KRONECKER PRODUCT MATRICES FOR COMPRESSIVE SENSING

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ABSTRACT

Compressive sensing (CS) is an emerging approach for acquisition of signals having a sparse or compressible representation in some basis. While CS literature has mostly focused on problems involving 1-D and 2-D signals, many important applications involve signals that are multidimensional. We propose the use of Kronecker product matrices in CS for two purposes. First, we can use such matrices as sparsifying bases that jointly model the different types of structure present in the signal. Second, the measurement matrices used in distributed measurement settings can be easily expressed as Kronecker products. This new formulation enables the derivation of analytical bounds for sparse approximation and CS recovery of multidimensional signals.

Index Terms— Data compression, multidimensional signal processing, signal reconstruction

1. INTRODUCTION

Compressive sensing (CS) is a new approach to simultaneous sensing and compression that enables a potentially large reduction in the sampling and computation costs at a sensor for a signal $\mathbf{x}$ having a sparse or compressible representation θ in some basis Ψ (i.e. $\mathbf{x} = \Psi\theta$) [2, 3]. The CS literature has mostly focused on problems involving single sensors and one-dimensional (1-D) or 2-D data. However, many important applications that hold the most promise for CS involve signals that are inherently multidimensional. The coordinates of these signals may span several physical, temporal, or spectral dimensions. Examples include hyperspectral imaging (with spatial and spectral dimensions), video acquisition (with spatial and temporal dimensions), and synthetic aperture radar imaging (with progressive acquisition in the spatial dimensions). Another class of promising applications for CS involves distributed networks or arrays of sensors, including for example environmental sensors, microphone arrays, and camera arrays.

Initial work on sparsity and compressibility of multidimensional signals and signal ensembles [4–10] has provided new sparsity and compressibility models for multidimensional signals. These models consider sections of the multidimensional data corresponding to a fixed value for a subset of the coordinates as separate signals; the correlations are defined between the values and locations of their sparse representations. The resulting models are rather limited in the types of structures admitted. For almost all of these models, theoretical guarantees on signal recovery have been provided only for strictly sparse signals, for noiseless measurement settings, or in asymptotic regimes. Additionally, almost all of these models are tied to ad-hoc recovery procedures.

In this paper, we show that *Kronecker product matrices* are a natural way to generate sparsifying and measurement matrices for CS of multidimensional signals, resulting in a formulation that we dub *Kronecker Compressive Sensing* (KCS). When the signal structure along each dimension can be expressed via sparsity, Kronecker product sparsity bases combine the structures for each signal dimension into a single matrix and representation. Similarly, Kronecker product measurement matrices for multidimensional signals can be implemented by performing a sequence of *separate measurements* obtained along each dimension. KCS enables the derivation of analytical bounds for recovery of compressible multidimensional signals from randomized or incoherent measurements.

This paper is organized as follows. Section 2 provides background material. Section 3 studies Kronecker product matrices for CS, and Section 4 studies wavelet-sparse signals as an example. Section 5 provides experimental results and Section 6 provides conclusions.

2. BACKGROUND

Compressive sensing: CS is a efficient signal acquisition framework for signals that are sparse or compressible in an appropriate domain. Let $\mathbf{x} \in \mathbb{R}^N$ be the signal of interest. We say that $\mathbf{x}$ is K-sparse or has sparsity K in a basis or frame Ψ if $\theta = \Psi^T\mathbf{x}$ obeys $\|\theta\|_0 = K$, with $K \ll N$ and Ψ^T denoting the transpose matrix of Ψ. Here, $\|\cdot\|_0$ denotes the ℓ_0 norm, which simply counts the number of nonzero entries in the vector. Similarly, we say that $\mathbf{x}$ is s-compressible in Ψ if the vector $\widetilde{\theta}$, containing the entries of θ sorted by absolute value, has entries with magnitudes that decay according to a power law $\left|\widetilde{\theta}(n)\right| \le Cn^{-s-1/2}$, for all $n = 1, \dots, N$, where $C < \infty$. Such vectors can be compressed by preserving only the coefficients with largest absolute magnitude.

The CS acquisition procedure consists of measuring the product of the signal against a measurement matrix $\Phi \in \mathbb{R}^{M\times N}$; the acquisiton procedure can be written as $\mathbf{y} = \Phi\mathbf{x} + \mathbf{n} = \Phi\Psi\theta + \mathbf{n}$, with the vector $\mathbf{y} \in \mathbb{R}^M$ containing the CS measurements and the vector $\mathbf{n}$ denoting the noise introduced in the measurement process. When the signal being observed is sparse enough, it can be estimated by solving

$$\widehat{\theta} = \arg\min \|\theta\|_1 \text{ s.t. } \|\mathbf{y} - \Phi\Psi\theta\|_2 \le \epsilon, \quad (1)$$

where ϵ is an upper bound on the Euclidean norm of the noise vector $\mathbf{n}$. In this case, $\|\cdot\|_1$ denotes the ℓ_1 norm, which is equal to the sum of the absolute values of the vector entries.

In particular cases, we do not have freedom to choose a measurement matrix to apply. Under this type of setup, we can assume

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that a basis $\Phi \in \mathbb{R}^{N\times N}$ is provided for measurement purposes, and we have the option to choose a subset of the signal's coefficients in this transform as measurements. That is, we let $\overline{\Phi}$ be an $N \times M$ submatrix of Φ that preserves the basis vectors with indices Γ and $\mathbf{y} = \overline{\Phi}^T \mathbf{x}$. In this case there exists a different metric that evaluates the performance of CS: the *mutual coherence* of the orthonormal bases $\Phi \in \mathbb{R}^{N\times N}$ and $\Psi \in \mathbb{R}^{N\times N}$ is the maximum absolute value for the inner product between elements of the two bases

$$\mu(\Phi, \Psi) = \max_{1\le i,j\le N} |\langle \phi_i, \psi_j\rangle|,$$

with ϕ_i and ψ_i denoting the i^{th} column of Φ and Ψ, respectively. The mutual coherence determines the value of M necessary for accurate recovery: if

$$M \ge CKN\mu^2(\Phi, \Psi)\log(N/\delta),$$

then with probability at least $1 - \delta$, θ is the solution to (1) [2]. Since $\mu(\Phi, \Psi) \in [N^{-1/2}, 1]$, the number of measurements required ranges from $O(K \log(N))$ to $O(N)$.

Kronecker products: The *Kronecker product* of two matrices A and B of sizes $P \times Q$ and $R \times S$, respectively, is defined as

$$A \otimes B := \begin{bmatrix} A(1,1)B & A(1,2)B & \ldots & A(1,Q)B \\ A(2,1)B & A(2,2)B & \ldots & A(2,Q)B \\ \vdots & \vdots & \ddots & \vdots \\ A(P,1)B & A(P,2)B & \ldots & A(P,Q)B \end{bmatrix}.$$

Thus, $A \otimes B$ is a matrix of size $PR \times QS$. Let Ψ_1 and Ψ_2 be bases for $\mathbb{R}^{N_1}$ and $\mathbb{R}^{N_2}$, respectively. Then one can find a basis for $\mathbb{R}^{N_1} \otimes \mathbb{R}^{N_2} \cong \mathbb{R}^{N_1N_2}$ as $\widetilde{\Psi} = \Psi_1 \otimes \Psi_2$.

Signal ensembles: In distributed sensing problems, we aim to acquire an ensemble of signals $\mathbf{x}_1, \ldots, \mathbf{x}_J \in \mathbb{R}^N$ that vary in time, space, etc. We assume that each signal's structure can be encoded using sparsity with an appropriate basis Ψ'. This ensemble of signals can be expressed as a $N \times J$ matrix $\mathbf{X} = [\mathbf{x}_1\ \mathbf{x}_2\ \ldots\ \mathbf{x}_J] = [\mathbf{x}^{1T}\ \mathbf{x}^{2T}\ \ldots\ \mathbf{x}^{NT}]^T$, where the individual signals $\mathbf{x}_1, \ldots, \mathbf{x}_j$ corresponding to columns of the matrix, and where the rows $\mathbf{x}^1, \ldots, \mathbf{x}^N$ of the matrix correspond to values of the signal ensembles at a given time, location, etc; thus, signal ensembles can also be posed as multidimensional signals.

3. KRONECKER PRODUCT MATRICES FOR MULTIDIMENSIONAL COMPRESSIVE SENSING

In this section, we describe our framework for the use of Kronecker product matrices in multidimensional CS. We call the restriction of a multidimensional signal to fixed indices for all but its d^{th} dimension a d-section of the signal. For example, for a 3-D signal $\mathbf{x} \in \mathbb{R}^{N_1\times N_2\times N_3}$, the subset $\mathbf{x}_{i,j,\cdot} := [\mathbf{x}(i,j,1)\ \mathbf{x}(i,j,2)\ \ldots\ \mathbf{x}(i,j,N_3)]$ is a 3-section of the signal $\mathbf{x}$.

Kronecker product sparsity bases: It is possible to simultaneously exploit the sparsity properties of a multidimensional signal along each of its dimensions to provide a new representation for their structure. We obtain a single sparsity basis for the entire multidimensional signal as the Kronecker product of the bases used for each of its d-sections. For multidimensional signals, this encodes all of the available structure using a single transformation. More formally, we let $\mathbf{x} \in \mathbb{R}^{N_1} \otimes \ldots \otimes \mathbb{R}^{N_D} = \mathbb{R}^{N_1\times\ldots\times N_D} \cong \mathbb{R}^{\prod_{d=1}^D N_d}$ and assume that each d-section is sparse or compressible in a basis Ψ_d. We then pose a sparsity/compressibility basis for $\mathbf{X}$ obtained from Kronecker products as $\overline{\Psi} = \Psi_1 \otimes \ldots \otimes \Psi_D$, and obtain a coefficient vector Θ for the signal ensemble so that $\overline{\mathbf{X}} = \overline{\Psi}\Theta$, where $\overline{\mathbf{X}}$ is a vector-reshaped representation of $\mathbf{X}$.

Kronecker product measurement matrices: We can also design measurement matrices that are Kronecker products; such matrices correspond to measurement processes that operate individually on a single d-section of the multidimensional signal. The resulting measurement matrix can be expressed as $\widetilde{\Phi} = \Phi_1 \otimes \ldots \otimes \Phi_D$. As an example with $D = 2$, if we have $\Phi_1 \in \mathbb{R}^{M_1\times N}$ and $\Phi_2 \in \mathbb{R}^{M_2\times J}$, we obtain $\Phi \in \mathbb{R}^{M_1M_2\times NJ}$. This results in a matrix that provides $M = M_1M_2$ measurements of the 2-D signal $\mathbf{X}$.

Consider the example of a signal ensemble where we obtain *distributed measurements*, in the sense that each measurement depends on only one of the signals. More formally, for each signal (or 1-section) $\mathbf{x}_{\cdot,j}$, $1 \le j \le J$ we obtain independent measurements $\mathbf{y}_j = \Phi_j\mathbf{x}_{\cdot,j}$ with an individual measurement matrix being applied to each 1-section. The structure of such measurements is often succinctly captured by Kronecker products. To compactly represent the signal and measurement ensembles, we denote $\mathbf{Y} = [\mathbf{y}_1^T\ \ldots \mathbf{y}_J^T]^T$ and $\widetilde{\Phi} = \mathrm{diag}(\Phi_1, \ldots, \Phi_J)$, where diag denotes a block-diagonal matrix with corresponding block entries. We then have $\mathbf{Y} = \widetilde{\Phi}\overline{\mathbf{X}}$. If a matrix $\Phi_j = \Phi'$ is used at each sensor to obtain its individual measurements, then we can express the joint measurement matrix as $\widetilde{\Phi} = \mathbf{I}_J \otimes \Phi'$, where $\mathbf{I}_J$ denotes the $J \times J$ identity matrix.

Mutual coherence for Kronecker product matrices: We now derive results for the coherence metric described in Section 2 applied to Kronecker product sparsifying and measurement matrices. This metric will determine the suitability of KCS for signal recovery. The following lemma provides a *conservation of mutual coherence* across Kronecker products.

Lemma 3.1. *[11, 12] Let Φ_d, Ψ_d be bases or frames for $\mathbb{R}^{N_d}$ for $d = 1, \ldots, D$. Then*

$$\mu(\Phi_1 \otimes \ldots \otimes \Phi_D, \Psi_1 \otimes \ldots \otimes \Psi_D) = \prod_{d=1}^{D} \mu(\Phi_d, \Psi_d).$$

Since the mutual coherence of each d-section's sparsity and measurement bases is upper bounded by one, the number of Kronecker product-based measurements necessary for successful recovery of the multidimensional signal is always lower than or equal to the corresponding number of necessary *partitioned measurements* that process only a portion of the multidimensional signal along its d^{th} dimension at a time, for some $d \in \{1, \ldots, D\}$. This reduction is maximized when the d-section measurement basis is Φ maximally incoherent with the d-section sparsity basis Ψ.

4. CS WITH MULTIDIMENSIONAL WAVELET BASES

Kronecker products are prevalent in the extension of wavelet transforms to multidimensional settings. There are several different wavelet basis constructions depending on the choice of basis elements involved in the Kronecker products. For these constructions, our interest is in the relationship between the compressibility of each d-section in a wavelet component basis and the compressibility of the multidimensional signal in the wavelet Kronecker product basis.

Multidimensional wavelets: Several different extensions exist for the construction of D-D wavelet basis elements as a Kronecker product of 1-D wavelets [1]. In each case, a D-D wavelet basis element is obtained from the Kronecker product of D 1-D wavelet basis elements:

$$\psi_{i_1,j_1,\ldots,i_D,j_D} = \psi_{i_1,j_1} \otimes \ldots \otimes \psi_{i_D,j_D}.$$

Many multidimensional bases can then be obtained through the use of appropriate combinations of 1-D wavelets in the Kronecker product. For example, *isotropic wavelets* arise when the same scale $j = j_1 = \ldots = j_D$ is selected for all wavelets involved, while *anisotropic wavelets* force a fixed factor between any two scales, i.e. $a_{d,d'} = j_d/j_{d'}$, $1 \le d, d' \le D$. Additionally, *hyperbolic wavelets* result when no restriction is placed on the scales $j_1, \ldots, j_D$. Therefore, the hyperbolic wavelet basis is obtained as the Kronecker product of the individual wavelet bases [1]. We denote the isotropic, anisotropic, and hyperbolic wavelet bases by Ψ_I, Ψ_A, and Ψ_H, respectively.

Besov spaces: A Besov space $B^s_{p,q}$ contains D-D signals that have (roughly speaking) s derivatives in $L_p(\Omega^D)$ in all directions; the parameter q provides finer distinctions of smoothness. One example is the class of natural images [13]. Signals that are in $B^s_{p,q}$ are s-compressible in a sufficiently smooth isotropic wavelet basis.

In applications other than natural image processing, such as PDEs, the type of structure present is different in each of the signal's dimensions. In these cases, anisotropic and hyperbolic wavelets can be used to achieve sparse and compressible representations for signals of this type. An anisotropic Besov space $B^{\overline{s}}_{p,q}$, where $\overline{s} = \{s_1, \ldots, s_D\}$, contains D-D signals that have (roughly speaking) s_d derivatives in $L_p(\Omega)$ for any d-section of the D-D function. Similarly to isotropic Besov spaces, signals in $B^{\overline{s}}_{p,q}$ are λ-compressible in a suitable anisotropic or hyperbolic wavelet basis, with $\lambda = D/\sum_{d=1}^{D} 1/s_d$ [1]. In contrast, such signals are ρ-compressible in a sufficiently smooth isotropic wavelet basis, with $\rho = \min_{1 \le d \le D} s_d$ [1].

The disadvantage of anisotropic wavelets, as compared with hyperbolic wavelets, is that they must have smoothness ratios between the dimensions that match that of the signal in order to achieve the optimal approximation rate [1]. Additionally, the hyperbolic wavelet basis is the only one out of the three basis types described that can be expressed as the Kronecker product of lower dimensional wavelet bases. Therefore, we use hyperbolic wavelets in the sequel and in the experiments of Section 5.

Performance of CS recovery: When Kronecker product matrices are used for measurement and transform coding of compressible signals, it is possible to compare the rates of approximation that can be obtained by using independent measurements of each signal snapshot (or signal). The following Theorem is proven in [12].

Theorem 4.1. *Assume that a D-D signal* $\mathbf{X} \in \mathbb{R}^{N_1 \times \ldots \times N_D}$ *is in* $B^{\overline{s}}_{p,q}$. *That is,* $\mathbf{X}$ *has* s_d*-compressible* d*-sections in sufficiently smooth wavelet bases* Ψ_d, $1 \le d \le D$. *Denote by* Φ_d, $1 \le d \le D$ *a set of measurement matrices that can be applied along each dimension of* $\mathbf{X}$. *If* M *measurements are obtained using a random subset of the columns of* $\Phi_1 \otimes \ldots \otimes \Phi_D$, *then with high probability the recovery error from these measurements has the property*

$$\|\mathbf{X} - \widehat{\mathbf{X}}\|_2 \le CM^{-\beta} \prod_{d=1}^{D} \mu(\Phi_d, \Psi_d)^{\beta},$$

where $\beta = \lambda - 1/4$, *while the recovery from* M *measurements equally distributed among sections of the signal in the* d^{th} *dimension has the property*

$$\|\mathbf{X} - \widehat{\mathbf{X}}\|_2 \le CM^{-\gamma_d} \mu(\Phi_d, \Psi_d)^{\gamma_d},$$

for $d = 1, \ldots, D$, *where* $\gamma_d = s_d/2 - 1/4$.

To summarize the theorem, as the number of measurements increases, the recovery error decay rate matches that of the signal's compressibility approximation error; however, there is an additional factor dependent on the inverse of the mutual coherences that affects the decay with the same exponential rate of decay. To put Theorem 4.1 in perspective, we consider the isotropic and extreme anisotropic cases. In the anisotropic case, when $s_d = \overline{s}$, $1 \le d \le D$, all approaches provide the same CS recovery approximation rate, i.e., $\beta = \gamma_d$, $1 \le d \le D$. In the extreme anisotropic case, i.e.,when $s_e \ll s_d$, $d \ne e$, the approximation rate of KCS recovery approaches $\beta \approx Ds_e$, while the approximation rate using standard CS on the sections of the signal along the d^{th} dimension is approximately $\gamma_d \approx s_d$. Thus, using KCS would only provide an advantage if the measurements are to be distributed along the e^{th} dimension.

5. EXPERIMENTAL RESULTS

We perform experiments to verify the compressibility properties of multidimensional hyperspectral signals in a hyperbolic wavelet basis. We also perform experiments that showcase the advantage of using Kronecker product sparsity bases and measurement matrices against schemes that operate on partitioned versions of the multidimensional signals. Additional results are available in [12].

Our first experiment considers synthetically generated signals of size $8 \times 8 \times 8$ (i.e., $N = 512$) that are $K = 10$-sparse in a Kronecker product basis, and compares three CS recovery schemes: the first one uses a single recovery from dense, *global* measurements; the second one uses a single *KCS recovery* from the set of measurements obtained independently from each 8×8 3-section; and the third one uses *independent recovery* of each 8×8 3-section from its individual measurements. We let the number of measurements M vary from 0 to N, with the measurements evenly split among the 3-sections in the independent and KCS recovery cases. For each value of M, we perform 100 iterations by generating K-sparse signals $\mathbf{x}$ with independent and identically distributed (i.i.d.) Gaussian entries and with support following a uniform distribution among all supports of size K, and generating measurement matrices with i.i.d. Gaussian entries for each 3-section as well. We then measure the probability of successful recovery for each value of M, where a success is declared if the signal estimate $\widehat{\mathbf{x}}$ obeys $\|\mathbf{x} - \widehat{\mathbf{x}}\|_2 \le 10^{-3}\|\mathbf{x}\|_2$. The overmeasuring factors M/K required for 95% success rate are 6, 15, and 30 for global measurements, KCS, and independent recovery, respectively; see [12] for details.

Our second experiment performs an experimental evaluation of the compressibility of a real-world hyperspectral datacube using independent spatial and spectral sparsity bases and compares it with a Kronecker product basis. The datacube for this experiment is obtained from the AVIRIS database. A $128 \times 128 \times 128$ voxel sample is taken, obtaining a signal of length $N = 2^{21}$ samples. We then process the signal through three different transforms: the first two (*Space, Frequency*) perform wavelet transforms along a subset of the dimensions of the data; the third one (*Kronecker*) transforms the entire datacube with a basis formed from the Kronecker product of a 2-D isotropic wavelet basis in space and a 1-D wavelet basis in frequency. In all cases the Daubechies-8 wavelet was used. For each one of these transforms, we measured normalized error magnitude when transform coding is used to preserve K coefficients of the data for varying values of K. The results are shown in Figure 1; the Kronecker transform provides the sparsest representation of the signal, outperforming the partial transforms. However, the rate of

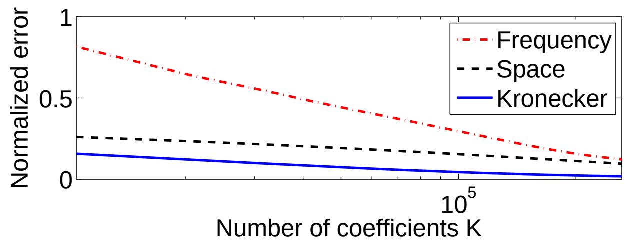

Fig. 1. *Performance of Kronecker product transform coding for hyperspectral imaging. The Kronecker product performs better than either basis independently. However, the rate of decay of the compression error using the Kronecker product basis is approximately the same as the lower rate obtained from the individual bases.*

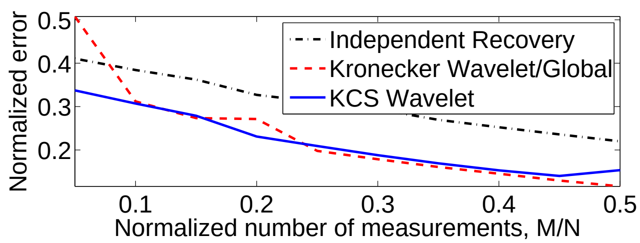

Fig. 2. *Performance of Kronecker product sparsity and measurements matrices for hyperspectral imaging. Recovery using the Kronecker product sparsifying basis outperforms separate recovery. Additionally, there is an advantage to applying distributed rather than global measurements when the number of measurements M is low.*

decay for the normalized error of the Kronecker transform is slightly higher than the minimum rate of decay among the individual transforms. Our analysis indicates that this result is due to the difference between the degrees of smoothness among the signal dimensions.

We also compare the performance of KCS to CS using standard bases to sparsify individual spectral frames. In our simulations we obtain CS measurements using a subsampled permuted Hadamard transform on each spectral frame. The datacube from the previous experiment was "flattened" to $128 \times 128 \times 16$ voxels to reduce the amount of computation required. Figure 2 shows the recovery error magnitude from several different setups: *independent* recovery operates on each spectral band independently using a wavelet basis to sparsify each spectral band. *KCS Wavelet* employs the Kronecker product formulations to perform joint recovery. We also obtain *global* CS measurements that depend on all the voxels of the datacube as a baseline; such measurements result in a fully dense measurement matrix Φ and therefore are difficult to obtain in real-world applications. In this case, we use the *Kronecker Wavelet* basis for sparsity and recover all frames at once. We see a strong advantage to the use of Kronecker product wavelet basis with joint recovery as compared to independent recovery. We also see an improvement on KCS over global measurements when the number of measurements M is small; as M increases, this advantage vanishes due to the availability of sufficient information.

6. CONCLUSIONS

In this paper we have developed the concept of KCS and presented initial analytical results on its performance. This theoretical framework is motivated by new sensing applications that acquire multidimensional signals in a progressive fashion, as well as by settings where the measurement process is distributed, such as sensor networks and arrays. We have also provided analytical results for the recovery of signals that live in anisotropic Besov spaces, where there is a well-defined relationship between the degrees of compressibility obtained using lower-dimensional wavelet bases on subsets of the signal and multidimensional anisotropic wavelet bases on the entire signal. Furthermore, because the formulation follows the standard CS approach of single measurement and sparsifying matrices, standard recovery algorithms that provide provable recovery guarantees can be used; this obviates the need to develop ad-hoc algorithms to exploit additional signal structure.

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