Compressive Time Delay Estimation using Interpolation

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***Abstract*—We show that compressive sensing (CS) applied to time delay estimation (TDE) simultaneously enables a reduction in the sampling frequency and preserves good estimation precision. With CS, we seek to recover signals and parameters from an under-determined system of linear equations by assuming sparsity in a known dictionary. A common problem in CS is that the observed signals may not be sparsely representable in the dictionary. This problem also occurs in TDE as the delay parameter is a continuous parameter. We remedy this issue by combining CS with interpolation.**

***Index Terms*—Compressive sensing, time delay estimation, parameter estimation, interpolation.**

I. Problem Formulation

Let the received time-domain analog signal be defined as

$$f(t;\boldsymbol{\alpha},\boldsymbol{\tau}) = \sum_{i=1}^{K} \alpha_i \cdot g(t-\tau_i) + n(t), \tag{1}$$

where $\boldsymbol{\alpha} = \{\alpha_1, \alpha_2, \cdots, \alpha_K\}$ are the unknown signal amplitudes, $\boldsymbol{\tau} = \{\tau_1, \tau_2, \cdots, \tau_K\}$ are the unknown signal delays in time, $g(t)$ is a known signal waveform and $n(t)$ is the noise. The task of the estimation algorithm is then to estimate $\boldsymbol{\alpha}$ and $\boldsymbol{\tau}$ from a sampled version of (1). Depending on the bandwidth of $g(t)$, the required sampling rate to estimate the delays to a desired precision may be high. If we assume that only a few signal components are active, i.e. K is small, we may use CS to achieve the desired precision at a lower sampling rate. With a CS receiver the received signal is $\mathbf{y} = \mathbf{\Phi f}$, where $\mathbf{f} \in \mathbb{C}^N$ is the Nyquist sampled version of (1), $\mathbf{y} \in \mathbb{C}^M$ is the received signal and $\mathbf{\Phi} \in \mathbb{R}^{M\times N}$ is the CS measurement matrix.

To enable reconstruction CS requires a sparsifying dictionary $\mathbf{\Psi} \in \mathbb{C}^{N\times N}$. In the case of TDE the dictionary is a circulant matrix of delayed waveforms. Since the delay parameter is continuous the received signal may not be sparsely representable by the dictionary, which may lower performance.

Our contribution is bridging the work on CS and interpolation to improve estimator precision in TDE while keeping the sampling frequency low. This is achieved by incorporating an interpolation step in a greedy algorithm. In each iteration of the algorithm, after finding the strongest correlating atom in the dictionary, we propose to use an interpolation function to improve the estimation precision. There are many possible choices of interpolation functions. In this work we compare two such functions: second order polynomial and polar interpolation based on a manifold model.

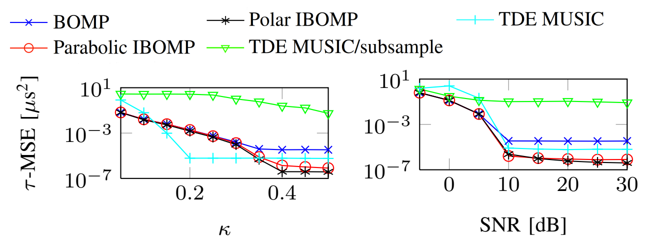

Fig. 1. τ-MSE vs. κ.

Fig. 2. τ-MSE vs. SNR, $\kappa = 0.5$.

II. Numerical Simulations

We compare five delay estimators: 1) *BOMP* is an existing greedy algorithm without interpolation, 2) *PaIBOMP* adds to BOMP parabolic interpolation, 3) *PoIBOMP* uses polar interpolation, 4) *TDE MUSIC* reconstructs the signal using ℓ_1-minimization and then estimates the delays using the MUSIC algorithm, and 5) *TDE MUSIC/subsample* directly downsamples the signal by a factor of N/M and estimates the delays using the MUSIC algorithm. The last algorithm shows that direct downsampling fails due to aliasing.

In the first experiments we assume a noise-free signal and vary the number of measurements $M = \kappa N$, where $\kappa \in [0, 1)$ is the CS subsampling rate. Fig. 1 shows the performance of the five estimators by computing the time delay mean squared error (τ-MSE) between the true and estimated value of the time delay. This corresponds to the sample variance of the estimators and is a measure of estimator precision. All four CS estimators allow for subsampling while maintaining good estimation precision. TDE MUSIC performs best for low κ, while the interpolation algorithms perform best as κ increases.

For the second experiment we include additive white Gaussian measurement noise in the signal model. We fix $\kappa = 0.5$ and vary the signal-to-noise ratio (SNR). Fig. 2 shows that the algorithms are affected by noise, but as SNR increases they converge towards the results for $\kappa = 0.5$ in Fig. 1.

These numerical results show that CS coupled with interpolation enables subsampling while maintaining a desired estimation precision. For full details, see our technical report [1] and our website www.sparsesampling.com/tde.

References

[1] K. Fyhn et al., "Compressive time delay estimation using interpolation," Tech. Rep., Joint work by Aalborg University, Denmark and University of Massachusetts Amherst, USA, 2013, Available at http://arxiv.org/abs/1306.2434.