Distributed Compressed Sensing

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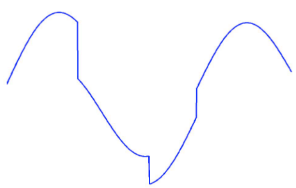
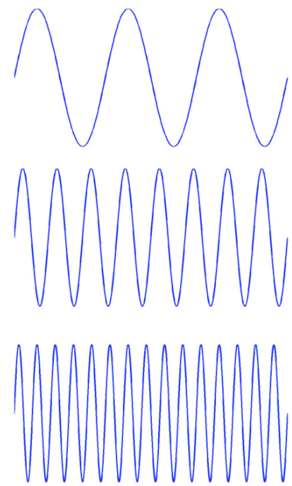
Compressed Sensing

Signal Representation

- Representation (basis, frame) $\{\psi_i\}$
  - spikes, Fourier sinusoids, wavelets, etc …



$$x = \sum_{i=1}^{N} \alpha_i \, \psi_i$$



- For orthonormal Ψ, coefficient α_i = projection (inner product) of x onto basis function ψ_i

$$\alpha_i = \langle x, \ \psi_i \rangle$$

Sparse Signal Representations

- For maximum *efficiency*, choose representation $\{\psi_i\}$ so that coefficients $\{\alpha_i\}$ are ***sparse*** (most close to 0)
  - smooth signals and Fourier sinusoids
  - piecewise smooth signals and wavelets, …

- Approximation – quantize/encode coeff sizes and locations

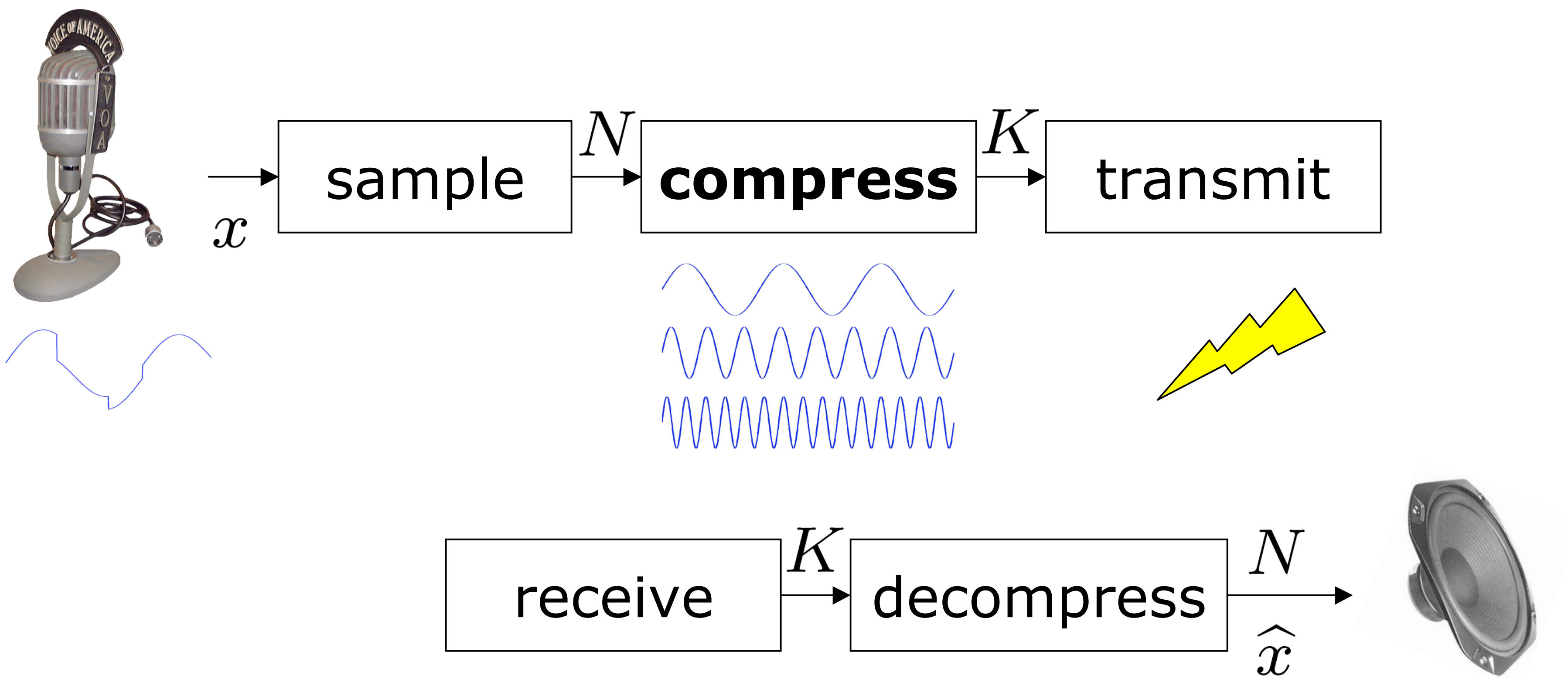
$$x = \sum_{i=1}^{N} \alpha_i \, \psi_i$$

$$\widehat{x} = \sum_{K \ll N \text{ largest terms}} \alpha_i^q \, \psi_i$$

- Transform coding examples: JPEG, MPEG, …

DSP Sensing

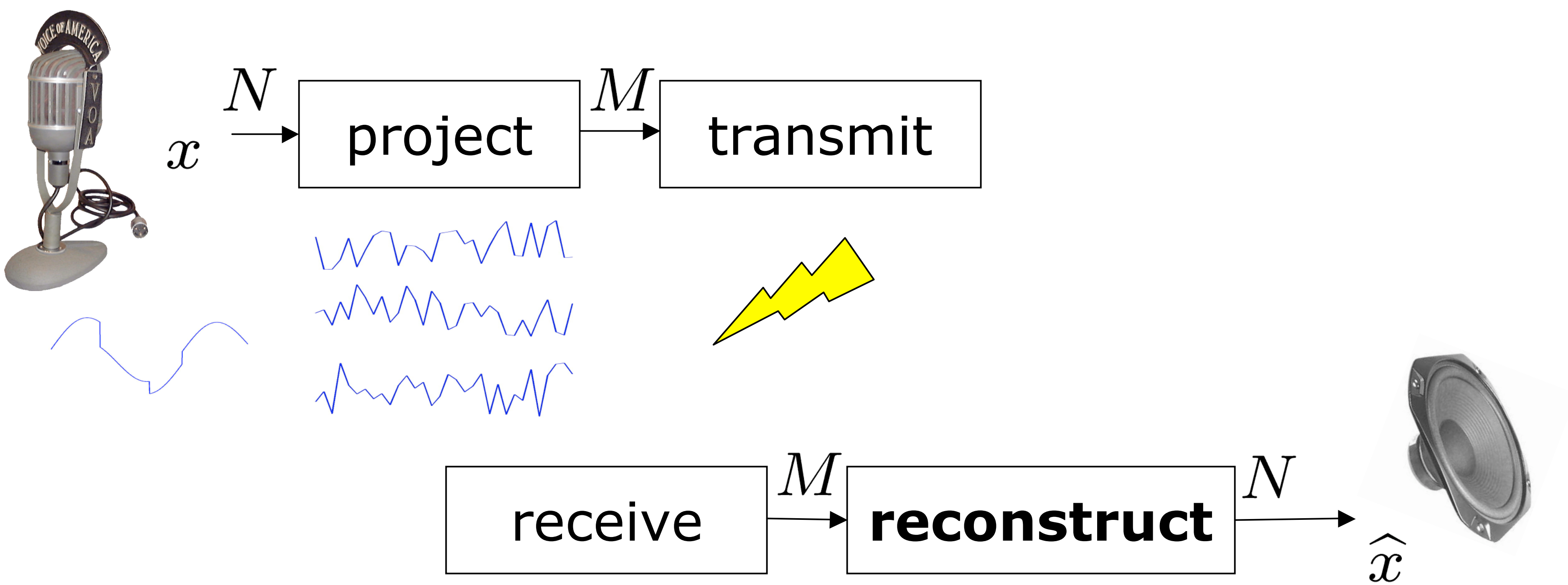
- The typical sensing/compression setup
  - compress = transform, sort coefficients, encode
  - most computation at *sensor*
  - *lots of work* to throw away >80% of the coefficients



Compressed Sensing (CS)

- Measure projections onto *incoherent* basis/frame
- Reconstruct via *optimization*
- Mild oversampling: $cK \leq M \ll N, \quad c \approx 3$
- Highly asymmetrical (most computation at *receiver*)

[Donoho; Candes, Romberg, Tao]



Compressed Sensing 101

- Foundation: Reconstruction from incoherent projections

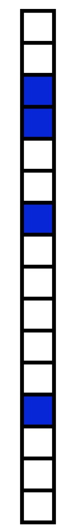
- Signal has *sparse representation* in some basis Ψ (ex: Fourier, wavelets, etc.)
  - WLOG assume signal is sparse in time domain $\Psi = I$

- Take second, incoherent basis Φ
  - elements of Φ are *not sparse* in Ψ
  - random Φ is incoherent with almost *all* Ψ

- Measure signal via *few linear projections*

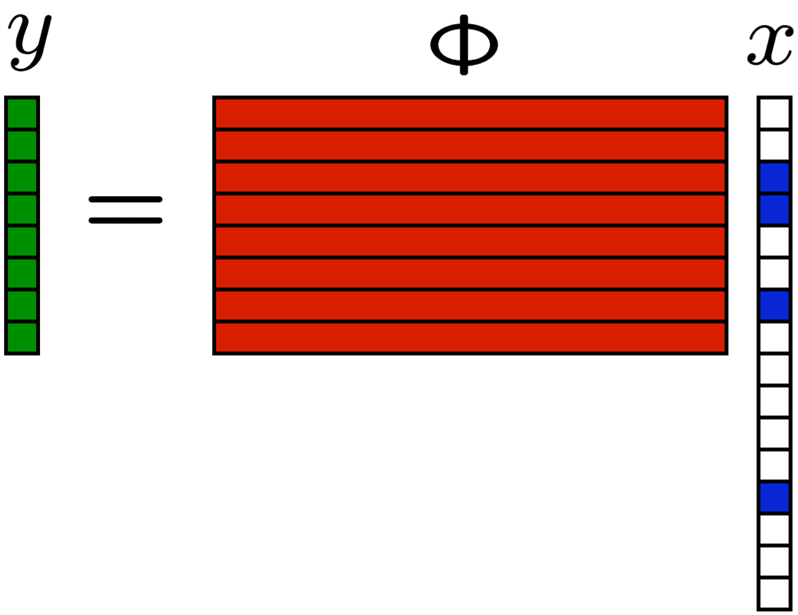
$$y = \Phi x$$

x



Before CS - ℓ_2

- Goal: Given measurements y find signal x
- Fewer rows than columns in measurement matrix Φ
- *Ill-posed*: infinitely many solutions $\widehat{x}$
- Classical solution: *least squares*



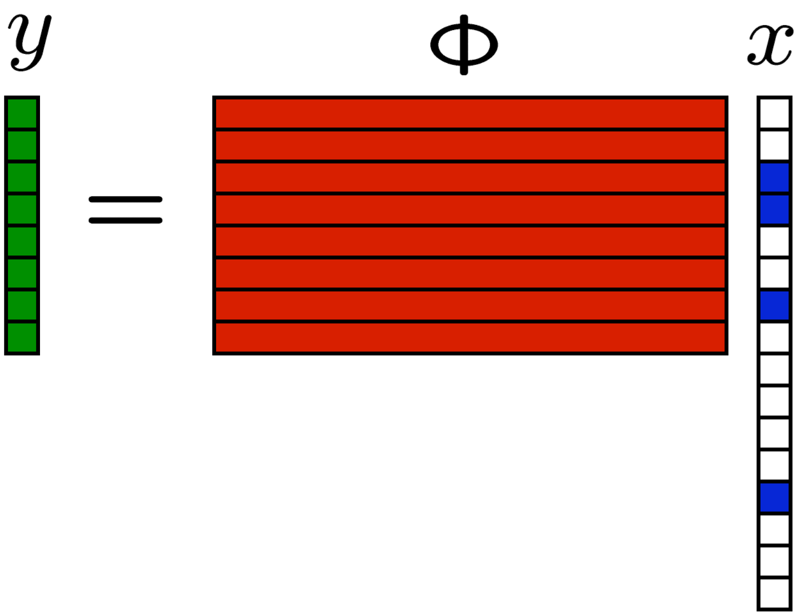
$$\widehat{x} = \arg\min_{y=\Phi x} \|x\|_2$$

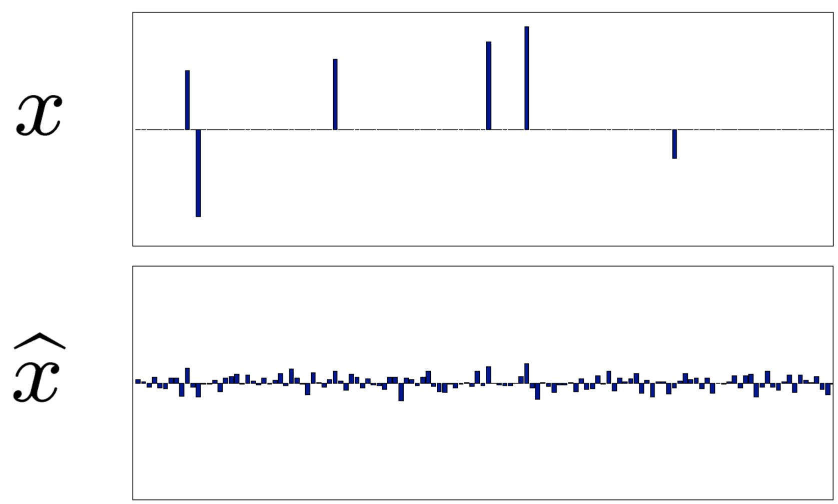
$$\sum_i |x_i|^2$$

$$\widehat{x} = (\Phi^T\Phi)^{-1}\Phi^T y$$

Before CS - ℓ_2

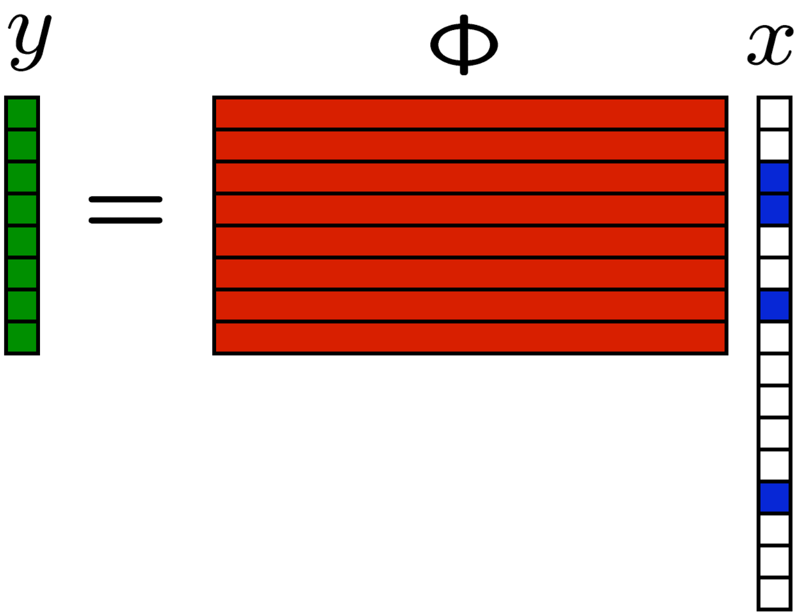
- Goal: Given measurements y find signal x
- Fewer rows than columns in measurement matrix Φ
- *Ill-posed*: infinitely many solutions $\widehat{x}$
- Classical solution: *least squares*
- Problem: *small L_2 doesn't imply sparsity*





CS – ℓ_0

- Modern solution: exploit sparsity of x
- Of the infinitely many solutions $\widehat{x}$ seek *sparsest* one

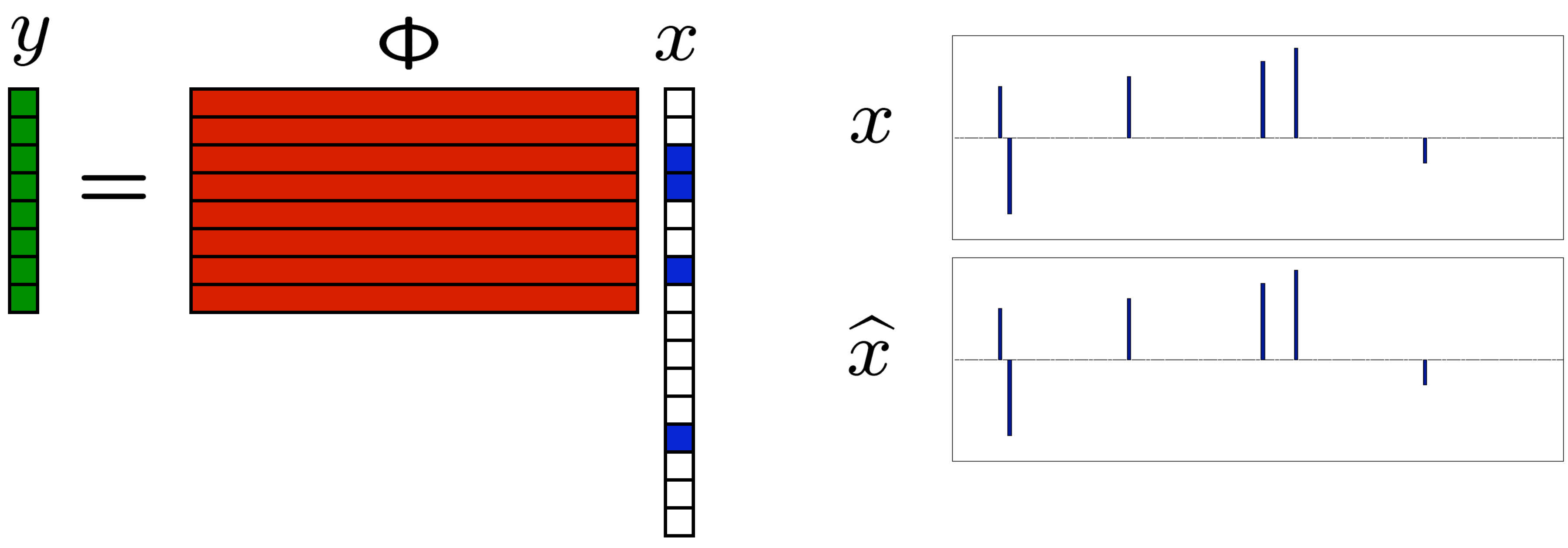


$$\widehat{x} = \arg\min_{y=\Phi x} \|x\|_0$$

number of nonzero entries

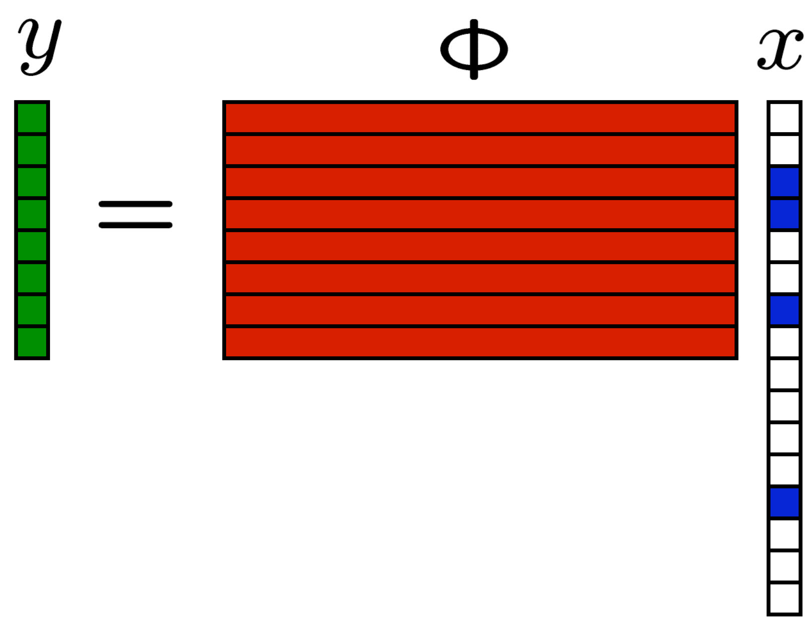
CS – ℓ_0

- Modern solution: exploit sparsity of x
- Of the infinitely many solutions $\widehat{x}$ seek *sparsest* one
- If $M \geq K + 1$ then ***perfect reconstruction*** w/ high probability
- But *combinatorial* computational complexity



The CS Miracle – ℓ_1

- Goal: Given measurements y find signal x
- Fewer rows than columns in measurement matrix Φ
- Modern solution: exploit sparsity of x
- Of the infinitely many solutions $\widehat{x}$ seek the one with smallest ℓ_1 norm



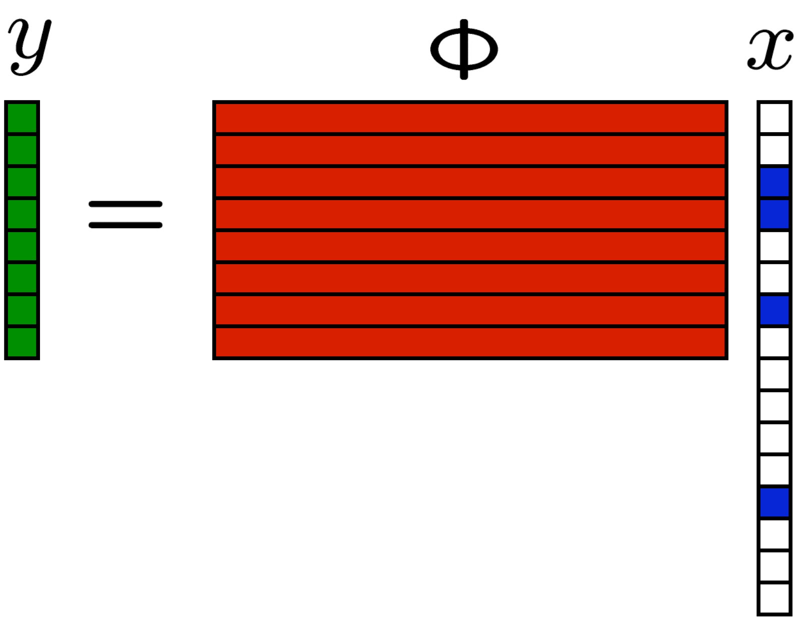
$$\widehat{x} = \arg\min_{y=\Phi x} \|x\|_1$$

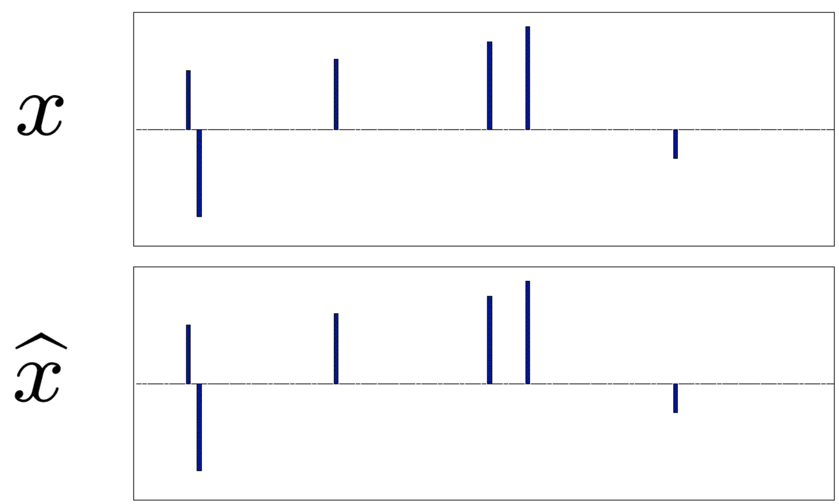
↑

$$\sum_i |x_i|$$

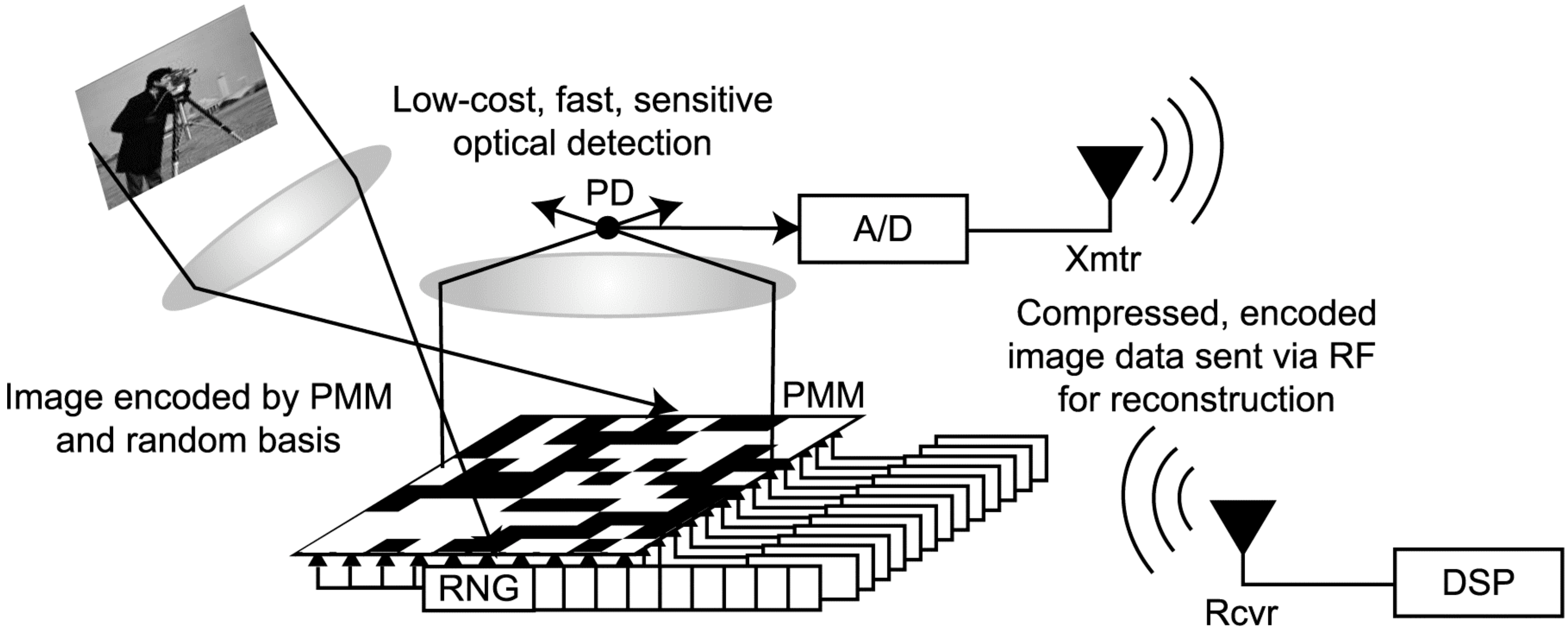
The CS Miracle – ℓ_1

- Goal: Given measurements y find signal x
- Fewer rows than columns in measurement matrix Φ
- $\exists c \approx 3$, if $M \geq cK$ then ***perfect reconstruction*** w/ high probability [Candes et al.; Donoho]
- *Linear programming* or other *sparse approximation* algorithms





CS Camera Architecture



joint work with Kevin Kelly, Yehia Massoud, Don Johnson, …

CS Reconstruction for Images

256x256 = 65536 pixels

CS Reconstruction for Images

26000 incoherent projections

CS Reconstruction for Images

6500 wavelet coefficients

Compressed Sensing Vision @ Rice

- CS changes the rules of the data acquisition game
  - changes what we mean by "sampling"
  - exploits a priori signal sparsity information
    (that the signal is compressible in some representation)

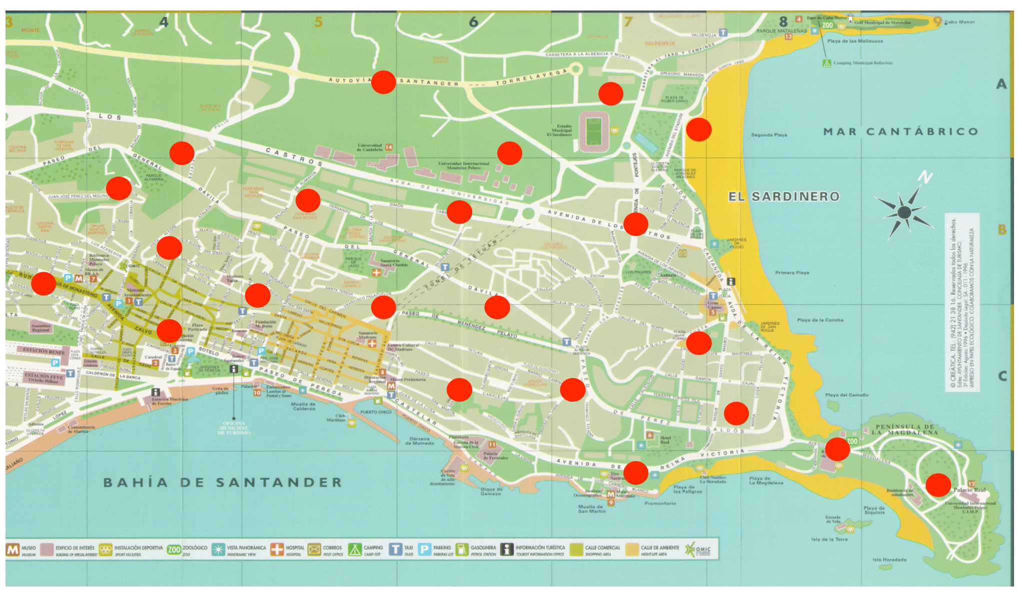
- Next generation data acquisition
  - new A/D converters (sub Nyquist)
  - new imagers and imaging algorithms
  - new distributed source coding algorithms (***today!***)

    …

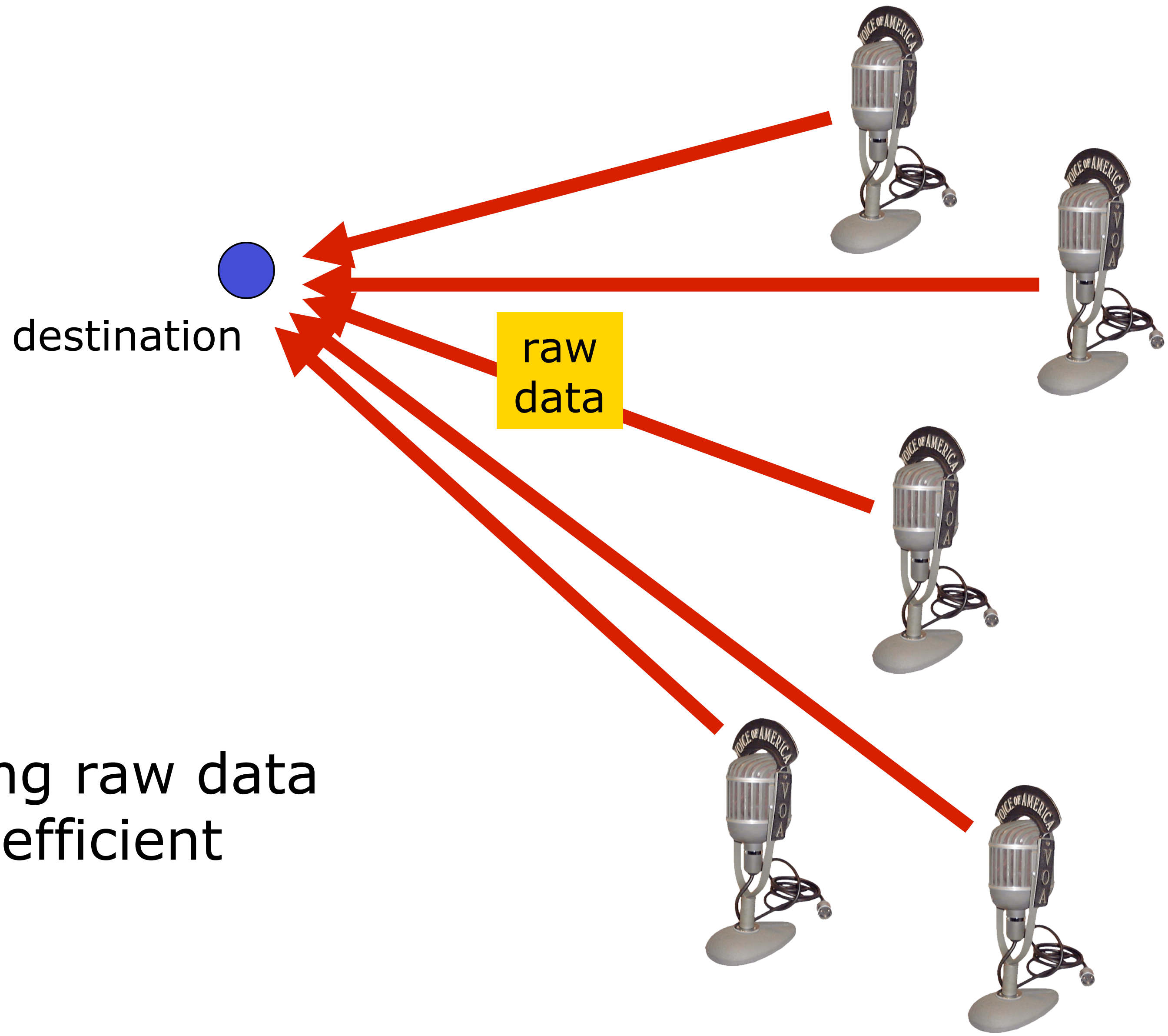
Distributed Compressed Sensing

Why Distributed?

- Networks of many *sensor nodes*
  - sensor, microprocessor for computation, wireless communication, networking, battery
  - can be spread over large geographical area
- Must be *energy efficient*
  - *minimize communication* at expense of computation
  - motivates *distributed compression*

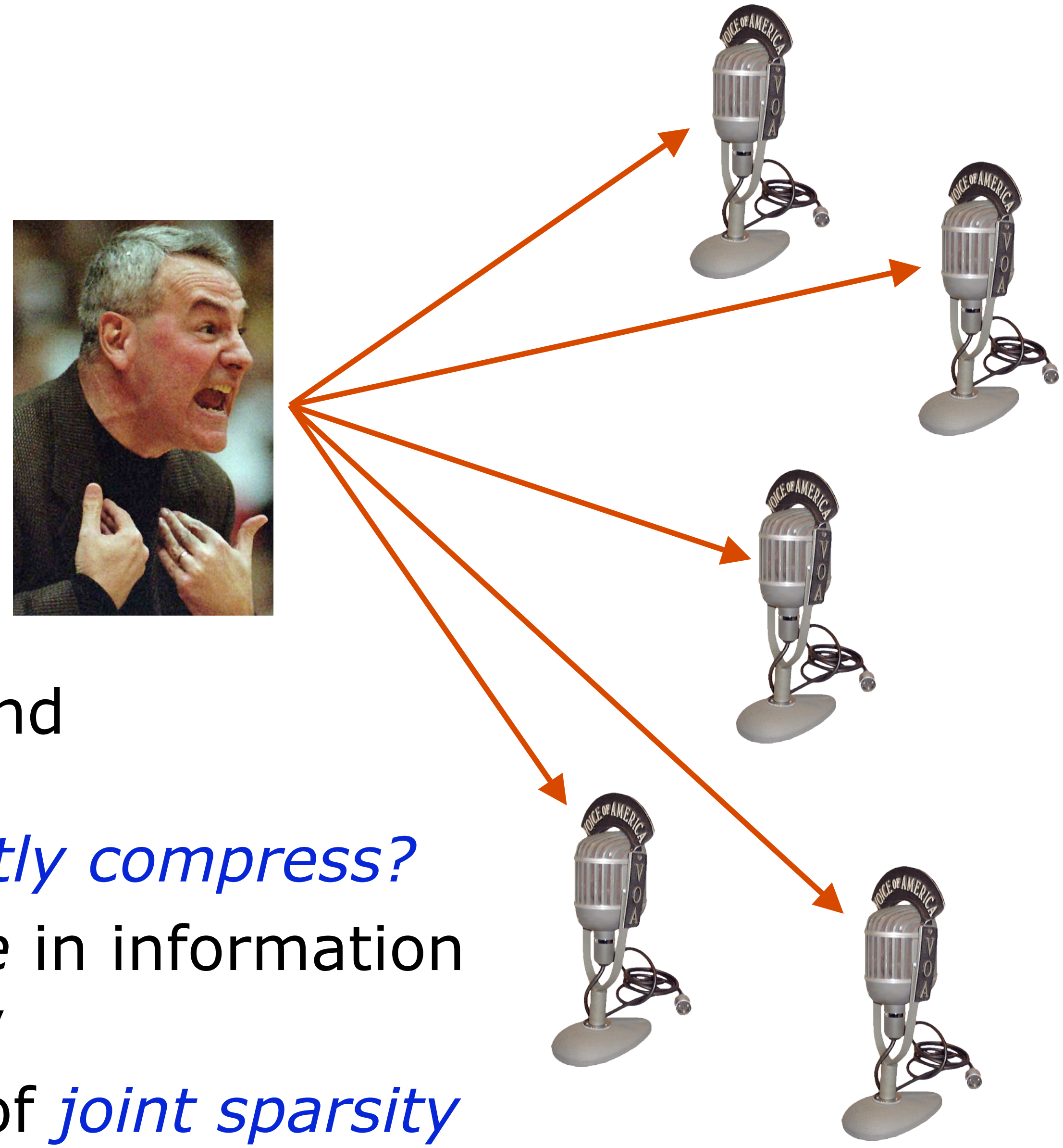


Separate Sensing



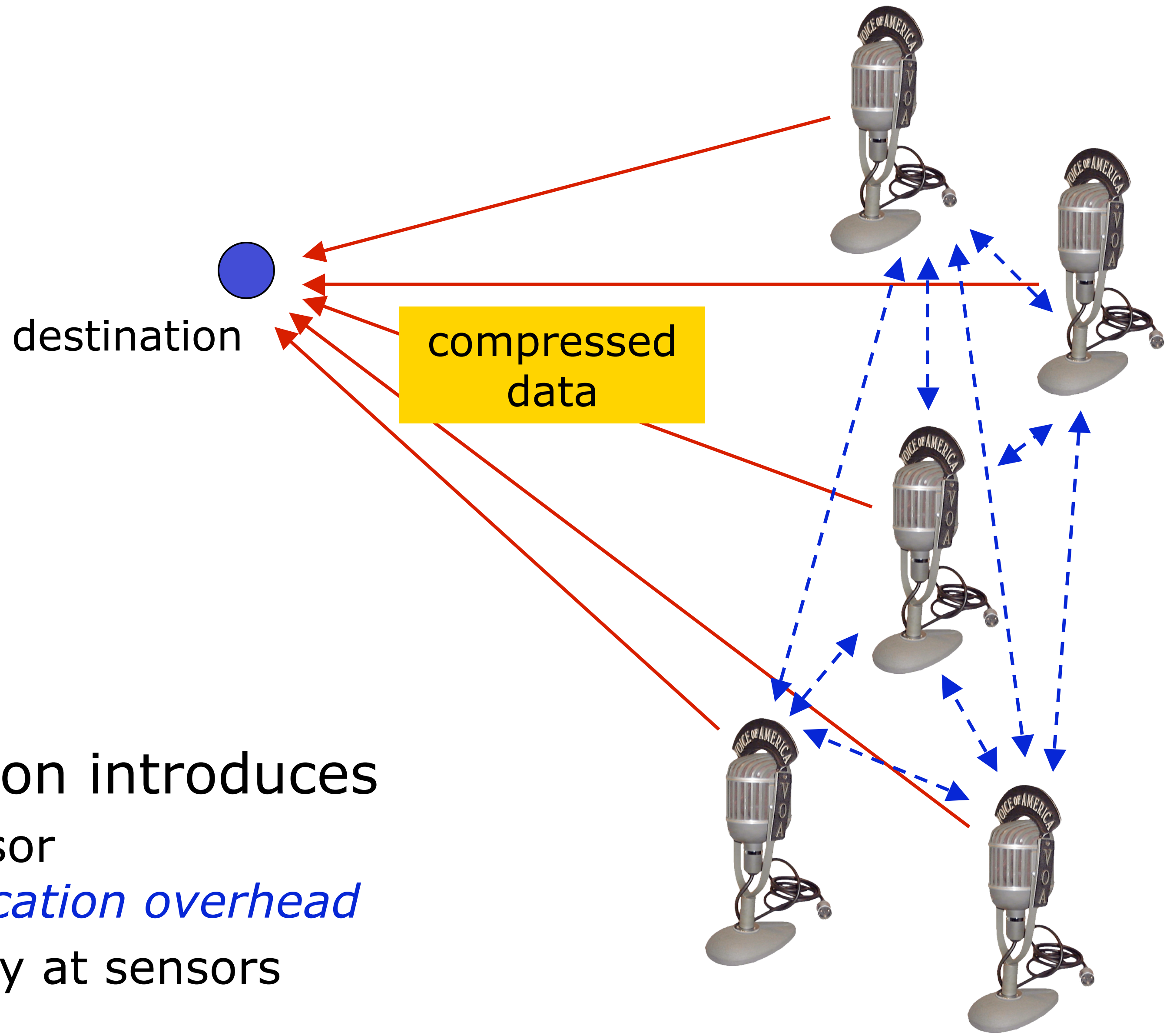
- Transmitting raw data typically inefficient

Correlation



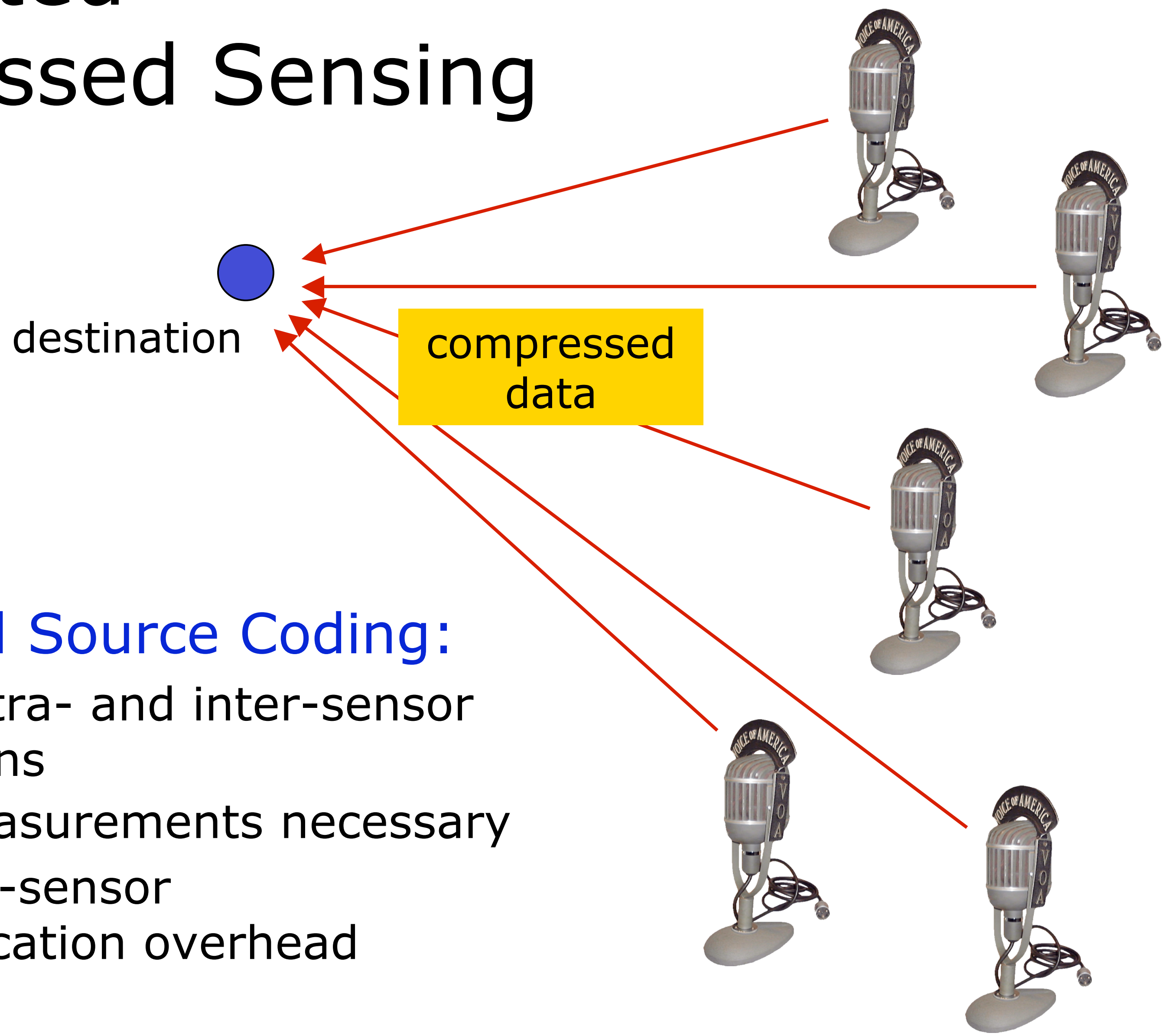
- Can we exploit
  intra-sensor and
  inter-sensor
  correlation to *jointly compress?*
- *Ongoing challenge* in information theory community
- Introduce notion of *joint sparsity*

Collaborative Sensing



- Collaboration introduces
  - inter-sensor *communication overhead*
  - complexity at sensors

Distributed Compressed Sensing (DCS)

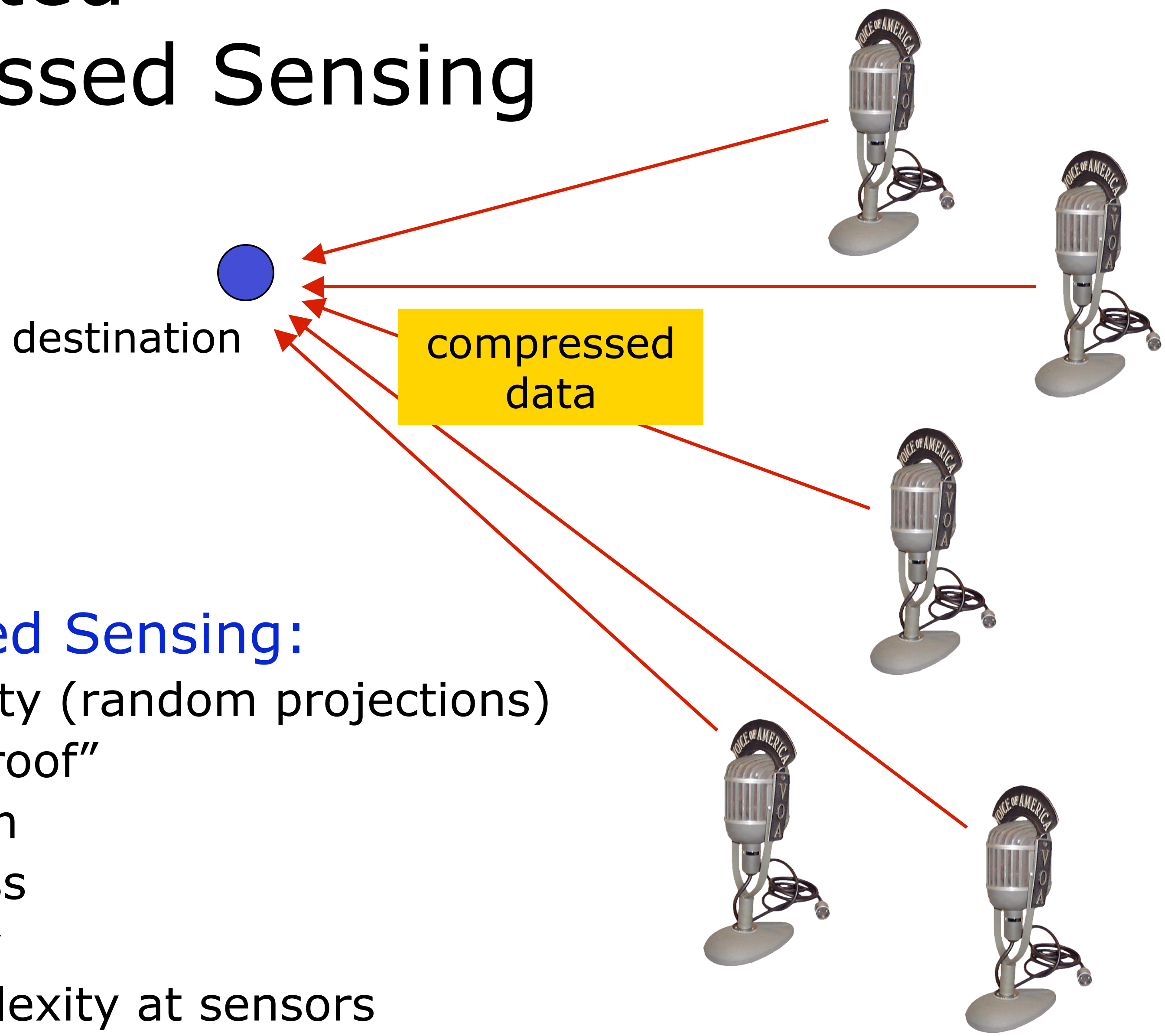


Benefits:

- Distributed Source Coding:
  - exploit intra- and inter-sensor correlations

  ⇒ fewer measurements necessary
  - zero inter-sensor communication overhead

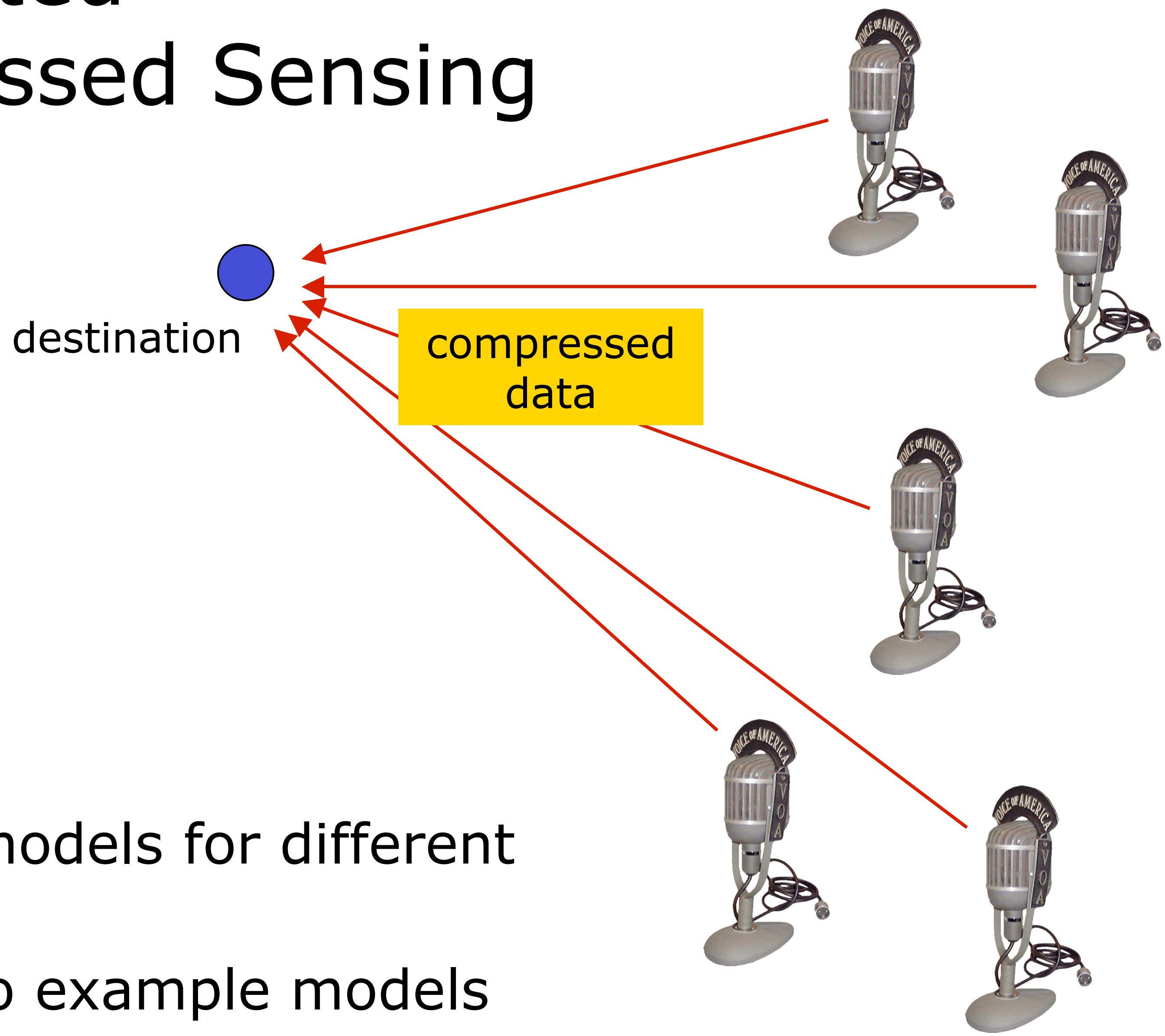
Distributed Compressed Sensing (DCS)



Benefits:

- Compressed Sensing:
  - universality (random projections)
  - “future-proof”
  - encryption
  - robustness
  - scalability
  - low complexity at sensors

Distributed Compressed Sensing (DCS)



- Different models for different scenarios
- Today: two example models

Model 1: Common + Innovations

Common + Innovations Model

- Motivation: sampling signals in a smooth field

- *Joint sparsity model*:
  - length-N sequences x_1 and x_2

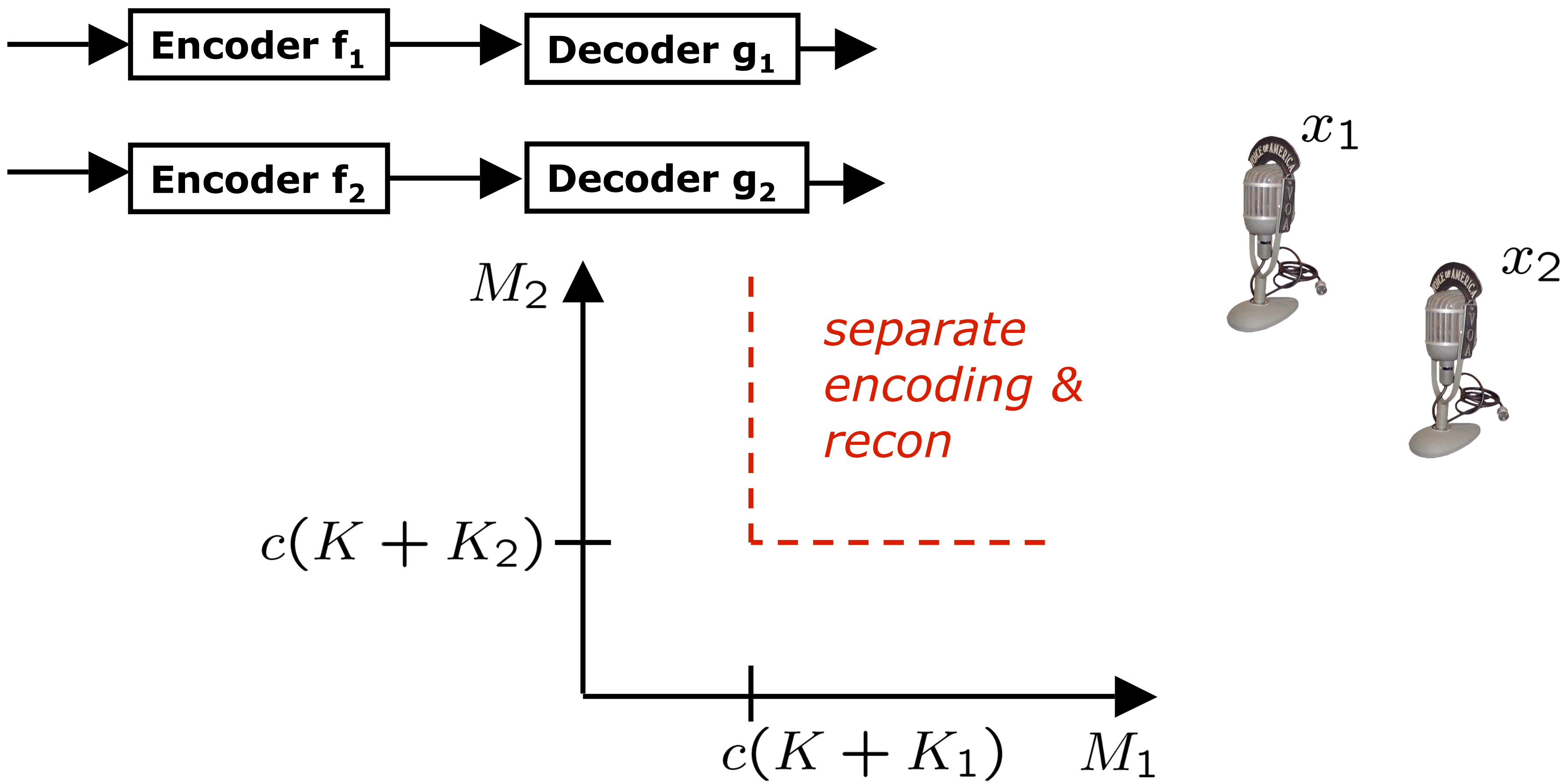
$$x_1 = z + z_1$$
$$x_2 = z + z_2$$

  - z is length-N *common* component
  - z_1, z_2 length-N *innovation* components
  - z has sparsity K
  - z_1, z_2 have sparsity K_1, K_2

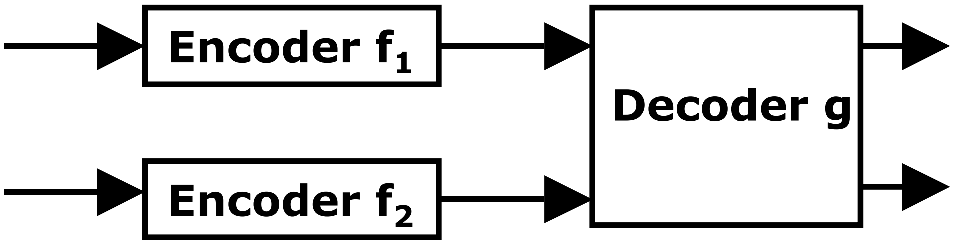
- Measurements

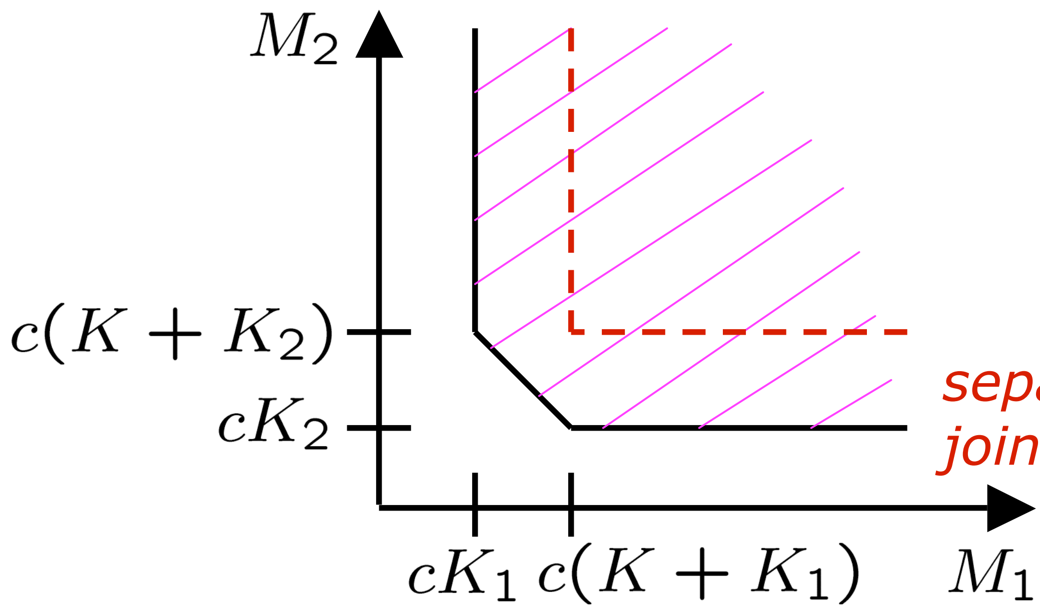
$$y_1 = \Phi_1 x_1$$
$$y_2 = \Phi_2 x_2$$

Measurement Rate Region with *Separate* Reconstruction



Goal: Measurement Rate Region with *Joint* Reconstruction





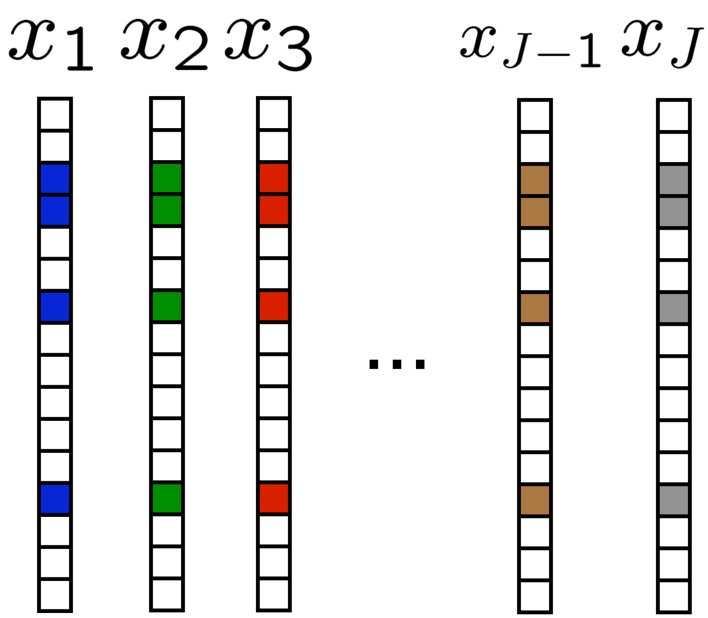
D. Baron, M. F. Duarte, M. B. Wakin, S. Sarvotham and R. G. Baraniuk, "*An Information Theoretic Approach to Distributed Compressed Sensing*", Allerton Conference on Communication, Control, and Computing 2005

Model 2:

Common Sparse Supports

Common Sparse Supports Model

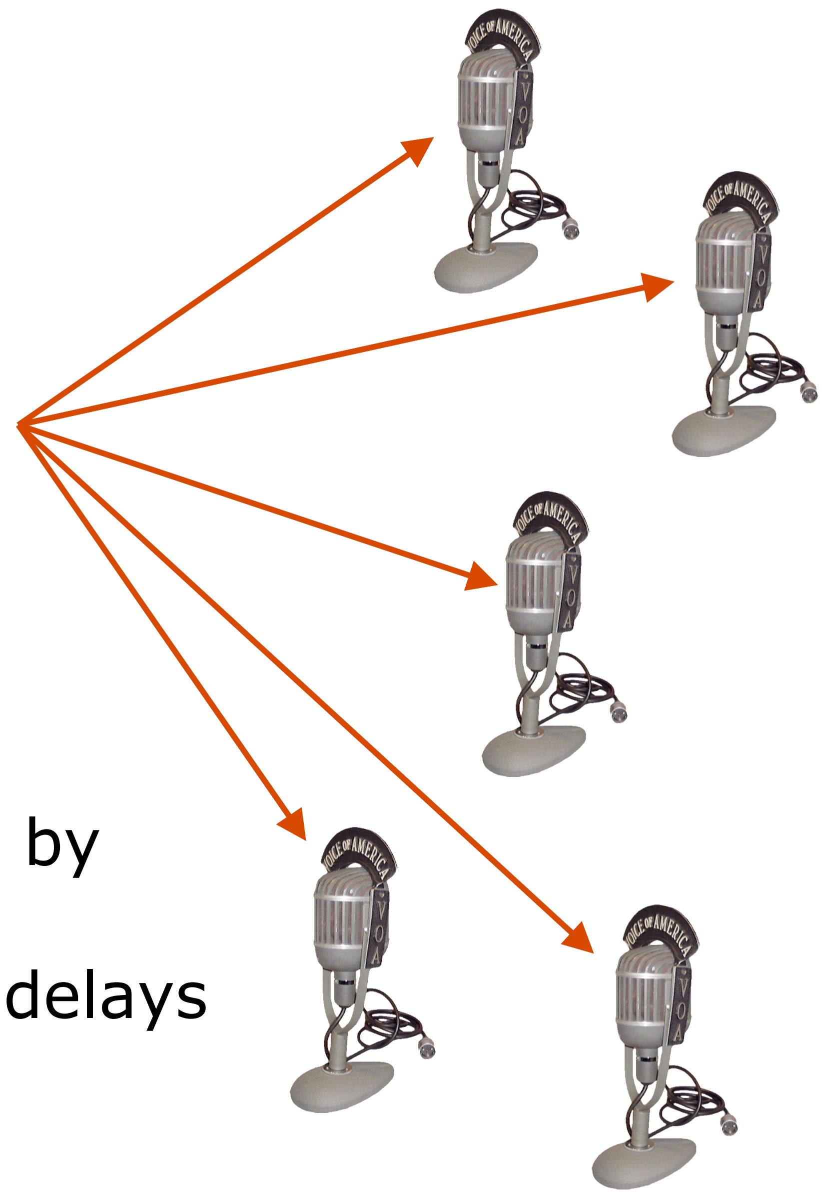
- *Joint sparsity model #2* (JSM-2):
  - measure J signals, each K-sparse
  - *signals share sparse components, different coefficients*



$$x_j = \sum_{\omega \in \Omega} x_{j,\omega} \psi_\omega,$$

$$|\Omega| = K$$

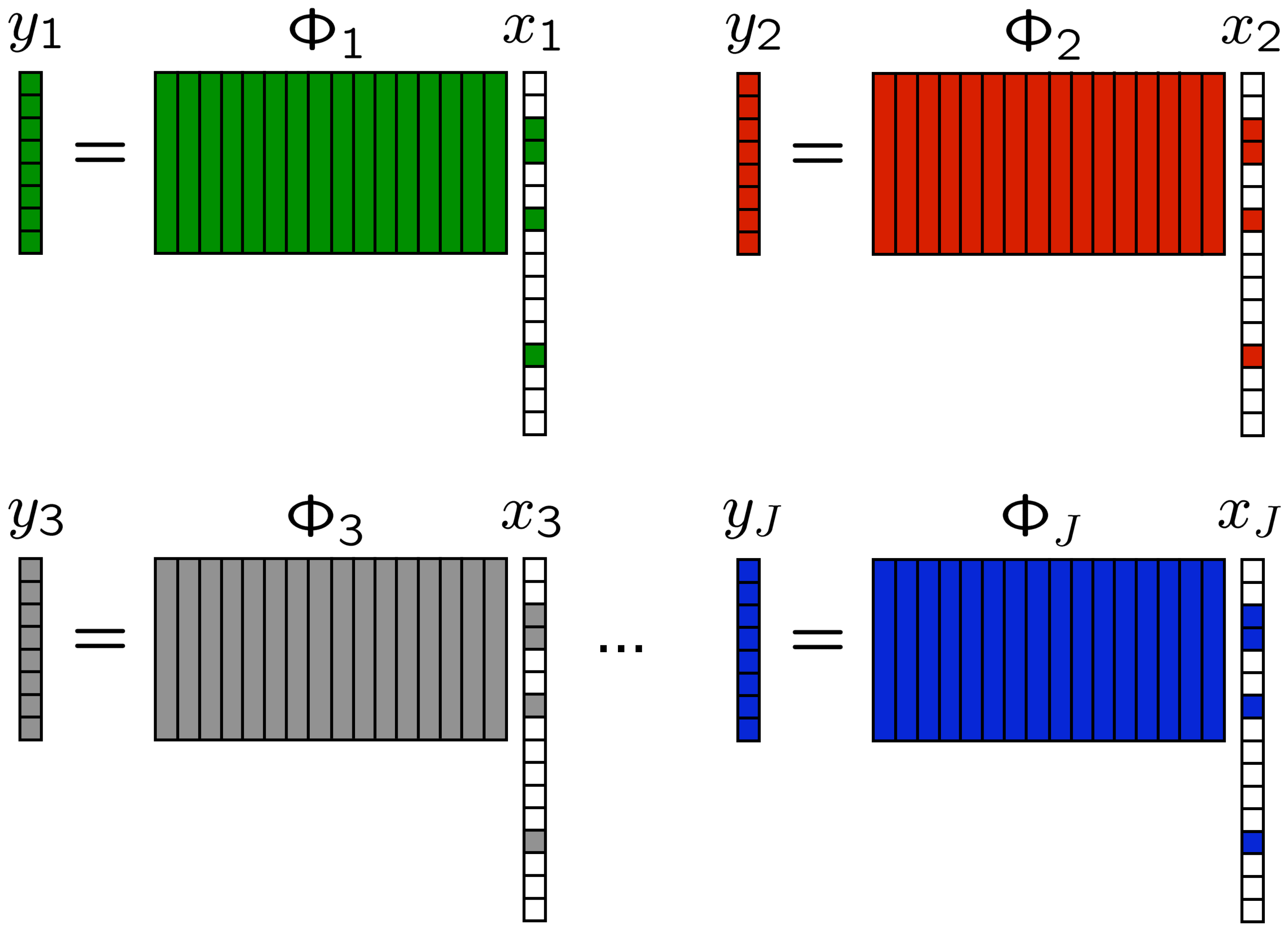
Common Sparse Supports Model



Audio Signals

- Sparse in Fourier Domain
- Same frequencies received by each node
- Different attenuations and delays (magnitudes and phases)

Common Sparse Supports Model



Common Sparse Supports Model: Reconstruction

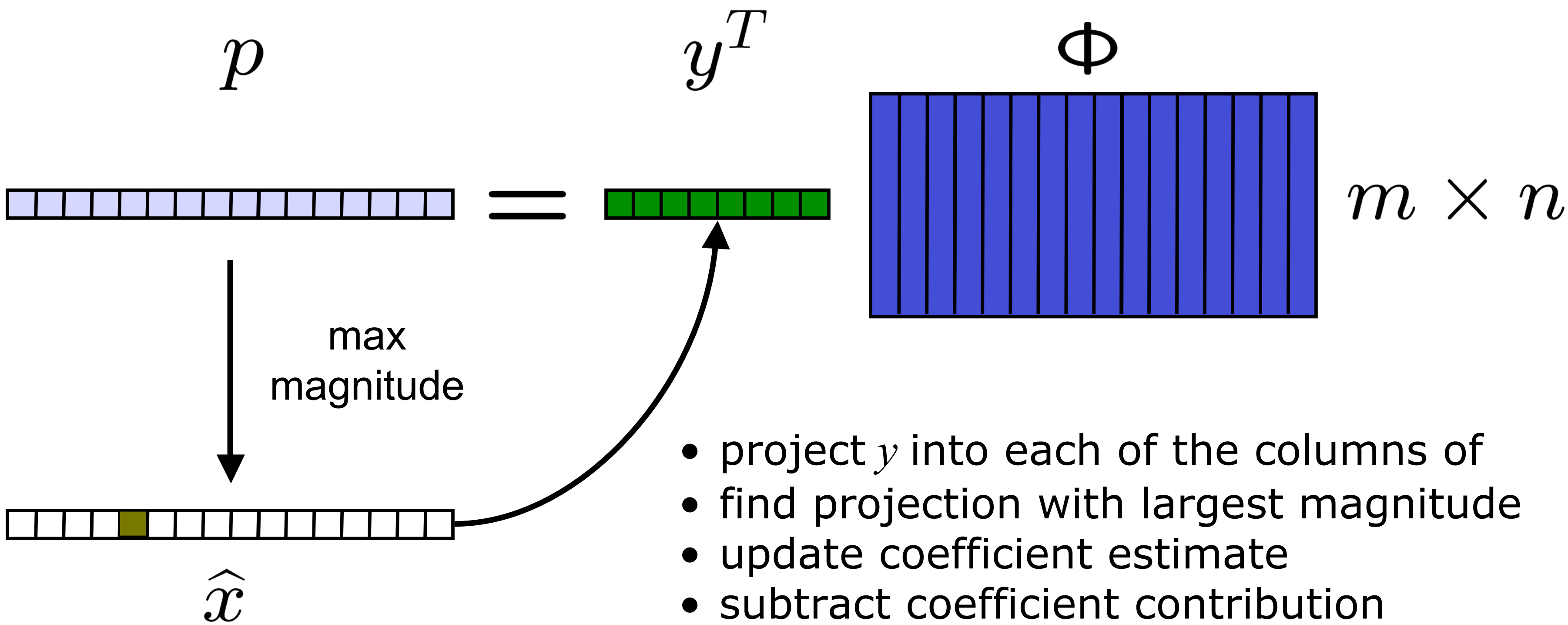
- Orthogonal Matching Pursuit
  - Estimate support of sparse signal using inner products between y_i and Φ_i

- Simultaneous Orthogonal Matching Pursuit
  - (Tropp, Gilbert, Strauss)
  - For signals with shared sparse support
  - Extend greedy algorithms to signal ensembles that share a sparse support

Simultaneous Sparse Approximation

Orthogonal Matching Pursuit

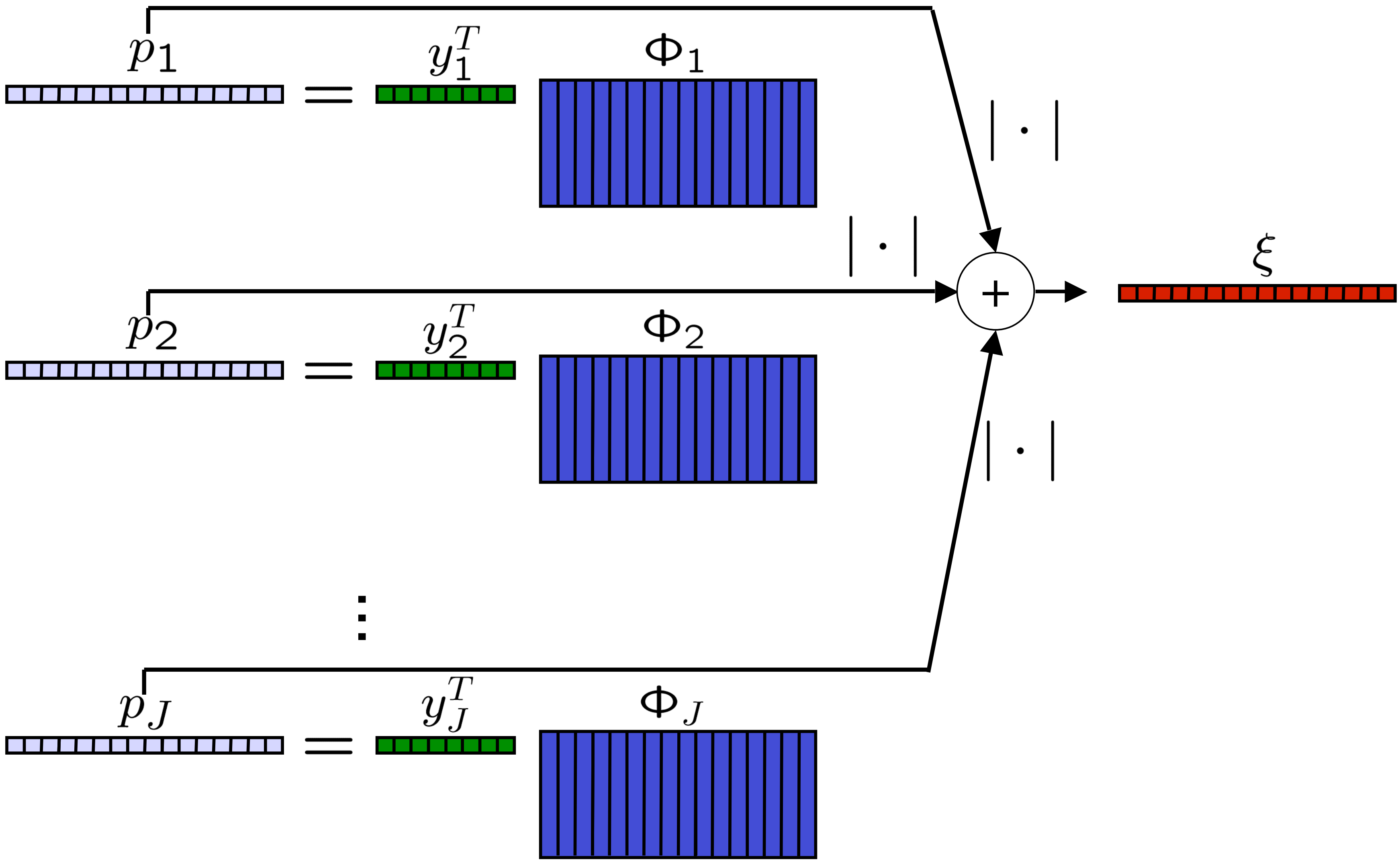
Approximation: $y = \Phi \widehat{x}$



- project y into each of the columns of
- find projection with largest magnitude
- update coefficient estimate
- subtract coefficient contribution
- orthogonalize all column vectors against chosen one
- repeat

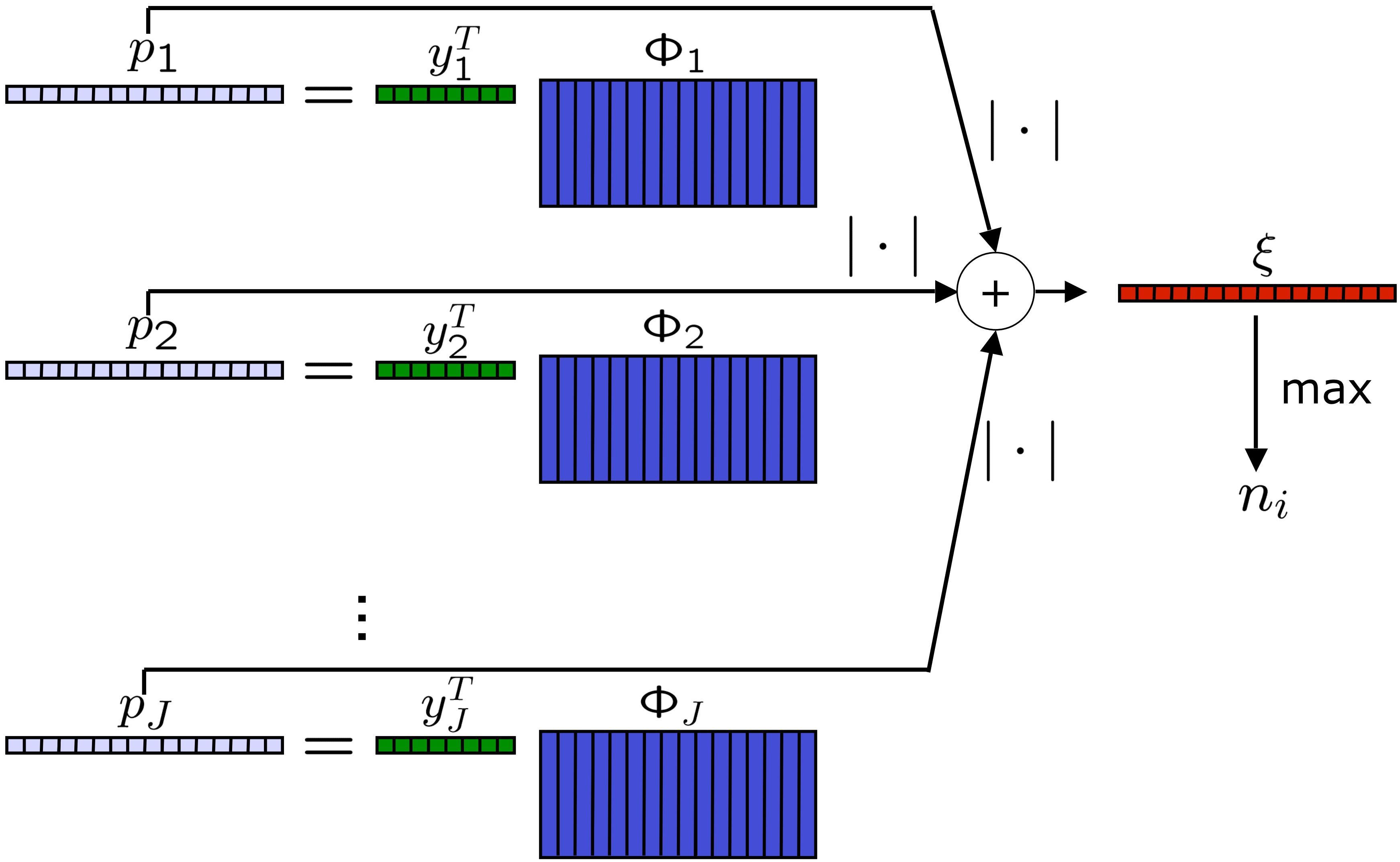
Simultaneous Orthogonal Matching Pursuit

Approximations: $y_j = \Phi_j \hat{x}_j$



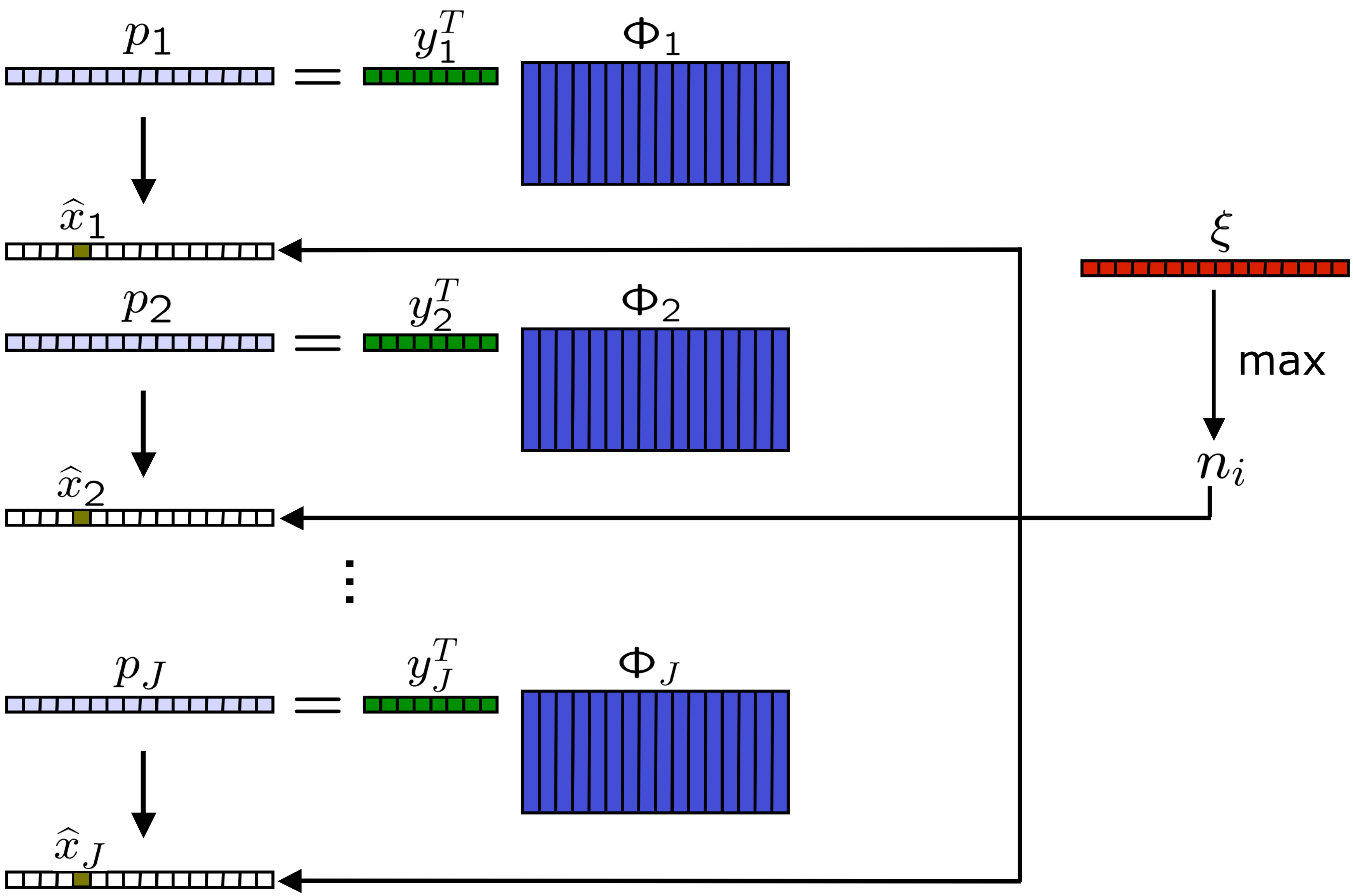
Simultaneous Orthogonal Matching Pursuit

Approximations: $y_j = \Phi_j \widehat{x}_j$



Simultaneous Orthogonal Matching Pursuit

Approximations: $y_j = \Phi_j \widehat{x}_j$

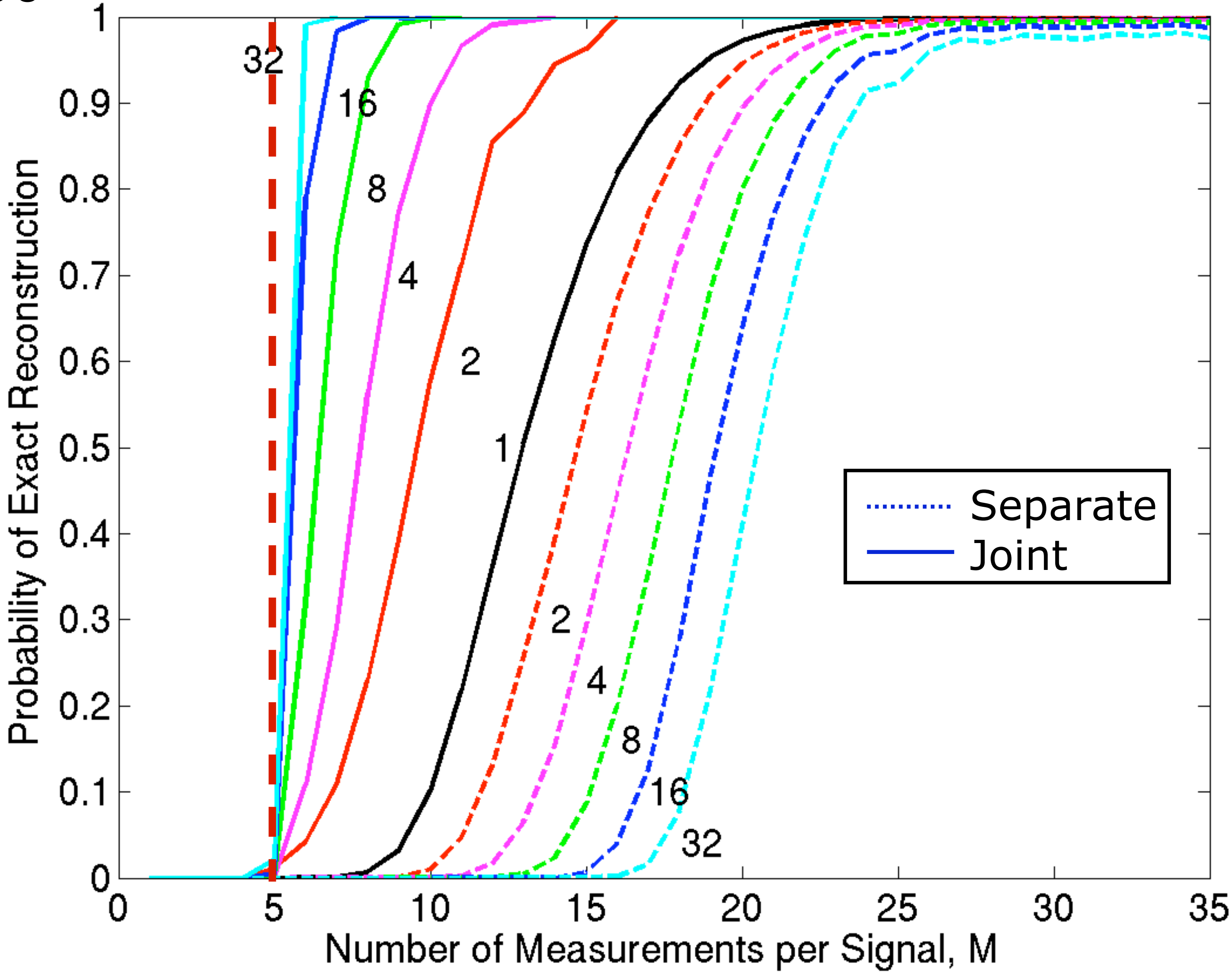


Common Sparse Supports Model: Reconstruction

- Performance (measurements per sensor):
  - ℓ_0 minimization: $K+1$
  - ℓ_1 minimization: cK
  - SOMP: ?

K=5
N=50

SOMP Results



Conclusions

- Theme: compressed sensing for multiple signals

- Distributed compressed sensing
  - new models for *joint sparsity*
  - suitable for sensor network applications
  - compression of sources w/ intra- and inter-sensor correlation

- More
  - additional joint sparsity *models*
  - real data
  - sensor networks

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Thanks

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