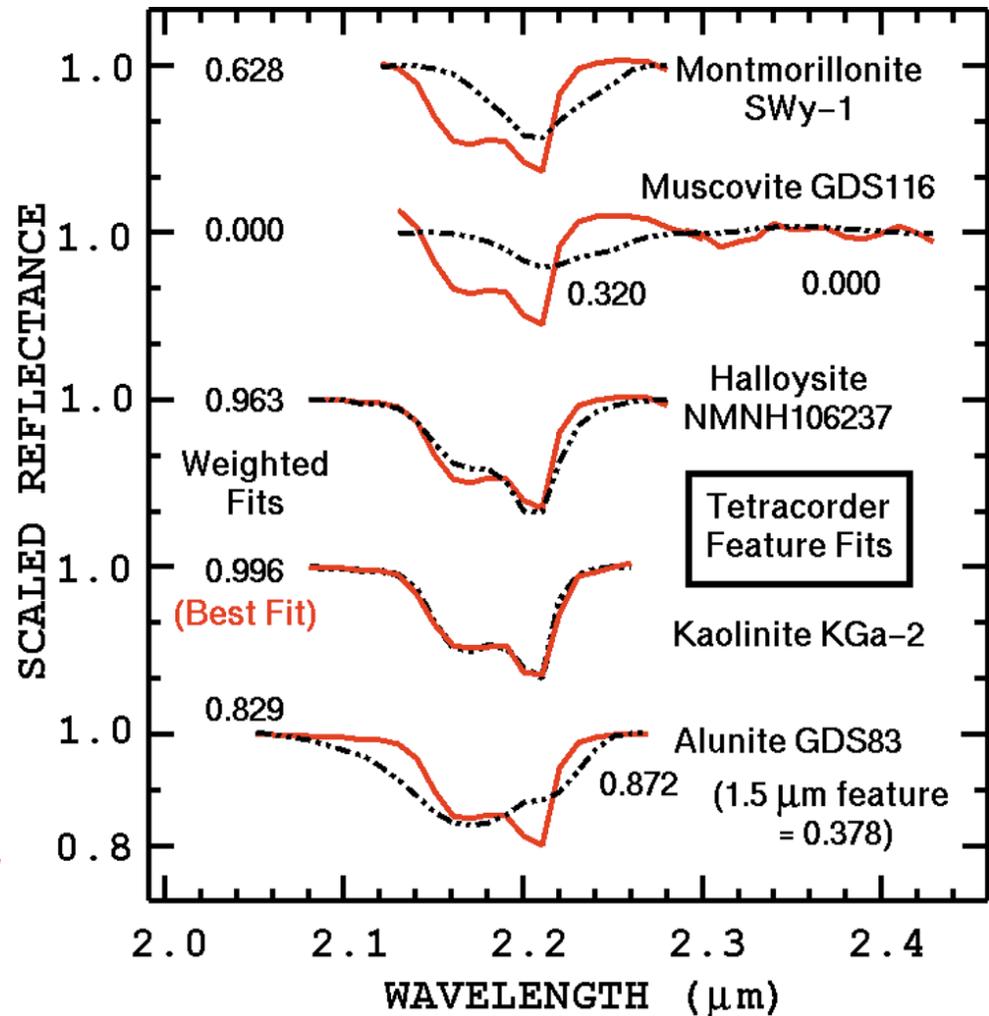
- **Tetracorder**: List of rules to identify spectra by shape
- Rules can be arbitrarily **complicated**
- New rules must be created for new materials
- “Difficult” cases need **experienced analyst**



# Hyperspectral Classification

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[Clark et al., USGS 2003]



# Hyperspectral Classification

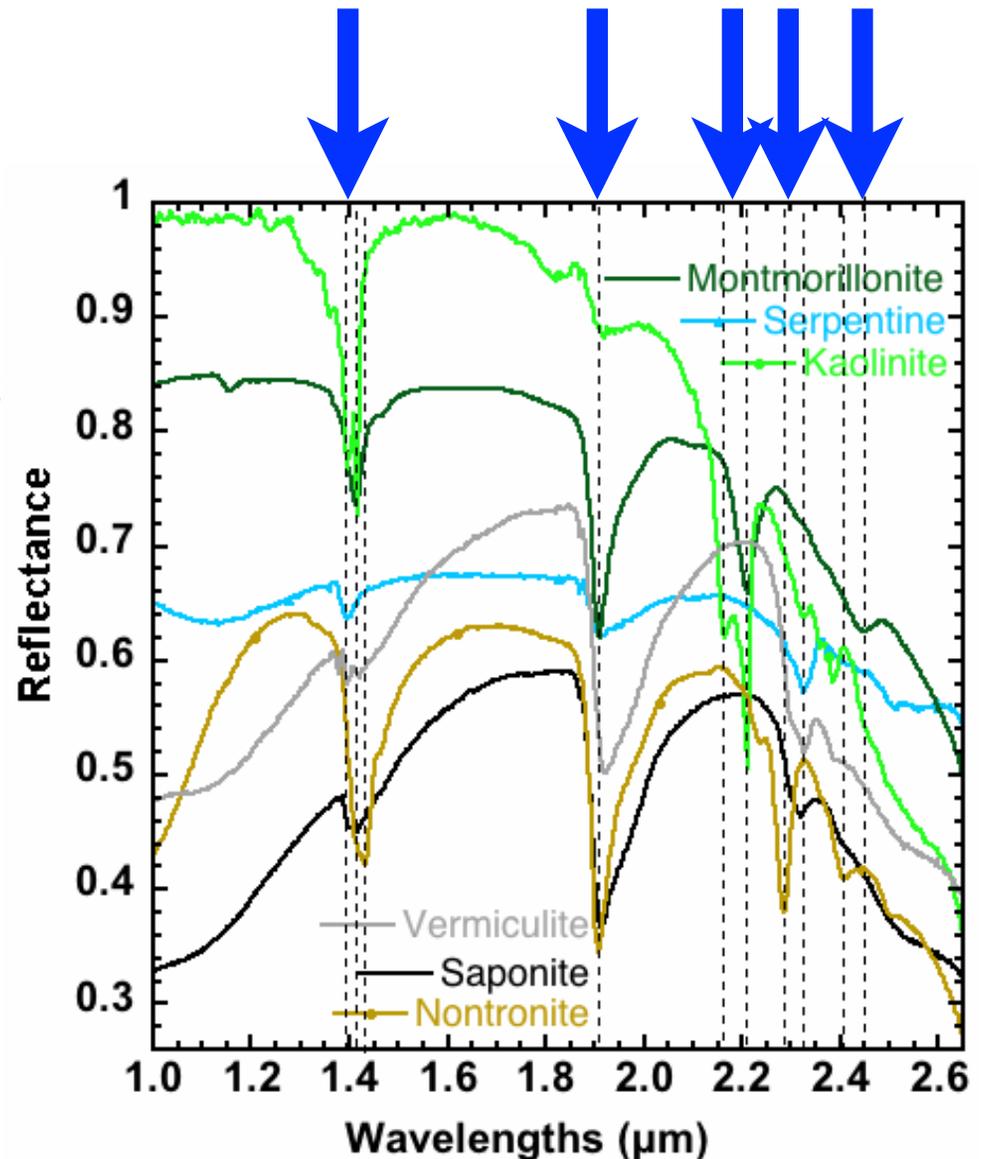
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[Clark et al., USGS 2003]

```
group 2 # algorithm: featfit1
# input library reference
spectrum #=TITLE=Alunite
GDS83 Na63
# channels to exclude (global
variable) Alunite GDS83 Na63
# 2 spectral features, 0 not
features Dw 2.048 2.078 2.247
2.277 ct .04
# continuum wavelengths,
threshold (ct) Dw 1.466 1.476
1.535 1.555 ct .05
# continuum wavelengths,
threshold (ct) FITALL > 0.5
# fit thresholds: if below 0.5,
reject
```

# Hyperspectral Classification

- **Specialized** distance metrics: spectral angle mapper, spectral divergence, etc.
  - aim to match shapes
  - **sensitive** to additional variations in signal from sample to sample
- How to successfully capture fluctuations in **punctuated, piecewise smooth** signals?



# Continuous Wavelet Transform

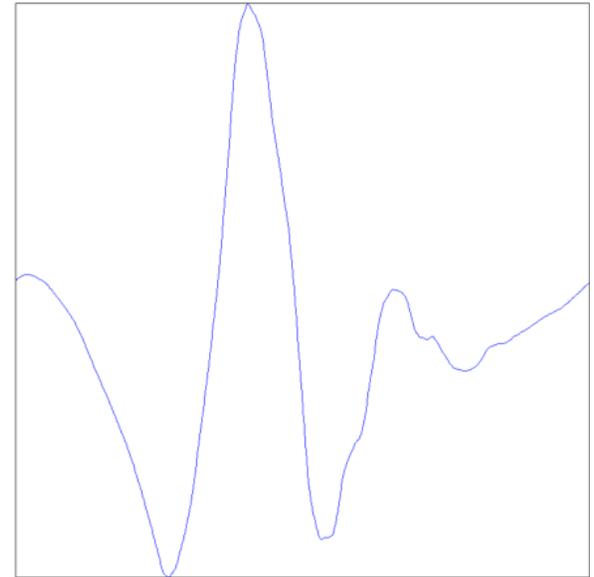
- Mother wavelet dilated to **scale**  $s$  and translated to **offset**  $u$ :

$$\psi_{s,u}(f) = \frac{1}{\sqrt{s}} \psi \left( \frac{f - u}{s} \right)$$

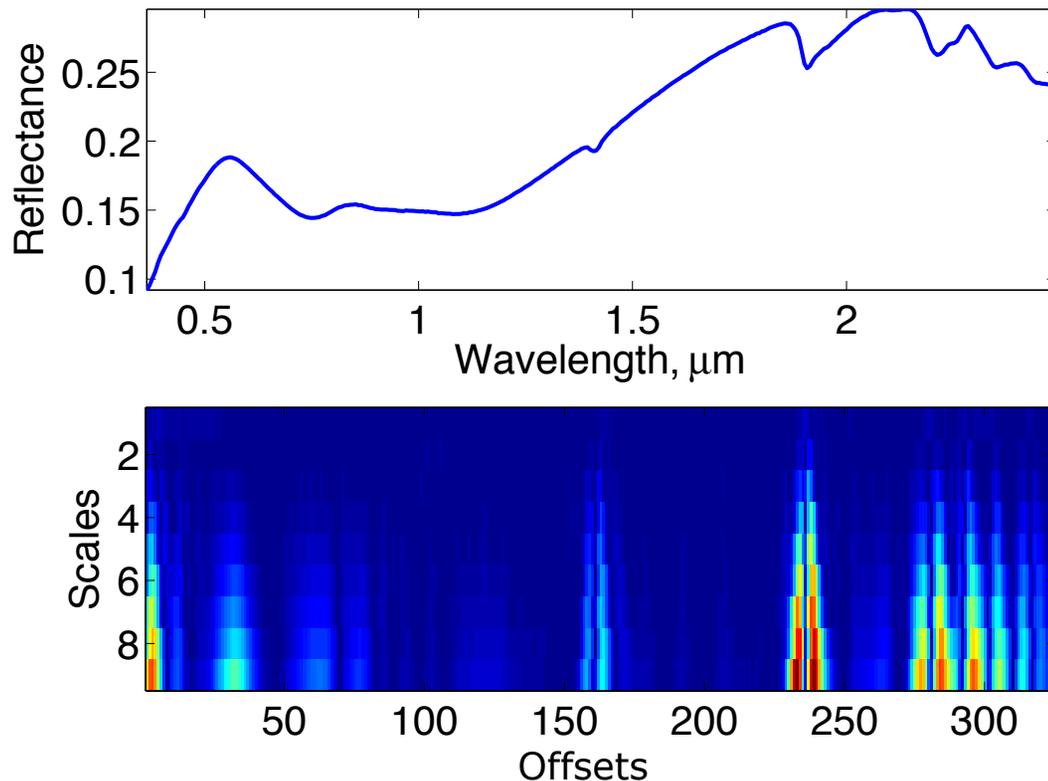
- CWT of a spectrum  $x(f)$ ,  $f \in [0, F]$ , composed of wavelet coefficients  $w_{s,u}$  at scales  $s = 1, \dots, S$ , offsets  $u = 0, F/N, 2F/N, \dots, F-F/N$ :

$$w_{s,u} = \int x(f) \psi_{s,u}(f) df$$

- Coefficient  $w_{s,u}$  acts as a “**detector**” of fluctuations of scale  $s$  at location  $f = u$

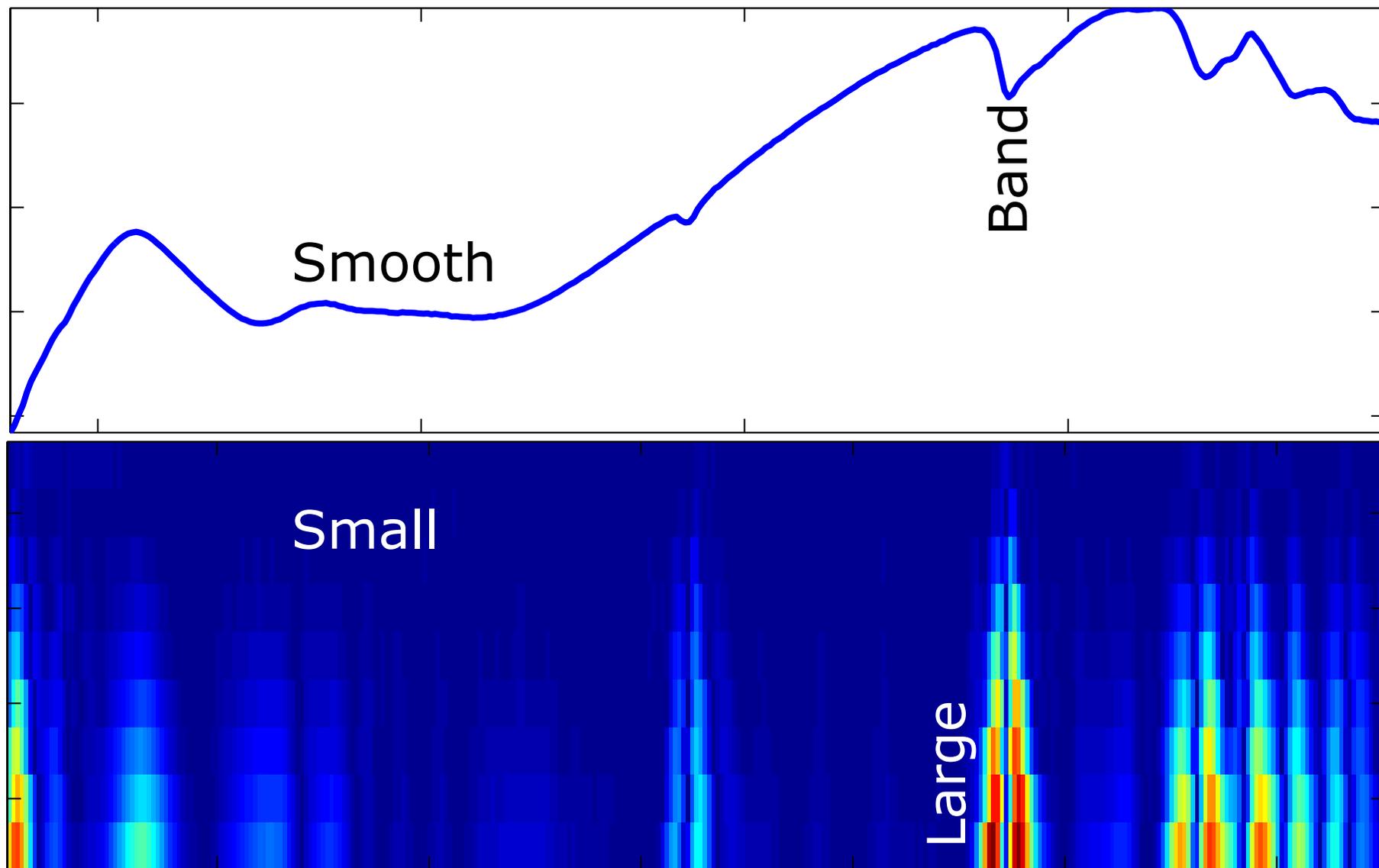


# Continuous Wavelet Transform

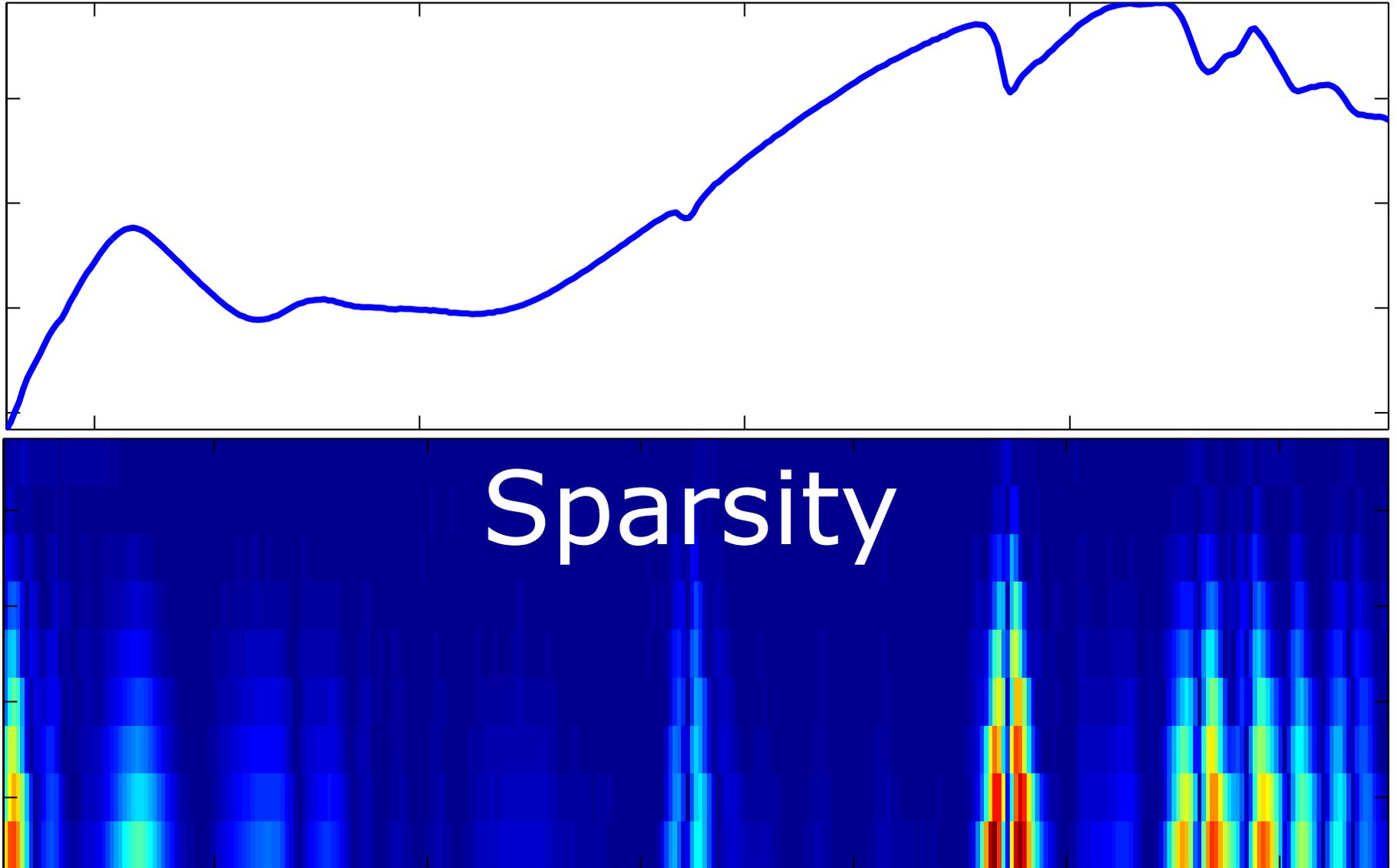


- Organize  $w_{s,u}$  in a 2-D array  $W \in \mathbb{R}^{S \times N}$ : rows are scales, columns are offsets.
- For simplicity, offset  $u = nF/N$  matched to index  $n = 0, 1, \dots, N-1$
- Wavelengths  $\lambda_u$  for indices  $n$  shown
- **Columns** of matrix representation give **chains** of parent/child wavelet coefficients

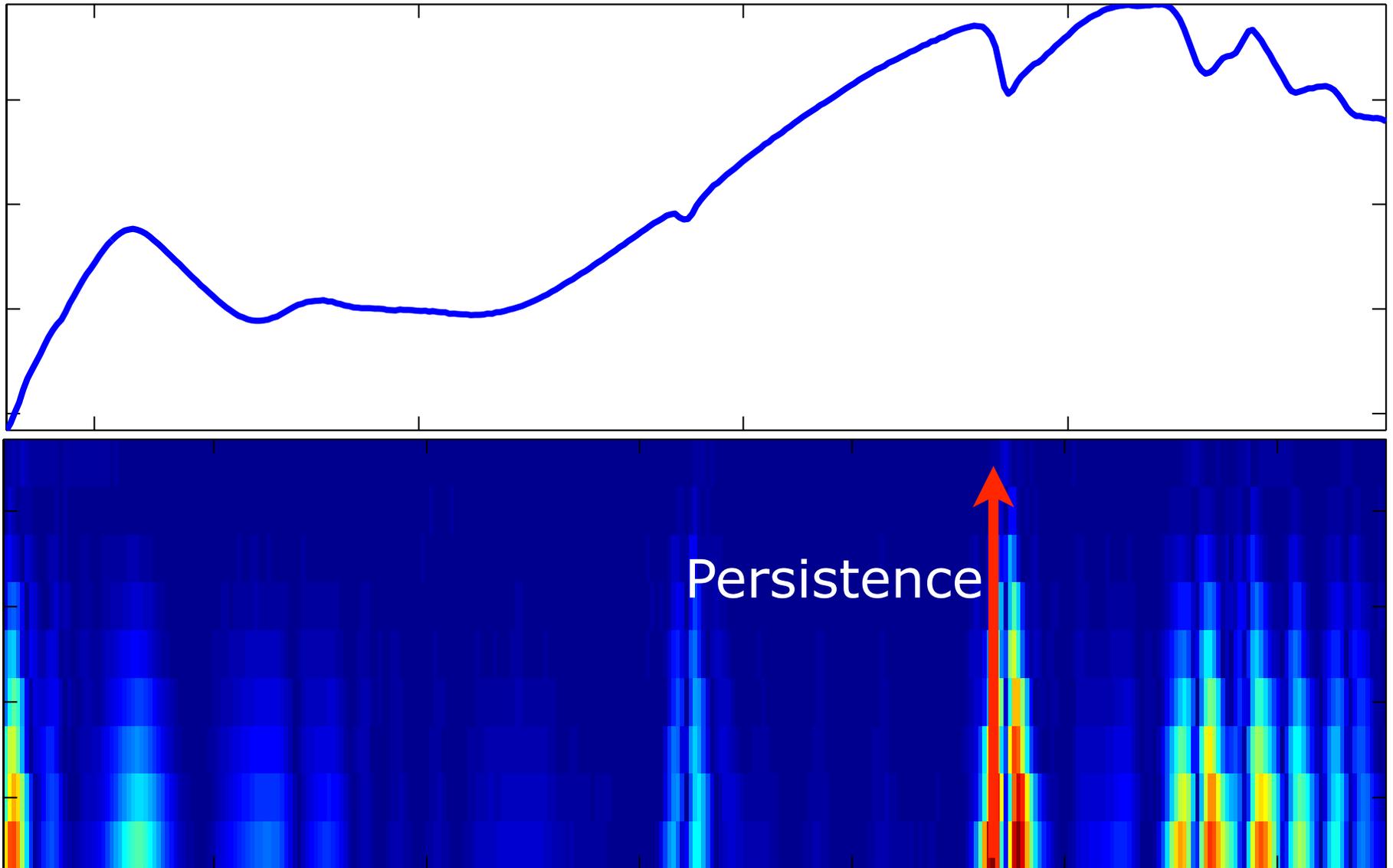
# Structure of CWT Coefficients



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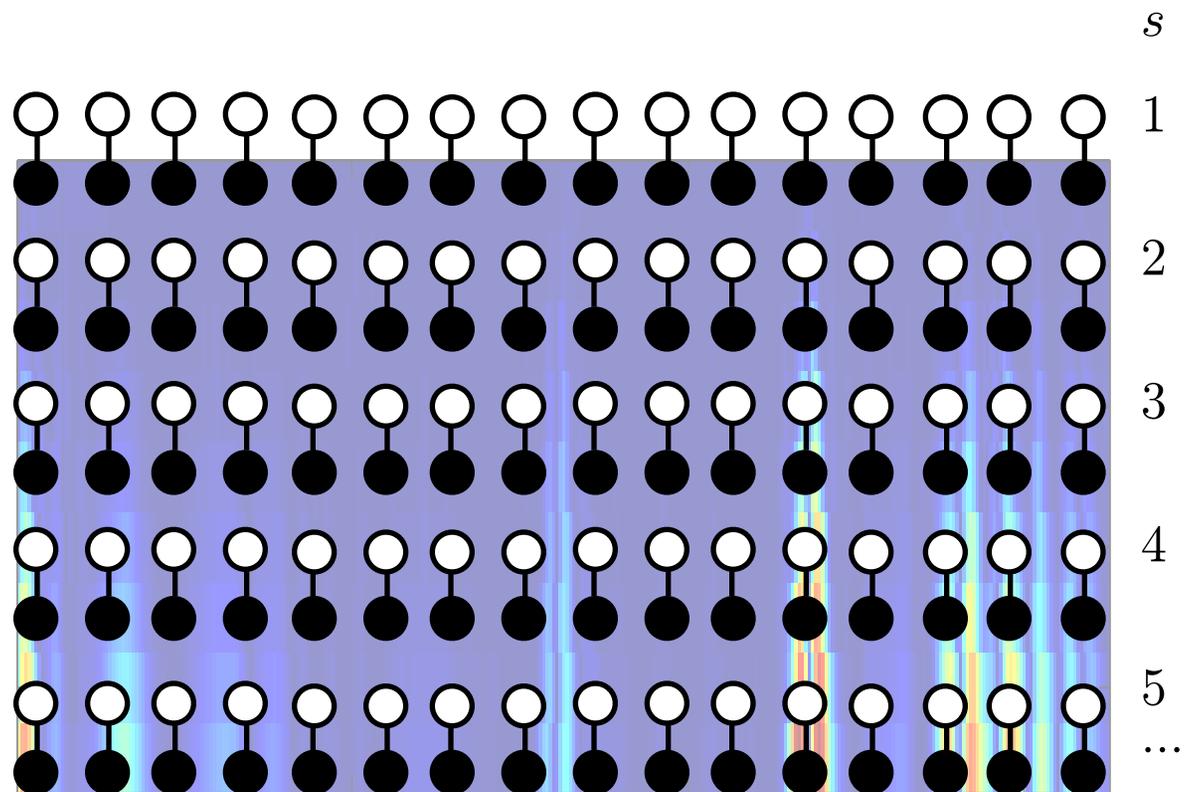
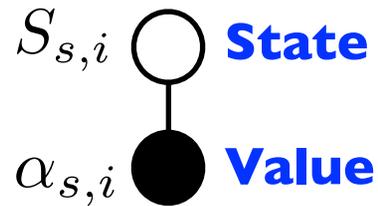


# Structure of CWT Coefficients



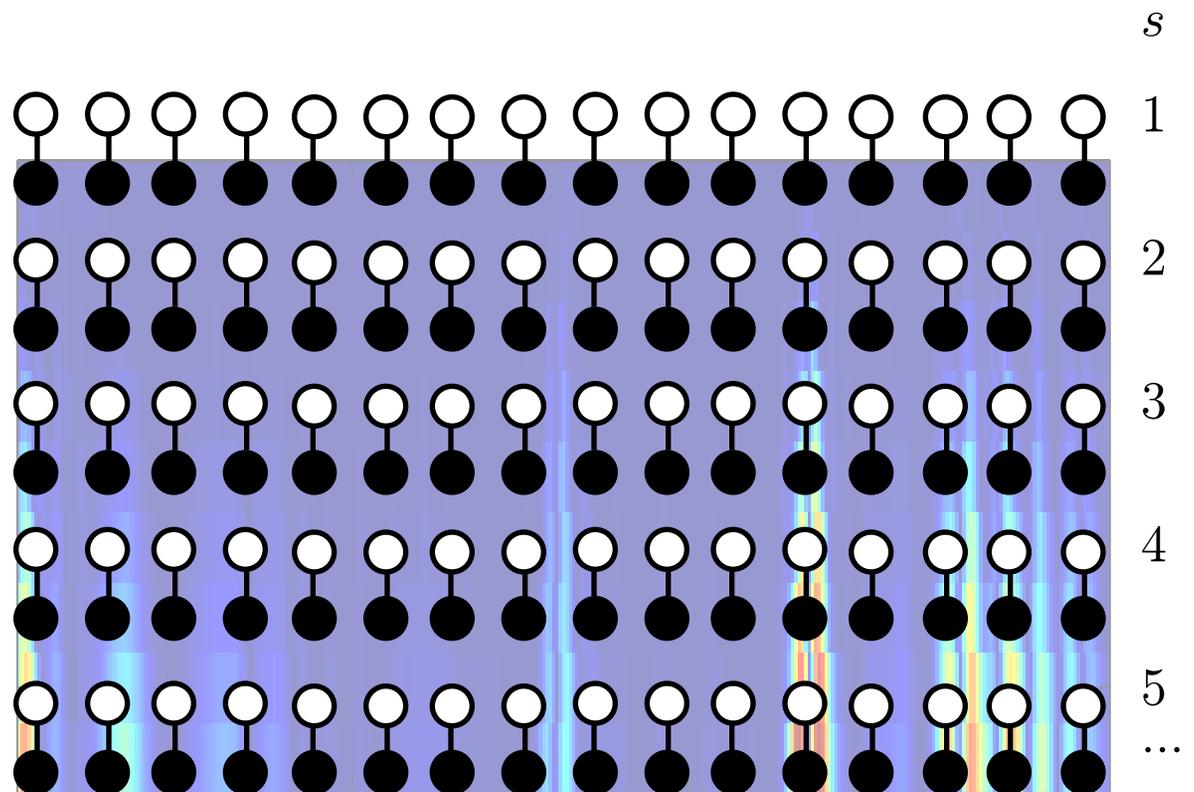
# Non-Homogeneous Hidden Markov Chains

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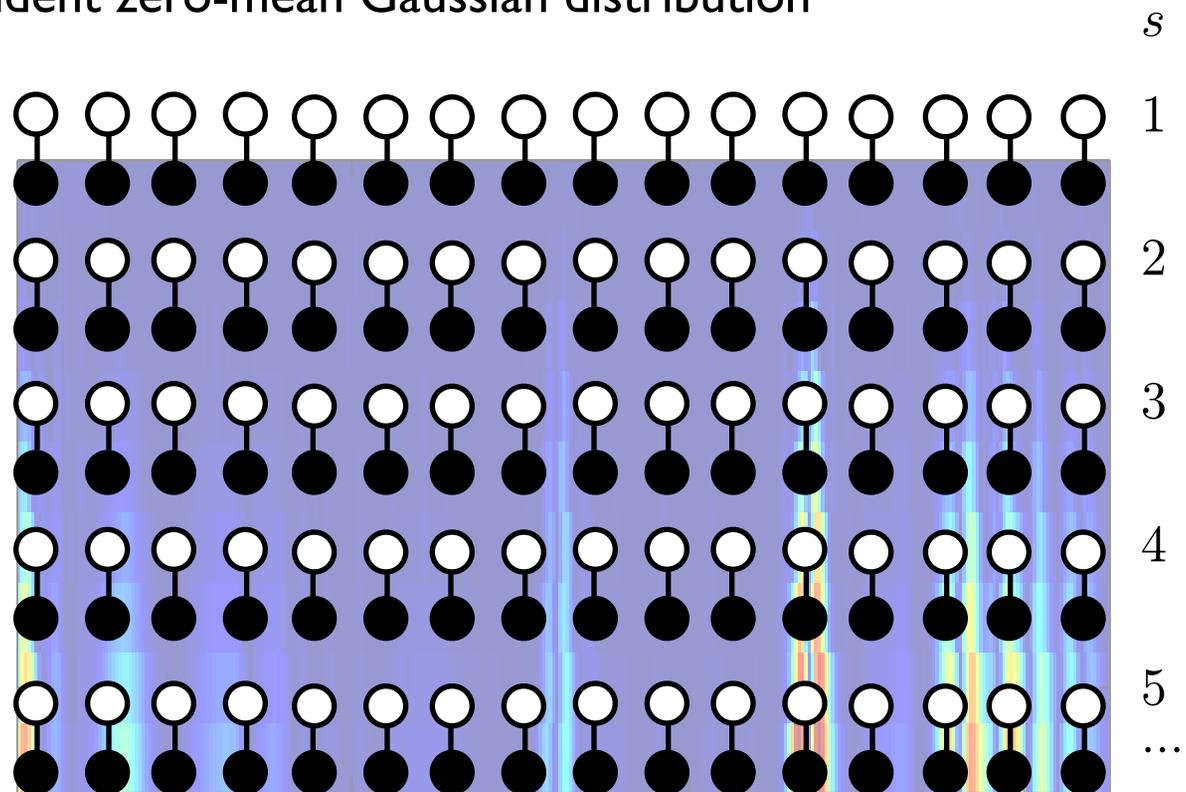
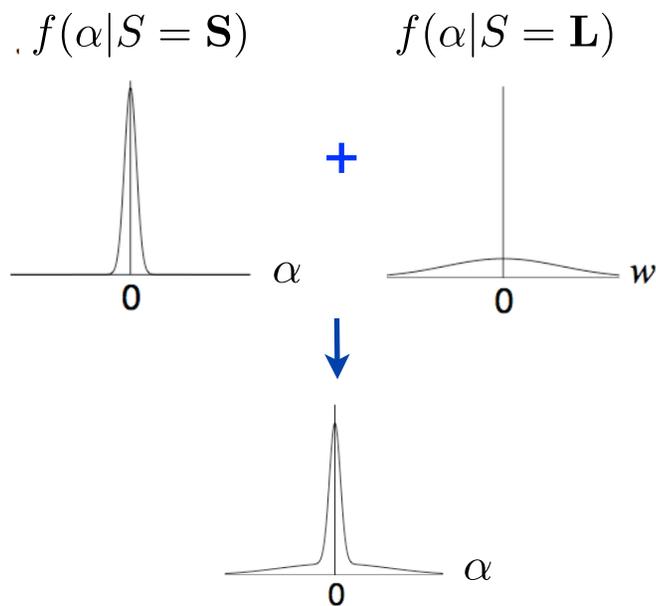


# Non-Homogeneous Hidden Markov Chains

- Stochastic model to encode structure of CWT coefficients

$S_{s,i}$  ○ **State:** Large, Small

$\alpha_{s,i}$  ● **Value:** State-dependent zero-mean Gaussian distribution

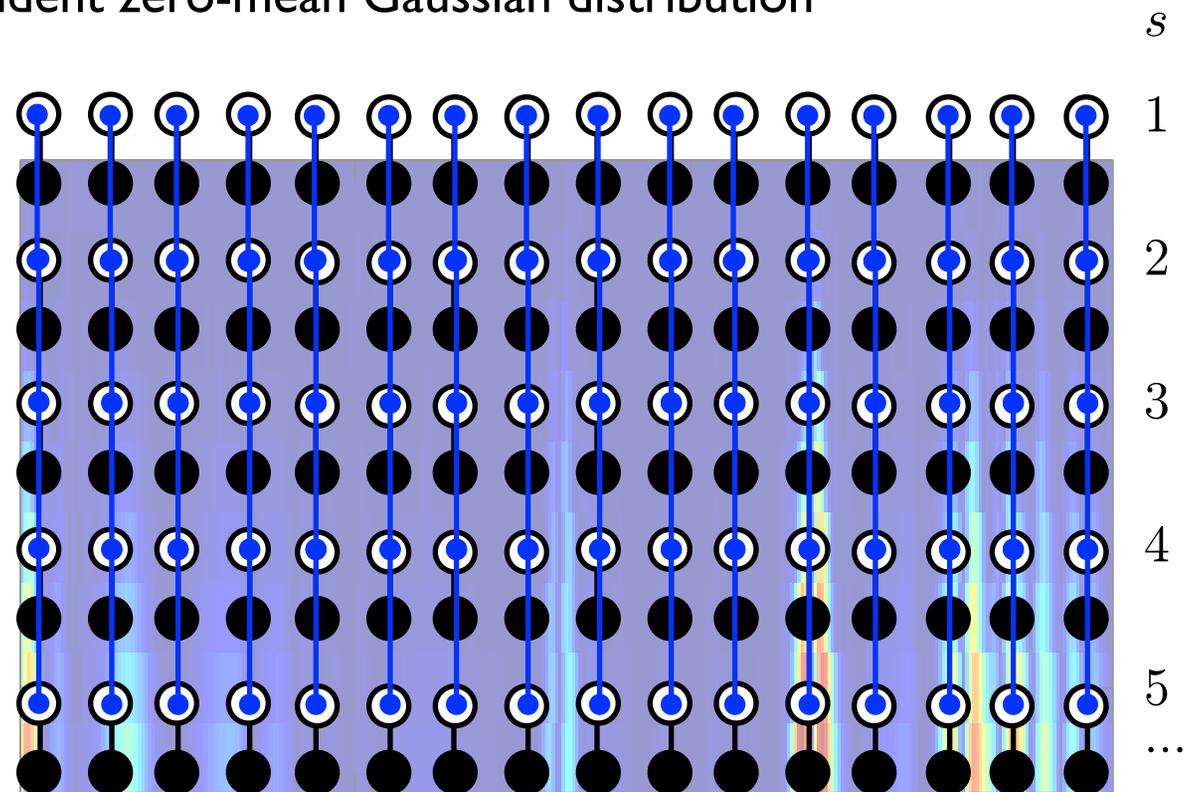
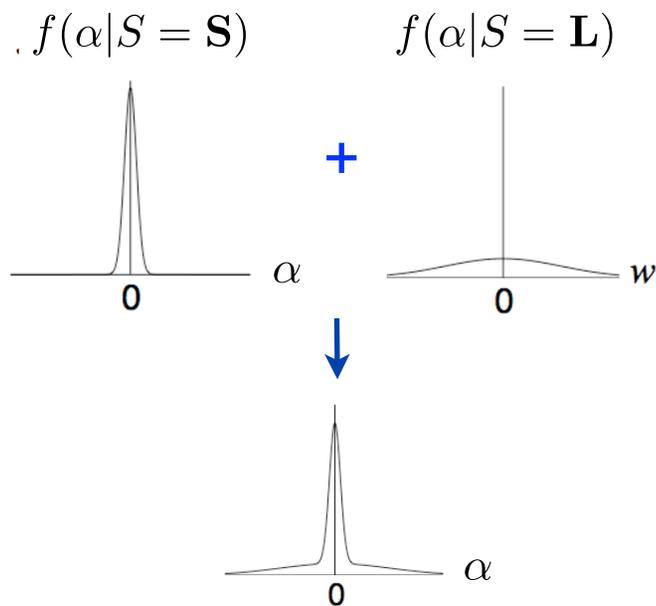


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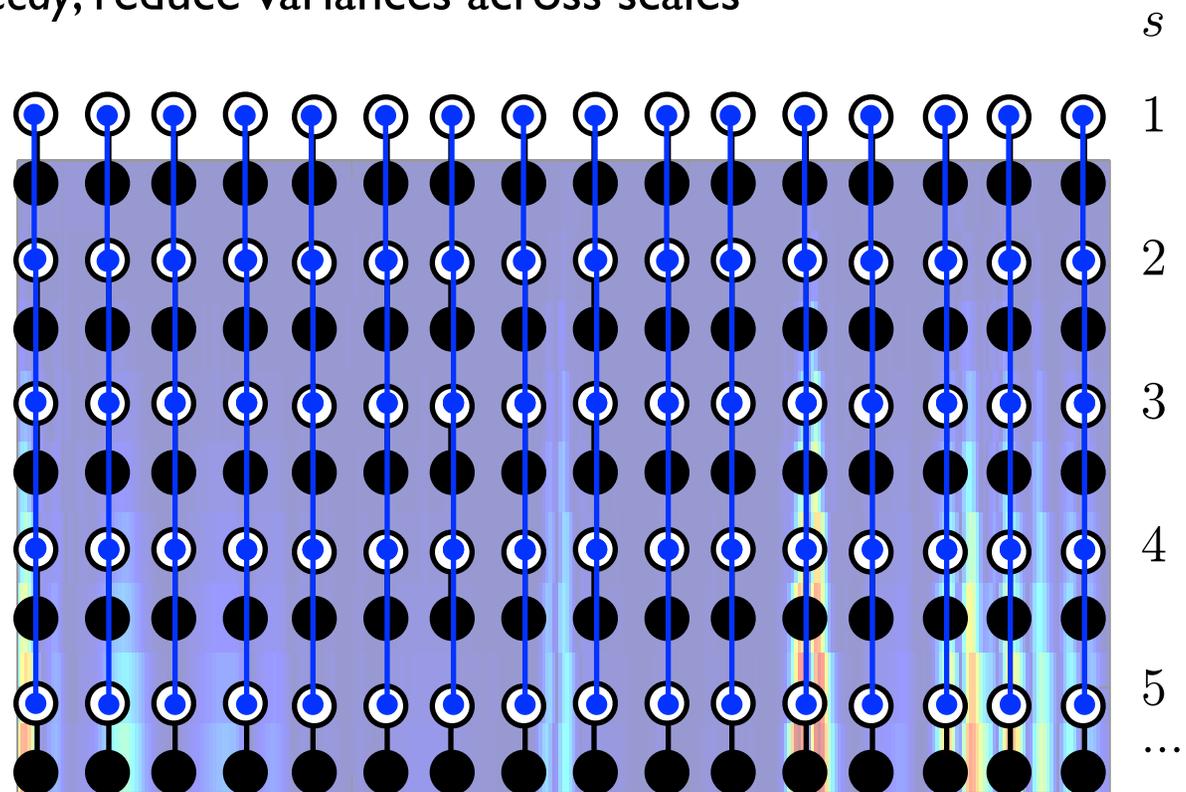
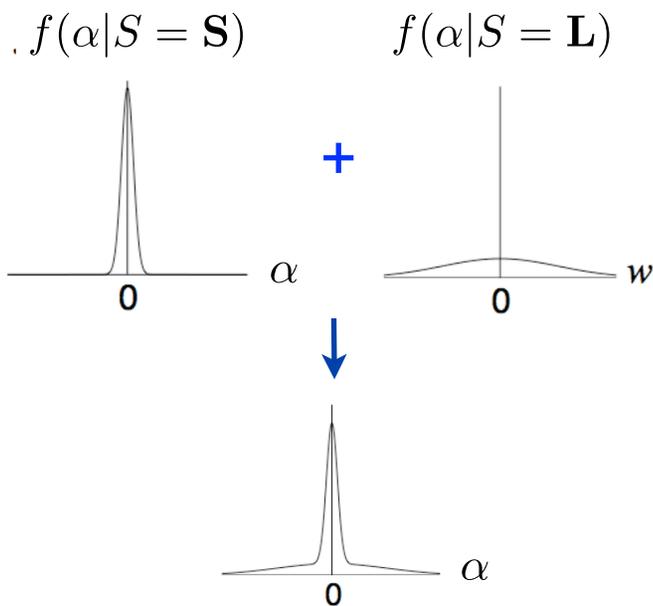
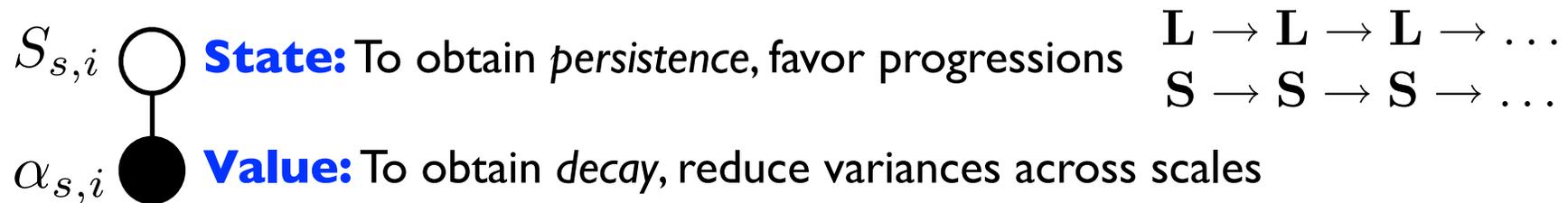
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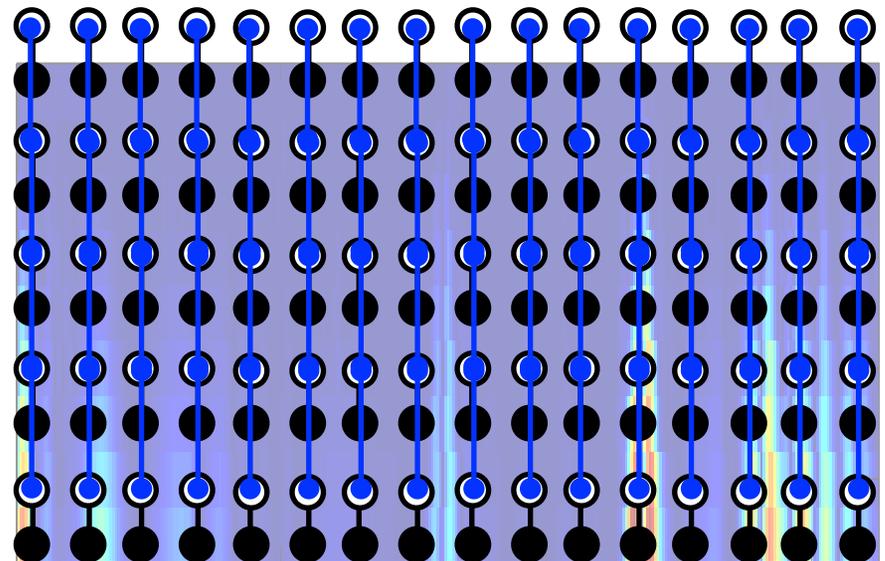
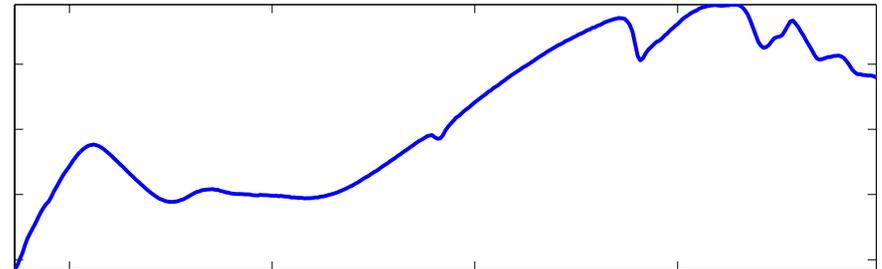
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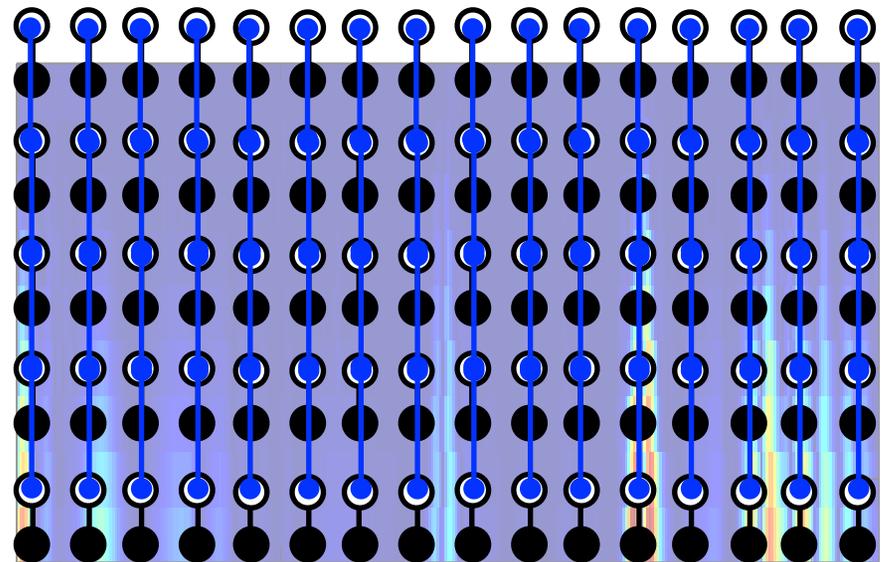
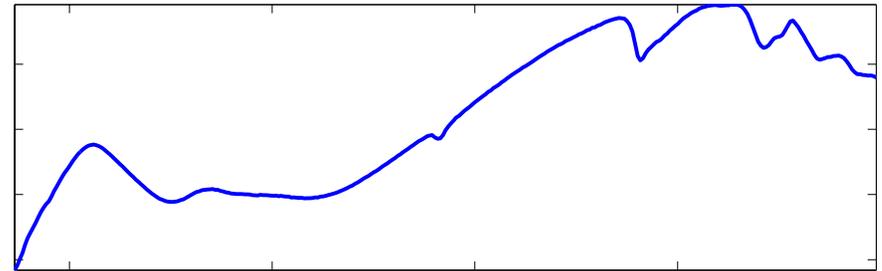
# Modeling Hyperspectral Datasets

- *Why use continuous/undecimated wavelets?*  
So that information at each scale is available for **each wavelength**
- *Why separate chains for each spectra?*  
Because the “size” of a relevant fluctuation is **relative to wavelength** (e.g., absorption bands appearing in all spectra)



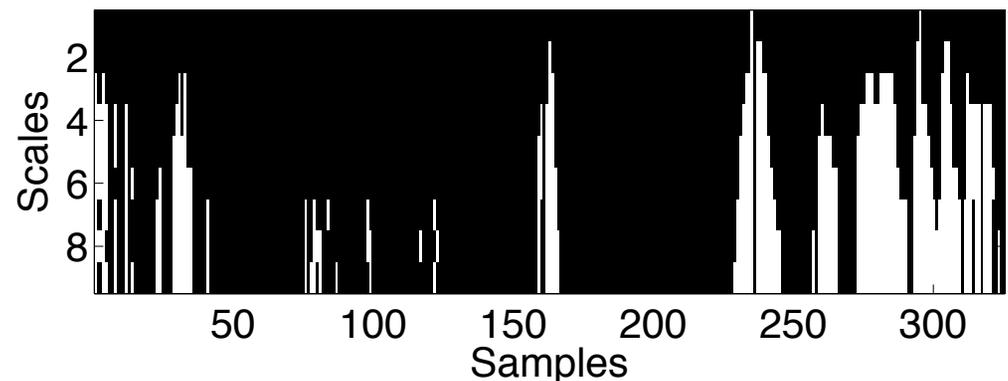
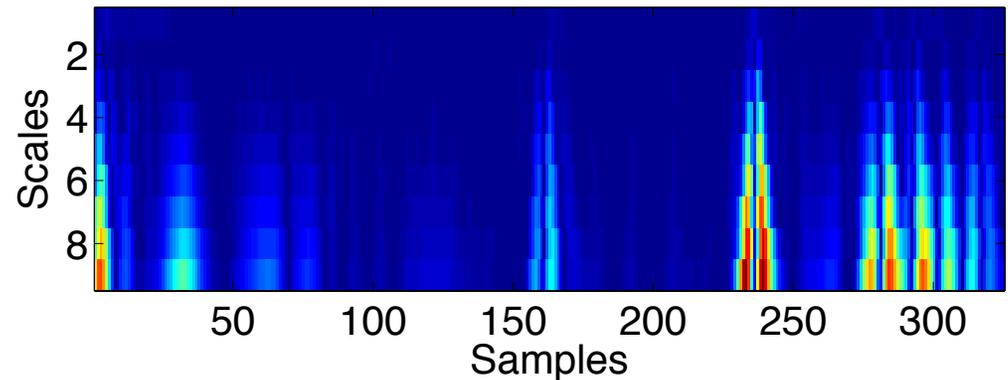
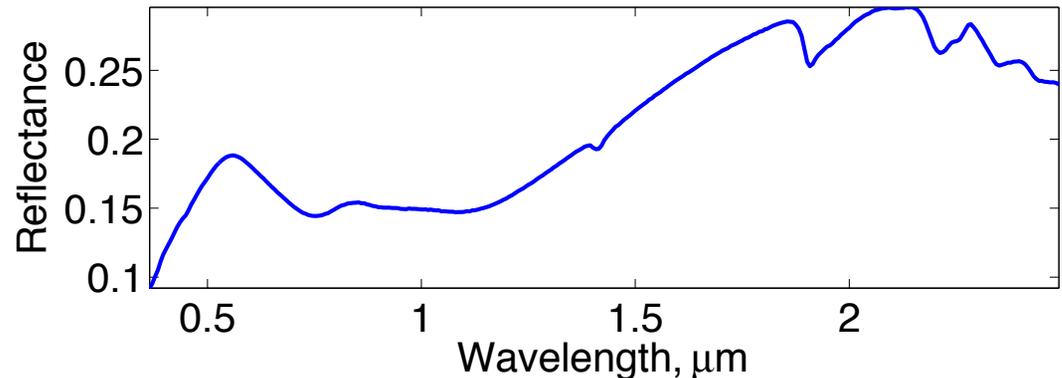
# Modeling Hyperspectral Datasets

- Collect representative (*universal*) library of hyperspectral signatures (e.g. USGS for minerals)
- Extract CWT coefficients for each hyperspectral signature; collect into 2-D array
- Train an NHMC on each of the  $N$  wavelengths (array columns) over the spectral library



# Modeling Hyperspectral Datasets

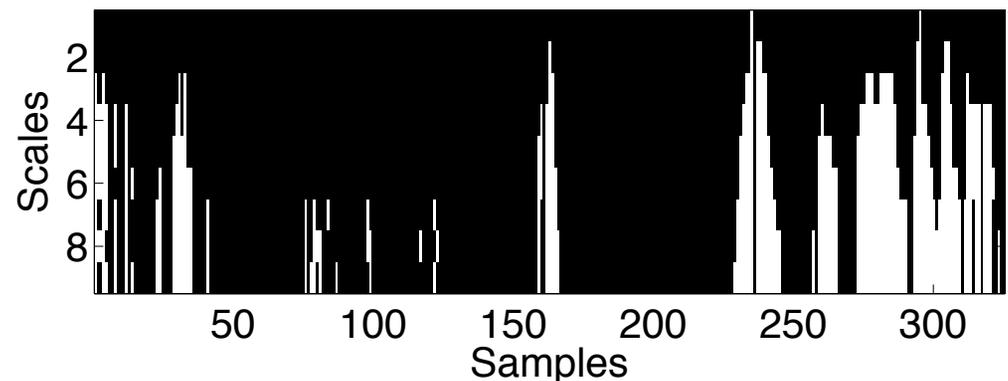
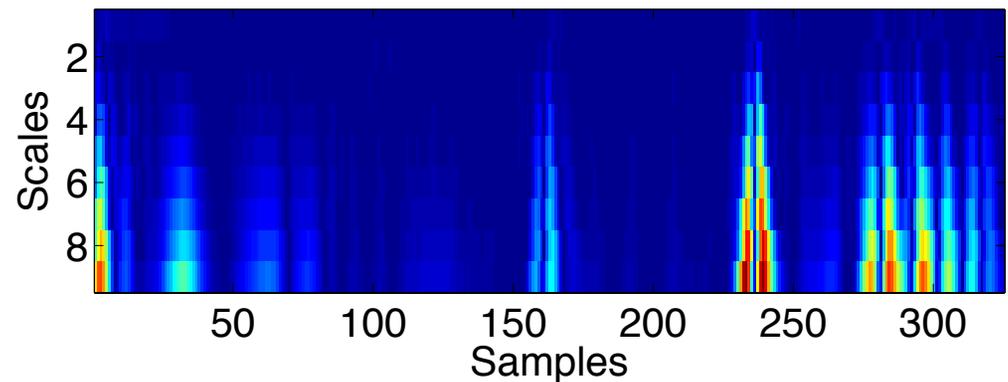
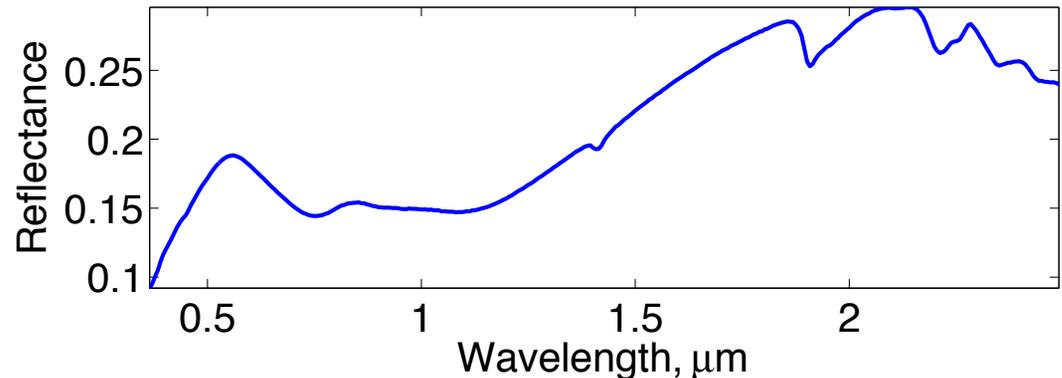
- Using learned NHMC model, generate **state probabilities/labels** for each hyperspectral signature in library
- State labels provide binary information on “**interesting**” parts of the signal
- Use as **features** in hyperspectral signature processing (e.g., classification)





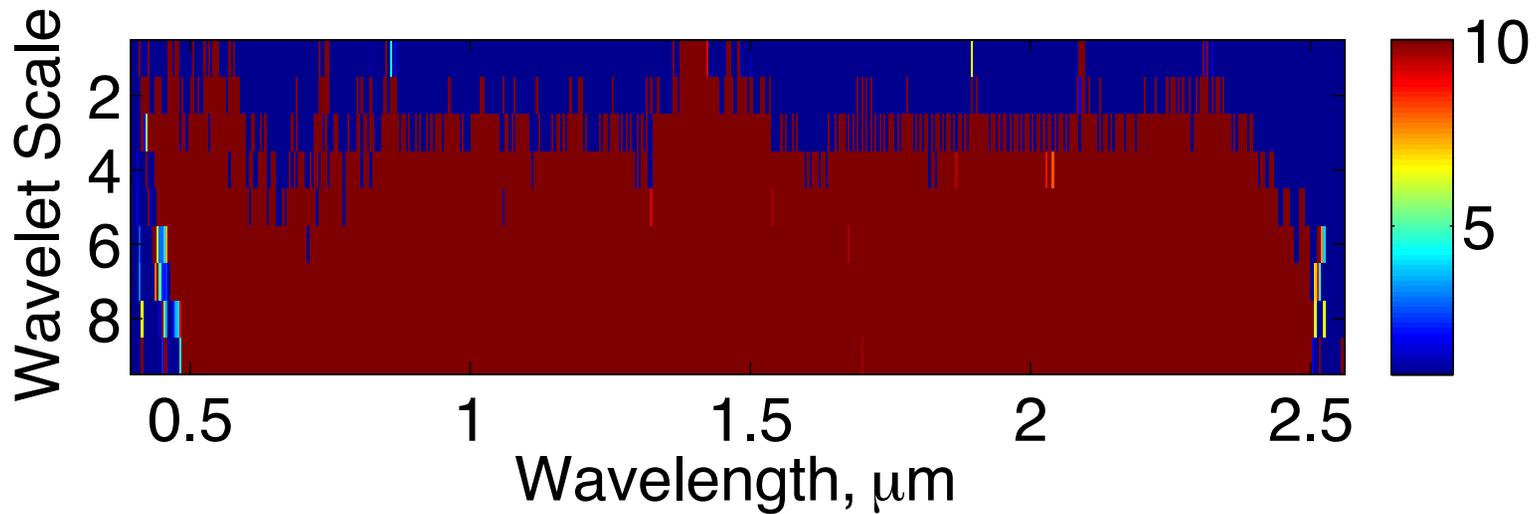
# The Power of "Big Data"

- Statistical modeling of coefficients across spectral sample provides *measures of relevance* of bands/smooth regions
- Model parameters can provide "*map*" of relevant scales, spectral bands, etc. for training dataset

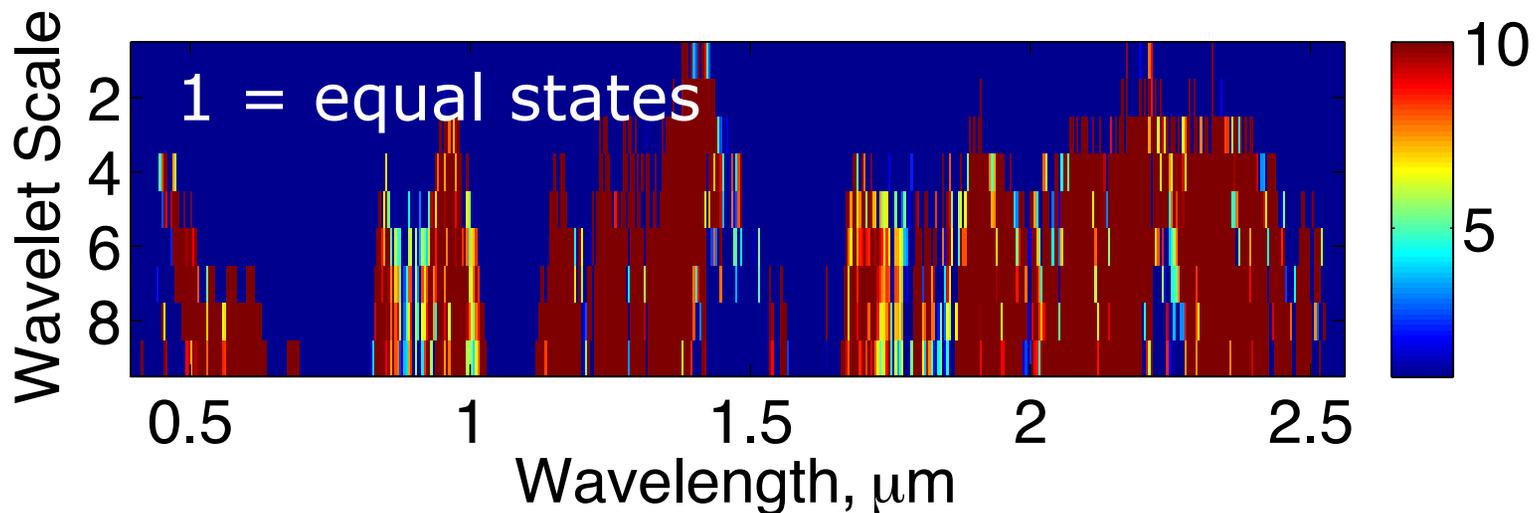


# The Power of "Big Data"

$\sigma_L^2/\sigma_S^2$ , training with all ENVI minerals

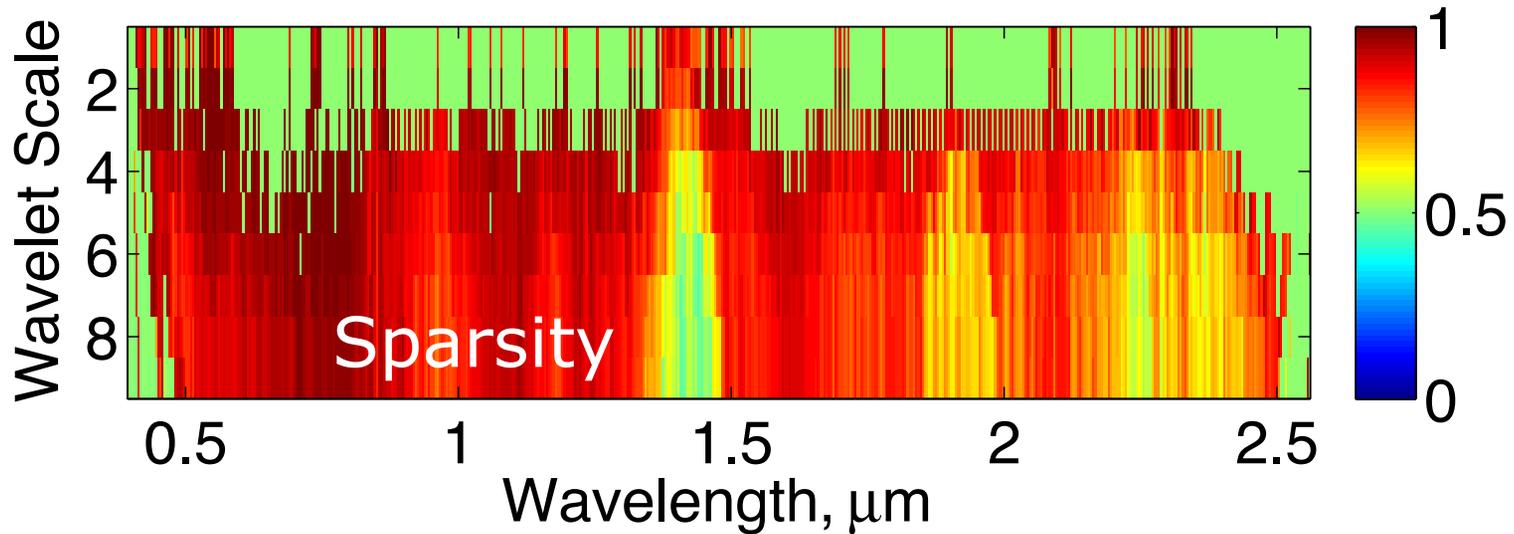


$\sigma_L^2/\sigma_S^2$ , training with ENVI clays only

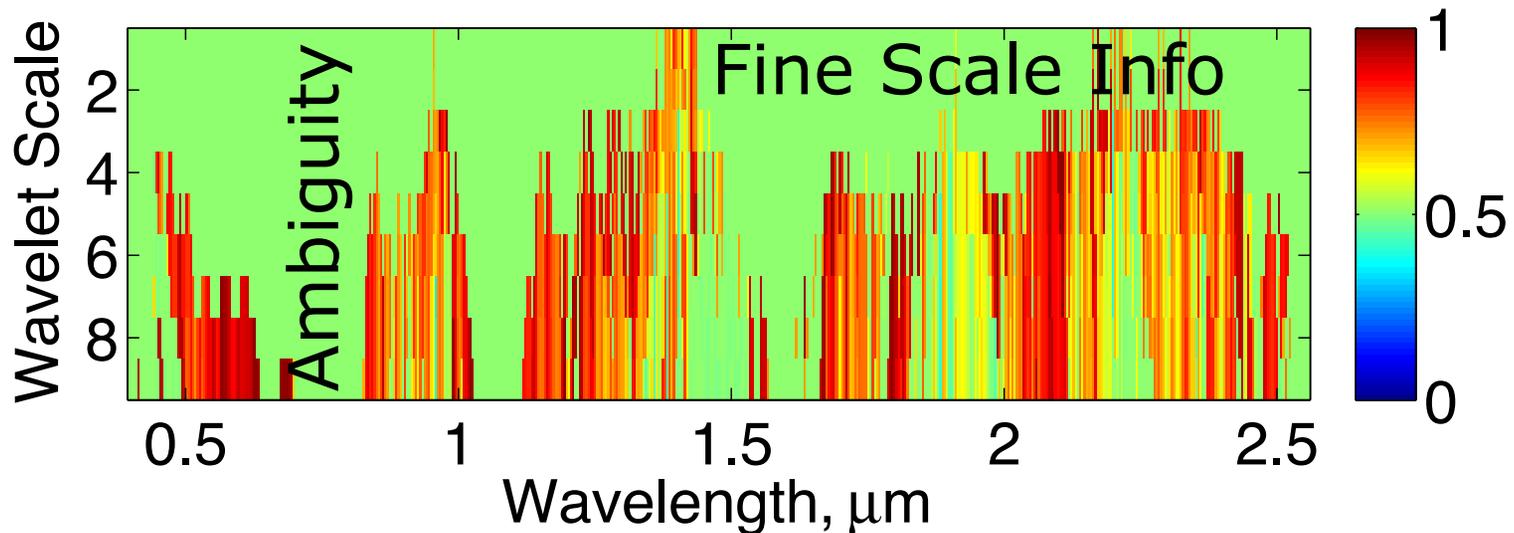


# The Power of "Big Data"

Probability of small state, training with all ENVI minerals

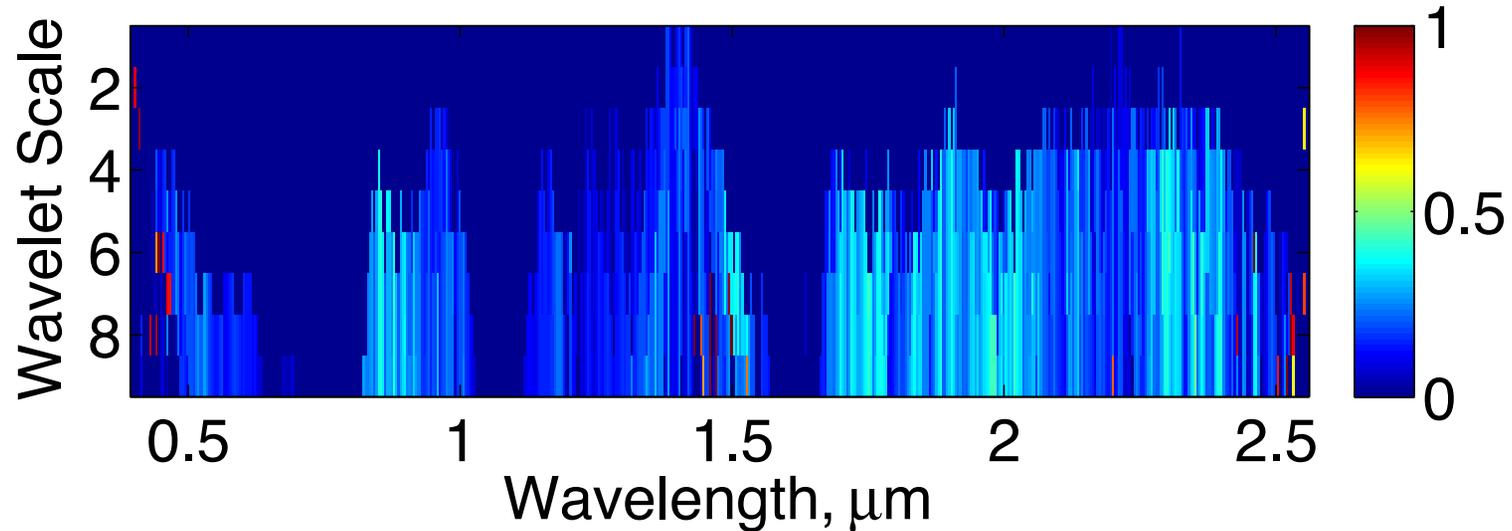


Probability of small state, training with ENVI clays only

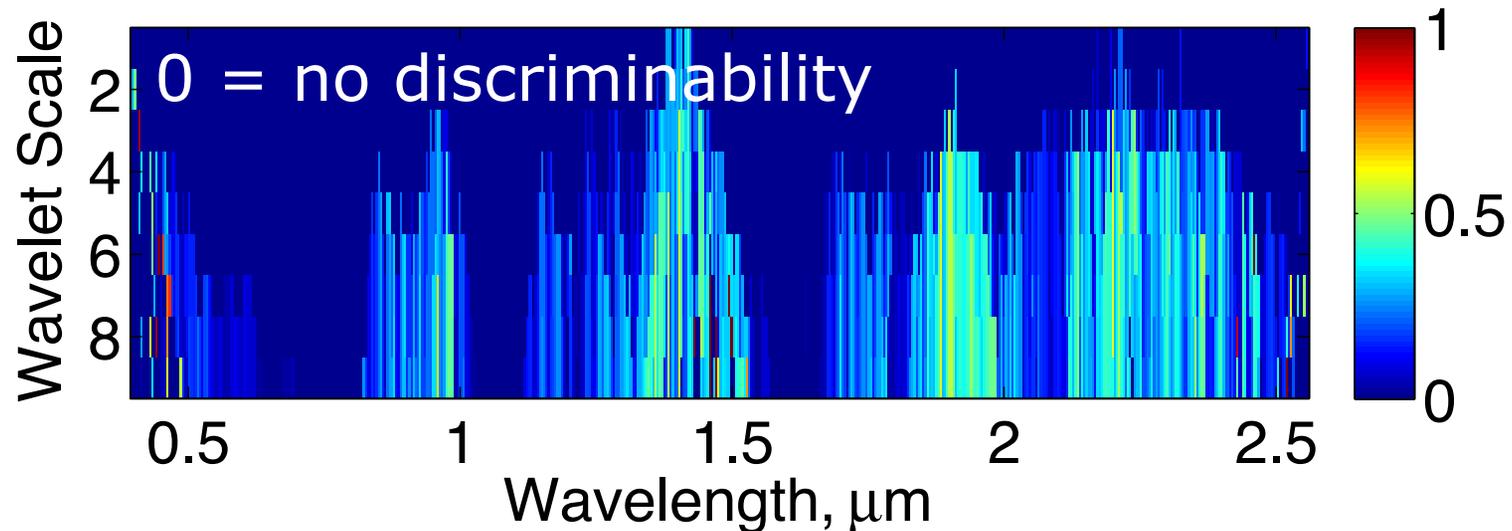


# The Power of "Big Data"

% samples labeled small, training with all ENVI minerals

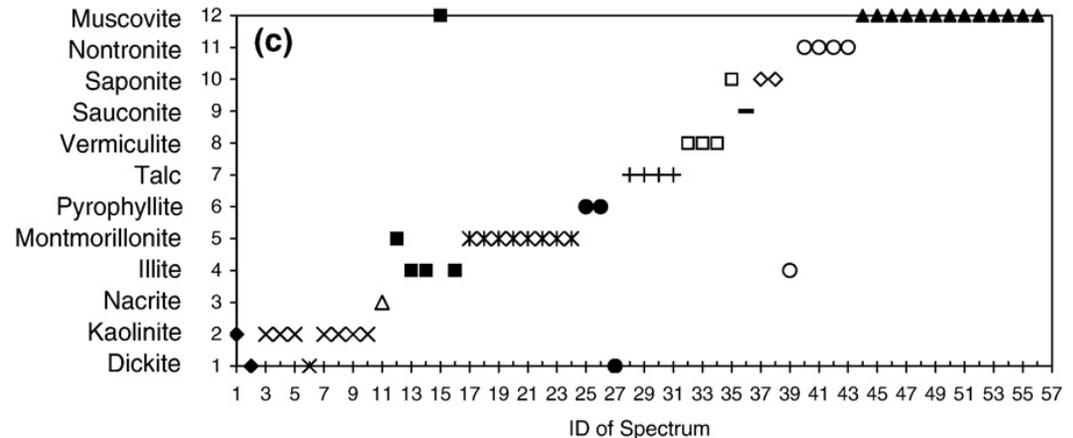


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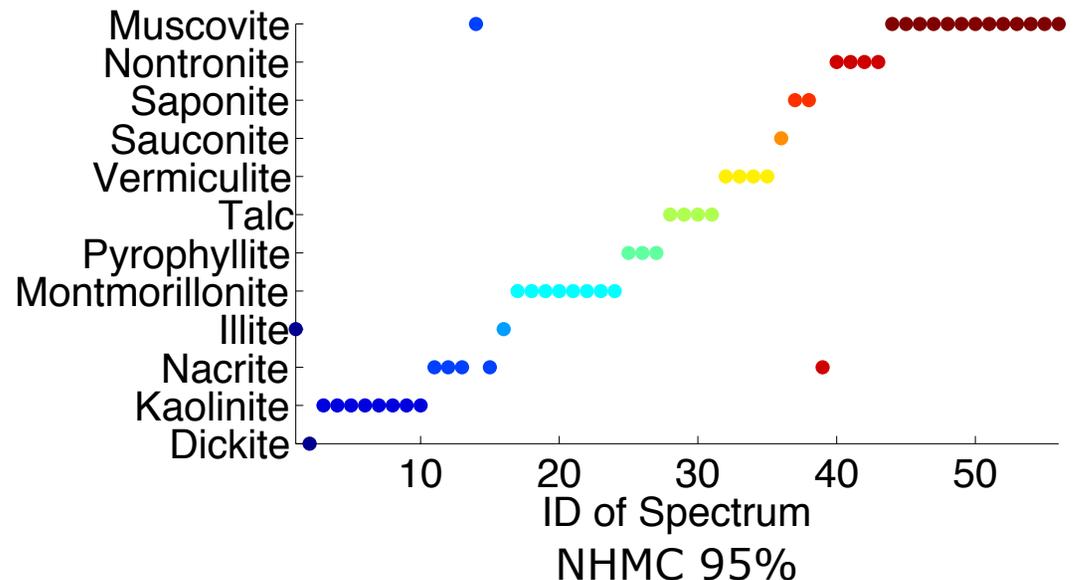


# Example: Mineral Classification

- Same example as before, but *subset of labels* selected according to three “discriminability” criteria
- For all metrics used, classification performance *matches* that obtained with all labels (95% success rate)



[Rivard et al., 2008] 89%



# Conclusions

- Goal: design hyperspectral signal models and features that can capture **semantic information** used by practitioners in remote sensing
  - **relevance** of absorption bands in tasks, e.g., classification
  - multiscale analysis studies a variety of spectral features
  - **robustness** to fluctuations in shape and location of bands
- Stochastic models (Non-Homogeneous Markov Chain) enable **robust** identification of **relevant features**
  - adaptive sampling, spectral sampling rate adjustments
  - identify non-informative absorption bands, universal features
- **Future work:**
  - Hyperspectral image applications: segmentation, unmixing, ...
  - Study robustness to signature fluctuations (lab & field datasets)

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