Signal Reconstruction from Noisy Compressive Measurements using Cross Validation

Petros Boufounos, Marco F. Duarte, Richard G. Baraniuk

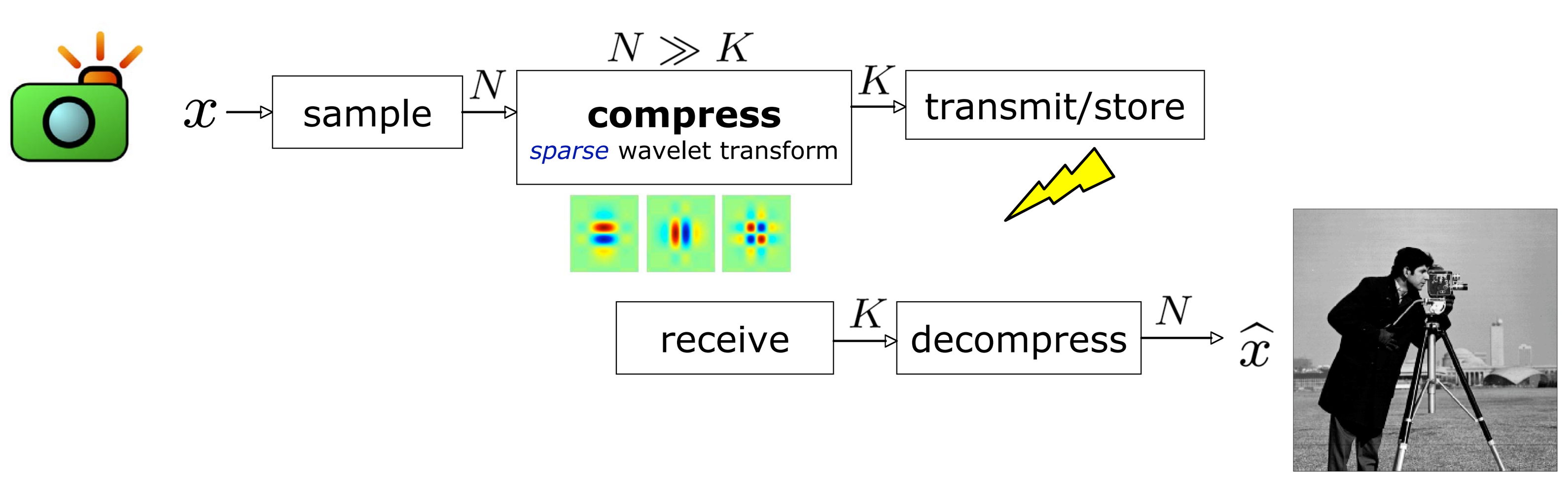
dsp.rice.edu/cs

Compressive Sensing

- Natural and man made signals often have **sparse** or **compressible** structure
- Traditional acquisition: sample then compress
- Compressive acquisition: jointly sample and compress

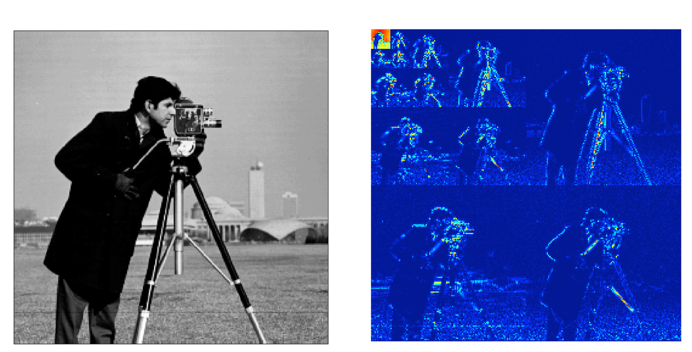
Sensing by Sampling

- ***Sample*** data at Nyquist rate
- ***Compress*** data (signal dependent, non-linear)



Sparsity/Compressibility

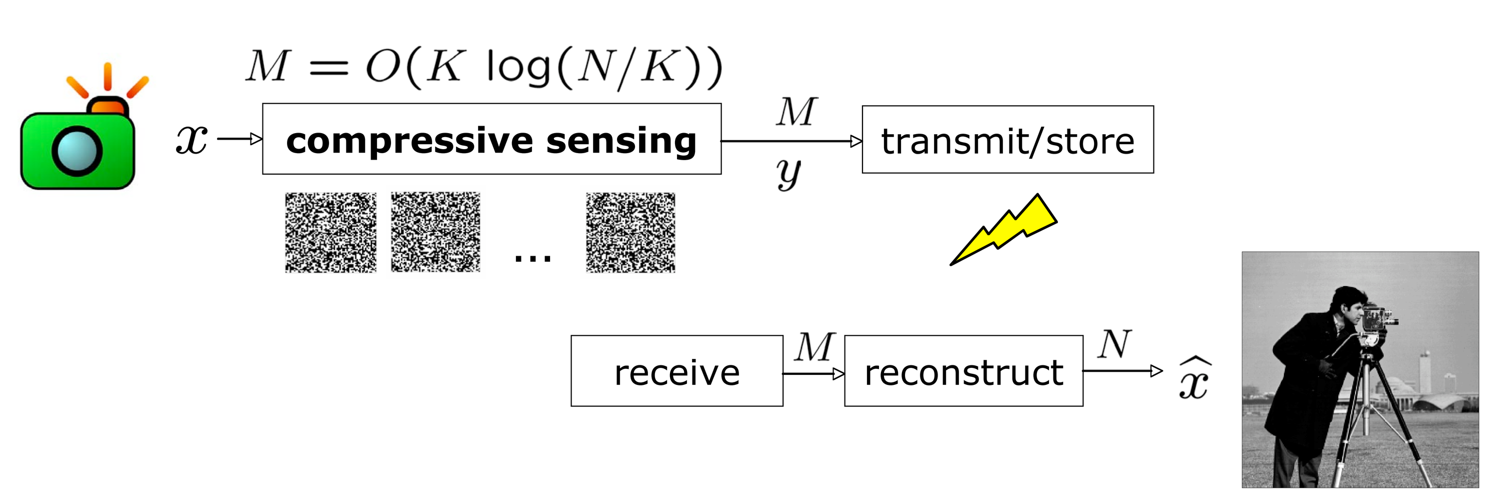
N pixel image



$K \ll N$ large wavelet coefficients

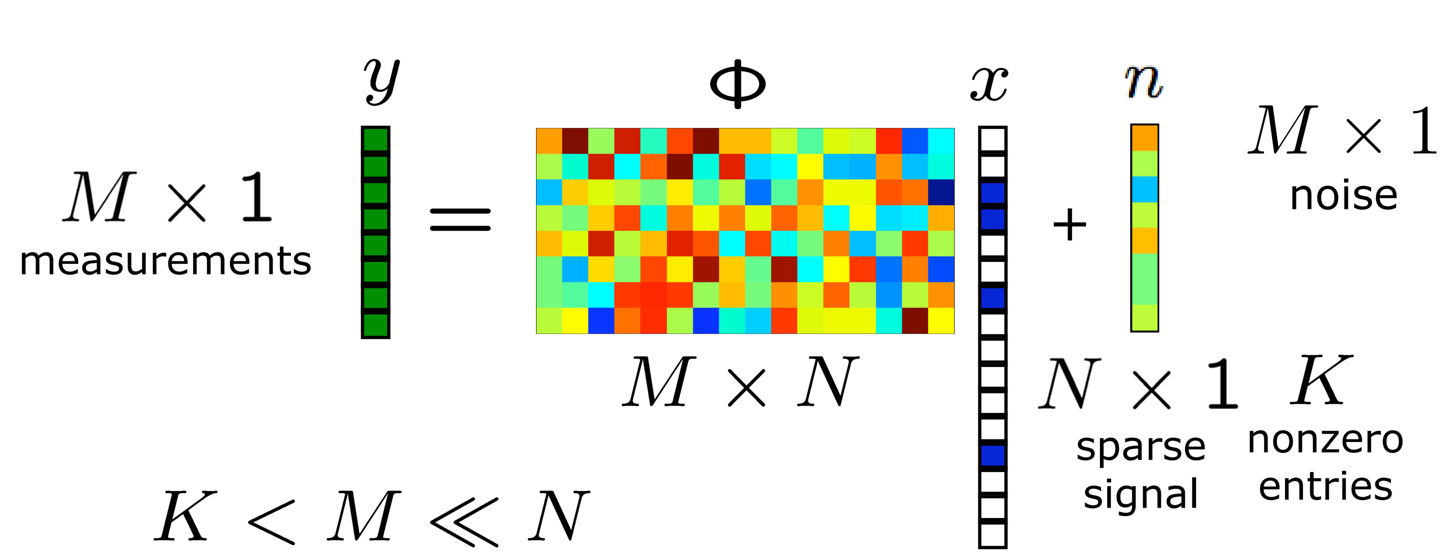
Compressive Sensing (CS)

- Directly acquire ***compressed measurements***



Mathematics of Compressive Sensing

Random projections for measuring $y = \Phi x + n$



$K < M \ll N$

Compressive Sensing in Noise

- Algorithms require setting a sensitivity parameter
- Ideal performance requires knowledge of noise characteristics
- LARS, Homotopy, OMP solve for all parameter values

Reconstruction from noisy measurements

Basis Pursuit Denoising
$$\widehat{x} = \arg\min_x \|x\|_1 + \lambda \|y - \Phi x\|_2$$

Lasso
$$\widehat{x} = \arg\min_x \|y - \Phi x\|_2 \text{ s.t. } \|x\|_1 < \delta$$

Basis Pursuit with Inequality Constrains
$$\widehat{x} = \arg\min_x \|x\|_1 \text{ s.t. } \|y - \Phi x\|_2 < \epsilon$$

Greedy Algorithms
$$\begin{aligned} c_k^{(i)} &= \langle \mathbf{r}^{(i-1)}, \phi_k \rangle, \\ \widehat{k} &= \arg\max_k |c_k^{(i)}|, \\ \mathbf{x}^{(i)} &= \mathbf{x}^{(i-1)} + c_{\widehat{k}}^{(i)} \delta_{\widehat{k}}, \\ \mathbf{r}^{(i)} &= \mathbf{r}^{(i-1)} - c_{\widehat{k}}^{(i)} \phi_{\widehat{k}}, \end{aligned}$$
Iterate if $\|\mathbf{r}^{(i)}\|_2 < \epsilon$

Parameter value $(\lambda, \delta, \epsilon)$ depends on ***magnitude*** of measurement noise

Solvers that require fixed parameter value

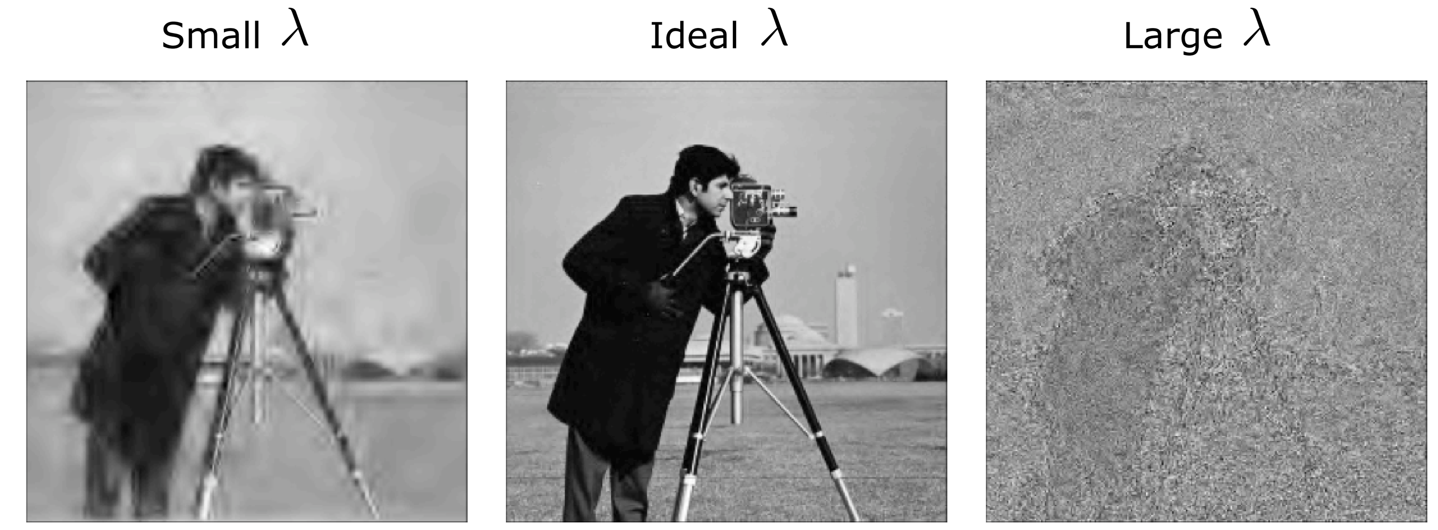
- Gradient Projection for Sparse Reconstruction
- ℓ_1 Regularized Least Squares
- Complexity Regularization
- Log Barrier Method

Solvers that obtain solutions for all parameter values

- Least Angle Regression
- Homotopy Continuation
- Orthogonal Matching Pursuit

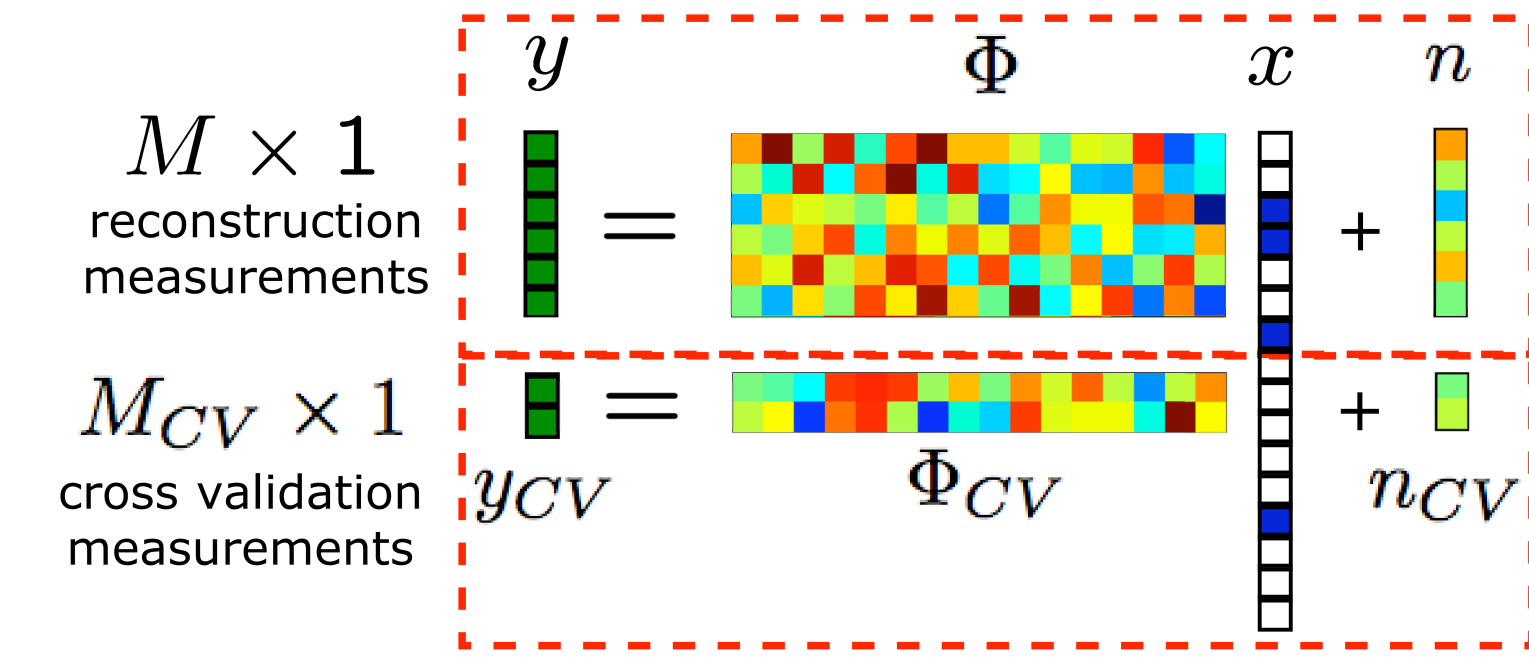
Solution can overfit noise or lack detail

Small λ | Ideal λ | Large λ

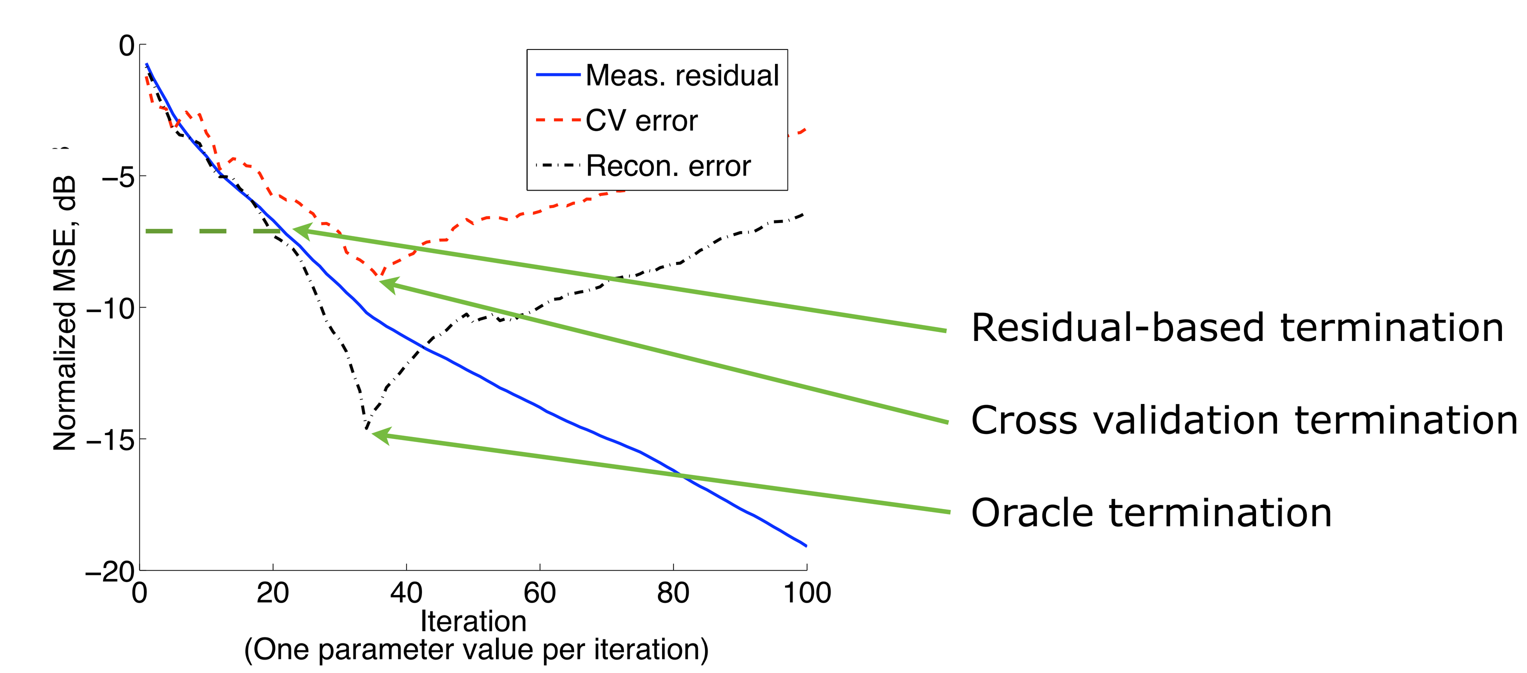


Cross Validation for CS Reconstruction

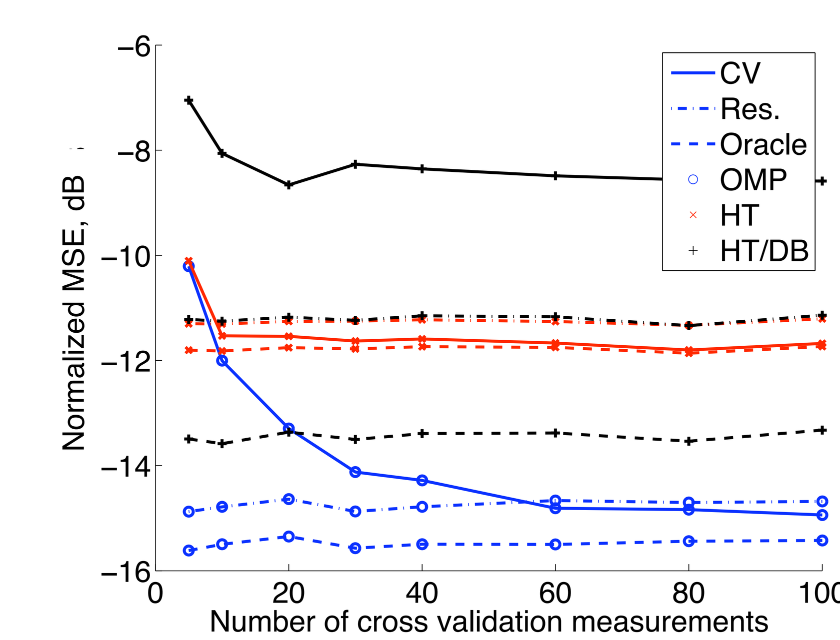
- Split measurement set into *reconstruction* measurements and *cross validation* measurements
- Validate reconstruction algorithm using performance in cross validation

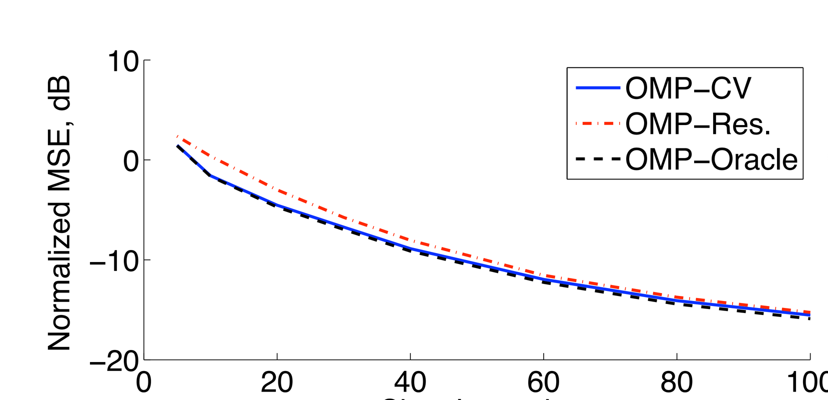


- ***Recover*** $\widehat{x}$ from y using reconstruction algorithm for a parameter value λ
- ***Validate*** parameter value λ using performance in cross validation measurements y_{CV}



- Debiasing (DB) solves least squares problem restricted to the support of the solution
- Homotopy (HT) improves with debiasing
- For large enough M_{CV}, cross validation outperforms residual criterion
- OMP outperforms Homotopy





- As M_{CV}/K grows, cross validation performance improves over residual criterion